UNIT-4

Propositional Logic & Predicate Calculus

Propositional sentence → a sentence which declare true or false but not both is known as propositional sentence.

Ex → it is monday today.
it is 5'o clock.
Please open the door. Not propositional
$x+y = y+x$

1 negation symbol → $\neg, \tilde{x}, \bar{x}$ used in boolean
2 conjuction (and)
3 disjunction (or)
4 Conditional (if then)
5 bi conditional (if and only if)
6 exclusive or (xor)
7 Nand
8 Nor

} Connectors

p → today is friday
$\neg p$ → today is not friday

① atleast 10 inches of rain fell today in Kanpur.
negation → atmost 10 inches of rain fell today in Kanpur
atleast 10 inches of rain did not fell today in Kanpur.

p	$\neg p$
T	F
F	T

② Conjunction → symbol → $\wedge$ (and)

$2^2 = 4$

p	q	$p \wedge q$
true	true	true
true	false	false
false	true	false
false	false	false

$p \wedge q \wedge r$
↓
$2^3 = 8$

Inclusive or :- The disjunction is true when atleast one of ~~of~~ the two proposition is true.

Ex→ 1 Roses are red (and) lotus are pink
2 Jack (and) Jill went up the hill
3 Jack and Jill are ~~coups~~ cousins ↗ not conjuction

→ let P: Roses are red
Q: lotus are pink
$P \wedge Q$ → Symbolic form

→ Jack went up the hill and Jill went up the hill.

③ Disjunction → symbol → $\vee$ (or)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

if exactly 20 or 30 were killed then it will disjunction

Ex→1 Roses are red or violets are blue.
2 Twenty (or) thirty animals were killed in fire today.
3 You can see the match at home (or) you can go to stadium. XOR ⊕ or $\bar{\vee}$
4 There is something wrong with the bulb (or) with the wiring. disjunction

④ Exclusive or → XOR symbol → ⊕ or $\bar{\vee}$

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

1 P: I bought a ticket
Q: I won the jackpot

$P \vee Q$ → I bought a ticket or I won the jackpot
$\neg P \wedge \neg Q$ → I did not buy a ticket and I did not won the jackpot
$\neg P \vee Q$ → I did not buy a ticket or I won the jackpot.

2 P: It is below freezing
Q: It is snowing

It is below freezing but not snowing → $P \wedge \neg Q$

⑤ Conditional → (symbol →) (if then)
A conditional statement is also called an implication.

$P \longrightarrow Q$ if P then Q
antecedent (P), consequent (Q)

$$P \rightarrow Q \equiv \neg P \vee Q$$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex→1 If I am elected then I will lower taxes.
$\underbrace{\text{I am elected}}_{P}$ then $\underbrace{\text{I will lower taxes}}_{Q}$

1 It rains : R
I will be wet : ω
I would stay at home : S
Picnic is cancelled : P

If it rains and the picnic is not cancelled or I don't stay at home then I will wet.

$$((R \wedge \neg P) \vee \neg S) \rightarrow \omega$$

2 P: swimming is allowed at the shore
Q: Sharks have been spotted near the shore

$P \rightarrow \neg Q$ If swimming is allowed at the shore then sharks have not been spotted near the shore

$\neg Q \rightarrow P$ If sharks have not been spotted near the shore then swimming is allowed at the shore.

$\neg P \rightarrow \neg Q$ If swimming is not allowed at the shore then sharks have not been spotted near the shore

Forms of Conditional →

1 If P then Q If you drive more than 400 miles then you need to buy gasoline
2 If P, Q If you drive more than 400 miles you need to buy gasoline
3 Q if P
4 Q when P
5 Q unless ¬P
6 P is sufficient for Q
7 A necessary condition for P is Q
8 Q whenever P
9 P only if Q
10 P implies Q.

Same 4, 8

Q whenever P → It rains whenever the wind blows from the east.

↓ Q consequent

↓ P antecedent

If P then Q → If the wind blows from the east then it rains.

Q if P → The apple trees will bloom if it stays warm for a week.

Q ... P

If P then Q → If it stays warm for a week then the apple trees will bloom.

P implies Q → That the pistons wins the championship implies that they beat the lakers.

P ... Q

If P then Q → If the pistons win the championship then they beat the lakers.

P only if Q → Your gaurantee is good only if you bought your CD player less than 90 days ago.

P ... Q

If P then Q → If your gaurantee is good then you must have bought your CD player less than 90 days ago.

Q is necessary for P → It is necessary to walk 8 miles to get to the top of long's peak.

↓ Q ... P ↓

If P then Q → If you get to the top of long's peak then you must have walked 8 miles.

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A sufficient condition for Q is P → To get tenure as a professor it is sufficient to be world famous
↓ Q ↓ P

If P then Q → If you are world famous then you will get tenure as a professor.

Q unless ¬P → John will go swimming unless the water is too cold
↑ Q ↓ ¬P

If P then Q → If the water is not too cold then John will go swimming.

Ex→1 The crop will be destroyed if there is a flood.
Q Q if P P

If P then Q → If there is a flood then the crop will be destroyed

$\boxed{P \rightarrow Q}$

2 You can access the internet from campus only if you are computer science major or you are not a freshman.

~~P only if Q~~

let I: you can access the internet from campus
C: you are a computer science major
F: you are a freshman

P only if Q

If P then Q → If you are accessing the internet from campus then you are computer science major or you are not a freshman.

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$$\boxed{I \to (C \lor \neg F)}$$

3 You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

$\neg C$ T   O

Q if P
Q unless $\neg P$

let C: You can ride the roller coaster.
T: You are under 4 feet tall.
O: You are ~~not~~ older than 16 years old.

If P then Q → If you are under 4 feet tall and You are not older than 16 years old then you cannot ride the roller coaster.

$$\boxed{(T \land \neg O) \to \neg C}$$

1 when you buy a new car you get \$2000 cashback or a car loan. Exclusive OR (XOR)

2 To take mathematics you must have taken calculus or a course in computer science. Inclusive OR (disjunction)

3 Dinner for 2 includes, 2 items from column A or 3 items from column B. Exclusive OR (XOR)

1 It is necessary to impress the boss to get promoted.
consequent (Q)   antecedent (P)

If P then Q → If you want to get promoted then you have to impress the boss.

2 I will remember to send you the address only if you send me an E-mail message.

P Q

If P then Q → If I will remember to send you the address then you must have send me an E-mail message.

3 To be a citizen of this country it is sufficient that you were born in US.

P Q

If P then Q → If you want to be citizen of this country then you must be born in US.

Converse, contrapositive and inverse :-

$P \rightarrow Q$ original

Converse $Q \rightarrow P$

contrapositive $\neg Q \rightarrow \neg P$

inverse $\neg P \rightarrow \neg Q$

$$\boxed{\begin{array}{l} P \rightarrow Q \equiv \neg Q \rightarrow \neg P \\ Q \rightarrow P \equiv \neg P \rightarrow \neg Q \end{array}}$$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$	$\neg P \rightarrow \neg Q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Q- The home team wins wherever it is raining.

Q P

$P \rightarrow Q$ If it is raining then the home team wins.

$Q \rightarrow P$ If the home team wins then it is raining.

$\neg Q \rightarrow \neg P$ If the home team does not win then it must not be raining.

$\neg P \rightarrow \neg Q$ If it is not raining then the home team must not win.

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Q- A positive integer is a prime only if has no divisor other than 1 or itself.

(P: A positive integer is a prime; Q: has no divisor other than 1 or itself)

$P \rightarrow Q$ If positive integer is a prime then it has no divisor other than 1 or itself.

$Q \rightarrow P$ If it has no divisor other than 1 or itself then a positive integer is a prime.

$\neg Q \rightarrow \neg P$ If it has divisor other than 1 or itself then a positive integer is not a prime.

$\neg P \rightarrow \neg Q$ If a positive integer is not a prime then it has ~~no~~ divisor other than 1 or itself.

⑥ Bi conditionals →

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \land (Q \rightarrow P)$$
$$\equiv (\neg P \lor Q) \land (\neg Q \lor P)$$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \land (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$$\boxed{P \leftrightarrow Q \equiv \neg(P \oplus Q)}$$

You can take the flight if and only if you buy a ticket.

(iff) keyword

if P then Q and if Q then P

27/08/2020

Algebra of Propositional →

Logically Equivalent

$A \equiv B$

$A \Leftrightarrow B$.

Rules →

(i) Idempotent Law →

$P \vee P \equiv P \qquad P \wedge P \equiv P$

(ii) Commutative Law →

$P \vee Q \equiv Q \vee P \quad | \quad P \wedge Q \equiv Q \wedge P$

(iii) Associative Law →

$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

(iv) Distributive Law →

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(v) Absorption Law →

$P \vee (Q \wedge P) \equiv P$

$P \wedge (Q \vee P) \equiv P$

(vi) De Morgan's Law →

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

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(vii) Identity law →

① $P \vee \neg P \equiv T$ $\quad P \wedge \neg P \equiv F$

② $P \vee T \equiv T$ $\quad P \wedge F \equiv F$

③ $P \wedge T \equiv P$ $\quad P \vee F \equiv P$

Tautology, Contradiction & Contigency →

Tautology → A propositional formula that is alway true no matter what the truth values of the propositional variables that occur in it, is called a tautology.

A compound proposition that is always false is called a contradiction.

Contigency → A compound propositiona that is neither a tautology nor a contradiction is called a Contigency.

Q- $\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q$

LHS

$\neg(P \vee (\neg P \wedge Q))$

$\equiv \neg((P \vee \neg P) \wedge (P \vee Q))$ Distributive law

$\equiv \neg(T \wedge (P \vee Q))$ Identity law

$\equiv \neg(P \vee Q)$ Identity law

$\equiv \neg P \wedge \neg Q$ De Morgan's law

RHS

Q- $(P \wedge Q) \to (P \vee Q)$ is a tautology

$\equiv \neg(P \wedge Q) \vee (P \vee Q)$ Conditional
$\equiv (\neg P \vee \neg Q) \vee (P \vee Q)$ De Morgan's law
$\equiv (\neg P \vee P) \vee (\neg Q \vee Q)$ Associative law
$\equiv T \vee T$ Identity law
$\equiv T$ Identity law

Q- $(\neg P \wedge (P \to Q)) \to \neg Q$

$(\neg P \wedge (\neg P \vee Q)) \to \neg Q$ Conditional
$\neg P \to \neg Q$ absorption law
$\neg(\neg P) \vee \neg Q$ Conditional
$P \vee \neg Q$

Contigency

31/08/2020

Rules of inference (Theory of inference)

Rules
Hypothesis
conclusion

Rules →

1 Modus Pones or MP Rule

$P \to Q$ statement 1
P statement 2
$\therefore Q$

2 Modus Tollens or MT Rule

$P \to Q$
$\neg Q$
$\therefore \neg P$

3 Hypothetical Syllogism or HS rule

$P \to Q$
$Q \to R$
$\therefore P \to R$

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4 Disjunctive Syllogism or DS Rule

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q \end{array}$$

5 Addition when we have lack of hypothesis means only 1 hypothesis is given.

$$\begin{array}{l} P \\ \hline \therefore P \vee Q \end{array}$$

6 Simplification

$$\begin{array}{l} P \wedge Q \\ \hline \therefore P \text{ or } Q \end{array}$$

7 Conjunction

$$\begin{array}{l} P \\ Q \\ \hline \therefore P \wedge Q \end{array}$$

8 Resolution

$$\begin{array}{l} P \vee Q \\ \neg P \vee R \\ \hline \therefore Q \vee R \end{array}$$

Q- Show that the premises "It is not sunny and it is colder than yesterday".
"we will go swimming only if it is sunny".
"If we do not go swimming then we will take a trip", and "If we take a trip, then we will be home by Sunset" lead to the conclusion "we will be home by sunset".

Let S: It is sunny
C: It is colder than yesterday
Sw: we will go swimming

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T: we will take a trip
H: we will be home by sunset

1. $\neg S \wedge C$
2. $Sw \rightarrow S$
3. $\neg Sw \rightarrow T$
4. $T \rightarrow H$ $\quad / \therefore H$

5	$\neg Sw \rightarrow H$	3,4	HS
6	$\neg S$	1	Simplification
7	$\neg Sw$	2,6	MT
8	T	3,7	MP
9	H	4,8	MP

Q- Show that the hypothesis "If you send me an e-mail message then I will finish writing the program." "If you do not send me an e-mail message, then I will go to sleep early" and "If I go to sleep early, then I will wake up feeling refreshed", lead to the conclusion "If I do not finish writting the program then I will wake up feeling refreshed".

Let P: You send me an e-mail message
Q: I will finish writing the program
R: I will go to sleep early
S: I will wake up feeling refreshed

Tremble Manohar

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1. $P \rightarrow Q$
2. $\neg P \rightarrow R$
3. $R \rightarrow S$ $/ \therefore \neg Q \rightarrow S$

4. $\neg P \rightarrow S$ 2, 3 HS
5. $\neg Q \rightarrow \neg P$ 1 Contrapositive
6. $\neg Q \rightarrow S$ 5,4 HS

Q-

1. $P \vee (Q \rightarrow S)$
2. $\neg R \rightarrow (S \rightarrow T)$
3. $P \rightarrow R$
4. $\neg R$ $/ \therefore Q \rightarrow T$

5 $\neg P$ 3,4 MT
6 $Q \rightarrow S$ 1, 5 DS
7 $R \vee (S \rightarrow T)$ 2 Conditional
8 $S \rightarrow T$ 7, 4 DS
9 $Q \rightarrow T$ 6, 8 HS

Q-

1. $(P \vee Q) \rightarrow (R \wedge S)$
2. $\neg R$ $/ \therefore \neg Q$

3. $\neg R \vee \neg S$ 2 addition
4. $\neg (R \wedge S)$ 3 De Morgan's
5. $\neg (P \vee Q)$ 1,4 MT
6. $\neg P \wedge \neg Q$ De Morgan's
7. $\neg Q$ Simplification

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Some advanced rules →

1. Constructive Dilemma CD Rule

$$(P \to Q) \land (R \to S)$$
$$P \lor R$$
$$\therefore\ Q \lor S$$

2. Destructive Dilemma DD Rule

$$(P \to Q) \land (R \to S)$$
$$\neg Q \lor \neg S$$
$$\therefore\ \neg P \lor \neg R$$

Q-

1. $(P \to Q) \land (R \to S)$
2. $(Q \land S) \to T$
3. $\neg T \quad /\therefore\ \neg(P \land R)$

4. $\neg(Q \land S)$ 2,3 MT
5. $\neg Q \lor \neg S$ 4 De Morgan's
6. $\neg P \lor \neg R$ 1,5 DD Rule
7. $\neg(P \land R)$ 6 De Morgan's

3. Rule of Conditional Proof (CP Rule)

1
2
3 $\quad /\therefore\ (\_\_\_) \to (\_\_\_)$

4

$/\therefore$

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we can add the antecedent part of conclusion to the list of statements and left the consequent part which is ~~is~~ next become the new conclusion.

Q-

1. $P \to Q$ $\quad / \therefore P \to (P \wedge Q)$

2. P $\quad / \therefore P \wedge Q$ $\quad$ 1 CP Rule
3. Q $\quad$ 1,2 MP Rule
4. $P \wedge Q$ $\quad$ 2,3 Conjunction

Q-

1. $(P \vee Q) \to ((R \vee S) \to T)$ $\quad / \therefore P \to (R \wedge S) \to T$

2. P $\quad / \therefore (R \wedge S) \to T$ $\quad$ 1 CP rule
3. $R \wedge S$ $\quad / \therefore T$ $\quad$ 2 CP rule
4. $P \vee Q$ $\quad$ 2 addition
5. $(R \vee S) \to T$ $\quad$ 1,4 MP
6. R $\quad$ 3 simplification
7. $R \vee S$ $\quad$ 6 addition
8. T $\quad$ 5,7 MP rule

03/09/2020 Rule of Indirect Proof or Proof by Contradiction →

In this method we first assume that the negation of conclusion is true i.e the negation of the conclusion become new premise then we start deduction from new premise ~~~~ including new premise then we reach our contradiction it means that the negation of the conclusion is true, was a wrong assumption.

{conclusion : In negation}

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Q- Show that the conclusion $\neg(P \vee R)$ follows logically from $(P \to Q) \wedge (R \to S)$, $(Q \vee S) \to T$ & $\neg T$

1 $(P \to Q) \wedge (R \to S)$
2 $(Q \vee S) \to T$
3 $\neg T$ $\quad /\therefore \neg(P \vee R)$

4 $\neg\neg(P \vee R)$ $\quad$ 1 ~~I~~P rule
5 $(P \vee R)$ $\quad$ 4 double negation
6 $(Q \vee S)$ $\quad$ 1,5 CD rule
7 T $\quad$ 2,6 MP rule
8 $T \wedge \neg T$ $\quad$ 3,7 conjunction
9 false. $\quad$ $\therefore$ our assumption is wrong so $\neg(P \vee R)$ is true.

Q- Show that the formula $(\neg Q \wedge (P \to Q)) \to \neg Q$ is a tautology.

$/\therefore (\neg Q \wedge (P \to Q)) \to \neg Q$

1. $(\neg Q \wedge (P \to Q))$ $\quad /\therefore \neg Q$ $\quad$ CP Rule
2. $\neg\neg Q$ $\quad$ 1 IP rule
3. Q $\quad$ 2 double negation
4. $\neg Q$ $\quad$ 1 simplification
5. $Q \wedge \neg Q$ $\quad$ 3,4 conjunction
6. false

$\therefore$ our assumption is wrong
so $\neg Q$ is true

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Fellacies →

Show that the following premises are inconsistant.

(i) If Jack misses many class through illness, then he fails high school.

(ii) If Jack fails high school, then he is uneducated.

(iii) If Jack reads a lot of books then he is not uneducated.

(iv) Jack misses the class through illness and reads a lot of books.

let P: ~~If~~ Jack misses many class through illness.

Q: he fails high school

R: he is uneducated

S: Jack reads a lot of books.

1. $P \rightarrow Q$
2. $Q \rightarrow R$
3. $S \rightarrow \neg R$ $\quad / \therefore P \wedge S$

4	$P \rightarrow R$	1, 2	HS rule
5	$\neg P \vee R$	4	conditional
6	$P \wedge \neg R$	5	De Morgan's
7	P	6	simplification
8	$\neg R$	6	simplification
9	R	4, 7	MP rule
10	$R \wedge \neg R$	9, 8	conjunction
11	F	10	Identity law.

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Quantifiers →

1 Universal Quantifier ($\forall$)

2 ~~Extential~~ Existential Quantifier ($\exists$)

1 Universal Quantifier → $\underline{\forall x\ P(x)}$ — x → subject, P → predicate → for all x P of x.

keywords → all of, for all, for arbitary, for every, for any, for each

Ex → $\underset{\forall}{\underline{\text{All}}}$ men are mortal.

let domain of dis course: All men in the world

m: man
M: are mortal

$\forall m\ M(m)$ → for every man in this world men are mortal

1 Q- 1 Every bird can fly $\forall b\ F(b)$
2 Every Koala can climb ~~$\forall K\ C(K)$~~ $\forall k\ C(k)$
3 Every one ~~student in this~~ in your class is friendly $\forall s\ F(s)$

1 let domain of dis course: all the bird in the world
b: bird
F: fly

Symbolic form: $\forall b\ F(b)$

let:
2 domain of dis course: All the Koala in the world.
k: Koala
C: can climb
Symbolic form: $\forall k\, C(k)$

3 let: domain of dis course: Every student in the class
s: student
F: is friendly
Symbolic form: $\forall s\, F(s)$

2 Existential Quantifier → $\exists x\, P(x)$

keywords → for some, for at least one, there is

Ex→ Some men are mortal.

let domain of discourse: all men in this world
Sub:
m: man
M: are mortal
Symbolic form: $\exists m\, M(m)$
↓
There is a man in this world such that the man is mortal.

Q- 1 Some of your classmates are perfect
2 There is a student in this class who owns a PC

1 let: domain of dis course: all ~~classmates of~~ classmate in the class
C: classmates

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P: are perfect

Symbolic form: $\exists c\, P(c)$ → There is a student in your class such that that student is perfect.

2 let: domain of discourse: all the students in this class
S: student P: who owns a PC

Symbolic form: $\exists s\, P(s)$ There is a student in this class such that that student who owns a PC.

Negating Quantified Expression:

① Everyone like sweets.
$\forall x L(x)$ → for all person x in this world, x like sweets

There is no one who does not like sweets.

$$\neg \exists x \neg L(x)$$

$$\boxed{\forall x L(x) \equiv \neg[\exists x \neg L(x)]}$$

09/09/2020

② Not everyone like sweets.
$\neg[\forall x L(x)]$ There is someone who does not like sweets

$$\exists x \neg L(x)$$

$$\boxed{\neg[\forall x L(x)] \equiv \exists x \neg L(x)}$$

③ Someone like sweets

$$\exists x\, L(x)$$

Not everyone dislike sweets

$$\neg[\forall x\, \neg L(x)]$$

$$\boxed{\exists x\, L(x) \equiv \neg[\forall x\, \neg L(x)]}$$

④ Not someone like sweets

$$\neg[\exists x\, L(x)]$$

Everyone does not like sweets

$$\forall x\, \neg L(x)$$

$$\boxed{\neg(\exists x\, L(x)) \equiv \forall x\, \neg L(x)}$$

① No one is perfect
② Everyone is perfect
③ All ignorant people are vain
④ There is a person who cannot speak hindi
⑤ All birds cannot fly.

⑤ Step 1 → all birds cannot fly
Step 2 → domain: all birds in this world
subject: a bird: b
Predicate: canfly: F

$$\forall b\, \neg F(b)$$

Step 3 → $\neg[\forall b\, \neg F(b)]$
Step 4 → $\exists b\, F(b)$

Inference theory of Predicate Calculus →

biggest possible domain
? predicate

Ex → all birds can fly

domain : all birds in this world
New domain : all creatures in this world

$\forall x [B(x) \longrightarrow F(x)]$

For every creature in this world if that creature is a bird then that creature can fly.

Rules of TOI (Predicate Calculus)

	Name	Rule
1	Universal Instantiation	$\dfrac{\forall x P(x)}{\therefore P(c)}$
2	Universal Generalization	$\dfrac{P(a) \text{ for any arbitary } a}{\therefore \forall x P(x)}$
3	Existential Instantiation	$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element}}$
4	Existential Generalization	$\dfrac{P(c) \text{ for some element}}{\therefore \exists x P(x)}$

Instantiation:
Predicate → Propositional $c \to$ specific

Generalization:
Propositional → Predicate

Q- "Everyone in their Discrete Maths class has taken a course in CS" and "Marla is the student in this discrete mathematics class."

"Marla has taken a course in CS" → Conclusion

Domain :- All students in this world.

Subject :- A student 'x'

Predicate :- DM(x) : x is a student in discrete mathematics class

for every student x in this world, if x is a student in DM class then x has taken a course in CS

CS(x) : x has taken a course in CS

1) $\forall x [DM(x) \rightarrow CS(x)]$
2) DM(Marla)

$\therefore$ CS(Marla)

3) DM(Marla) $\rightarrow$ CS(Marla) — 1) UI
4) CS(Marla) — 3,2 MP

Q- "A student in this class has not read the book"
"Everyone in this class passed the first exam"
"Someone who passed the first exam has not read the book" $\rightarrow$ conclusion

Domain :- All students in this world
Subject :- A student 'x'
Predicate :- C(x) : x is a student in this class
R(x) : x has read the book.
P(x) : x has passed the first exam

P(a) $\wedge \neg$R(a)

1) $\exists x (C(x) \wedge \neg R(x))$
2) $\forall x [C(x) \rightarrow P(x)]$

$\therefore \exists x (P(x) \wedge \neg R(x))$

3) $C(a) \wedge \neg R(a)$ — 1) EI
4) $C(a) \rightarrow P(a)$ — 2) UI
5) $C(a)$ — 3) simplification
6) $P(a)$ — 4,5 MP
7) $\neg R(a)$ — 3) simplification

8) $P(a) \wedge \neg R(a)$ 6, 7 conjunction
9) $\exists x (P(x) \wedge \neg R(x))$ 8 EG

Nested Quantifiers →

$\forall x \forall y P(x, y)$

$\forall x \exists y P(x, y)$

$\exists x \forall y P(x, y)$

$\exists x \exists y P(x, y)$

Ex → $\forall x \forall y (x+y = y+x)$

domain: Real number

for every real number x and for every real number y $x+y = y+x$.

$\forall x \forall y (((x>0) \wedge (y<0)) \rightarrow (xy<0))$

The product of a positive and a negative real number is always negative.

↑ conversiont into simplest

for every real no x and for every real no y if $x>0$ and $y<0$ then $xy<0$

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Q-1 The sum of two positive integers is always positive. $\rightarrow \forall x \forall y ((x>0) \wedge (y>0) \rightarrow (x+y>0))$

domain: integers

Q-2 Every real no except zero has a multiplicative inverse. domain: real no

$\forall x \forall y ((x \neq 0) \rightarrow (xy=1))$ $\forall x ((x \neq 0) \rightarrow \exists y (xy=1))$

Q-3 Every student in your school has a computer or has a friend who has a computer.

domain: every student in school

$\forall x \, (C(x) \vee \exists y (C(y) \wedge F(x,y)))$

$C(x)$: x has a computer

$C(y)$: y has a computer

$F(x,y)$: x is friend of y

If a person is a female and is a parent, then this person is someone's mother.

domain: all the person in this world

$\forall x (F(x) \wedge P(x)) \rightarrow \exists y \, M(x,y)$

Everyone has exactly one best friend.

$\forall x \exists y (B(x,y) \wedge \forall z ((z \neq y) \rightarrow \neg B(x,z)))$

domain: every person in this world

Sheet 6 c) The disjunction of two contigencies can be a tautology.

domain: all propositional formula

$C(x)$: x is a contigency
$C(y)$: y is a contigency

$T(x \vee y)$: disjunction of x & y is a tautology

$\exists x \exists y((C(x) \wedge C(y)) \rightarrow T(x \vee y))$

for some propositional formula x & for some propositional formula y if x is a contigency and y is a contigency then disjunction of x & y is a tautology

Satisfiability → [for Propositionals]

→ make a truth table of given Propositional formula
→ find all values and if we get one or more true value then formula is satisfiable at that values of truth table.

9 b) whenever there is an active alert, all queued messages are transmitted.

p

q

all messages

$\exists x(A(x)) \longrightarrow \forall m(Q(m) \rightarrow T(m))$

for some alert x if there is an active alert x then for all messages if there are queued messages then messages will be transmitted queued messages

$\exists x (A(x)) \longrightarrow \forall m\, T(m)$

for some alert x if there is an active alert x then for every queued messages ~~there is a transmitted~~ messages are transmitted.

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Unit-1

→ Set Theory
→ Relation
→ Functions
→ Mathematical Induction

Set → A set is an unordered, well defined collection of ~~discrete of~~ distinct objects.

There are two methods to represent set →

1 Roaster / set builder method
2 Tabular method

$P = \{x \mid x \text{ is a positive integer and } x < 10\}$
$P = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Some common sets are:-

$N = \{0, 1, 2, 3, ----\}$ the set of natural numbers
$Z = \{---- -2, -1, 0, 1, 2 ----\}$ the set of integers
$Z^+ = \{1, 2, 3, ----\}$ the set of positive integers
$Q = \{P/q \mid P \in Z; q \in Z, \text{and } q \neq 0\}$, the set of rational numbers
$R = \{$ the set of real numbers $\}$

Finite set → If a set contains a finite number of distinguishable element.

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Infinite set → If a set contain an infinite number of elements. Ex → The set of +ve integers.

NOTE → $S = \{0, \{1,2\}, \phi, \{9\}\}$ is also a set which contains set in it

Equality set → Two sets will be equal if both the sets are subsets of each other.

Two sets are equal if and only if they have same elements. That is, if A and B are set of the +ve integers then A & B are equal if and only if $\forall x\ (x \in A \leftrightarrow x \in B)$, $A = B$, A and B are equal.

→ A is subset of B $\equiv \forall x (x \in A \rightarrow x \in B)$ is true.

Empty set (null set) → The set that has no elements, denoted by ϕ or $\{\}$.

Eg:- set of all +ve integers greater than their square is an empty set.

Finite set → let S be a set. If there are exactly n distinct elements in S where n is a non-negative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by $|S|$.

Eg:- Null set has no elements, $|\phi| = 0$.

Singleton set → If a set consists of one element, then it is said to be singleton set.

Power set → A set which contains all the subsets of a given set.

Eg → $\{1, 2, 3\}$

Power set $= \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

Power set of a null set : $\{\phi\}$

$$P(\phi) = \{\phi\}$$

$$P(\{\phi\}) = \{\phi, \{\phi\}\}$$

If a set has n elements, its power set has 2^n elements.

Properties of subset :-

1. Null set is the subset of every set.
2. Every set is the subset of itself.

Cardinality → Total no. of elements in a set.

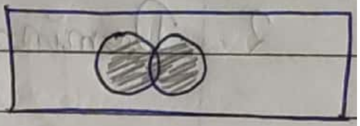
Proper Subset →

$$\boxed{A \subset B \Leftrightarrow A \subseteq B \wedge A \neq B}$$

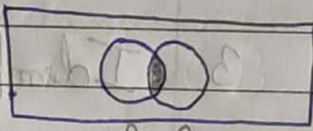
Set operations →

Union $A \cup B = \{x \mid x \in A \vee x \in B\}$

$A \cup B$

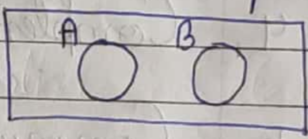


Intersection $A \cap B = \{x \mid x \in A \wedge x \in B\}$



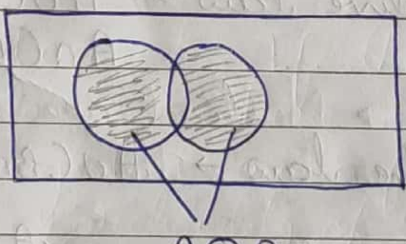
$A \cap B$

disjoint set when no common element is present
$A \cap B = \phi$



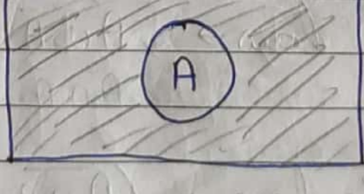
Set difference $\oplus$

$A \oplus B = (A - B) \cup (B - A)$
$= \{x \mid x \in (A-B) \vee x \in (B-A)\}$



$A \oplus B$

Complement $\bar{A} = \{x \mid x \notin A\}$ $\bar{A} = U - A$



T : U
F : ϕ or { }
$\wedge$: $\cap$
$\vee$: $\cup$

Power set → Collection of all subset of a given set

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Set identities →

1 Identity law → $A \cup \phi = A$
$A \cap U = A$

2 Domination law → $A \cup U = U$
$A \cap \phi = \phi$

3 Idempotent law → $A \cup A = A$
$A \cap A = A$

4 Complementation law → $(\bar{\bar{A}}) = A$

5 Commutative law → $A \cup B = B \cup A$
$A \cap B = B \cap A$

6 Associative law → $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

7 Distributive law → $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

8 Absorption law → $A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$

9 Complement law → $A \cup \bar{A} = U$
$A \cap \bar{A} = \phi$

10 DeMorgan's law → $(\overline{A \cup B}) = \bar{A} \cap \bar{B}$
$(\overline{A \cap B}) = \bar{A} \cup \bar{B}$

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Q- $A \cup (B-A) = A \cup B$

LHS

$$A \cup (B-A) = \{x \mid x \in (A \cup (B-A))\}$$
$$= \{x \mid x \in A \text{ or } x \in (B-A)\}$$
$$= \{x \mid x \in A \text{ or } (x \in B \text{ and } x \notin A)\}$$
$$= \{x \mid (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \notin A)\}$$ distributive
$$= \{x \mid x \in (A \cup B) \text{ and } x \in (A \cup \bar{A})\}$$
$$= \{x \mid x \in (A \cup B) \wedge x \in U\}$$ complement
$$= \{x \mid x \in (A \cup B) \wedge U\}$$
$$= \{x \mid x \in (A \cup B) \cap U\}$$
$$= \{x \mid x \in (A \cup B)\}$$ Identity

$$(A \cup (B-A)) \subseteq (A \cup B) \quad — \text{①}$$

$$(A \cup B) \subseteq (A \cup (B-A)) \quad — \text{②}$$

To prove this eq^n take RHS and just go in reverse direc^n of proof of RHS.

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Q- $A - \phi = A$

LHS

$A - \phi = \{x \mid x \in (A - \phi)\}$

$= \{x \mid (x \in A) \text{ and } (x \notin \phi)\}$

$= \{x \mid (x \in A) \cap (x \notin \phi)\}$

$= \{x \mid x \in (A \cap \overset{U}{\cancel{\phi}})\}$

$= \{x \mid x \in A\}$

$A - \phi \subseteq A$ — ①

$A \subseteq A - \phi$ — ② → Same method.

Multiset → Multiset are the set in which an element can occur more than one.

$A = \{a, a, a, b, b, c\}$

$= \{3 \cdot a, 2 \cdot b, 1 \cdot c\}$

↓ ↓ ↓

multiplicity of the element

Operations of multiset → $P = \{4 \cdot a, 2 \cdot b, 1 \cdot c\}$

$Q = \{3 \cdot a, 3 \cdot b, 2 \cdot d\}$

$P \cup Q = \{4 \cdot a, 3 \cdot b, 1 \cdot c, 2 \cdot d\}$

~~Mult~~

$P \cap Q = \{3 \cdot a, 2 \cdot b\}$

$P - Q = \{1 \cdot a, 1 \cdot c\}$

$P + Q = \{7 \cdot a, 5 \cdot b, 1 \cdot c, 2 \cdot d\}$

Generalized union & Intersection →

$A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n = \sum_{i=1}^{n} \cup A_i$

$= \bigcup_{i=1}^{n} A_i$

$A_1 \cup A_2 \cup A_3 \cdots \cup A_n \cdots \cup \cdots = \bigcup_{i=1}^{\infty} A_i$

$A_1 \cap A_2 \cap A_3 \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$

$A_1 \cap A_2 \cap A_3 \cdots\cdots = \bigcap_{i=1}^{\infty} A_i$

Q- Let $A_i = \{i, i+1, i+2, \cdots\}$

$\bigcup_{i=1}^{n} A_i = ?$ & $\bigcap_{i=1}^{n} A_i = ?$

Solⁿ $i = 1$

$A_1 = \{1, 2, 3 \cdots\}$

$A_2 = \{2, 3, 4 \cdots\}$

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$A_3 = \{3, 4, 5 \text{ -----}\}$

$A_n = \{n, n+1, n+2 \text{ ----}\}$

$\bigcup_{i=1}^{n} A_i = \{1, 2, 3, 4, 5 \text{ -----} n, n+1, \text{ ----}\}$

$= Z^+$

$\bigcap_{i=1}^{n} A_i = \{n, n+1, n+2 \text{ ----} \infty\}$

$= A_n$

Q- find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ for $i \in Z^+$

a) $A_i = \{-i, -i+1, \text{ ----} -1, 0, 1, \text{ ---} i-1, i\}$

b) $A_i = \{-i, i\}$

c) $A_i = [-i, i]$ that is the set of real numbers x with $-i \leq x \leq i$

d) $A_i = [i, \infty)$ that is the set of real number x with $x \geq i$.

a) $\bigcup_{i=1}^{\infty} A_i = \{-i, -i+1, \text{ ----} -1, 0, 1, \text{ ----} i-1, i\}$

$= Z$

$$\bigcap_{i=1}^{\infty} A_i = \{-1, 0, 1\}$$

b) $$\bigcup_{i=1}^{\infty} A_i = \{-i, -i+1, \dots -1, 1, \dots i-1, i\}$$

$$= Z - \{0\}$$

$$\bigcap_{i=1}^{\infty} A_i = \phi$$

C $$\bigcup_{i=1}^{\infty} A_i = R$$

$$\bigcap_{i=1}^{\infty} A_i = [-1, 1]$$

d) $$\bigcup_{i=1}^{\infty} A_i = R^+$$

$$\bigcap_{i=1}^{\infty} A_i = [1, \infty)$$

Cartesian Product →

$A = \{1, 2\}$
$B = \{a, b\}$

$$A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$$

$A \times B = B \times A$ if (i) $A = B$ or (ii) either $A = \phi$ or $B = \phi$

Properties of Cartesian Product →

1 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2 $A \times (B \cap C) = (A \times B) \cap (A \times C)$

3 $A \times (B - C) = (A \times B) - (A \times C)$

4 $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

1 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

let $\{(a,b) \mid (a,b) \in A \times (B \cup C)\}$

$= \{(a,b) \mid (a \in A) \text{ and } b \in (B \cup C)\}$

$= \{(a,b) \mid (a \in A) \text{ and } (b \in B \text{ or } b \in C)\}$

$= \{(a,b) \mid (a \in A \text{ and } b \in B) \text{ or } (a \in A \text{ and } b \in C)\}$

$= \{(a,b) \mid (a,b) \in A \times B \text{ or } (a,b) \in A \times C\}$

$= \{(a,b) \mid (a,b) \in ((A \times B) \cup (A \times C))\}$

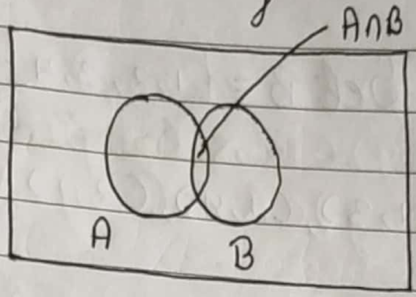
$= A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — 1

$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ — 2

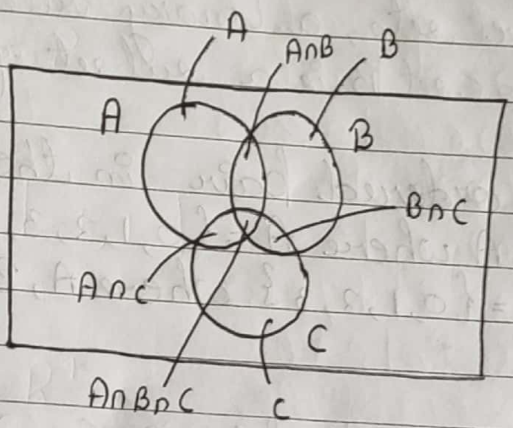
↓

Same method.

Venn diagram →



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

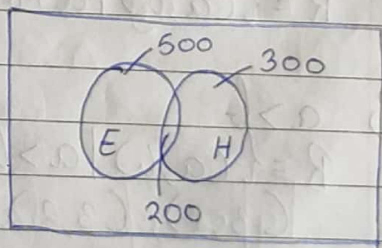


$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) \cancel{- n(C \cap A)} - n(A \cap C) + n(A \cap B \cap C)$$

Q-

$\cancel{n(E) = 700}$
$\cancel{n(H) = 500}$
$\cancel{n(E \cup H) = 1000}$

People = 1000
English = 700
Hindi = 500



$$n(E \cup H) = n(E) + n(H) - n(E \cap H)$$
$$1000 = 700 + 500 - n(E \cap H)$$
$$n(E \cap H) = 200$$

Relation →

Set $A = \{1, 2\}$ Set $B = \{1, 2, 3, 4\}$

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

$R = \{(A, B) \mid A^2 = B\} = \{(1,1)(2,4)\}$ such that

Let A & B be sets, a binary relation from a set A to set B is a subset of A×B.

Q- List the ordered pair in the relation R from set A where $A = \{0, 1, 2, 3, 4\}$ to set B where $B = \{0, 1, 2, 3\}$ where $A, B \in R$ iff

i) $a = b$

$R = \{(a, b) \mid a = b\} = \{(0,0), (1,1), (2,2), (3,3)\}$

ii) $a + b = 4$

$R = \{(a, b) \mid a + b = 4\} = \{(1,3), (2,2), (3,1), (4,0)\}$

iii) $a > b$

$R = \{(a, b) \mid a > b\} = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$

iv) a / b a mod b = a divides b

$R = \{(a, b) \mid a \mid b\} = \{(1,1), (1,2), (1,3), (2,2), (3,3), (0,1), (0,2), (0,3), \cancel{(0,0)}\}$

v) $\gcd(a, b) = 1$

$R = \{(a, b) \mid \gcd(a, b) = 1\} = \{(2,3), (3,2), (1,2), (2,1), (4,3), (1,1), (4,1), (3,1), (2,1)\}$

vi) $LCM(a,b) = 2$

$R = \{(a,b) \mid LCM(a,b) = 2\} = \{(2,2), (1,2), (2,1)\}$

Domain & Range →

$R = \{(a,b) \mid gcd(a,b) = 1\} \Rightarrow \{(2,3), (3,2), (4,3), (1,2), (1,1), (1,3), (4,1), (2,1), (3,1)\}$

Domain $= \{1, 2, 3, 4\}$

Range $= \{1, 2, 3\}$

The set A $\{a \in A \mid (a,b) \in R \; \exists \; b \in B\}$ is called the domain of R and set denoted by DOM(R)

The set B $\{b \in B \mid (a,b) \in R \; \exists \; a \in A\}$ is called the range of R and denoted by Ran(R).

Q- The relation R on set $\{1,2,3,4,5\}$ is defined by the rule $(x,y) \in R$ if 3 divides $x-y$ find the element of R

$R = \{(x,y) \mid 3 \text{ divides } x-y\}$

$= \{(1,4), (2,5), (1,1), (2,2), (3,3), (4,4), (5,5)\}$

$R^{-1} = \{(y,x) \mid 3 \text{ divides } y-x\}$

$= \{(4,1), (5,2), (1,1), (2,2), (3,3), (4,4), (5,5)\}$

$DOM(R) = \{1,2,3,4,5\}$ $\quad$ $Ran(R) = \{1,2,3,4,5\}$

$DOM(R^{-1}) = \{1,2,3,4,5\}$ $\quad$ $Ran(R^{-1}) = \{1,2,3,4,5\}$

Operations on Relation →

$R \cup S = \{(x,y) \mid (x,y) \in R \vee (x,y) \in S\}$

$R \cap S = \{(x,y) \mid (x,y) \in R \wedge (x,y) \in S\}$

$R - S = \{(x,y) \mid (x,y) \in R \wedge (x,y) \notin S\}$

$\bar{R} = \{(x,y) \notin R\}$

$R \Delta S \mid R \oplus S = \{(x,y) \mid (x,y) \in R - S \vee (x,y) \in S - R\}$

Q- Let the set of real numbers are

$R_1 = \{(a,b) \in R_1 \mid a > b\}$

$R_2 = \{(a,b) \in R_2 \mid a \geq b\}$

$R_3 = \{(a,b) \in R_3 \mid a < b\}$

$R_4 = \{(a,b) \in R_4 \mid a \leq b\}$

$R_5 = \{(a,b) \in R_5 \mid a = b\}$

$R_6 = \{(a,b) \in R_6 \mid a \neq b\}$

a) $R_2 \cup R_4$

b) $R_3 \cap R_6$

c) $R_6 - R_3$

d) $R_3 \oplus R_5$

a) $R_2 \cup R_4 = \{(a,b) \mid (a,b) \in (a \geq b) \vee (a,b) \in (a \leq b)\}$

b) $R_3 \cap R_6 = \{(a,b) \mid (a,b) \in (a < b) \wedge (a,b) \in (a \neq b)\}$

c) $R_6 - R_3 = \{(a,b) \mid (a,b) \in a \neq b \wedge (a,b) \notin (a < b)\}$

d) $R_3 \oplus R_5 = \{(a,b) \mid (a,b) \in ((a,b) \in a < b \wedge (a,b) \notin a = b) \vee (a,b) \in ((a,b) \in a = b \wedge (a,b) \notin a < b)\}$

Types of relation →

a) Inverse relation → $R^{-1} = \{(b,a) \mid (a,b) \in R\}$

Eg: R = a is father of b

R^{-1} = b is son/daughter of a

b) Identity relation → $I_A = \{(a,a) \mid a \in A\}$

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c) Congruence Module Relation →

$a \equiv b \bmod m$

$a - b = k \cdot m$

Ex → $7 \equiv 1 \bmod 3$

$7 - 1 = k \cdot 3$

$\boxed{6 = 2 \cdot 3}$

Q- Let R_1 and R_2 be the "congruent modulo 3" and "the congruent modulo 4" relations, respectively, on the set of integers that is $R_1 = \{(a,b) \mid (a,b) \in a \equiv b \bmod 3)\}$ and $R_2 = \{(a,b) \mid (a,b) \in a \equiv b (\bmod 4)\}$
find →

a) $R_1 \cup R_2 = \{(a,b) \mid (a,b) \in a \equiv b (\bmod 3) \vee (a,b) \in a \equiv b (\bmod 4)\}$

b) $R_1 \cap R_2 = \{(a,b) \mid (a,b) \in a \equiv b (\bmod 3) \wedge (a,b) \in a \equiv b (\bmod 4)\}$

c) $R_1 - R_2 = \{(a,b) \mid (a,b) \in a \equiv b (\bmod 3) \wedge (a,b) \notin a \equiv b (\bmod 4)\}$

d) $R_1 \oplus R_2 = \{(a,b) \mid (a,b) \in ((a,b) \in a \equiv b (\bmod 3) \wedge (a,b) \notin a \equiv b (\bmod 4)) \vee ((a,b) \in a \equiv b (\bmod 4) \wedge (a,b) \notin a \equiv b (\bmod 3))\}$

Q- Let R be the relation $R = \{(a,b) \mid a \text{ divides } b\}$ on the set of positive integers.

find a) $R^{-1} = \{(b,a) \mid (b,a) \in b \text{ divides } a\}$

b) $\bar{R} = \{(a,b) \mid (a,b) \notin a \text{ divides } b\}$

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Properties of relation →

1 **Reflexive relation** → A relation R on a set A is called reflexive, if $(a, a) \in R \ \forall a \in A$

$R = \{(a,a) \mid a \in A\}$

Eg → $A = \{1, 2, 3\}$

$I_A = \{(1,1), (2,2), (3,3)\}$ ↑ compulsary

Reflexive relation $= \{\underline{(1,1), (2,2), (3,3)}, \underline{(1,2), (2}$
$\underline{(3,2)}\}$
↓ not compulsary

Every I_A is reflexive but every reflexive is not necessary I_A.

Q- $R = \{(a,b) \mid a/b\} \ \forall a, b \in Z$

Reflexive relation

But not Identity relation

2 **Irreflexive relation** →

$R = \{(a,b) \notin R \mid \forall a \in A\}$

3 **Non-Reflexive relation** → A relation R on a set A is called non-reflexive if it is reflexive and irreflexive both.

Eg → $R = \{\underbrace{(1,1), (2,2)}_{\text{reflexive}}, \underbrace{(2,3)}_{\text{irreflexive}}\}$

4 **Symmetric relation** →

$\forall a \ \forall b \{(a,b) \in R \rightarrow (b,a) \in R\} \ \forall a, b \in A$

Eg $A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,1), (3,1)\}$ Not symmetric

$R = \{(1,1), (1,2), (2,1), (3,1), (1,3)\}$ symmetric

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5 Asymmetric relation →

$\forall a \forall b \{(a,b) \in R \rightarrow (b,a) \notin R\}$ where $a,b \in A$

6 Transitive relation →

$\forall a \forall b \forall c \{((a,b) \in R \wedge (b,c) \in R) \rightarrow (a,c) \in R\}$

PTO

Antisymmetric →
$\forall a \forall b (((a,b) \in R \wedge (b,a) \in R) \rightarrow a = b)$

Ex → $R = \{(1,2), (2,1), (1,1), (2,2)\}$

Equivalence relation → A relation is said to be equivalent relation if it is reflexive, symmetric, transitive.

Q- Let m be a positive integer with $m > 1$.
Show that the relation
$$R = \{(a,b) \mid a \equiv b \pmod m\}$$
is an equivalence relation on the set of integer.

Sol^n → $Z \times Z$
$$R = \{(a,b) \mid a \equiv b \pmod m\}$$

$$a \equiv b \bmod m$$
$$\boxed{a - b = K.m}$$

① Reflexivity: $\forall a \{(a,a) \in R \mid a \in A\}$

$$a - b = K.m$$
$$a - a = K.m$$
$$0 = K.m$$
$$\boxed{0 = 0.m}$$

② Symmetricity: $\forall a \forall b \{\underbrace{(a,b) \in R}_{\text{let this is true}} \rightarrow \underbrace{(b,a) \in R}_{\text{To prove}}\}$

let $(a,b) \in R$

$$a - b = K.m$$
$$-(a-b) = -Km$$
$$(b-a) = (-K)m$$
$$(b,a) \in R$$

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③ Transitivity: $\forall a \forall b \forall c \{((a,b) \in R \wedge (b,c) \in R) \rightarrow (a,c) \in R\}$

let $(a,b) \in R$ and $(b,c) \in R$

$a - b = k_1 m$ — ①

$b - c = k_2 m$ — ②

adding ① & ②

$a - c = (k_1 + k_2) m$

$(a,c) \in R$.

Q- Let R and S be a relation from A to B, show that -

(i) if $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$

(ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

(iii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

(i) let $R \subseteq S$

if $(a,b) \in R^{-1}$ then $(b,a) \in R$ and $(b,a) \in S$

then $(a,b) \in S^{-1}$

$\therefore$ $\boxed{R^{-1} \subseteq S^{-1}}$

(ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

LHS → $(a,b) \in (R \cap S)^{-1}$

$(b,a) \in (R \cap S)$

$(b,a) \in R$ and $(b,a) \in S$

$(a,b) \in R^{-1}$ and $(a,b) \in S^{-1}$

$(a,b) \in R^{-1} \cap S^{-1}$

$(R \cap S)^{-1} \subseteq R^{-1} \cap S^{-1}$ — ①

$R^{-1} \cap S^{-1} \subseteq (R \cap S)^{-1}$ — ②

Q- let R be a relation on A, Prove that,

① if R is reflexive, so is R^{-1}
let prove

② R is symmetric if and only if $R = R^{-1}$

③ R is antisymmetric if and only if R

① If R is reflexive
$\forall a \ (a,a) \in R$
$(a,a) \in R^{-1}$

② If R is symmetric, $\forall a \forall b \ (aRb \rightarrow bRa)$

let $(a,b) \in R^{-1}$ – ①
$(b,a) \in R$
$(a,b) \in R$ – ②

using ① & ② $\boxed{R^{-1} \subseteq R}$ – (i) let $(a,b) \in R$

$(b,a) \in R^{-1}$

$(b,a) \in R$

$\boxed{R \subset R^{-1}}$ — (ii)

Composite Relations :-

Reflexive
symmetric
transitive

D R	D R	
$A \times \underline{B}$	$\underline{B} \times C$	$A \times C$
R	S	$S \circ R$
		S of R

If range of one relation is same as the domain of another relation, then they will get cancelled out.

Eg → $A = \{1,2\}$, $B = \{1,2,3\}$, $C = \{1,2,3,4\}$

$A \times B$

$R = \{(1,1), (2,3), (1,3)\}$

$B \times C$

$S = \{(1,2), (2,3), (2,4), (3,2), (3,4)\}$

$S \circ R = \{(1,2), (2,2), (2,4), (1,2), (1,4)\}$

→ Relations can be expressed in matrix form.

$A = \{1, 2, 3\}$ $\quad B = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,3), (2,4), (3,3), (3,4)\}$

$A \times B$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$M_{R_1} = \begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{matrix} \qquad M_{R_2} = \begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

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Composite relations → Let R be a relation from a set A to set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pair (a, c), where $a \in A$, $c \in C$ and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$. We denote the composite of R and S by $R \circ S$.

Ex → What is the composite of the relations R and S, where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1,0); (2,0), (3,1), (3,2), (4,1)\}$

Sol^n → $S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$

Let R be a relation on the set A. The powers R^n, $n = 1, 2, 3 \ldots$ are defined recursively by

$R^1 = R$, $R^2 = R \circ R$, $R^3 = R^2 \circ R$ ---- $R^{n+1} = R^n \circ R$.

The Matrix representation composition → Suppose that R is a relation from A to B and S is a relation from B to C. Suppose that m, n and P are the elements of A, B, C respectively. The ordered pair (a_i, c_j) belongs to $S \circ R$ if and only if there is an element b_k such that (a_i, b_k) belongs to R and (b_k, c_j) belongs to S from the defination.

from this defination $\boxed{M_{S \circ R} = M_R \odot M_S}$

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Q- find the matrix representing the Relation R^2, where the matrix represent R is

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 \end{array} \quad \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 \end{array} =$$

(1,2) (1,3)

Matrix representing union & intersection →

$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$ and $M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$

Representing relations using Matrices :-

A relation between finite sets can be represented using a zero-one matrix. Suppose that R is a relation from $A = \{a_1, a_2, \ldots a_m\}$ to $B = \{b_1, b_2, b_3 \ldots b_n\}$ the relation R can be represented by its matrix

$M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

Example 1 → Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$ and $a > b$.

What is the matrix representing R if $a_1 = 1$, $a_2 = 2$ and $a_3 = 3$ and $b_1 = 1$, $b_2 = 2$?

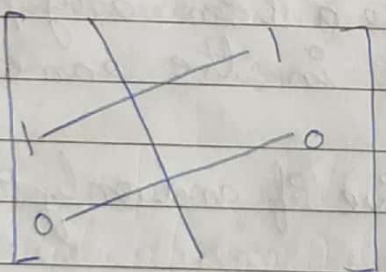
Sol^n because $R = \{(2,1), (3,1), (3,2)\}$, the matrix for R

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

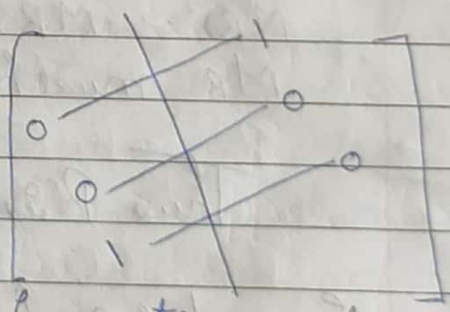
Reflexive relation → R is reflexive if and only if $(a_i, a_i) \in R$ for $i = 1, 2, \ldots n$. Hence, R is reflexive iff $m_{ii} = 1$ for $i = 1, 2, \ldots n$.
In ~~other~~ other words, R is if reflexive if all the elements on the main diagonal of M_R are equal to 0.

$$M_R = \begin{bmatrix} 1 & \cdots & \cdots \\ 0 & 1 & \cdots \\ 0 & \cdots & 1 \end{bmatrix}$$

Symmetric Relation Matrix → The relation R is a symmetric if $(a, b) \in R$ implies that $(b, a) \in R$. Consequently, the relation R on a set $A = \{a_1, a_2 \ldots a_n\}$ is symmetric if and only if $m_{ij} = 1$ whenever $m_{ji} = 1$. Consequently, R is symmetric iff $m_{ij} = m_{ji}$, for all pair of integers i and j with $i = 1, 2 \ldots n$ and $j = 1, 2, \ldots n$.



a) symmetric



b) antisymmetric.

Antisymmetric :-
The relation R is antisymmetric iff $(a, b) \in R$ and $(b, a) \in R$ imply that $a = b$. Consequently, the matrix of an antisymmetric relation has the property that if $m_{ij} = 1$ with $i \neq j$, then $m_{ij} = 0$ or in other words, either $m_{ij} = 0$ or $m_{ji} = 0$ when $i \neq j$

Example → Suppose that the relation R on a set is represented by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ is R reflexive, symmetric and/or antisymmetric?

Solⁿ → For reflexivity, because all the diagonal of this matrix are equal to 1, R is reflexive. More over, because M_R is symmetric, it follows that R is symmetric but R is not ~~sym~~ antisymmetric.

Transitive → A relation R is transitive if and only if its matrix $M_R = [m_{ij}]$ has the property if $m_{ij} = 1$ and $m_{jk} = 1$, then $m_{ik} = 1$. Thus statement simply means R is transitive if $M_R \cdot M_R$ has 1 position i.k. Thus, the transitivity of R means that if $M_R^2 = M_R \cdot M_R$ has a 1 in any position then M_R must have a 1 in the same position ~~then~~

Thus R is transitive if and only if $M_R^2 + M_R = M_R$

Eg → Let $A = \{1, 2, 3, 4\}$ and let ; R be a relation on A whose matrix is:

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$ Show that R is transitive.

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Solⁿ→ $M_R^2 = M_R \cdot M_R$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^2 + M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$= M_R$

∴ The relation R is transitive.

Closure of Relations→

Reflexive closure→ The reflexive closure $R^{(r)}$ of a relation R is the smallest reflexive relation that contained R as a subset. Given a relation R on a set A, the reflexive closure of R can be formed by adding to R all pairs of the form (a, a) with A∈R not already in R

$$R^{(r)} = R \cup I_A$$

where, $I_A = \{(a,a) : a \in A\}$

Example→ A relation $R = \{(1,2),(2,1),(1,1),(2,2)\}$ is defined on a set $S = \{1, 2, 3, 4\}$. find the reflexive closure of R.

Solⁿ→ $R^{(r)} = R \cup I_A$

$= \{(1,2),(2,1),(1,1),(2,2)\} \cup \{(1,1),(2,2),(3,3),(4,4)\}$

Date:

$$R^{(r)} = \{(1,1),(1,2),(1,1),(2,2),(3,3),(4,4)\}$$

Example → what is the reflexive closure of the relation $R = \{(a,b) \mid a<b\}$ on the set of integers?

Solⁿ → $R \cup I_A = \{(a,b) \mid a<b\} \cup \{(a,a) \mid a \in Z\} = \{(a,b) \mid a \le b\}$

Symmetric Closure → The symmetric closure $R^{(s)}$ is the smallest symmetric relation that contains R as a subset. A symmetric relation contains (x,y) if it contains (y,x). Since the inverse relation R^{-1} contains (y,x) if (x,y) is in R, the symmetric closure of relation is

$$R^{(s)} = R \cup R^{-1}$$

where $R^{-1} = \{(y,x) : (x,y) \in R\}$

Example → If $R = \{(1,2),(4,3),(2,2),(2,1),(3,1)\}$ be a relation on $S = \{1,2,3,4\}$. Find the symmetric closure.

Solⁿ → $R^{(s)} = R \cup R^{-1}$

$R^{-1} = \{(3,4),(1,3)\}$

$R^{(s)} = \{(1,2),(2,1),(4,3),(3,4),(3,1),(1,3),(2,2)\}$

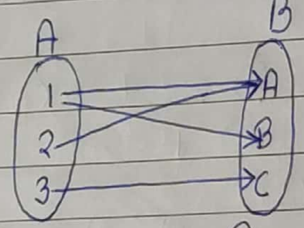
FUNCTIONS →

let these be two sets A & B :-

$A = \{1, 2, 3\}$ $B = \{A, B, C\}$

$A \times B = \{(1,A), (1,B), (1,C), (2,A), (2,B), (2,C), (3,A), (3,B), (3,C)\}$

$f = \{(1,A), (1,B), (2,A), (3,C)\}$



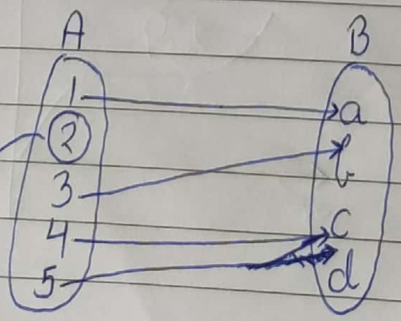
Domain Range

{ Not a function because more than one image of 1 is present }

* No element from set of domain should be left out. It should have an image.

* Elements can have same image.

Ex → (4, c) ; (5, c)



No image is present
∴ not a function

Every function is a relation, but every relation might not be a function.

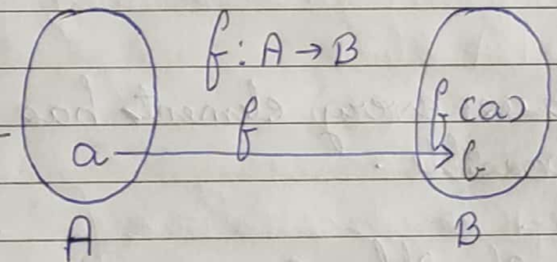
Defination of Functions :-

* Given any set A, B, a function f from (or "mapping") A to B ($f : A \to B$) is an assignment of exactly one element $f(x) \in B$ to each element $x \in A$.

$\boxed{\text{Conditions}}$:-

- $\forall a \in A$ must be related to $\exists b \in B$
- $(a, b) \in f \wedge (a, c) \in f \Rightarrow b = c$

Pictorial representation



$f(a) = b$

f such that A mapped to B and b is the image of a
A ↓ domain
B ↓ codomain

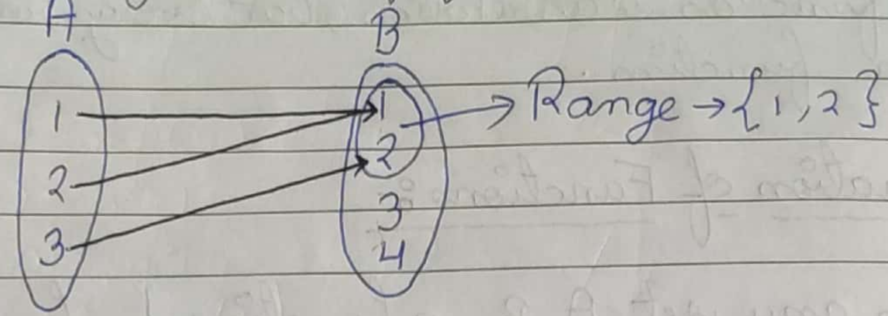
☆ Some functions terminology

☆ If $f : A \to B$, and $f(a) = b$ (where $a \in A$ & $b \in B$), then,

→ A is domain of f
→ B is the codomain of f
→ b is the image of a under f
→ a is the pre-image of b under f.

- In general, b may have more than one pre-image.

→ The range $R \subseteq B$ of f is $\{ b \mid \exists a\, f(a) = b \}$



Q- from the set of integers to the set of real numbers.

Determine whether f is a function from Z to R if

a) $f(n) = \pm n$, (b) $f(n) = \sqrt{n^2+1}$

c) $f(n) = \dfrac{1}{n^2-4}$

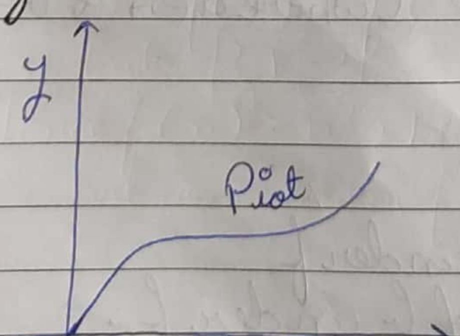
a) Not a function, as every element has two images, which is not possible.

b) Not a function.

c) If $n = 2$, then it will give ∞, therefore, there is an element 2 in the domain, for which there is no domain, which is not possible.

$\therefore$ Not a function.

Graphical Representation :-



$f(a) = b$

preimage of b — image of a

Q- Determine whether f is a function from R to R if a) $f(x)=\frac{1}{x}$, b) $f(x)=\sqrt{x}$, c) $f(x)=\pm\sqrt{x^2+1}$

a) Not a function

b) Not a function } because two images will be formed
c) Not a function }

Types of functions:-

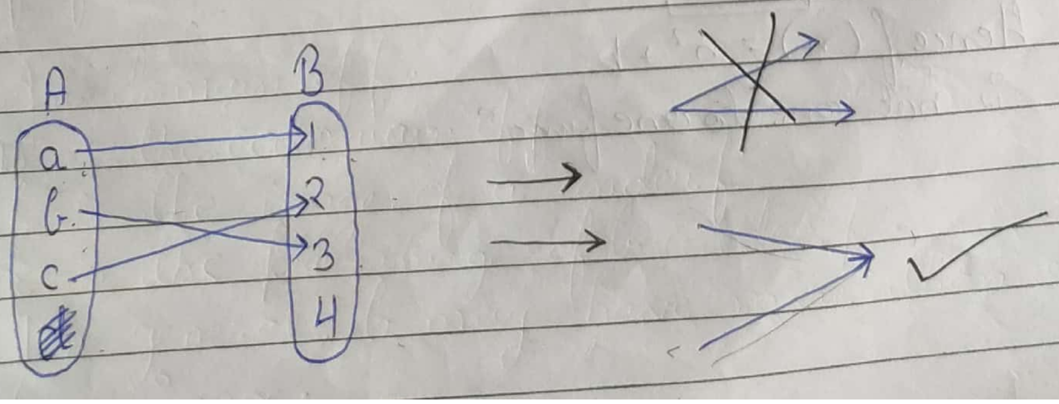
1 One to one
2 Onto
3 One-one-onto
4 Many to one

1 One to one function:- other name is injective function

$\forall a \forall b ((f(a)=f(b)) \to a=b)$ ← to disprove one-one

$\forall a \forall b (a \neq b \to (f(a) \neq f(b)))$ ← to prove one-one

where the universe of discourse is the domain of function.
a, b both are elements of A



Q- Determine whether each of these functions from Z to Z is one-to one.

a) $f(n) = n-1$
b) $f(n) = n^2+1$
c) $f(n) = n^3$
d) $f(n) = \lceil n/2 \rceil$

a) $f: Z \to Z$
$f(n) = n-1$
$\forall a \forall b (a \neq b \to (f(a) \neq f(b)))$
let $a, b \in Z$
$a \neq b$ is true — ①
subtracting ① from both sides in eq^n ①
$a-1 \neq b-1$
$f(a) \neq f(b)$

Hence $f(n) = n-1$ is one to one f^n.

b) $f(n) = n^2+1$

$\forall a \forall b ((f(a) = f(b)) \to a = b)$
$a, b \in Z$
let $f(a) = f(b)$ is true — ①
since $f(2) = f(-2) = 5$
but $\boxed{2 \neq -2}$
Hence $f(n) = n^2+1$
is not one to one func^n

not a one to one bcz for 1 image is 2 and for -1 image is 2, so NO.

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let $a, b \in Z$
$a \neq b$ is true — ①
subtracting ① from both sides in eq^n ①
$a-1 \neq \cancel{a} b-1$
$f(a) \neq f(b)$

Hence $f(n) = n-1$ is one to one f^n.

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$\forall a \forall b ((f(a) = f(b)) \to a = b)$
$a, b \in Z$
let $f(a) = f(b)$ is true — ①
since $f(2) = f(-2) = 5$
but $\boxed{2 \neq -2}$
Hence $f(n) = n^2+1$
is not one to one func^n

not a one to one bcz for 1 image is 2 and for -1 image is 2, so NO.

c) $f(n) = n^3$

$\forall a \forall b (a \neq b \rightarrow (f(a) \neq f(b)))$

$a, b \in Z$

$a \neq b$ is true

$\llcorner$ ①

cubing both sides in eq^n ① :-

$a^3 \neq b^3$

$f(a) \neq f(b)$

Hence $f(n) = n^3$ is one to one f^n.

$\lceil \; \rceil$ or $\lfloor \; \rfloor$

ceiling floor

to convert real numbers into integers

$\lceil 8.1 \rceil = 9$

$\lfloor 8.999 \rfloor = 8$

d) $f(n) = \lceil n/2 \rceil$

$\forall a \forall b \cdot ((f(a) = f(b)) \rightarrow a = b)$

let $f(a) = f(b)$

since $\lceil 7.1 \rceil = \lceil 7.9 \rceil = 8$

but $7.1 \neq 7.9$

Hence $f(n) = \lceil n/2 \rceil$

is not one - one f^n

Increasing & Decreasing function :-

<u>Increasing</u> $f(x) \leq f(y)$ whenever $x<y$ where x & y are in one domain

<u>Decreasing</u> $f(x) \geq f(y)$ whenever $x<y$

Strictly increasing & strictly decreasing function

$f(x) < f(y)$ whenever $x<y$
$f(x) > f(y)$ whenever $x<y$

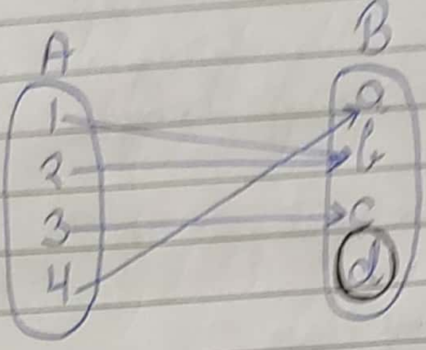
<u>Q-</u> let $f: R \to R$ and let $f(x) > 0 \ \forall x \in R$ Show that $f(x)$ is strictly decreasing if and only if function $g(x) = 1/f(x)$ is strictly increasing.

<u>Sol^n</u> → let $f(x)$ is strictly decreasing
Given :- $f(x) > f(y)$ whenever $x<y$
taking reciprocal :-

$$\frac{1}{f(x)} < \frac{1}{f(y)} \quad \text{whenever } x<y$$

$$g(x) < g(y).$$

Onto function → other name is surjective function



Not onto f^n bcz in codomain d is left.

Range $\subseteq$ codomain

$\boxed{\text{If Range} = \text{codomain}}$

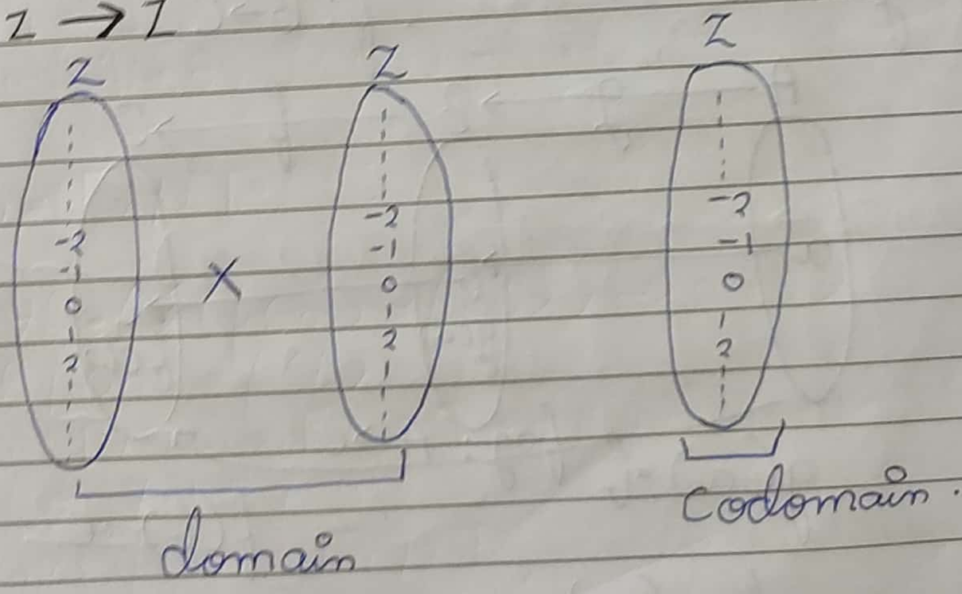
↓ f^n is onto

$f(1) = b$
$f(2) = b$
$f(3) = c$
$f(4) = a$

$\forall y \; \exists x \; (f(x) = y)$
where x is the element of domain and y is the element of co-domain

for every element of codomain & for some element of domain ~~there is~~ that $f(x) = y$

Q→ $f : Z \times Z \rightarrow Z$



a) $f(m, n) = m + n$
b) $f(m, n) = m^2 + n^2$
c) $f(m, n) = m$
d) $f(m, n) = |m|$
e) $f(m, n) = m - n$
f) $f(m, n) = m + n + 1$
g) $f(m, n) = |n|$
h) $f(m, n) = m^2 - 4$

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a) $f(m,n) = m+n$

$f(1,2) = 3$

$f(-1,-1) = -2$

$f(-1,1) = 0$

$\forall y\, \exists x\, (f(x) = y)$

Since, $f(m,n) = m+n$

let $m+n = y$

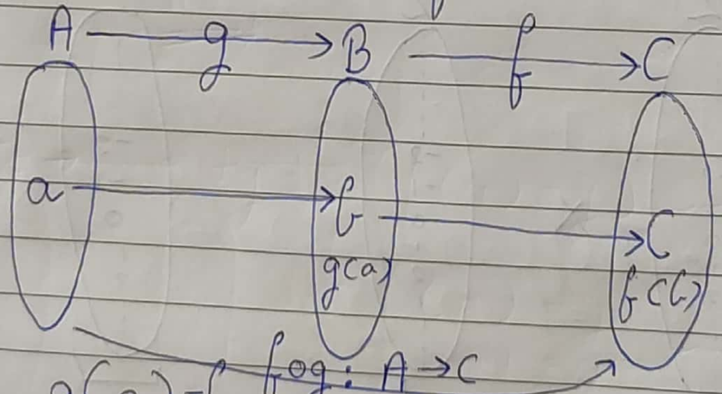
$\forall y \in \mathbb{Z}$ there are m & $n \in \mathbb{Z}$

such that $m+n = y \in \mathbb{Z}$

One-one-onto → other name is bijective or bijection function.

Composition of function →

$g : A \to B$ $\quad\quad f : B \to C$



$g(a) = b$ $\quad f \circ g : A \to C$

$f(b) = c$

$f \circ g\,(a) = c$

$f \circ g(a) = f[g(a)]$

$= f[b]$

$= c$

Q- $f(x) = 2x + 3$
$g(x) = 3x + 2$

find fog & gof

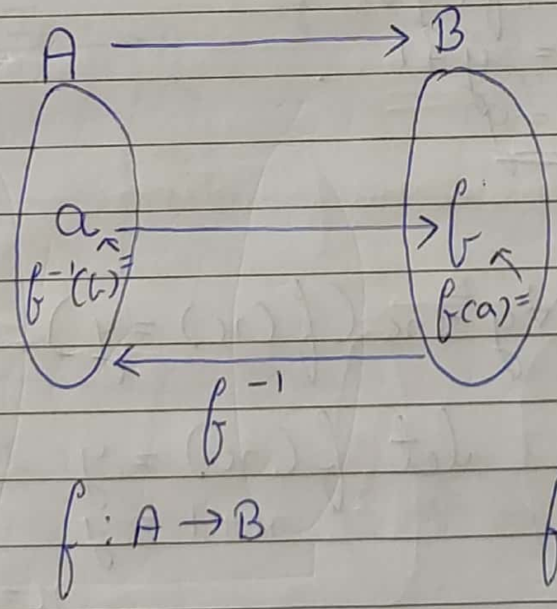
$fog = f[g(x)]$
$= f[3x+2]$
$= 2(3x+2) + 3$
$= 6x + 7$

$gof = g[f(x)]$
$= g[2x+3]$
$= 3(2x+3)+2$
$= 6x + 11$

$$\boxed{fog \neq gof}$$

Inverse function →

$f : A \to B$



$f : A \to B$　　　　$f^{-1} : B \to A$

$f(a) = b$.　　　　$f^{-1}(b) = a$.

<u>NOTE</u> → A one-one-onto correspondence is called invertible.

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Q- $f: Z \to Z$

$f(x) = x+1$ is it invertible = ? what is its inverse = ?

first prove one-one & onto then it is invertible

$f(x) = y \quad f^{-1}(y) = x$

$$\boxed{f^{-1}(y) = y-1}$$

$x+1 = y \Rightarrow x = y-1$

one-one →

$\forall x \forall y \, ((f(x) = f(y)) \to \overset{x=y}{\cancel{x=y}})$

let $f(x) = f(y)$

$x+1 = y+1$

$\underline{\underline{x = y}}$

onto → $\forall y \, \exists x \, (f(x) = y)$

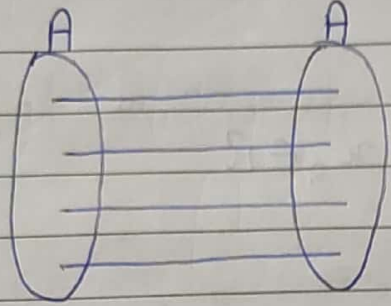
let $f(x) = y$

$x+1 = y$

$\underline{\underline{x = y-1}}$

Identity function →

$i_A : A \to A$

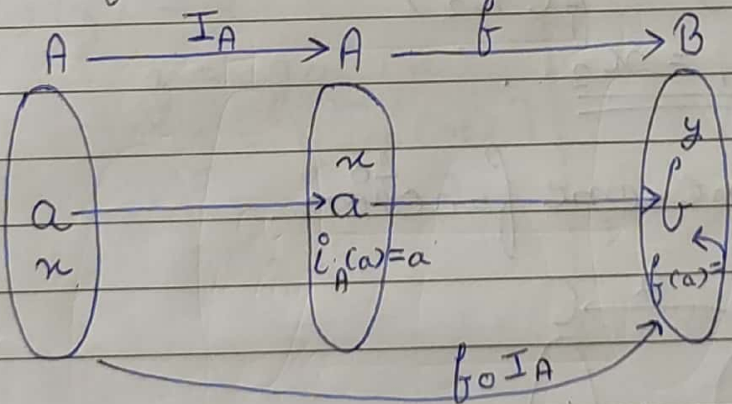


identity function is always one-one onto.

$i_A(x) = x$

Theorem: The composition of any function with identity function is the function itself.

i.e., $f \circ I_A(x) = I_B \circ f(x) = f(x)$



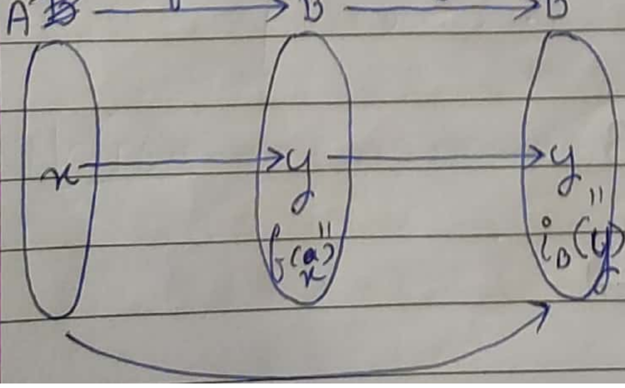
$i_A(x) = x$

$f(x) = y$

$f \circ I_A(x)$

$\Rightarrow f[I_A(x)]$

$\Rightarrow f(x)$



$I_B \circ f(x)$

$\Rightarrow I_B[f(x)]$

$\Rightarrow I_B[y]$

$\Rightarrow y$

Q. If $f: A \to B$ and $g: B \to x$ be one-one onto function. then gof is also one-one onto and $(gof)^{-1} = f^{-1} o g^{-1}$.

Since f is one-one

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2 \quad \forall x_1, x_2 \in R$

g is one-one

$g(y_1) = g(y_2)$

$\Rightarrow y_1 = y_2$

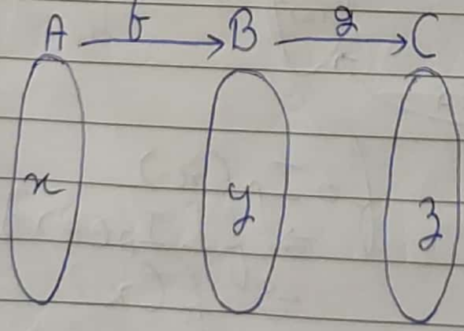
$gof(x_1) = gof(x_2)$

$g[f(x_1)] = g[f(x_2)]$

$f(x_1) = f(x_2)$

$\underline{\underline{x_1 = x_2}}$.

$\therefore$ gof is one-one function.



Since g is onto
for $\forall z \in C$ there is $y \in B$
and since f is onto
for $\forall y \in B$ there is $x \in A$

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$z = g(y)$
$z = g[f(x)]$
$z = g \circ f(x)$

$\therefore$ $g \circ f$ is onto function.

To prove, $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

$g \circ f : A \to C$, $(g \circ f)^{-1} : C \to A$
$f : A \to B$, $g : B \to C$
$f^{-1} : B \to A$, $g^{-1} : C \to B$

$f^{-1} \circ g^{-1} = C \to A$

$(g \circ f)^{-1}(z) = x$
$z = g \circ f(x)$
$z = g[f(x)]$
$z = g[y]$
$z = z$

$y = g^{-1}(z)$
$f^{-1}(y) = x$
$f^{-1}[g^{-1}(z)] = f^{-1} \circ g^{-1}($
$f^{-1} \circ g^{-1}(z) = x$

Q-1 $f : R \to R$, $f(x) = x^2$
find $f^{-1}(4)$ and $f^{-1}(-4)$

Not one-one and onto but still we c find value of inverse because it is not aske about one-one &on

$f^{-1}(4) = \{x \in R, f(x) = 4\}$
$= \{x \in R, x^2 = 4\}$
$= \{x \in R, x = \pm 2\}$

$f^{-1}(4) = \{-2, 2\}$

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$f^{-1}(-4) = \{x \in R, f(x) = -4\}$
$= \{x \in R, x^2 = -4\}$
$= \{x \in R, x = \pm 2i\}$
$= \phi$

$f^{-1}(-4) = \phi$

Q-2
$f : R \to R$
$f(x) = \{3x-4, x>0$
$-3x+2, \cancel{x} x \le 0\}$

find $f(0), f(2/3), f(-2), f^{-1}(0), f^{-1}(2), f^{-1}(-7)$

$f(0) = -3 \times 0 + 2$
$= 2$

$f(2/3) = 3 \times \frac{2}{3} - 4$
$= -2$

$f(-2) = -3 \times (-2) + 2$
$= 8$

$f^{-1}(0) = \{x \in R \mid f(x) = 0\}$

$= \{x \in R \mid x > 0 \text{ and } 3x-4=0\} \cup$
$\{x \in R \mid x \le 0 \text{ and } -3x+2=0\}$

$= \{x \in R \mid x>0 \text{ and } x = 4/3\} \cup$
$\{x \in R \mid x \le 0 \text{ and } x = 2/3\}$

$f^{-1}(0) = \frac{4}{3} \cup \phi = \frac{4}{3}$

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$f^{-1}(2) = \{x \in R \mid f(x) = 2\}$

$= \{x \in R \mid x > 0 \text{ and } 3x - 4 = 2\} \cup$
$\{x \in R \mid x \leq 0 \text{ and } -3x + 2 = 2\}$

$= \{x \in R \mid x > 0 \text{ and } x = 2\} \cup$
$\{x \in R \mid x \leq 0 \text{ and } x = 0\}$

$f^{-1}(2) = 2 \cup 0$
$f^{-1}(2) = \{2, 0\}$

$f^{-1}(7) = \{x \in R \mid f(x) = 7\}$ ϕ ??

$= \{x \in R \mid x > 0 \text{ and } 3x - 4 = 7\} \cup$
$\{x \in R \mid x \leq 0 \text{ and } -3x + 2 = 7\}$

$= \{x \in R \mid x > 0 \text{ and } x = 11/3\} \cup$
$\{x \in R \quad x \leq 0 \text{ and } x = -5/2\}$

Mathematical Induction →

Piano's Axioms :-

Let P(n) be the formula

Step 1 → Base step To prove P(1) is true

Step 2 → Inductive step a) let P(K) is true

b) To Prove P(K+1) is true

variety of question

→ Summation

→ Divisibility

→ InEquality

→ Strong MI

Summation →

Q- Use MI to prove that the sum of first 'n' natural numbers is $n(n+1)/2$

Sol^n → Let $P(n) : 1+2+3 ---- n-1+n$

Base step → To prove $P(1): 1 = \frac{1(1+1)}{2} = 1$

Inductive step → let, $P(K): 1+2+3 ---- (K-1)+K$ —①

$= \frac{K(K+1)}{2}$ is true

adding (K+1) in eq ① on both side

$1+2+3 ---- K+(K+1) = \frac{K(K+1)}{2} + (K+1)$

$$= (k+1)\left[\frac{k}{2}+1\right]$$

$$= \frac{(k+1)(k+2)}{2}.$$

Q-1 $1+2+2^2+\cdots+2^n = 2^{n+1}-1$

Q-2 $1.2+2.3+3.4+\cdots+n.(n+1) = \frac{n(n+1)(n+2)}{3}$

Q-3 $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+\frac{1}{2^n} = 1-\frac{1}{2^n}$

Q-4 Show that ~~If~~ the sum of n terms of AP is $\frac{n}{2}[2a+(n-1)d]$ where $a+(n-1)d$ is the n^{th} term of AP.

Q-5 Prove that for all natural numbers n

$$1+\sum_{k=1}^{n} k.k! = (n+1)!$$

Q-6 Prove that the sum of cubes of the first n natural numbe[rs] is

$$\left\{\frac{n(n+1)}{2}\right\}^2, \forall\, n \geq 1$$

Q-7 Prove that the sum of squares of the first n natural numbers is

$$\frac{n(n+1)(2n+1)}{6}, \forall\, n \geq 1$$

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Divisibility →

Q- Prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer using MI.

Sol^n → $P(n): 6^{n+2} + 7^{2n+1} \;/ 43$

Base step: for $n = 1$

$P(1): 6^{1+2} + 7^{2\times1+1}$

$= 6^3 + 7^3$

$= 216 + 343$

$= 559 / 43$ divisible by 43

Inductive step: for $n = k$

let $P(k): 6^{k+2} + 7^{2k+1}$ is divisible by 43

~~$: 6^{k+3} + 7^{2k+3}$~~

for $n = k+1$

$P(k+1): 6^{(k+1)+2} + 7^{2(k+1)+1}$

$= 6^{k+3} + 7^{2k+3}$

$= 6^{(k+2)+1} + 7^{2k+1+2}$

$= 6^{(k+2)} \cdot 6 + 7^{2k+1} \cdot 49$

$= 6^{k+2} \cdot 6 + 7^{2k+1} \cdot (6+43)$

$= 6\underbrace{(6^{k+2} + 7^{2k+1})}_{P(k)} + 7^{2k+1} \cdot 43$

$\therefore$ $P(k+1)$ is also divisible by 43.

Q- use MI to show that $11^{n+2} + 12^{2n+1}$ is divisible by 133.

Sol^n → $P(n): 11^{n+2} + 12^{2n+1} \;/ 133$

Base step: for $n = 1$

$P(1) = 11^{1+2} + 12^{2\times1+1}$

$= 11^3 + 12^3$

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$= 1331 + 1728$

$= 3059 \;/ 133$ divisible by 133

Inductive step → for $n = K$

let $P(K): 11^{K+2} + 12^{2K+1}$ is divisible by 133

for $n = K+1$

$P(K+1): 11^{(K+1)+2} + 12^{2(K+1)+1}$

$= 11^{K+3} + 12^{2K+3}$

$= 11^{(K+2)+1} + 12^{2K+1+2}$

$= 11^{(K+2)} \cdot 11 + 12^{2K+1} \cdot 144$

$= 11^{(K+2)} \cdot 11 + 12^{2K+1} \cdot (11+133)$

$= 11\underbrace{(11^{K+2} + 12^{2K+1})}_{P(K)} + 12^{2K+1} \cdot 133$

$\therefore P(K+1)$ is also divisible by 133

Q-1 Show that $x^{n-1} - 1$ is divisible by $x-1$ for all $n \geq 0$.

Q-2 use MI to show that $n^3 + 2n$ is divisible by 3 $\forall n \in Z^+$

Solⁿ 1 → $P(n): x^{n-1} - 1 \;/ x-1$

Base step: for $n = 1$

$P(1) = x^{1-1} - 1$

$= 1 - 1$

$= 0 \;/ x-1$ divisible by 0

Inductive step: for $n = K$

let $P(K): x^{K-1} - 1$ is divisible by $x - 1$

for $n = K+1$

$P(K+1): x^{(K+1)-1} - 1$

$= x^K - 1 + x - x$

$= x^k - x + x - 1$

$= x[x^{k-1} - 1] + x - 1$

$\therefore$ P(K+1) is divisible by $x-1$

Sol^n 2 → $P(n): n^3 + 2n \,/\, 3$

Base step: for $n = 1$

$P(1) = 1^3 + 2$

$= 1 + 2 \times 1$

$= 3 \,/\, 3$ divisible by 3

Inductive step: for $n = K$

let $P(K): K^3 + 2K$ is divisible by 3

for $n = K+1$

$P(K+1): (K+1)^3 + 2(K+1)$

$= K^3 + 1 + 3K(K+1) + 2K + 2$

$= K^3 + 2K + 3K(K+1) + 3$

$= \underbrace{K^3 + 2K}_{P(K)} + 3(K^2 + K + 1)$

$\therefore$ P(K+1) is divisible by 3

Inequality →

Q- Show that $n! > 2^n$ for any number $n \geq 4$

let $P(n): n! > 2^n$

Base step: $P(4): 4! > 2^4$

$24 > 16$ is true

Inductive step:

let $P(K): K! > 2^K$ is true — ①

for $n = K+1$

$P(K+1):$

Multiply eq ① by 2 ⇒ $2 \cdot K! > 2^K \cdot 2$

$2 \cdot K! > 2^{K+1}$

$5 \cdot K! > 2^{K+1}$

$(4+1)K! > 2^{K+1}$ $K = 4$

$(K+1)K! > 2^{K+1}$

$(K+1)! > 2^{K+1}$

Q-1 $2^n > n^2$ $\forall n \geq 5$

Q-2 $2^n > n!$ $\forall n \geq 10$

Sol^n 1 → let $P(n): 2^n > n^2$

Base step: $P(5): 2^5 > 5^2$

$32 > 25$ is true

Inductive step: let $P(K): 2^K > K^2$ is true — ①

for $n = K+1$

$P(K+1):$

Multiply eq ① by 2 $\Rightarrow$ $2 \cdot 2^K > K^2 \cdot 2$

$$2^{K+1} > K^2 \cdot 2$$

$$2^{K+1} > (K+1)^2 \qquad [2K^2 > (K+1)^2]$$

<u>Hence Proved</u>

Q- $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{-----} + \frac{1}{\sqrt{n}} > \sqrt{n} \qquad n \geq 2$

$P(2): \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$

$1 + 0.707 > 1.414$

$1.707 > 1.414$ <u>Proved</u>

$P(K): \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{-----} + \frac{1}{\sqrt{K}} > \sqrt{K} \qquad K \geq 2$ is true

adding $\frac{1}{\sqrt{K+1}}$

$P(K+1): \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{-----} + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \sqrt{K} + \frac{1}{\sqrt{K+1}}$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{----} + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \frac{\sqrt{K}\sqrt{K+1} + 1}{\sqrt{K+1}}$$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{-----} + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \frac{\sqrt{K^2+K} + 1}{\sqrt{K+1}}$$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \text{-----} + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \sqrt{K+1}$$

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Strong MI →

Step 1 → To prove $P(1)$ is true

Step 2 → Let $P(2), P(3), P(4) ---- P(K-1), P(K)$ all are true

Step 3 → To prove $P(K+1)$ is true.

Q-1 Consider the fibonacci sequence of number 1, 1, 2, 3, 5, 8 - each term in the sequence from the third term is obtained by adding the previous two terms.
If F_n is the n^{th} term, then the fibonacci series is defined by
$F_1 = 1$, $F_2 = 1$, ~~xxx~~, & $F_n = F_{n-1} + F_{n-2}$ $\forall\ n \geq 3$

Show that for each +ve integer n, the n^{th} fibonacci in $F_n < 2^n$.

$F_{K+1} < 2^{K+1}$

$F_{K+1} < 2^{K-1} \times 4$

Sol^n → Base step →
$P(3) = 2 < 2^3$
$2 < 8$ is true

Inductive step → let $P(4), P(5) ---- P(K-1), P(K)$ are true

$P(K+1) = F_{K+1} = F_K + F_{K-1} < 2^K + 2^{K-1}$

$F_{K+1} < 2^{K-1}(2+1)$

$F_{K+1} < 2^{K-1} \times 3$

$\Rightarrow F_{K+1} < 2^{K+1}$

Q-2 Show that the set of n elements has exactly 2^n subsets for any non negative integer n.

Sol^n → Base step → $P(0) = 2^0 = 1$ is true

if set is having zero elements then there is only one subset

Inductive step →

let $P(1), P(2), P(3) \ldots P(k-1), P(k)$ are true

$$P(k+1) = P($$

Q- Show that if n is an integer greater than 1, then n canbe the product of primes.

Base step: $P(2)$: $2 = 2\times1$ is true

Inductive step: $P(3), P(4), P(5) \ldots P(k-1), P(k)$ are true

$P(k+1)$

$k+1$ — Prime no

$k+1$ — Composite no

$$k+1 = a \times b$$

$$2 \le a \le b < k+1$$

$2 = 2\times1$	$10 = 5\times2$
$3 = 3\times1$	$11 = 11\times1$
$4 = 2\times2$	$12 = 2\times2\times3$
$5 = 5\times1$	$13 = 13\times1$
$6 = 3\times2$	$14 = 2\times7$
$7 = 7\times1$	
$8 = 2\times2\times2$	
$9 = 3\times3$	

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Q- let $x_1, x_2 ---- x_n$ be n sets. Then prove by M I that

$$\overline{\left(\bigcap_{i=1}^{n} x_i\right)} = \bigcup_{i=1}^{n} \overline{x_i} \quad \forall\, n \geq 1$$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

Base step: $P(1)$: $\overline{x_1} = \bar{x}_1$ ✗

$P(2)$: $\overline{x_1 \cap x_2} = \bar{x}_1 \cup \bar{x}_2$ ✓

Inductive step: let $P(K)$ is true

$$\overline{\left(\bigcap_{i=1}^{K} x_i\right)} = \left(\bigcup_{i=1}^{K} \overline{x_i}\right)$$

for $n = K+1$

To prove

$$\overline{\left(\bigcap_{i=1}^{K+1} x_i\right)} = \left(\bigcup_{i=1}^{K+1} \overline{x_i}\right)$$

$$= \overline{\left(\bigcap_{i=1}^{K} x_i \cap X_{K+1}\right)}$$

$$= \bigcup_{i=1}^{K} \overline{x_i} \cup \overline{X_{K+1}}$$

$$= \left(\bigcup_{i=1}^{K+1} \overline{x_i}\right)$$

29/10/2020

Unit 3 (I)

Posets & Lattices
Boolean Algebra → truth table, K Map, Circuit diagrams, Circuit minimization.

→ In relations - Partial order relation sets

set $A \times B$, R
$\{(-,-), (-,-), (-,-) \dots\}$

reflexive, ~~symm~~ transitive, Antisymmetric

Poset $\{(-,-), (-,-), (-,-), \dots\}$

collection of sets satisfying relation R.

A relation R on a set A is called partial ordering if it is reflexive, antisymmetric & transitive

Ex → $(Z^+, |)$
divisibility

$a \in Z^+$
reflexive → $a|a$
transitive → $a|b \wedge b|c \Rightarrow a|c$
antisymmetric → $\forall a \forall b (((a, b) \in R \wedge (b, a) \in R) \to a = b)$
$a|b$

∴ it is a Poset.

1. $(Z, =)$ → Poset
2. $(Z, \neq)$ → Not a Poset ∵ not antisymmetric
3. $(Z, \geq)$ → Poset

Comparable elements →

Partial order

The element a and b of a Poset $(S, \leq)$ are called comparable if either $a \leq b$ or $b \leq a$
when $a \nleq b$ and $b \nleq a$ then a and b are called incomparable.

$Ex \rightarrow (Z^+, |)$ $\{(2,4), (5,7) \ldots\}$

$2|4$, $4 \nmid 2$ comparable

$5 \nmid 7$, $7 \nmid 5$ incomparable

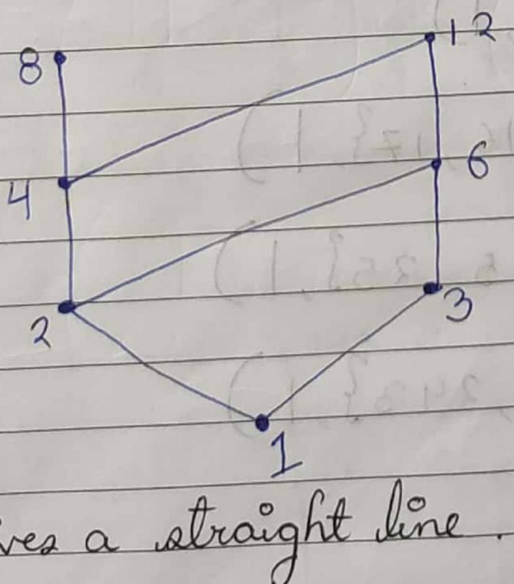
Toset → (Totally ordered set) every element ~~a set~~ ~~is a Poset~~. is comparable

$a \leq b$ or $b \leq a$.

$Ex \rightarrow (Z, \leq)$

Hasse Diagram → diagram to represent Poset & Toset

1. $(\{1, 2, 3, 4, 6, 8, 12\}, |)$



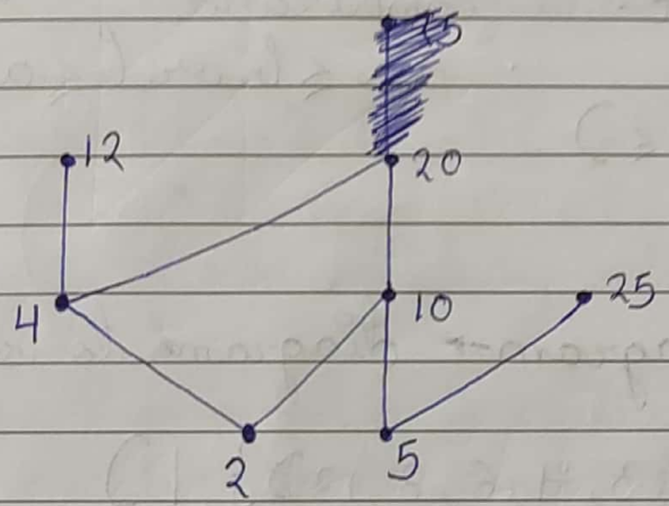
$\boxed{*}$ Toset gives a straight line.

PTO

2 ({1, 3, 9, 27, 81}, 1)

Toset

3 ({2, 4, 5, 10, 12, 20, 25}, 1)



Q-1 ({1, 2, 3, 4, 5, 6}, 1)

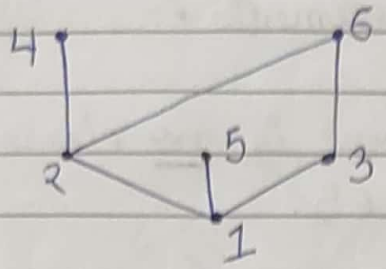
Q-2 ({3, 5, 7, 11, 13, 16, 17}, 1)

Q-3 ({2, 3, 5, 10, 11, 15, 25}, 1)

Q-4 ({1, 3, 9, 27, 81, 243}, 1)

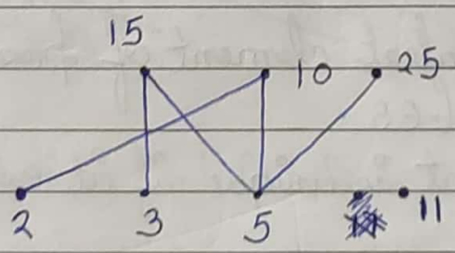
PTO

1



2

3



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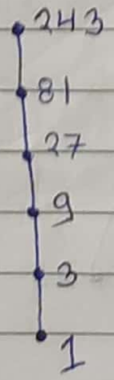

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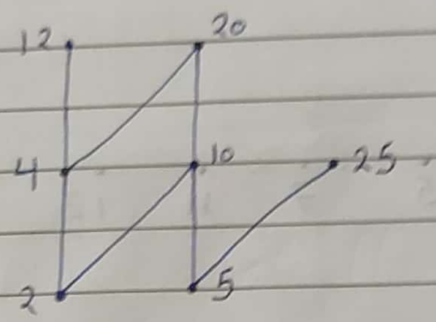
Special elements of Hasse Diagram →

1 Minimal & Maximal elements →

'a' is minimal if there is <u>no</u> element $b \in S$ such that $b \leq a$ for $a, b \in S$.

'a' is maximal if there is no element $b \in S$ such that $a \leq b$ for $a, b \in S$



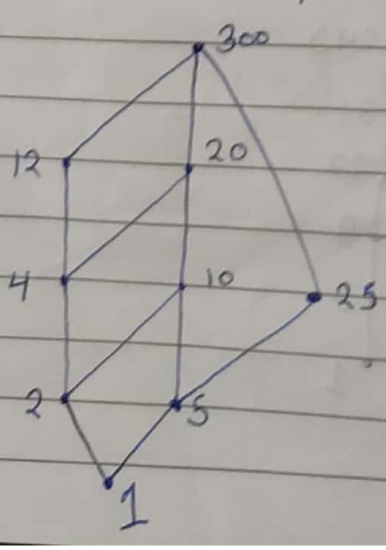
minimal → 2, 5
maximal → 12, 20, 25

greatest → x
least → x

2 Greatest & Least elements →

'a' is the greatest element of poset $(S, \leq)$ if $b \leq a \;\forall\, b \in S$.
Greatest element is unique if it exists.

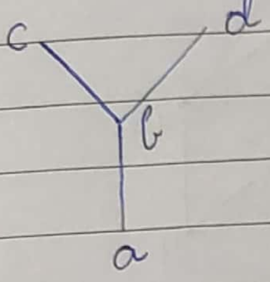
'a' is the least element of poset $(S, \leq)$ if $a \leq b \;\forall\, a \in S$.
Least element is unique if it exists.

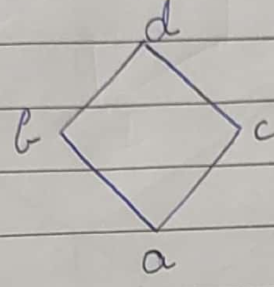


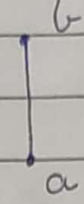
greatest → 300
least → 1

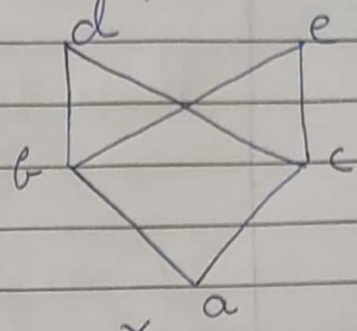
* Trick → if your figure is closed or straight line then there will be greatest & least element possible. upar se open to greatest nhi milega aur

agar neeche se open to least nhi milega.







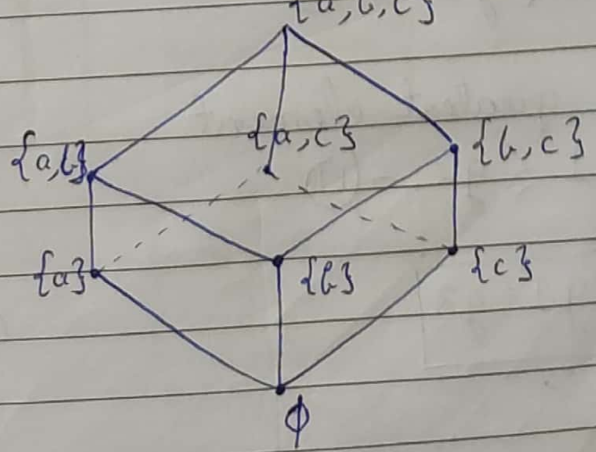


greatest →	x	d	b	x
least →	a	a	a	a
maximal →	c,d	d	b	d,e
minimal →	a	a	a	a

Q- $(P(S), \subseteq)$, $S = \{a, b, c\}$

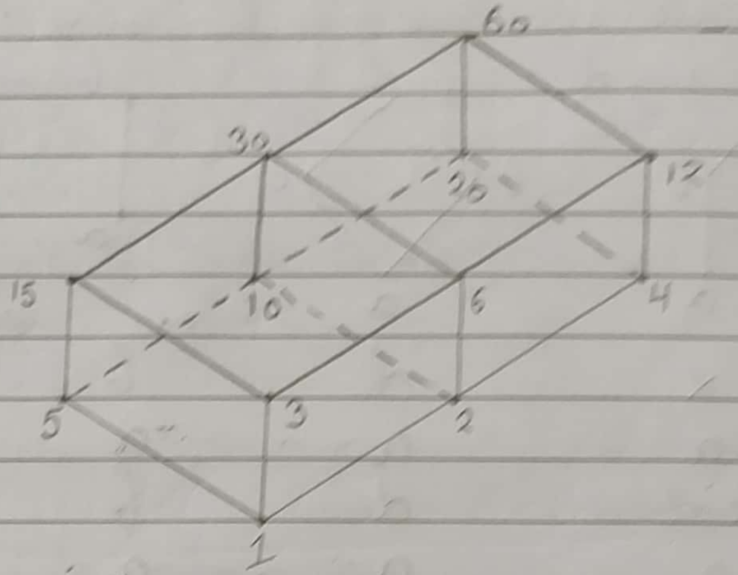
~~P(S) = { {a}, {b}, {c}, {a,b}, {b,c}, {c,a}, {a,b,c} }~~

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$



$D_{60} \rightarrow$ factors of 60

$(\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}, |)$



Theorem $\rightarrow$ Show that there is exactly one greatest element of a poset if such an element exists.

let $g1$ and $g2$ be two greatest elements of poset $(S, \leq)$

where $g1, g2 \in S$

If $g1$ is the greatest element

$g2 \leq g1$ — (I)

If $g2$ is the greatest element

$g1 \leq g2$ — (II)

$\boxed{g1 = g2}$

$a | b$ and $b | a$

$\boxed{a = b}$

$a \leq b$ and $b \leq a$

$\boxed{a = b}$

Theorem → Show that there is exactly one least element of a poset if such an element exists.

let $l1$ and $l2$ be two least element of poset $(S, \leq)$
where $l1, l2 \in S$

If $l1$ is the least element
$l1 \leq l2$ — ①

If $l2$ is the least element
$l2 \leq l1$ — ⑪

$\boxed{l1 = l2}$

Q- If a poset that has - ⓐ a minimal element but not maximal element
ⓑ a maximal element but not minimal element
ⓒ neither a minimal element nor a maximal element.

a) ~~N,≤~~ $(N, \leq)$ less greater than, $(Z^+, \leq)$, $(R^+, \leq)$

b) $(Z^-, \leq)$, $(R^-, \leq)$

c) $(Z, \leq)$, $(R, \leq)$

Product order :-

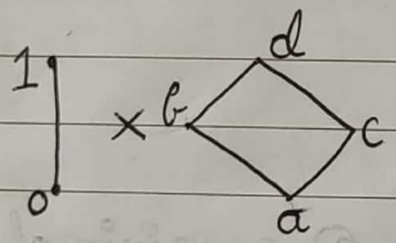
$(A, \leq_1)$ and $(B, \leq_2)$ are posets

Then $(A \times B, \leq)$ is also a poset.

If $a \leq_1 a'$ in A
and $b \leq_2 b'$ in B

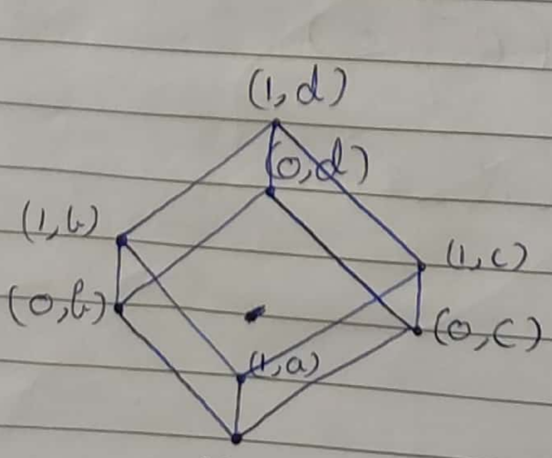
then $(a, b) \leq (a', b')$ in $A \times B$

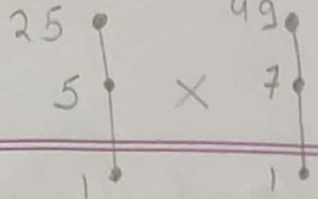
Q-



$\{0, 1\} \times \{a, b, c, d\}$

$\{(0,a), (0,b), (0,c), (0,d)$
$(1,a), (1,b), (1,c), (1,d)\}$

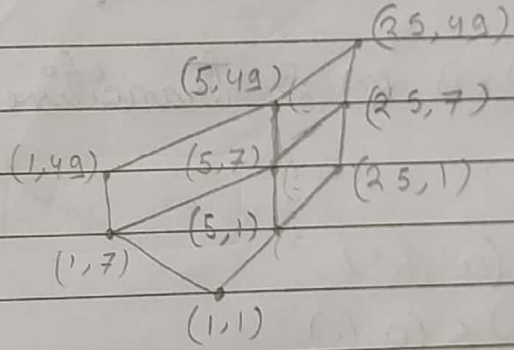




Q- $D_{25} \times D_{49}$

$$(\{1, 5, 25\}, |) \times (\{1, 7, 49\}, |)$$

$$\{(1,1), (1,7), (1,49)$$
$$(5,1), (5,7), (5,49),$$
$$(25,1), (25,7), (25,49)\}$$



Theorem → If $(A, \leq)$ and $(B, \leq)$ be posets then $(A \times B, \leq)$ is also a poset.

$(A, \leq_1)$ $(B, \leq_2)$

$a, a' \in A$ $b, b' \in B$

a'' b''

$(A \times B, \leq)$

$(a, b) \leq (a, b)$

$a \leq_1 a$

$b \leq_2 b$

Hence $(A \times B, \leq)$ is reflexive.

$(a, b), (a', b'), (a'', b'')$

If $(a, b) \leq (a', b')$
and $(a', b') \leq (a'', b'')$
then $(a, b) \leq (a'', b'')$

$a, a', a'' \in A$ $\quad$ $b, b', b'' \in B$

$a \leq_1 a'$ and $\quad$ $b \leq_2 b'$ and

$a' \leq_1 a''$ $\quad$ $b' \leq_2 b''$

$\Rightarrow a \leq_1 a''$ $\quad$ $\Rightarrow b \leq_2 b''$

Hence $(A \times B, \leq)$ is transitive

If $(a, b) \leq (a', b')$
and $(a', b') \leq (a, b)$
then $(a, b) = (a', b')$

$2 \leq 4$

$4 \leq 2$

$a \leq_1 a'$ and $a' \leq_1 a$
$\Rightarrow \quad a = a'$

$b \leq_2 b'$ and $b' \leq_2 b$
$\Rightarrow b = b'$

Hence $(A \times B, \leq)$ is antisymmetric

$\therefore (A \times B, \leq)$ is a poset.

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Upperbounds & Lowerbounds →

Let B be a subset of a poset $(A, \leq)$ and element u belongs to A is called an upperbound of B if u succeeds every element of B.

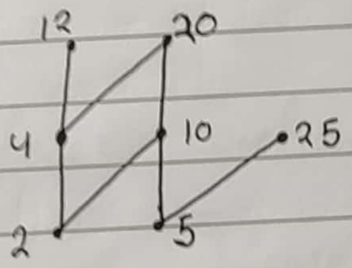
Let B be a subset of a poset $(A, \leq)$ and element l belongs to A is called an lower bound of B if l precedes every element of B.

$(A, \leq)$
$(B \subseteq A)$
$u \in A$
$x \leq u$
$\forall x \in B$
u is the upperbound

$(A, \leq)$
$(B \subseteq A)$
$l \in A$
$l \leq x$
$\forall x \in B$
l is the lower bound

upperbound → {5, 12} kisse relation rakh rha hai
lowerbound → {5, 12} se kon relation rakh rha hai

x →



$\{2, 4, 5, 10, 12, 20, 25\}$

ϕ is the lower bound of $\{5, 12\}$
ϕ is the upper bound of $\{5, 12\}$

$\{2\}$ is the lower bound of $\{2, 4\}$
$\{4, 12, 20\}$ is the upper bound of $\{2, 4\}$

Least ~~Lower~~ upper Bound (LUB): Supremum
Greatest Lower Bound (GLB): Infimum

$\{u_1, u_2, u_3, u_4\}$
$u_3 \leq u_1$
$u_3 \leq u_2$
$u_3 \leq u_4$

$\{l_1, l_2, l_3, l_4\}$
$l_2 \leq l_1$
$l_3 \leq l_1$
$l_4 \leq l_1$

LUB: let we are having upper bound $\{u_1, u_2, u_3, u_4\}$ and suppose u_3 is LUB then:

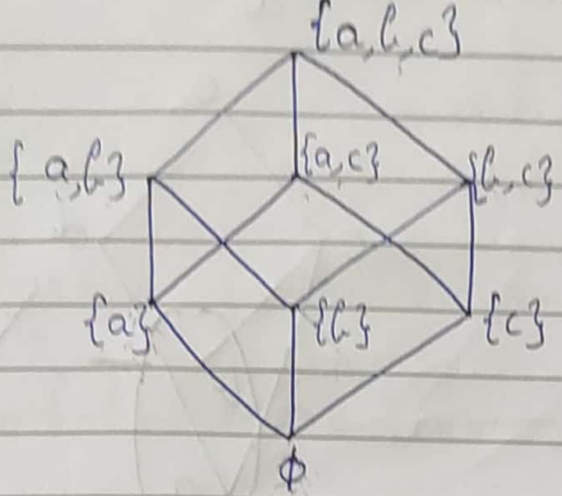
$u_3 \leq u_1$ if any of the condition fails it
$u_3 \leq u_2$ will not be the LUB.
$u_3 \leq u_4$

GLB: let we are having lower bound $\{l_1, l_2, l_3, l_4\}$, and suppose l_3 is GLB, then:

$l_1 \leq l_3$
$l_2 \leq l_3$
$l_4 \leq l_3$

LATTICES →

A poset in which every pair of element has both LUB and GLB is known as lattice.



$\phi \leq \{\phi, \{a\}\} \leq \{a\}$

$\{\phi, \{b\}\}$

$\{\phi, \{c\}\}$

$\{\phi, \{a,b\}\}$

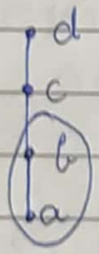
$\{\phi, \{b,c\}\}$

$\{\phi, \{a,c\}\}$

$\{\phi, \{a,b,c\}\}$

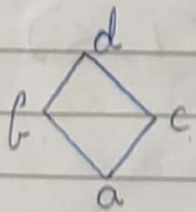
NOTE:- $a \vee b$ = a join b {LUB of a & b}
$a \wedge b$ = a meet b {GLB of a & b}

Suppose we are having any poset →



If we take $\{a,b\} \Rightarrow$ LUB = {b}
GLB = {a}

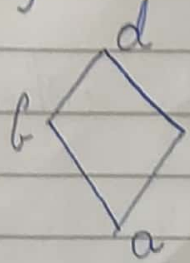
$\{b,c\} \Rightarrow$ LUB = {c}
GLB = {b}



→ LUB = d
GLB = a

Examples:-

✓



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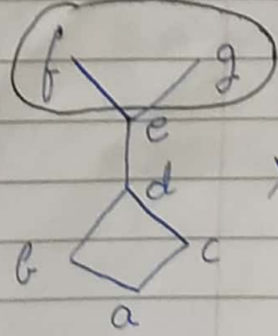
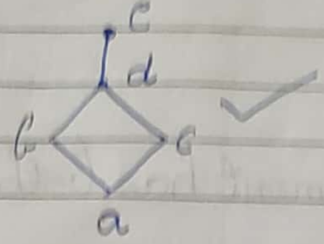
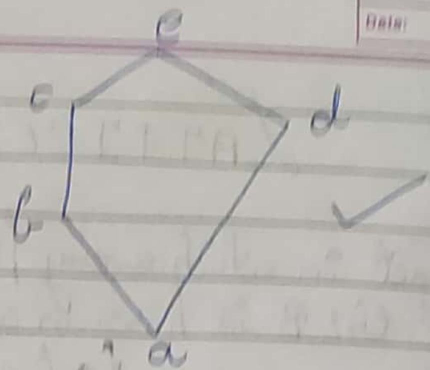
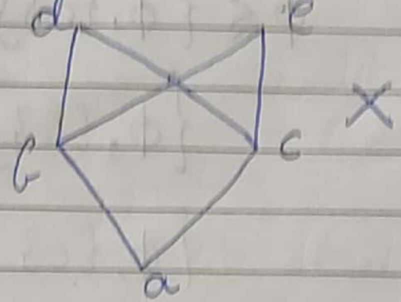


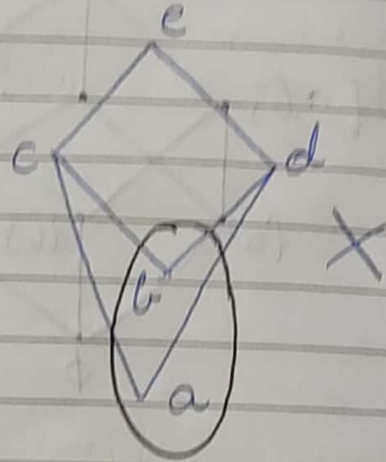
✗

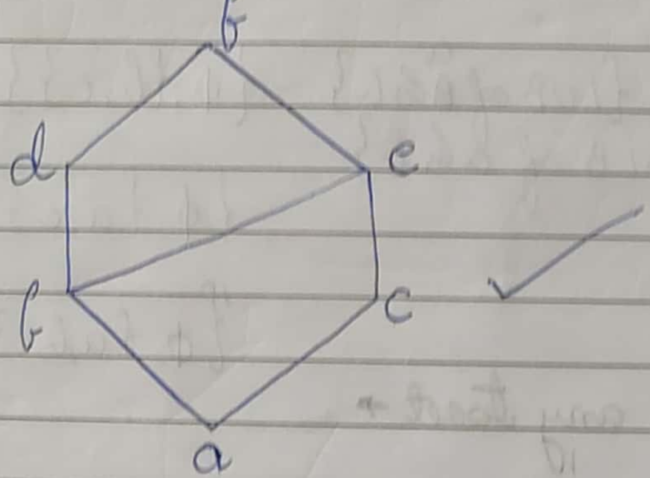
✗







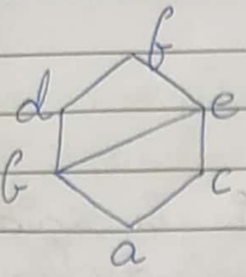


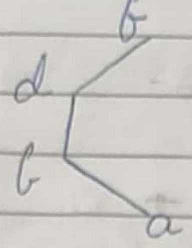


Sub-lattices :-

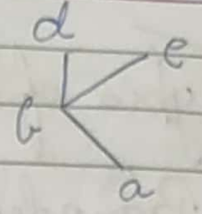
A non empty subset 's' of lattice 'L' is called a sub-lattice of lattice 'L' if $(L, \leq)$ $S \subseteq L$ $(S, \leq)$

$a \vee b \in S$
and
$a \wedge b \in S$ whenever $a, b \in S$

Eg → 



is a sublattice



not a sub lattice as d & e having no LUB.

Properties of lattices :-

1 Idempotent law :- $a \vee a = a$, $a \wedge a = a$

2 Commutative law :- $a \vee b = b \vee a$, $a \wedge b = b \wedge a$

3 Associative law :- $(a \vee b) \vee c = a \vee (b \vee c)$
$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

4 absorption law :- $a \vee (a \wedge b) = a$, $a \wedge (a \vee b) = a$

Theorem :- for any a, b, c, d in a lattice $(L, \leq)$

1 $a \leq a \vee b$ and $a \wedge b \leq a$

2 If $a \leq b$ and $c \leq d$ then $a \vee c \leq b \vee d$ and
$a \wedge c \leq b \wedge d$

2 $a \leq b$
$b \leq b \vee d$
$d \leq b \vee d$
$a \leq b \vee d$
$c \leq b \vee d$
$\boxed{a \vee c \leq b \vee d}$

~~$c \leq d$~~
~~$b \wedge d \leq b$~~
~~$b \wedge d \leq d$~~

$a \leq b$ $\quad$ $a \wedge c \leq a$

$c \leq d$ $\quad$ $a \wedge c \leq c$ $\quad \therefore \; a \wedge c \leq b \wedge d$

$a \wedge c \leq b$

$a \wedge c \leq d$

Types of lattices :-

1 **Bounded lattice** → a lattice is said to be bounded if it has a greatest element & a least element.

$$0, a, 1 \in L$$

$$0 \leq a \leq 1$$

identity property →

$$a \vee 1 = 1, \quad a \vee 0 = a$$

$$a \wedge 1 = a, \quad a \wedge 0 = 0$$

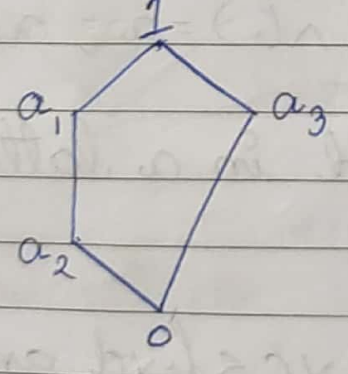
2 **Distributive lattice** → A lattice $(L, \leq)$ is said to be distributive if $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$

$$a \vee (b \wedge c)$$

$$a \wedge (b \vee c)$$

$$(a \wedge b) \vee c$$

Ex →



$$a_1 \wedge (a_2 \vee a_3)$$

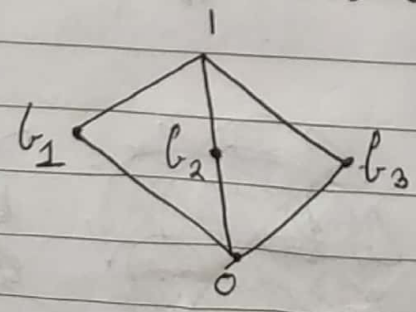
$$= (a_1 \wedge a_2) \vee (a_1 \wedge a_3)$$

$$a_1 \wedge 1 = a_2 \vee 0$$

$$a_1 \neq a_2$$

Not a distributive lattice,

Ex →



Theorem → In a distributive lattice $(L, \leq)$ if $a \vee b = a \vee c$ & $a \wedge b = a \wedge c$ imply that $b = c$.

$$
\begin{aligned}
b &= b \wedge (b \vee a) && \text{absorption} \\
&= b \wedge (a \vee c) && \text{given} \\
&= (b \wedge a) \vee (b \wedge c) && \text{distributive} \\
&= (a \wedge c) \vee (b \wedge c) && \text{given} \\
&= (a \vee b) \wedge c && \text{distributive} \\
&= (a \vee c) \wedge c && \text{given} \\
&= c && \text{absorption}
\end{aligned}
$$

Hence Proved

3 **Modular lattice →** a lattice $(L, \leq)$ is said to be modular if $a \vee (b \wedge c) = (a \vee b) \wedge c$ whenever $a \leq c$

Theorem → every distributive lattice is Modular.

$c = a \vee c$

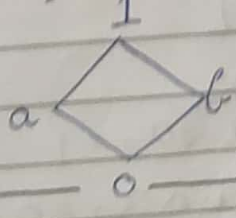
$a = a \wedge c$

$$
\begin{aligned}
a \vee (b \wedge c) &= (a \vee b) \wedge (a \vee c) \\
&= (a \vee b) \wedge c
\end{aligned}
$$

4 **Complemented lattice →** It is a lattice in which every element has a one complement.

$(L, \wedge, \vee, 0, 1)$

$b, a \in L$



$0 \vee 1 = 1$

$0 \wedge 1 = 0$

$a \vee b = 1$

$a \wedge b = 0$

$0 \vee b = b$ ×

$0 \wedge b = 0$

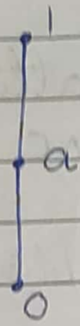
$a \vee 0 = 0$ ×

$a \wedge 0 = 0$ ✓

1] $0 \leq a \leq 1$

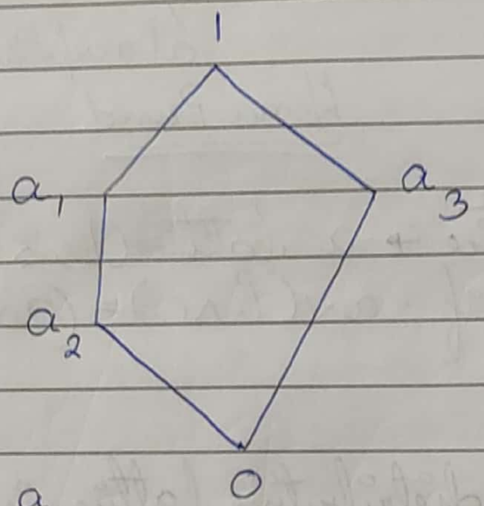
2] $a \vee b = 1$
$a \wedge b = 0$

$$\boxed{\begin{matrix} (a)' = b & (0)' = 1 \\ (b') = a & (1)' = 0 \end{matrix}}$$



$1 \vee 0 = 1$
$1 \wedge 0 = 0$
$0 \vee a = a$ ✗
$0 \wedge a = 0$
$a \vee 1 = 1$
$a \wedge 1 = a$ ✗

Not a complementary lattice because there is no complement of a.



$a_1 \vee a_3 = 1$
$a_1 \wedge a_3 = 0$
$a_2 \vee a_3 = 1$
$a_2 \wedge a_3 = 0$
$a_2 \vee 0 = a_2$ ✗
$a_1 \vee 0 = a_1$ ✗
$a_3 \vee 0 = a_3$ ✗
$0 \vee 1 = 1$
$0 \wedge 1 = 0$

$(a_3)' = a_2, a_1$
$(a_1)' = a_3$
$(a_2)' = a_3$
$(0)' = 1$
$(1)' = 0$

Theorem → in a distributive lattice if an element has a complement then this complement is unique.

$a, a_1, a_2 \in L$

$a \vee a_1 = 1$ $\quad$ $a \vee a_2 = 1$
$a \wedge a_1 = 0$ $\quad$ $a \wedge a_2 = 0$

$a_1 = a_1 \wedge 1$ identity
$a = a_1 \wedge (a \vee a_2)$ given

$= (a_1 \wedge a) \vee (a_1 \wedge a_2)$

$= \cancel{a \times} 0 \vee (a_1 \wedge a_2)$

$= (a \wedge a_2) \vee (a_1 \wedge a_2)$

$= (a \vee a_1) \wedge a_2$

$= 1 \wedge a_2$

$= a_2$.

Theorem → In a complemented and distributive lattice

from i) prove ii)

ii)	iii)
iii)	iv)
iv)	i)

i) $a \leq b$

ii) $a \wedge b' = 0$

iii) $a' \vee b = 1$

iv) $b' \leq a'$

(ii) $b = a \vee b$

$a = a \wedge b$

$a \vee b = b$

$(a \vee b) \wedge b' = b \wedge b'$

$(a \wedge b') \vee (b \wedge b') = b \wedge b'$

$(a \wedge b') \vee 0 = 0$

$(a \wedge b') = 0$.

(iii) $(a \wedge b') = 0$

$(a \wedge b')' = (0)'$

$a' \vee b = 1$

A $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

$a \leq a \vee b$ | $a \leq a \vee c$

$b \wedge c \leq b \leq a \vee b$ | $b \wedge c \leq c \leq a \vee c$

$\to a \leq (a \vee b) \wedge (a \vee c)$

$b \wedge c \leq (a \vee b) \wedge (a \vee c)$

$\Rightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

$a \vee (b \wedge c) \leq (a \vee b) \wedge c$

if $a \leq c$

$a \vee c = c$

Unit 3 (II) Boolean Algebra.

★ Duality law :-

U_4 , U_3 , boolean
lattice

P, Q	a, b	x, y
$\vee$	$\vee$ join	$+$ sum
$\wedge$	$\wedge$ meet	$\cdot$ product
T	1 greatest	1
F	0 least	0
$\neg$	$'$	$'$, $\overline{}$

$$\neg P \wedge (Q \rightarrow R) \equiv \neg P \wedge (\neg Q \vee R)$$
$$\equiv \neg P \vee (\neg Q \wedge R) \ *$$

$$x \cdot y\bar{z} + \bar{y}z \equiv (x+y+\bar{z}) \cdot (\bar{y}+z)$$

Truth table

	x	y	z	$\bar{z}$	$\bar{y}$	$xy\bar{z}$	$\bar{y}z$	$xy\bar{z}+\bar{y}z$
0	0	0	0	1	1	0	0	0
1	0	0	1	0	1	0	1	1
2	0	1	0	1	0	0	0	0
3	0	1	1	0	0	0	0	0
4	1	0	0	1	1	0	0	0
5	1	0	1	0	1	0	1	1
6	1	1	0	1	0	1	0	1
7	1	1	1	0	0	0	0	0

PTO

$x+y+\bar{z}$	$\bar{y}+z$	$(x+y+\bar{z})\cdot(\bar{y}+z)$
1	1	1
0	1	0
1	0	0
1	1	1
1	1	1
1	1	1
1	0	0
1	1	1

Laws :-

1 $x+x=x$, $x\cdot x=x$] Idempotent

2 $x+y=y+x$, $x\cdot y=y\cdot x$] Commutative

3 $x+(y+z)=(x+y)+z$, $x\cdot(\cancel{x}y\cdot z)=(x\cdot y)\cdot z$] associative

4 $x\cdot(y+\cancel{z}^{x})=x$, $x+(y\cdot x)=x$] absorption

5 $\overline{x+y}=\bar{x}\cdot\bar{y}$, $\overline{x\cdot y}=\bar{x}+\bar{y}$] De Morgan's

6 $\bar{\bar{x}}=x$ double complement

7 $x+\bar{x}=1$, $x\cdot\bar{x}=0$] identity law

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8 $x + 1 = 1$] → unit property
$x \cdot 0 = 0$] → zero property

9 $x + 0 = x$
$x \cdot 1 = x$] domination law

10 $x \cdot (y + z) = x \cdot y + x \cdot z$
$x + (y \cdot z) = (x+y) \cdot (x+z)$] → distributive

☆ Canonical SOP & POS :-

min = $x \cdot y \cdot \bar{z}$, $x\bar{y}z$ → sum → SOP

max = $x + y + \bar{z}$, $x + \bar{y} + z$ → Product → POS

<u>eg</u> :- $f(x, y, z) = \bar{x}$

$= \bar{x} + 0$ domination
$= \bar{x} + (y \cdot \bar{y})$ identity
$= (\bar{x} + y) \cdot (\bar{x} + \bar{y})$ distributive
$= (\bar{x} + y + 0) \cdot (\bar{x} + \bar{y} + 0)$
$= (\bar{x} + y + z\bar{z}) \cdot (\bar{x} + \bar{y} + z\bar{z})$
$= \underbrace{(\bar{x} + y + z) \cdot (\bar{x} + y + \bar{z}) \cdot (\bar{x} + \bar{y} + z) \cdot (\bar{x} + \bar{y} + \bar{z})}$

↓
POS form

$= \bar{x} \cdot 1$
$= \bar{x} \cdot (y + \bar{y})$
$= \bar{x} \cdot y + \bar{x} \cdot \bar{y}$
$= \bar{x} \cdot y \cdot 1 + \bar{x} \cdot \bar{y} \cdot 1$
$= \bar{x} \cdot y \cdot (z + \bar{z}) + \bar{x} \cdot \bar{y} \cdot (z + \bar{z})$

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$$= \bar{x}\cdot y\cdot z + \bar{x}\cdot y\cdot\bar{z} + \bar{x}\cdot\bar{y}\cdot z + \bar{x}\cdot\bar{y}\cdot\bar{z}$$

↓

SOP form

eg:- $(x+y)\cdot z$

SOP →
$$= \cancel{(x\cdot 1)}\cdot(x\cdot 1 + y\cdot 1)\, z$$
$$= (x\cdot(y+\bar{y}) + y\cdot(x+\bar{x}))\, z$$
$$= (x\cdot y + x\cdot\bar{y} + y\cdot x + y\cdot\bar{x})\, z$$
$$= (xy + x\bar{y} + y\bar{x})\, z$$
$$= xyz + x\bar{y}z + \bar{x}yz$$

POS →
$$= (x+y+0)\cdot(z+0+0)$$
$$= (x+y+z\bar{z})\cdot(z + x\cdot\bar{x} + y\cdot\bar{y})$$

$$= (x+y+z)(x+y+\bar{z})\cdot(z+x+y)(z+\bar{x}+y)$$
$$\cdot\,(z+x+\bar{y})\cdot(\bar{x}+z+\bar{y})$$

Functionally Complete set of Connective :-

$\{+, '\}$ $\quad$ $\{\cdot, '\}$

$\downarrow$ nor $\quad$ $\uparrow$ nand $\quad$ $\downarrow \rightarrow$ nor, $\uparrow \rightarrow$ nand

Q- Show that a) $\overline{x} = x \downarrow x = x \uparrow x$

$(x \uparrow y) \uparrow (x \uparrow y) =$ b) $xy = (x \downarrow x) \downarrow (y \downarrow y)$

$(x \uparrow x) \uparrow (y \uparrow y) =$ c) $x + y = (x \downarrow y) \downarrow (x \downarrow y)$

a) $\overline{x} = \overline{x + x}$ $\quad$ [$x = x + x$ idempotent]

$\overline{x} = x \downarrow x$

$\overline{x} = \overline{x \cdot x}$ $\quad$ [idempotent]

$\overline{x} = x \uparrow x$

b) $xy = \overline{\overline{x \cdot y}}$

$= \overline{\overline{x} + \overline{y}}$

$= \overline{x} \downarrow \overline{y}$

$= \overline{x + x} \downarrow \overline{y + y}$

$= (x \downarrow x) \downarrow (y \downarrow y)$.

$\cancel{xy = \overline{\overline{x \cdot y}} = \overline{x \uparrow y} = \overline{x \cdot x} \uparrow \overline{y \cdot y} = (x \uparrow x) \uparrow (y \uparrow y)}$

$xy = \overline{\overline{xy}}$

$= \overline{\overline{(xy) + (xy)}}$

$= \overline{\overline{(xy)} \cdot \overline{(xy)}}$

$= \overline{xy} \uparrow \overline{xy}$

$= (x \uparrow y) \uparrow (x \uparrow y)$

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c) $x+y = \overline{\overline{x+y}}$

$= \overline{\bar{x}\cdot\bar{y}}$

$= \bar{x}\uparrow\bar{y}$

$= \overline{x\cdot x}\uparrow\overline{y\cdot y}$

$= (x\uparrow x)\uparrow(y\uparrow y)$

$x+y = \overline{\overline{x+y}}$

$= \overline{(x+y)\cdot(x+y)}$

$= \overline{(\overline{\overline{x+y}})+(\overline{\overline{x+y}})}$

$= (\overline{x+y})\downarrow(\overline{x+y})$

$= (x\downarrow y)\downarrow(x\downarrow y)$

$x, y, z \to 1$

$\bar{x}, \bar{y}, \bar{z} \to 0$

KMAP →

$f(x,y) =$

	$\bar{y}$	y
$\bar{x}$	00 (0)	01 (1)
x	10 (2)	11 (3)

$f(x,y,z) =$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{x}$	000 (0)	001 (1)	011 (3)	010 (2)
x	100 (4)	101 (5)	111 (7)	110 (6)

$\Sigma \rightarrow SOP \rightarrow 1$

$\pi \rightarrow POS \rightarrow 0$

↑ octat
quad
pair
single

$f(x, y, z, w) =$

$xy \backslash zw$	$\bar{z}\bar{w}$	$\bar{z}w$	zw	$z\bar{w}$
$\bar{x}\bar{y}$	0000 (0)	0001 (1)	0011 (3)	0010 (2)
$\bar{x}y$	0100 (4)	0101 (5)	0111 (7)	0110 (6)
xy	1100 (12)	1101 (13)	1111 (15)	1110 (14)
$x\bar{y}$	1000 (8)	1001 (9)	1011 (11)	1010 (10)

Q- $f(x, y, z, w) = \Sigma\ 0, 1, 2, 3, 7, 8, 11, 12$

	$\bar{z}\bar{w}$	$\bar{z}w$	zw	$z\bar{w}$
$\bar{x}\bar{y}$	1 (0)	1 (1)	1 (3)	1 (2)
$\bar{x}y$	(4)	(5)	1 (7)	(6)
xy	1 (12)	(13)	(15)	(14)
$x\bar{y}$	1 (8)	(9)	1 (11)	(10)

1 quad + 3 pair

$\bar{x}\bar{y} + x\bar{z}\bar{w} + \bar{x}zw + \overset{\bar{y}}{\cancel{x}}zw$

UNIT-5

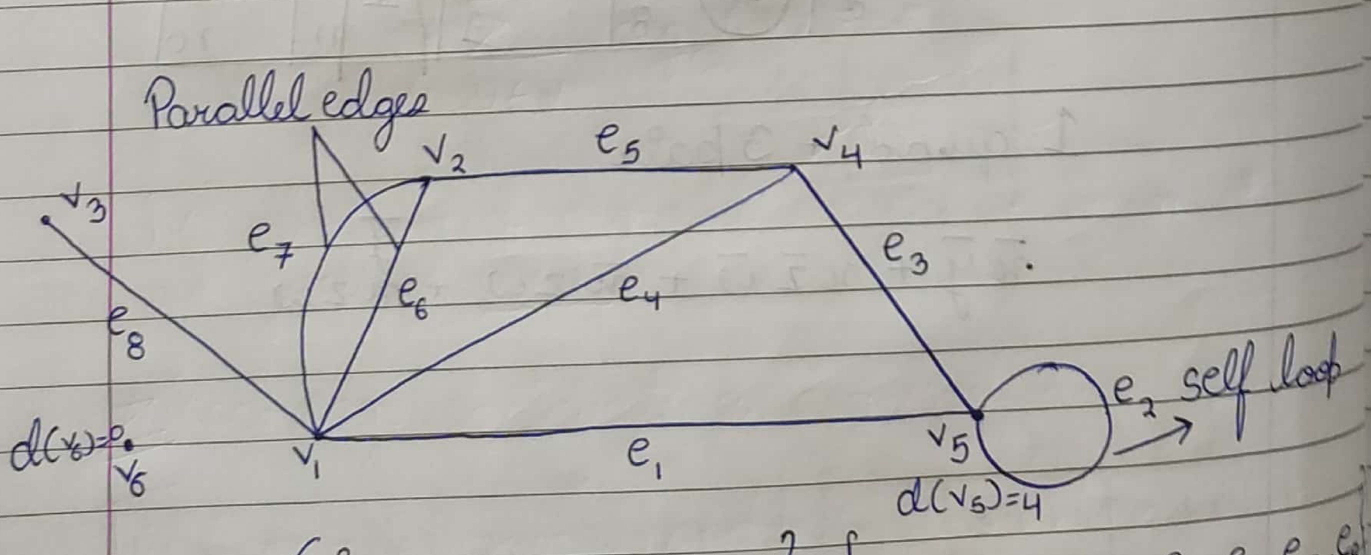
1. Graph Theory →
2. Tree
3. Counting Principles → Sum rule
   Product rule
   Inclusion - Exclusion
   Pigeonhole Principle
   $P_n C$
4. Advance Counting Principles → Recurrence relation
   Generating function

Graph Theory → [represent with G]

$G = (V, E)$

V: set of vertices   E: set of edges

Graph → directed graph (with direction)
→ undirected graph (with no direction)



$G = (\{v_1, v_2, v_3, v_4, v_5\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

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adjicent vertices → having common edge
Ex → v_3 & v_1 etc.

adjicent edges → having common vertex
Ex → e_1 & e_3 etc.

order of graph O(G) → no of vertices present in a graph. O(G) = 5

size of graph S(G) → no of edges in a graph.
S(G) = 8

Type of edges →

1 Self loop → an edge which is having same end points.

2 Parallel edges → having same end points for two edges.

Types of Graph →

1 Simple Graph → having no self loops & no parallel edges.

2 Multi Graph → having parallel edges but no self loops

3 Pseudo Graph → Having both parallel edges & self loops.
→ only self loops also come in Pseudo Graph.

Degree of a vertex → no of edges incident on a vertex.

representation
↓
$d(v)$ or $deg(v)$

$d(V_1) = 5$ $d(V_3) = 1$
$d(V_2) = 3$ $d(V_4) = 3$
$d(V_5) = 4$

Type of vertices →

1 Isolated node → a vertex with degree zero
Ex → V_6

2 Pendant node → a vertex with degree 1
Ex → V_3

Handshaking Theorem →

If $G = (V, E)$ be an undirected graph with e edges.
Then,

$$\sum_{v \in V} deg_G(v) = 2e$$

i.e, the sum of degree of the vertices in an undirectional graph is even.

☆ each edge contribute a degree of 2.

This theorem is known as handshaking theorem because in a handshake there is requirement of 2 people and similarly in this theorem an edge requires 2 vertices.

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Theorem → Show that the degree of a vertex of a simple graph 'G' on '~~no~~ n' vertices cannot exceed $n-1$.

If a graph consists of n vertices and if we take out 1 vertex then that vertex will be connected maximum to $n-1$ vertices. Therfore, we can say that the degree of a vertex of a simple graph 'G' on n vertices cannot exceed $n-1$.

The minimum degree for a vertex is 0 i.e. isolated node.

degree of any vertex will flow within this range ← $\boxed{0 \leq deg_G(v) \leq n-1}$ where n is no. of ~~edges~~ vertices.

Theorem → Show that the maximum number of edges in a simple graph with n vertices is $e = \frac{n(n-1)}{2}$

$$\sum_{i=1}^{n} deg_G(v_i) = 2e$$ [handshaking Theorem]

$$deg(v_1) + deg(v_2) + deg(v_3) + \cdots + deg(v_n) = 2e$$

max degree of any vertex is $n-1$

$$n-1 + n-1 + n-1 + \cdots\cdots n-1 = 2e$$

$$n(n-1) = 2e$$

$$\boxed{e = \frac{n(n-1)}{2}}$$

Theorem:- In an undirected graph the total no of odd degree vertices is even.

$$\sum_{i=1}^{n} \deg_G (v_i) = \underbrace{2e}_{\text{even}}$$

$$\underbrace{U}_{\substack{\text{no of vertices} \\ \text{with even} \\ \text{degree}}} + \underbrace{W}_{\substack{\text{no of vertices} \\ \text{with odd} \\ \text{degree.}}} = n$$

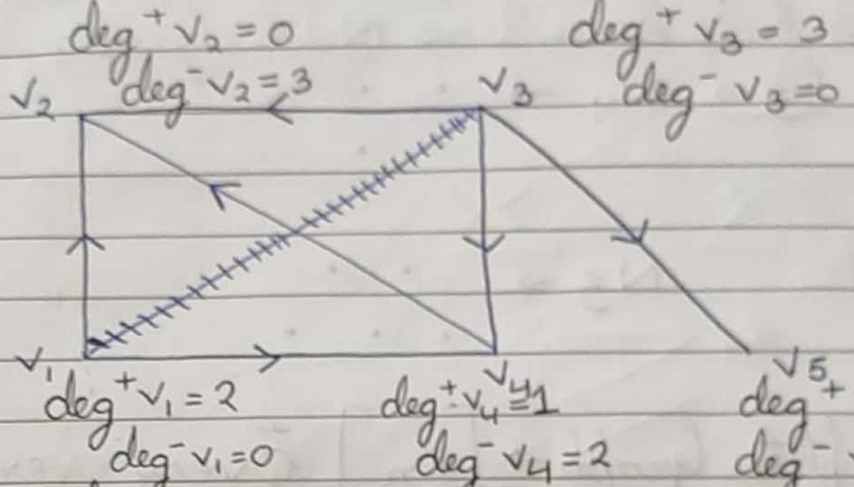
$$\underbrace{\sum_{i=1}^{U} \deg_G v(i)}_{\text{even}} + \sum_{i=1}^{W} \deg_G (v_i) = \underbrace{2e}_{\text{even}}$$

$$\sum_{i=1}^{W} \deg_G (v_i) = 2e - \sum_{i=1}^{U} \deg_G (v_i)$$

$$= \text{even} - \text{even}$$

$$\boxed{= \text{even}}$$

Directed Graph → The graph in which the edges have direction.



outdegree(+) → from where the edge is going to start

indegree(-) → from where the edge is going to end.

source → where indegree is zero.
sink → where outdegree is zero.

Theorem → If $G = (V, E)$ be a directed graph with e edges then

$$\sum deg^+_G v = \sum deg^-_G v = e$$

Each edge contributes with 2 vertex and also contributes to 1 indegree and 1 outdegree, edge always ~~Hence~~ starts from any point (out degree) & ends at a point (in degree)

Hence total no of indegree = total no of outdegree = e

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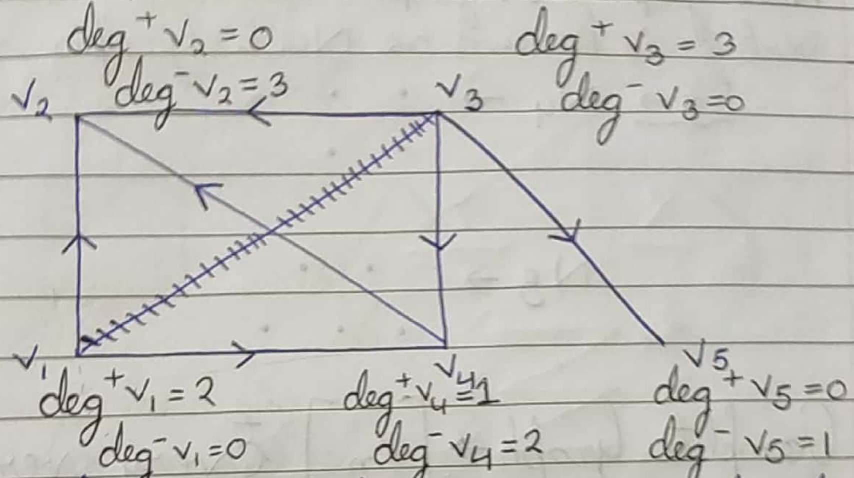
Directed Graph → The graph in which the edges have direction.



outdegree(+) → from where the edge is going to start

indegree(−) → from where the edge is going to end.

Source → where indegree is zero.
Sink → where outdegree is zero.

Theorem → If $G = (V, E)$ be a directed graph with e edges then

$$\sum deg^+_G v = \sum deg^-_G v = e$$

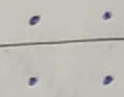
Each edge contributes with 2 vertex and also contributes to 1 indegree and 1 outdegree, edge always ~~[illegible]~~ starts from any point (out degree) & ends at a point (in degree)

Hence total no of indegree = total no of outdegree = e

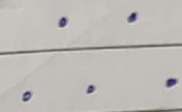
Type of Graphs →

1 Null Graph → $[N_n]$ A graph with the collection of isolated vertex is known as Null Graph.

Ex → N_4 →

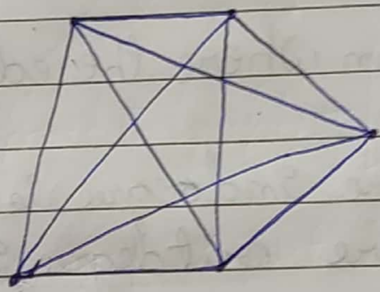


N_5 →



2 Complete Graph → $[K_n]$ Every vertex is adjacent to every other vertex.

Ex → K_5



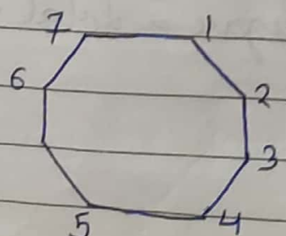
If there are 'n' vertex then total no of edges = $\frac{n(n-1)}{2}$

3 Regular Graph → $[R_n]$ A graph in which every vertex has same degree i.e. 'r'

If there are 'n' vertex of degree 'r' then total no of edges = $\frac{n \times r}{2}$.

Ex →

vertex → 7
degree → 2



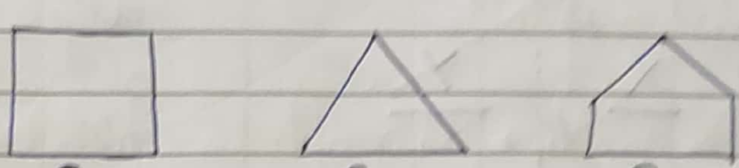
$e = \frac{7 \times 2}{2} = 7$.

Complete Graph & Null Graph are example of regular graph.

4 Cycle → $[C_n]$ to make a cycle $n \geq 3$.

every vertex has degree 2 and adjacent to its neighbour

Ex →

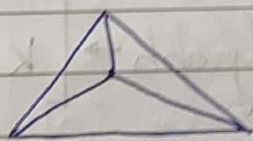


C_4 C_3 C_5

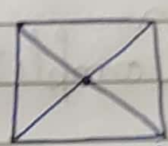
it should has a closed figure
no of edges = no of vertex.

5 Wheel $[W_n]$ →

vertex is 4 W_3
but we write W_3

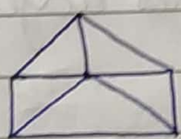


W_4



vertex is 5 but we write W_4

W_5



vertex is 6 but we write W_5

center vertex has degree $= n$ & rest vertex has degree $= 3$.

Total no of edges $= 2n$.

6 ~~Bor~~ Bipartite Graph → A graph $G = (V, E)$ is bipartite if the vertex set V can be partitioned into two disjoint subsets V_1 & V_2 such that every edge in E connects a vertex in V_1 and a vertex in V_2 so that no edge in G connects either two vertices in V_1 or two vertices in V_2 is called a bipartition of G.

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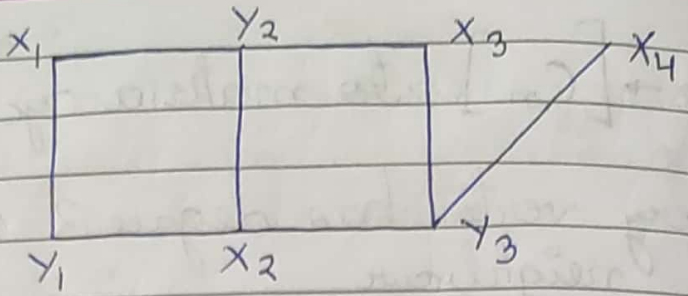
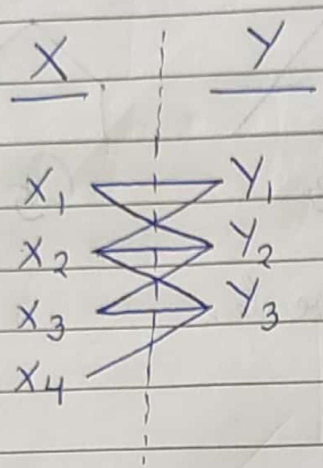
Ex →

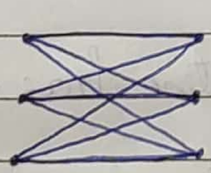




7 Complete Bipartite Graph → $[K_{m,n}]$

every vertex of X set is adjecent to every vertex of Y set.

Ex → $K_{3,3}$



8 Connected Graph → An undirected graph G is called connected if there is atleast one path between every pair of verties of G otherwise G is disconnected

☆ Null graph with more than one vertex is disconnected

☆ If there is only one vertex than it is connected graph.

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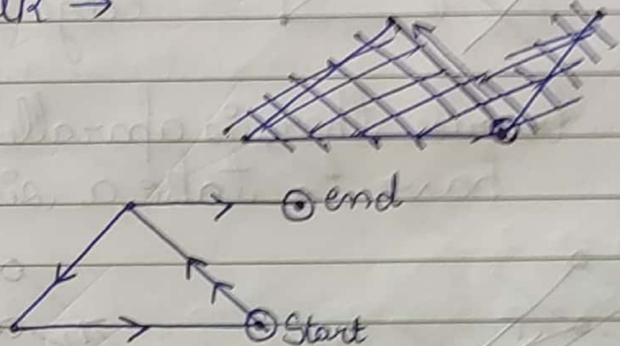
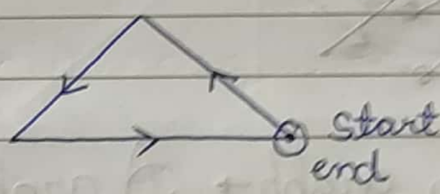
Repeated vertices	Repeated edges	open	close	Name
Yes	Yes	Yes		open walk
Yes	Yes		Yes	closed walk
Yes	No	Yes		Trail
Yes	No		Yes	Circuit
No	No	Yes		Path
No	No		Yes	Cycle

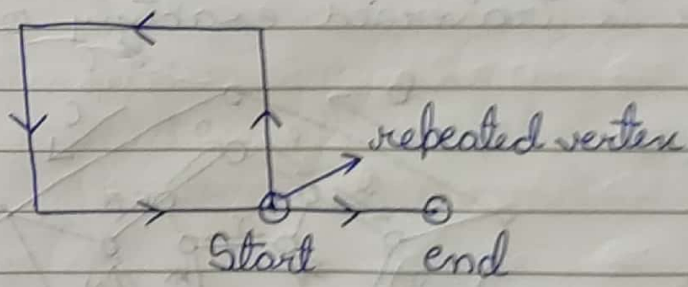
Open walk →



Closed walk →



Trail →



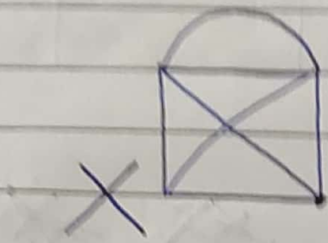
Two type of connected Graphs:-

1 Euler Graph → circuit available

2 Hamiltonian Graph → Cycle

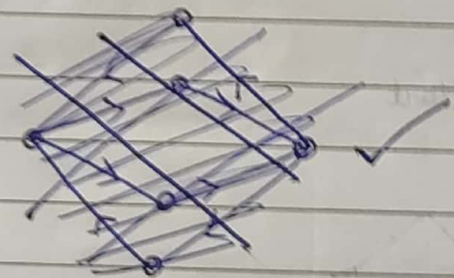
1 Euler graph → A graph in which every edge is traversed exactly once.

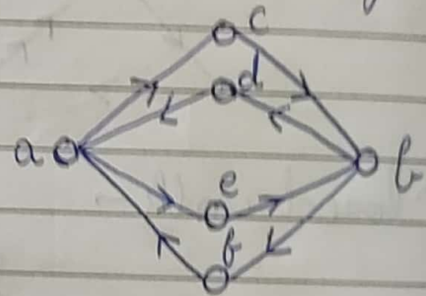
Ex→



vertices may repeat but we cannot repeat edge.

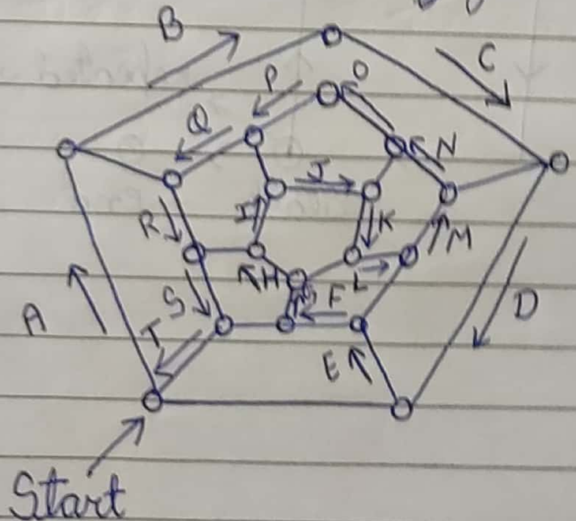
because there is a parallel edge & we have to take a simple graph





2 Hamiltonian graph → A graph in which every vertex is traversed exactly once.

Ex→

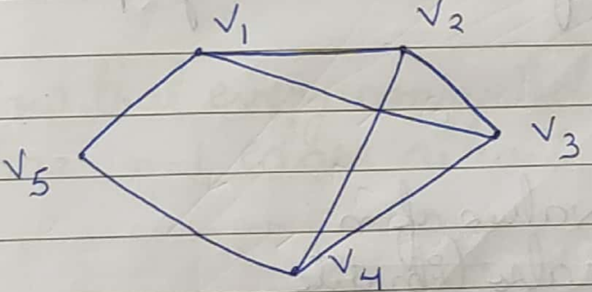


Q:- does there exist a simple graph with 5 vertices of the following degree

i) 3, 3, 3, 3, 2 → Yes
ii) 1, 2, 3, 4, 5 → No
iii) 1, 2, 3, 4, 4 → No
iv) 0, 1, 2, 2, 3 → Yes
v) 1, 1, 1, 1, 1 → No

(i) $3+3+3+3+2 = 14$ [handshaking theorem is valid]

$\frac{14}{2} = 7$ ↓ no of edges

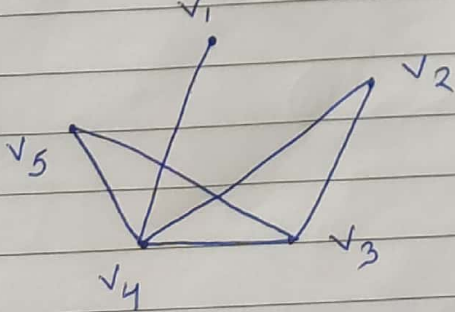


(ii) $1+2+3+4+5 = 15$ [handshaking theorem is not valid]

$\frac{15}{2} = 7.5$ ↓ 7.5 edges are not possible

(iii) $1+2+3+4+4 = 14$ [handshaking theorem is valid]

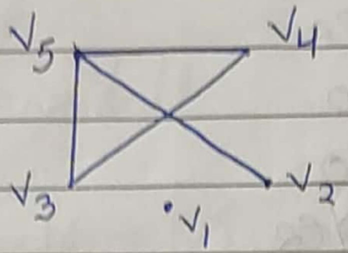
$\frac{14}{2} = 7$



we want v_5 with degree 4 so if we connect it with v_1 & v_2 then degree of v_1 & v_2 will increase.

(iv) $0+1+2+\overset{2}{\cancel{3}}+3 = 8$ [Handshaking theorem is valid]

$\frac{8}{2} = 4$



v) $1+1+1+1+1 = 5$ [Handshaking theorem is not valid]

$\frac{5}{2} = 2.5$

Q- for which value of n are these graphs are bipartite

a) $K_n \Rightarrow n = 1, 2$

b) $C_n \Rightarrow$ even values of n

c) $W_n \Rightarrow$ ~~xx~~ is also bipartite

~~d)~~ ~~$K_{m,n}$~~

Q- for which value of n are these graphs are regular

a) $K_n \Rightarrow n \geq 2$ degree will be $n-1$

b) $C_n \Rightarrow n \geq 3$

c) $W_n \Rightarrow n = 3$

d $K_{m,n} \Rightarrow m = n$

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Q- how many vertices does a regular graph of degree 4 with 10 edges have

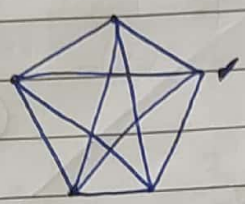
from handshaking theorem,

$$4+4+4+\cdots+n = 10\times 2$$

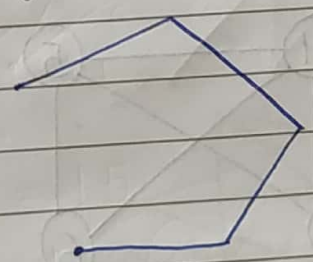
$$4n = 10\times 2$$

$$n = 5$$

$\therefore$ no of vertices $= 5$



Q- Show that every connected graph with n vertices has $n-1$ edges atleast



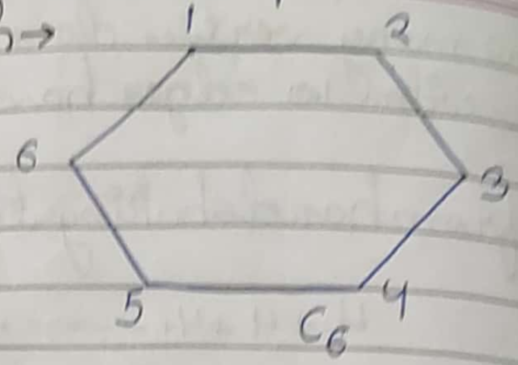
we can reach any of the vertex from any where.
so, for $n=5$ vertex we need atleast 4 edges.

Q- for which value of n are these graphs are Euler

a) $K_n \Rightarrow$
b) C_n
c) W_n
d) $K_{m,n}$

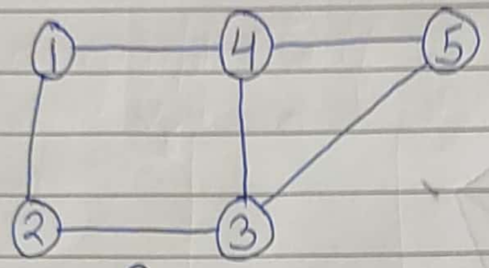
C_6 ($\bar{C_6}$ complement)

Operations on Graph →

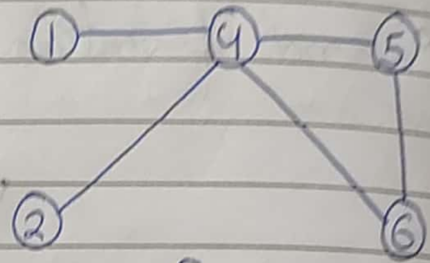


union
intersection
difference
set difference
complement

$G = (V, E)$

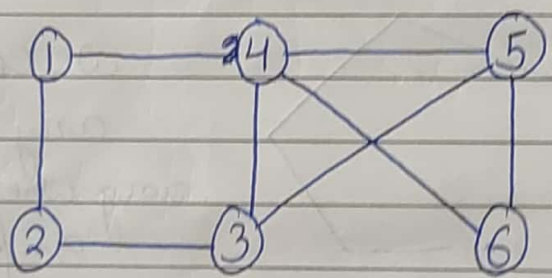


G_1

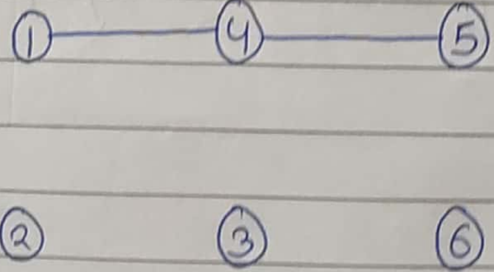


G_2

$G_1 \cup G_2$ →



$G_1 \cap G_2$ →



$G_1 - G_2 \rightarrow$ (3)

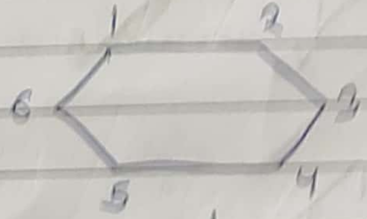
$G_2 - G_1 \rightarrow$ (6)

$G_1 \Delta G_2 = (G_1 - G_2) \cup (G_2 - G_1)$

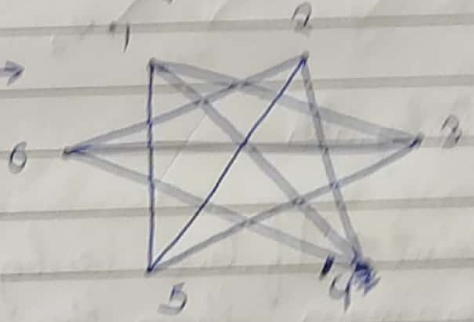
$= $ (3) (6)

Complement $\rightarrow$ remove the edges which are currently in a graph & draw all the other edges.

Ex $\rightarrow$ C_6



$C_6' \rightarrow$



Q- If G is a simple graph with 15 edges & G' has 13 edges. How many ~~edges~~ vertices does G have.

$$G + G' = K_n = \frac{n(n-1)}{2}$$

$$15 + 13 = \frac{n(n-1)}{2}$$

$$n^2 - n = 56$$

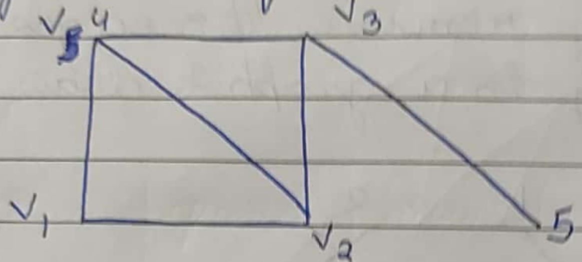
$$n^2 - 8n + 7n - 56 = 0 \qquad \boxed{n = 8}$$

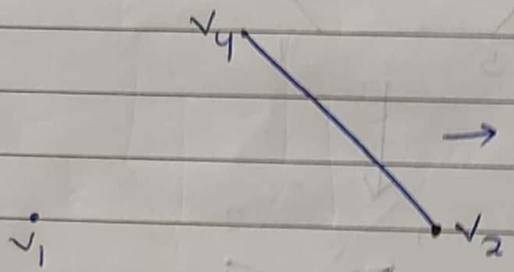
Q- If the simple graph G has (v,e) how many edges does G' have

$$\frac{v(v-1)}{2} - e$$

Subgraphs → A part of a graph is said to a subgraph.

☆ a graph itself is also a subgraph



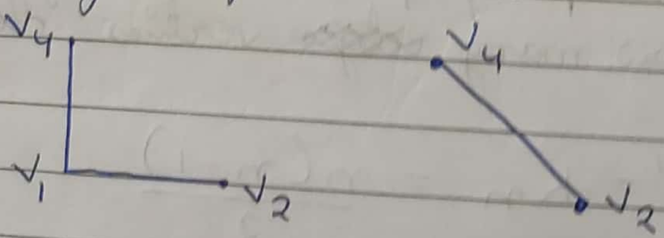


→ Subgraph

↓ Subgraph

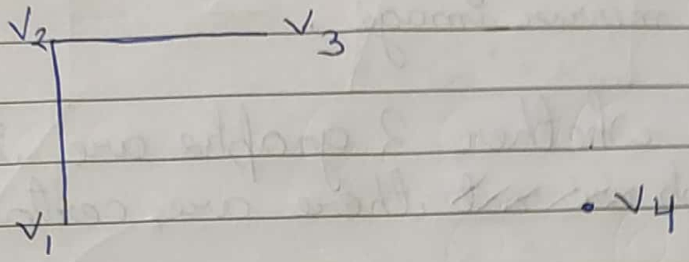
Edge disjoint → If two subgraphs have no edge in common then the subgraph are said to be edge disjoint.

Ex →



Vertex disjoint → If two subgraphs have no vertex in common then the subgraphs are said to be vertex disjoint.

Ex →

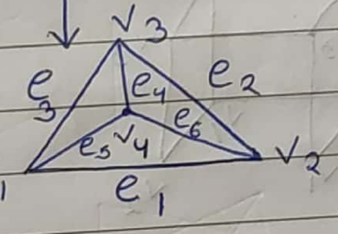


This is both edge & vertex disjoint because if any graph is vertex disjoint then it will always be edge disjoint.

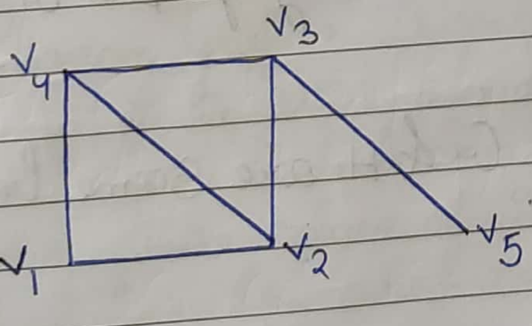
Q- How many subgraphs with atleast one vertex does K_3 & W_3 have.

K_3
↓
7

W_3
↓



Adjacency Matrix →



$$\begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1 & 0 & 1 & 0 \\ v_2 & 1 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 1 & 0 & 1 & 1 \\ v_4 & 1 & 1 & 1 & 0 & 0 \\ v_5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Isomorphisms of Graph →

Isomorphisms means mirror image.

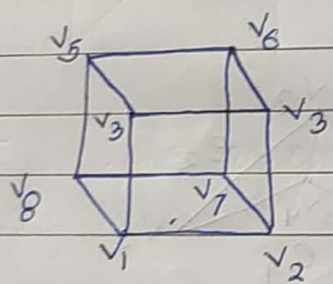
To check whether 2 graphs are isomorphism of graph ~~or not~~ there are certain conditions

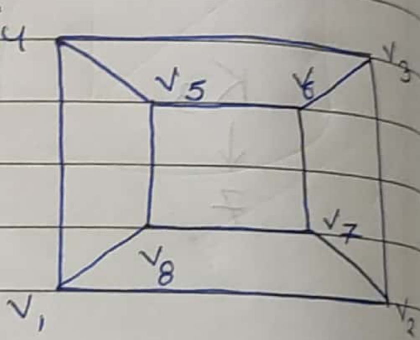
1 no of vertices should be same

2 see the edges between the vertex whether ~~they are same~~ if same than isomorphism

3 degree sequence must be same

Ex →





Q- Suppose that G & H are isomorphic simple graph show that their complementary graphs G' & H' are also isomorphic

The no of vertex of G & H are same because they are isomorphic.

no of vertex of G & G' are same & H & H' are same because they are complementary graph.

~~So~~, so the no of vertex of H' & G' will also same vertex

$G \rightarrow n$

$H \rightarrow n$

$G' \rightarrow n$

$H' \rightarrow n$

$$G + G' = K_n$$

$$\bar{G} = \frac{n(n-1)}{2} - e$$

$$\bar{H} = \frac{n(n-1)}{2} - e$$

$\therefore$ no of edges of G' & H' will also same – ②

all the edges present in G will be absent in G'.
all the edges present in H will be absent in H'

$$\deg G = \deg H$$

$$\therefore \deg G' = \deg H'$$

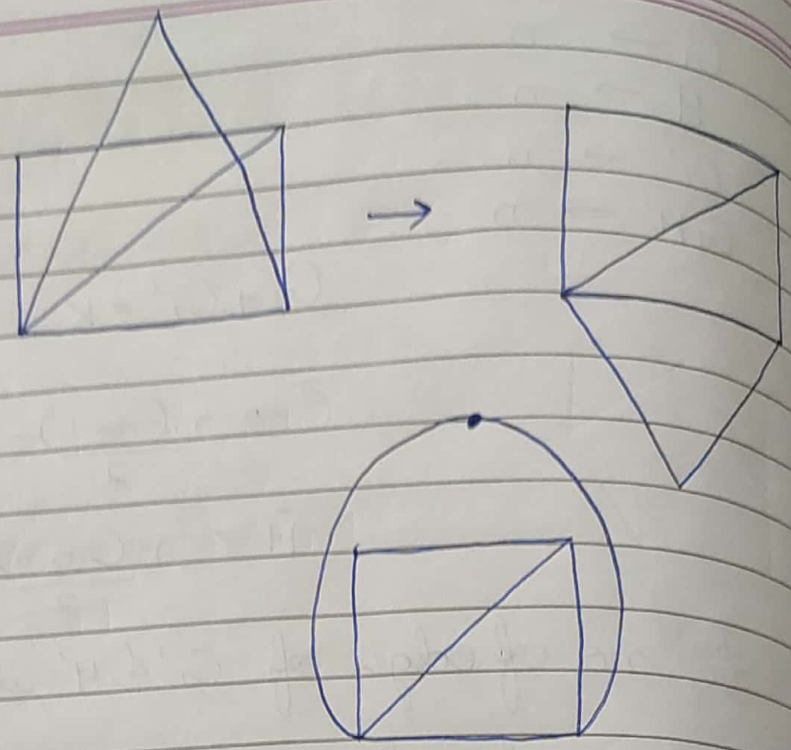
Hence, degree sequence of G' & H' is also same – ③

from ①, ② & ③ the graph G' & H' are isomorphic

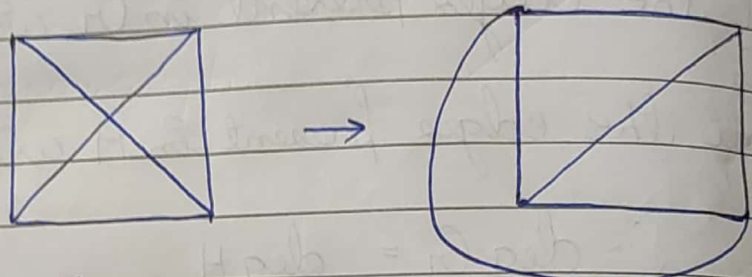
Planar Graph → A graph G is said to be planar if their exist some geometric representation of G which can be drawn on a plane such that no two edges of it's, intersect. The point of intersection are called crossover.

A graph that can be drawn on a plane without a crossover b/w its edges is called a planar graph.

Ex →



K_4 →



It is a planar graph.

There are two theorems to check whether the graph is planar ~~or not~~ ~~if any one~~ or not.

1 If G is a connected planar simple graph then G has the vertex of degree not exceeding with 5.

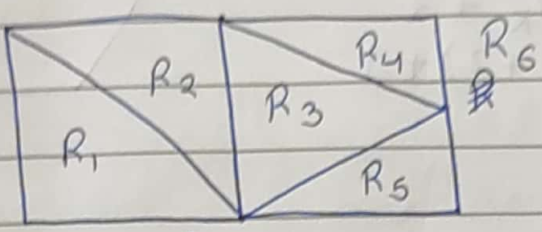
2 If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then,
$\boxed{e \leq 3v-6}$

K_4, K_5, $K_{3,3}$

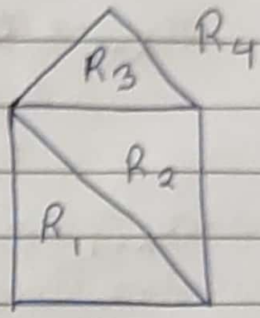
K_4: Planar

K_5, $K_{3,3}$: Not Planar

Euler's Theorem :- Planar Graph



(i)



(ii)

Euler's formula → let G be a connected planar simple graph with 'e' edges and 'v' vertices. Let there are 'r' regions.

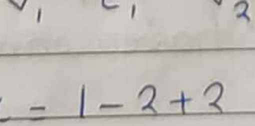
$$r = e - v + 2$$

in fig (i) $r = e - v + 2$
$= 11 - 7 + 2$
$= 6$

in fig (ii) $r = 7 - 5 + 2$
$= 4$.

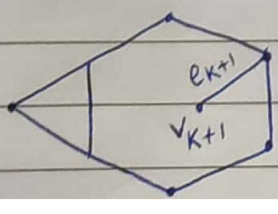
Prove by induction :-

Base Step :-



$r = 1 - 2 + 2$
$= 1$

Inductive Step :- let $r_K = e_K - v_K + 2$ is true



$e_{K+1} = e_K + 1$

$r_{K+1} = r_K + 1$

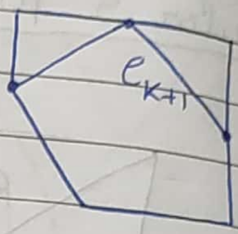
$r_{K+1} = e_{K+1} - v_{K+1} + 2$

∴ (extending vertex or edge won't effect no of regions)

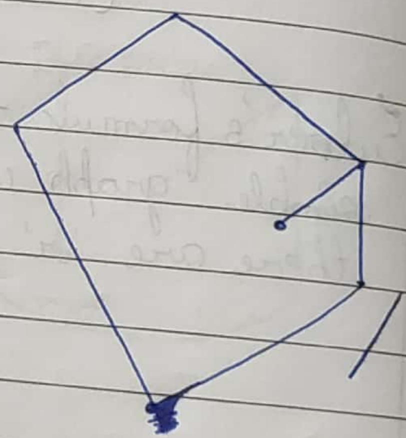
$$= (e_K + 1) - (v_K + 1) + 2$$

$$r_{K+1} = e_K - v_K + 2 = r_K$$

$$r_{K+1} = e_{K+1} - v_{K+1} + 2$$
$$= e_{K+1} - v_K + 3$$
$$= (e_K - v_K + 2) + 1$$
$$r_{K+1} = r_K + 1$$



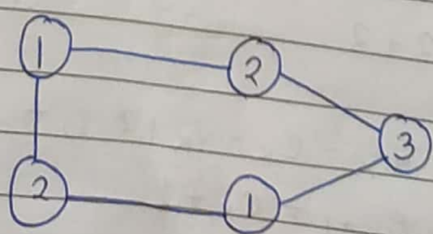
It is not a connected planar graph



Graph Coloring :-

- → edge coloring (not in our syllabus)
- → vertex coloring ✓

Chromatic number → Minimum number of color required to color the vertices of a graph in such a way that no adjacent vertices have same color.



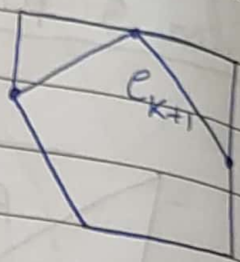
$\chi = 3$

Scanned with CamScanner

$$= (e_K + 1) - (V_K + 1) + 2$$

$$r_{K+1} = e_K - V_K + 2 = r_K$$

$$r_{K+1} = e_{K+1} - V_{K+1} + 2$$
$$= e_{K+1} - V_K + 2$$
$$= (e_K - V_K + 2) + 1$$
$$r_{K+1} = r_K + 1$$



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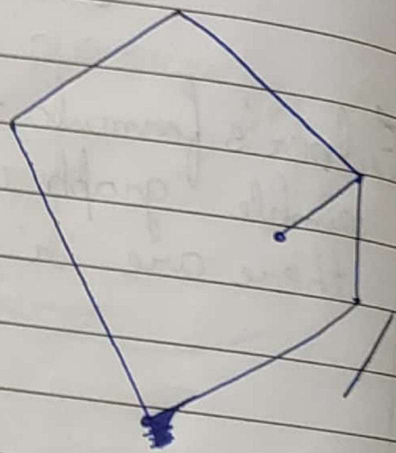

Graph Coloring :-

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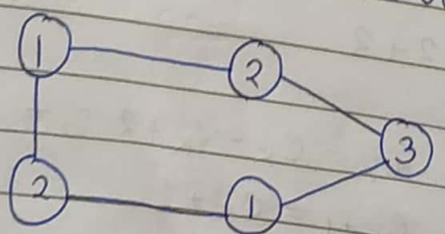


$$\chi_K = 3$$

χ_K

$K_n \longrightarrow n$

$W_n \longrightarrow$ 3 for odd, 4 for even

$K_{m,n} \longrightarrow 2$

$C_n \longrightarrow$ 2 for even, 3 for odd

☆ TREE →

Tree is a simple connected unidirected graph with no cycles, self loop and parallel edges. It is a particula type of graph.

root node → a node without

Theorem :- Let 'T' be a tree of n-nodes where $n>0$ then it has exactly 'n-1' edges.

every node except root node has exactly one parent so total edges $= n-1$.

If we consider $n-2$ edges then one of the node will be disconnected & it will not be a tree.

Theorem:- A full m-ary tree with 'i' internal vertices contains $n = mi + 1$ vertices.

~~internal node → every node only except leaf~~
~~Since each of the 'i' every internal node vertices has 'm' children~~
~~total children = mi & on~~

every vertex except leaf vertex is called internal
Since each of the 'i' internal vertices has 'm' children
so there are mi vertices in the tree other than the root.
∴ the tree contains $n = mi + 1$ vertices.

Theorem:- A full m-ary tree with

i) n vertices has $i = (n-1)/m$ internal vertices &
$l = [(m-1)n + 1]/m$

(ii) 'i' internal vertices has $l = (m-1)i + 1$ leaves

(iii) l leaves has $n = (ml - 1)/(m-1)$ vertices &
$i = (l-1)/(m-1)$ internal leaves

given $\begin{cases} n = mi + 1 \\ n = i + l \end{cases}$

i → internal vertices, l → leaf vertices

(i) ∴ $n = mi + 1$
$n - 1 = mi$
$i = \dfrac{(n-1)}{m}$

as we know $n = i + l$
so, $l = n - i$

$l = n - \frac{(n-1)}{m}$

$l = [mn - n + 1]/m$

$l = [(m-1)n + 1]/m$

(ii) $l = n - i$

$l = mi + 1 - i$ $\qquad \boxed{n = mi + 1}$

$l = i(m-1) + 1$

(iii) $n = mi + 1$

$i = \frac{n-1}{m} \quad -①$

$n = i + l$

$i = n - l \quad -②$

from ① & ②

$\frac{n-1}{m} = n - l$

$n - 1 = mn - ml$

$ml = mn - n + 1$

$\frac{ml-1}{n} = (m-1)$

$\boxed{n = (ml-1)/(m-1)}$

$n = mi + 1 \quad -①$

$n = i + l \quad -②$

$mi + 1 = i + l$

$mi - i = l - 1$

$i(m-1) = l - 1$

$\boxed{i = (l-1)/(m-1)}$

~~Counting Principles~~

Q- There are atmost m^h leaves in an m-ary tree of height 'h' $\quad l = m^h$

Induction

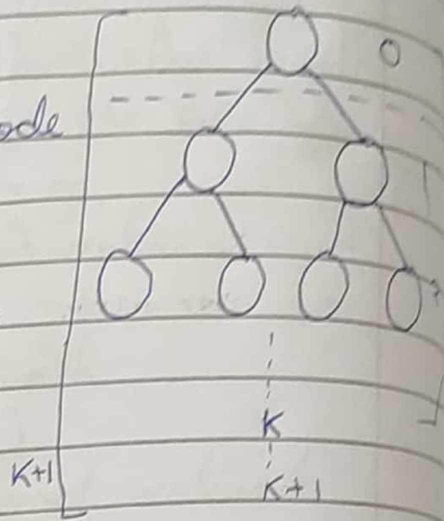
base step $h=0$

$m^0 = 1$ 0 root node

Inductive step : $h=K$

let m^K ~~have~~ leaves are there at $h=K$ for $h=K+1$

$m \cdot m^K$ leaves.



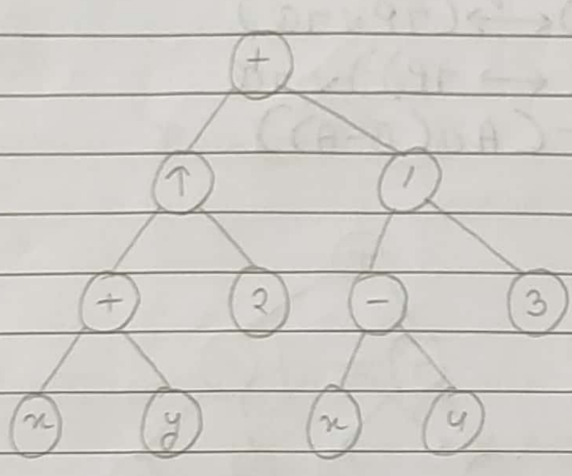
Theorem→ If an m-ary tree of height 'h' has l leaves then $h >= \lceil \log_m l \rceil$.

inequality for unbalanced. $\quad l \leq m^h$

$\log_m l \leq h$

ceil function to get integer value ~~for~~ from log. $\longrightarrow \lceil \log_m l \rceil \leq h$

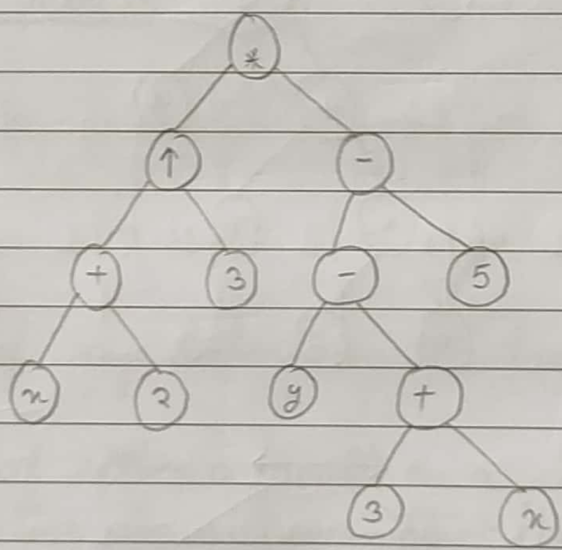
Q-1 $((x+y)\uparrow 2) + ((x-4)/3)$



(Root L R) preorder → $+ \uparrow + x\, y\, 2\, / - x\, 4\, 3$

(L R root) Post order → $x\, y + 2 \uparrow x\, 4 - 3 / +$

Q-2 $((x+2)\uparrow 3) * ((y-(3+x))-5)$



3 $(x + xy) + (x/y)$

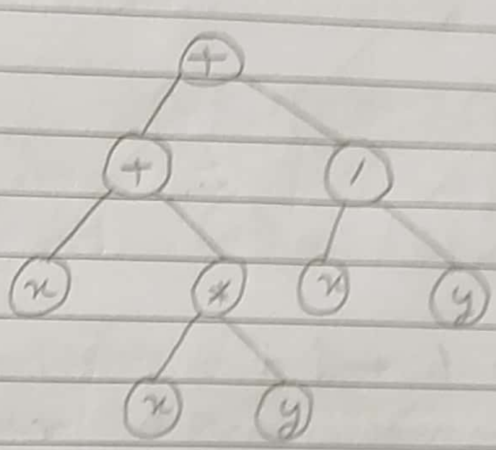
4 $x + ((xy + x)/y)$

5 $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

6 $(\neg P \wedge (Q \leftrightarrow \neg P)) \vee \neg Q$

7 $(A \cap B) - (A \cup (B - A))$

3



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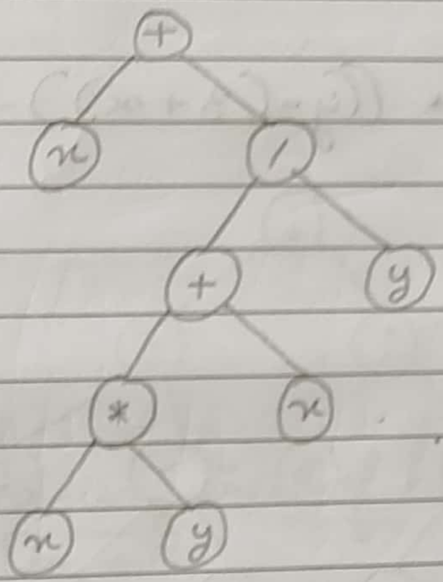

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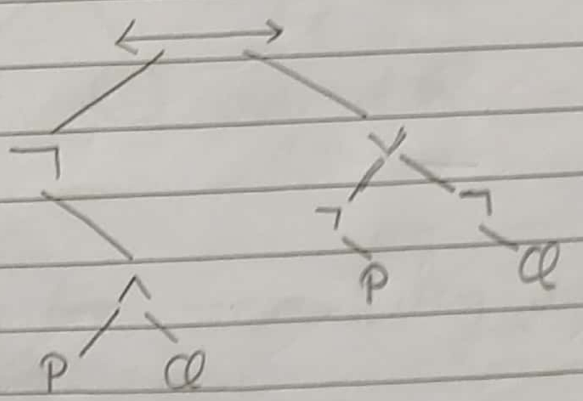

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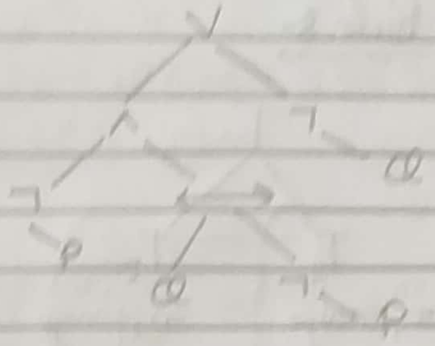

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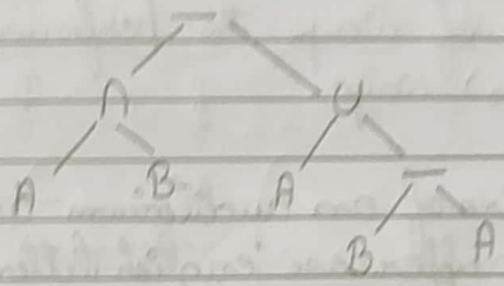
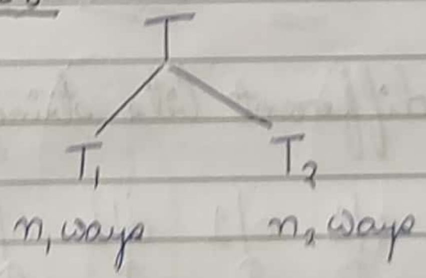


The Sum Rule :-



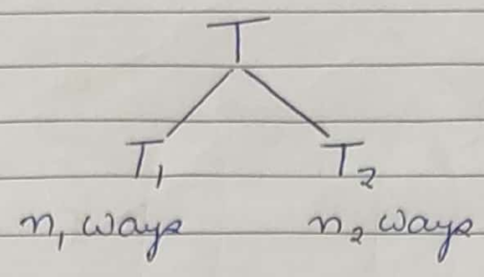
OR

Total no of ways to perform the task T is $(n_1 + n_2)$ ways.

Q- Suppose that either a faculty or a student is chosen as a representative to a committee. How many different choices are there for this representation, if there are 37 faculties and 83 students.

$37 + 83 = 130$ Ans.

The product Rule :-



AND.

Total no. of ways to perform the task T is $(n_1 \times n_2)$ ways.

Q- The chairs of an auditorium are to be labelled with an uppercase english letter, followed by a positive integer not exceeding 100. What is the largest no. of chairs that can be differently.

$26 \times 100 = \underline{\underline{2600}}$ **Ans**

Q- How many different bits strings of length 7 are there.

0/1	0/1	0/1	0/1	0/1	0/1	0/1
↑	↑	↑	↑	↑	↑	↑
2	2	2	2	2	2	2

$= 2^7 = \underline{\underline{138}}$ Ans

Q- How many strings are there of lower case letters of length 4 or less, not counting the empty string.

26^1 26^2 26^3 26^4

_ ↑ 26   _ _ ↑ 26 ↑ 26   _ _ _ ↑ 26 ↑ 26 ↑ 26   _ _ _ _ ↑ 26 ↑ 26 ↑ 26 ↑ 26

$\therefore\ 26^1 + 26^2 + 26^3 + 26^4$

$26 + \cancel{xxx}\,676 + 17576 + 456976$

$= \underline{475,254}$ Ans

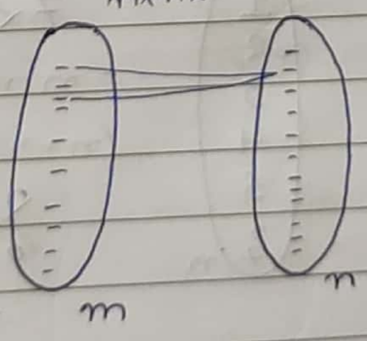
Q- How many bit strings of length 10 both begin and end with 1.

$$\underline{1}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underset{\substack{\uparrow \\ 2}}{\underline{0|1}}\ \ \underline{1}$$

$$2^8$$ Ans

Q- How many functions are there from a set with m elements to a set with n elements.

$n \times n \times n \times n \times - - - - - \times n$

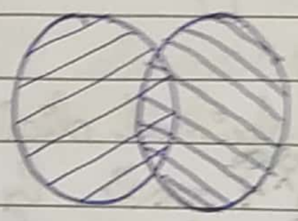


for every element of 'm' there are 'n' choices

n^m

$\therefore\ \underline{n^m}$ Ans

Subtraction Rule :- [Inclusion | Exclusion Rule]



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Q- How many bit string of length 8 either start with 1 or end with 00

$2^7 + 2^6 - 2^5$

Q- How many +ve integer b/w 50 & 100 are (included)

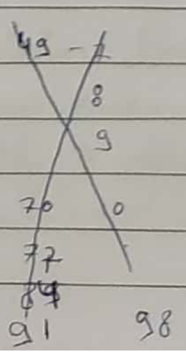
① are divisible by 7
② are divisible by 11
③ are divisible by both 7 & 11

Total no of +ve integers = 51

a) $\frac{51}{7} = 7 \ldots\ldots$ ~~7.28~~ $= 7$

b) $\frac{51}{11} = 4 \ldots\ldots \simeq 5$

c) $\frac{51}{77} = 0 \ldots \simeq 1 \Rightarrow$ 77 is the only value



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900 total values b/w 100 to 999

Q- How many +ve integers b/w 100 & 999 inclusive
① are divisible by 7 $900/7$
② are odd $900/2 = 450$
③ have the same 3 decimal digit
④ are not divisible by 4
⑦ are divisible by 3 but not 4
⑧ are divisible by 3 and 4
~~⑤ are not divisible by 4 or 3~~
⑤ are divisible by either 4 or 3
⑥ not divisible by either 4 or 3.

③ 9

④ $900 - \frac{900}{4}$

⑤ $\frac{900}{3} + \frac{900}{4} - \frac{900}{12} = 450$

⑥ $900 - 450 = 450$

⑦ $\frac{900}{3} - \frac{900}{12}$

⑧ $\frac{900}{12}$.

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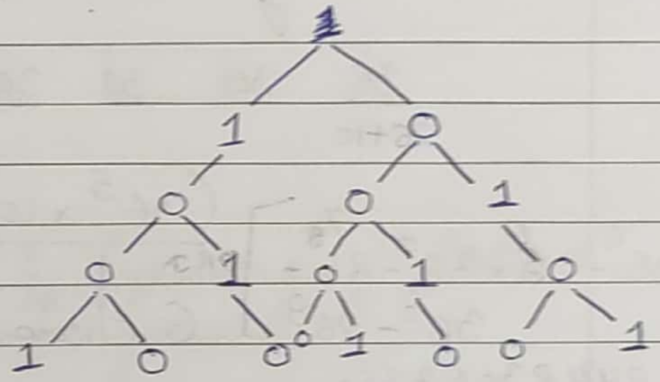
Tree diagram:

Q- How many bit strings of length 4 do not have two consecutive ones?

$\overline{①}\ \overline{②}\ \overline{③}\ \overline{④}$

0001   0100
1001   0010
0101   1010
0000
1000



Total 8 strings are possible.

Q- How many strings of 3 decimal digits.

a) do not contain the same digit thrice.
b) begin with a odd digit. $5 \times 10 \times 10 = 500$
c) Have exactly two digits that are 4's.
$(1 \times 1 \times 9) + (9 \times 1 \times 1) + (1 \times 9 \times 1) = 27$

$10 \times 10 \times 10 - \{000, 111, 222, 333, 444, 555, 666, 777, 888, 999\}$

$= 990$

Q- How many strings of 4 decimal digits.

a) do not contain the same digit twice $10 \times 9 \times 8 \times 7 = 5040$
b) end with an even digit $10 \times 10 \times 10 \times 5 = 5000$
c) have exactly 3 digits that are 9.
$(1 \times 1 \times 1 \times 9) + (1 \times 1 \times 9 \times 1) + (1 \times 9 \times 1 \times 1) + (9 \times 1 \times 1 \times 1)$
$= 36$

Maple

$P[6] := 36^6 - 26^6;$ 1867866560   add$(36^i - 26^i, i=6..8);$

$P[7] := 36^7 - 26^7;$ 70332353920

$P[8] := 36^8 - 26^8;$ 2612282842880   OR

$P := P[6] + P[7] + P[8];$ 2684483063360

Q- Each user of a comp. system has a password which is 6-8 characters long where each character is an uppercase letter or a digit each password must contain atleast 1 digit. How many possible passwords are there.

$\underset{26+10}{\underline{36}}$ $\underline{36}$ $\underline{36}$ $\underline{36}$ $\underline{36}$ $\underline{10}$

$\left[36^6 - 26^6 + 36^7 - 26^7 + 36^8 - 26^8\right]$ OR $\underbrace{(36^5 \times 10)}_{6 \text{ character}} + \underbrace{(36^6 \times 10)}_{7 \text{ character}} + \underbrace{(36^7 \times 10)}_{8 \text{ character}}$

2684483063360

Permutation & Combination :-

Permutation → arrangement

Combination → to select or to choose

Permutation $\rightarrow {}^nP_r = \dfrac{n!}{(n-r)!}$

Q- Consider a main memory of a system has 4 frame of fixed size 5 KB each if 4 processes of 3 KB, 2 KB, 4 KB & 1 KB enters the system then in how many ways the process can allocate the memory.

4 frames → n

4 process can allocate memory → r

5   5   5   5

$4 \times 3 \times 2 \times 1 = 4! = {}^4P_4 = \dfrac{4!}{(4-4)!} = 4$

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add (i, i=1..10); $\underline{55}$

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In how many ways 2 processes of 3KB & 4KB can enter the system

$$^4P_2 = \frac{4!}{2!} = \frac{4\times3\times\cancel{2!}}{\cancel{2!}} = 12.$$

$4\times3 = 12$

Q- Let $S=\{a, b, c, d, e, f\}$, how many distinct words of 4 letters can be formed.

$^6P_4$ Ans

Q- In how many different ways can be ~~award~~ award ~~prizes~~ prizes 1^{st}, 2nd & 3rd among 24 teams if there are no ties.   12144

$\overline{24} \times \overline{23} \times \overline{22}$ $^{24}P_3$

$$= \frac{24!}{21!} = {}^{24}P_3$$

Q- How many different strings from the letters of P, Q, R, S, T, U, V, W, X, Y, Z can be formed that contains the substring X, Y, Z

$$\underbrace{P, Q, R, S, T, U, V, W}_{8}, \underbrace{X, Y, Z}_{1}$$

$(8+1)! = 9!$

Permutation with repetition →

$$\underset{4}{\uparrow} \times \underset{3}{\uparrow} \times \underset{2}{\uparrow} \times \underset{1}{\uparrow} = 4! \qquad {}^nP_r = \frac{n!}{(n-r)!}$$

$$4 \times 4 \times 4 \times 4 = 4^4 = n^r$$

make first person to sit so total seats are 4 the he stand up & leave that seat now second person also get 4 total seat to sit same for third pearson & for 4^{th} person.

Q- $S = \{a, b, c, d, e, f\}$ how many 4 letter word will form using repetation

$$\overline{6}\ \overline{6}\ \overline{6}\ \overline{6} = 6^4$$

Q- How many distinct permutation of the letters in the word JAGARAN can be formed?

$\underset{r_1}{J}\ \underset{r_2}{A}\ \underset{r_3}{G}\ \underset{r_4}{R}\ \underset{r_5}{N}$

$$\frac{n!}{r_1! \times r_2! \times \cdots \times r_n!}$$

$$= \frac{7!}{1! \times 3! \times 1! \times 1! \times 1!} = \frac{7!}{3!}$$

Q- ENGINEERING

$$\frac{11!}{3! \times 3! \times 2! \times 2!}$$

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Combination :-

$$^nC_r = \frac{n!}{r! \times (n-r)!}$$

Q- How many bit string of length n contains exactly r 1s?

^nC_r Ans

Q- In How many ways can a team of 11 players be selected from a pool of 15 players to play the matches at national level.

$^{15}C_{11}$

Q- In how many ways can a committee of of 3 girls & 4 boys be formed from a class of 25 girls & 40 boys?

3 girls or 4 boys

$^{25}C_3 + {}^{40}C_4$

$^{25}C_3 \times {}^{40}C_4$

Q- How many strings of length n can be formed from the alphabet set {0,1} that Contains exactly m no of zeroes.

^nC_m

Q- How many bit string of length 10 contain

a) exactly 4 1's $^{10}C_4$

b) atmost 4 1's $^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

c) atleast 4 1's $^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$

d) equal no of 0's & 1's $^{10}C_5$

Q- How many bit string of length 10 have

a) exactly 3 0's $^{10}C_3$
b) more 0's than 1's $^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$
c) atleast 7 1's $^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$
d) atleast 3 1's $^{10}C_3 + {}^{10}C_4 + \dots + {}^{10}C_{10}$

Q- How many Permutations of the letters a,b,c,d,e, f,g contain

a) The string b,c,d $^5P_5 = 5!$ bcd
b) The string c,f,g,a $^4P_4 = 4!$ cfga
c) The string b,a & g,f $^5P_5 = 5!$ ba gf
d) The string a,b,c & d,e $^4P_4 = 4!$ abc de
e) The string a,b,c & c,d,e $^3P_3 = 3!$
f) The string c,b,a & b,e,d 0

e) $\underbrace{abcde}_{1}$, f, g $\rightarrow 3!$

Combination with repetition →

$$C(n+r-1, r) = \frac{(n+r-1)!}{(n+r-1-r)! \times r!} = \frac{(n+r-1)!}{(n-1)! \, r!}$$

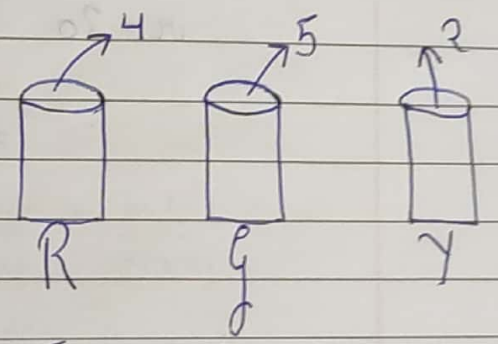
n → total no of objects
r → no of objects to choose

Q- How many solution does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, x_3 are non negative integers

$n = 3$

$r = 11$

Ex →



$n =$ Total container → 3

$r =$ balls selected → 11

$$\frac{(n+r-1)!}{(n-1)! \times r!} = \frac{(3+11-1)!}{(3-1)! \times 11!}$$

$$= \frac{13!}{2! \times 11!}$$

$$= 13 \times 6$$

$$= 78$$

Now, if $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$

we have already selected 1 value from x_1
2 value from x_2
3 value from x_3

So, remaining no to be selected i.e., $r = 5$

$n = 3$, $r = 5$

$$\frac{(3+5-1)!}{(3-1)! \times 5!} = \frac{7!}{2! \times 5!} = 7 \times 3 = 21$$

Q- How many different outcomes are possible if 10 coins are tossed.

$n = 2$, $r = 10$

$$\frac{(n+r-1)!}{(n-1)! \times r!} = \frac{(2+10-1)!}{(2-1)! \times 10!} = \frac{11!}{10!} = 11$$

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Q- How many ways can 20 similar chair be chosen by 5 different people -

$n = 5$ $\qquad \frac{(n+r-1)!}{(n-1)! \times r!}$

$r = 20$

$$= \frac{(5+20-1)!}{(5-1)! \times 20!} = \frac{24!}{4! \times 20!}$$

$$= \frac{24 \times 23 \times 22 \times 21}{4 \times 3 \times 2} = 10626$$

Q- How many bit string contain exactly 8 zeroes & 10 ones if every zero must be immediately followed by a 1.

_ 01 _ 01 _ 01 _ 01 _ 01 _ 01 _ 01 _ 01 _ 1 1

~~out of 10 one's we have to choose 2 one's~~ $^{10}C_2$ Ans

Total entities are 10 & we have to select 2 entities

Q- How many bit string must contain exactly 5 0's & 14 one's if every zero must be immediately followed by 2 one's.

$^{9}C_4$ Ans

~~$^{14}C_4 = \frac{14!}{(14-4)! \times 4!}$~~

011 011 011 011 011 1 1 1 1

~~$= \frac{14!}{10! \times 4!} = \frac{14 \times 13 \times 12 \times 11}{4 \times 3 \times 2} = 1001$ Ans~~

Total entities are 9 & we have to select 4 entities

How many bit string of length 10 contain atleast 3 ones & 3 zeroes?

3 0s 7 1's + 4 0's 6 1's + 5 0's 5 1's + 6 0's 4 1's + 7 0's 3 1's

$$^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7$$

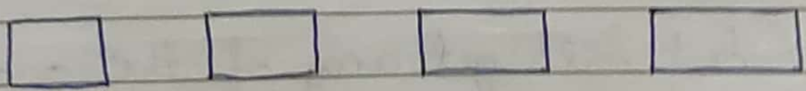
$$\frac{10!}{7! \times 3!} + \frac{10!}{6! \times 4!} + \frac{10!}{5! \times 5!} + \frac{10!}{4! \times 6!} + \frac{10!}{7! \times 3!}$$

$$\frac{10 \times 9 \times 8}{3 \times 2} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} + \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} + 210 + 120$$

$$120 + 210 + 252 + 210 + 120$$
$$= 912 \quad \text{Ans}$$

Pigeonhole Principle :-

If there are k pigeonholes and k+1 pigeons then there is atleast one pigeonhole which will keep atleast more than one pigeon in it.



best → suppose there are 11 pigeon & 10 Ph and we have kept all 11 pigeon in one Pigeonhole only $\boxed{11}$ rest are empty

~~worst~~ average → keeping pigeons randomly

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ - - - -

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worst → [1] [1] ------ [2]

last Pigeonhole will contain only 2 pigeons.

Q- How many students must be in a class to guarentee that atleast two students recieve the same score on the final exam if the exam is graded ~~f~~ on a scale from 0-100 points.

So total 102 student

102 students

↑

↓ ↓ ↓ ↓
0 1 2 ------ 100 Total 101 students & one student may get any grad

Q- what is the min no of students required in a class to be sure that atleast 6 will recieve the same grade if there are 5 possible grade A, B, C, D, F.

A	B	C	D	F
↑	↑	↑	↑	↑
↑	↑	↑	↑	↑
↑	↑	↑	↑	↑
↑	↑	↑	↑	↑
↑	↑	↑	↑	↑

→ 25

26th student will get any of the grade.

26 students Ans

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Q- How many cards must be selected from a standard deck of 52 cards to guarentee that (i) atleast 3 cards are choosen from same deck
(ii) atleast 3 hearts are selected.

(i)

H	S	D	C
↑	↑	↑	↑
↑	↑	↑	↑

→ 8

9^{th} card will be from any of 4

Ans 9.

(ii)

H	S	D	C
3	13	13	13

= 42 Ans

Q- (i) I have 7 pair of socks in my drawer one for each color of the rainbow how many socks do I have to draw out in order to guarentee that I have grabbed atleast 1 pair.
(ii) what if there are likewise coloured the pair of gloves and I cannot tell the difference b/w gloves & socks, and I want a matching set pair of (Socks + gloves)

$13 \times 3 = 39$

(i)

↑ ↑ ↑ ↑ ↑ ↑ ↑ → 7

8^{th} socks will be of any colour to make a pair.

Ans → 8

(ii)

↑	↑	↑	↑	↑	↑	↑
↑	↑	↑	↑	↑	↑	↑
↑	↑	↑	↑	↑	↑	↑

→ 21

22^{th} socks or glove will be of any colour to make a matching set.

Ans → 22

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Advance Pigeonhole Principle →

If there are m pigeonholes and n ~~p~~ pigeons where $n > m$ ~~xxxxx~~ then there must be atleast one pigeonhole in which we can keep $\lceil \frac{n}{m} \rceil$ pigeons in it.

Q- How many different rooms are needed to assign 500 lectures if there are 45 time slots in university time table that are available.

$n = 500$

$m = 45$

$$\left\lceil \frac{500}{45} \right\rceil = \underline{\underline{12}}$$

Q- There are 5 cargo's in a ship yard & are total 232 containers in the cargo's how many containers are min needed to fill 1 cargo

$n = 232$

$m = 5$

$$\left\lceil \frac{232}{5} \right\rceil = \underline{\underline{47}}$$

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Advance Pigeonhole Principle →

If there are m pigeonholes and n ~~p~~ pigeons where n > m ~~xxxxxx~~ then there must be atleast one pigeonhole in which we can keep $\left\lceil \frac{n}{m} \right\rceil$ pigeons in it.

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$n = 500$

$m = 45$

$\left\lceil \frac{500}{45} \right\rceil = 12$

Q- There are 5 cargo's in a ship yard & are total 232 containers in the cargo's how many containers are min needed to fill 1 cargo

$n = 232$

$m = 5$

$\left\lceil \frac{232}{5} \right\rceil = 47$

Advanced Counting Principles OR Techniques →

Recurrence Relation →
- → To form] → given in the question
- → To solve
  - iteration method ✗ ← NOT in syllabus
  - characteristic root method ✓
  - generating function method ✓

Q- The no. of bacteria in a colony doubles every hour. If a colony begins with 5 bacteria. How many will be present in n hours.

n^{th} hour → a_n no of bacteria

$$\boxed{a_n = 2a_{n-1}} \quad n \geq 1$$ recurrence relation.

when a term is composed of its previous terms then that relation is called as recurrence relations

$$\boxed{a_0 = 5}$$ boundary condition OR initial condition.

fibonacci series → $\boxed{f_n = f_{n-1} + f_{n-2}}$

$f_0 = f_1 = 1 \leftarrow n \geq 2$

$f_1 = f_2 = 1 \leftarrow n \geq 3$

factorial → $f(n)! = f(n-1)! \times f(n)$.

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Q- find the four terms for each of the following relation.

a) $a_k = 2a_{k-1} + k$
$k \geq 2,\ a_1 = 1$

$a_2 \quad a_3 \quad a_4 \quad a_5$
$4, 11, 26, 57.$

b) $a_k = a_{k-1} + 3a_{k-2}$
$k \geq 2,\ a_0 = 1,\ a_1 = 2$

$a_2 \quad a_3 \quad a_4 \quad a_5$
$5, 11, 26, 59$

c) $a_k = k(a_{k-1})^2$
$k \geq 1,\ a_0 = 1$

$a_1 \quad a_2 \quad a_3 \quad a_4$
$1, 2, 12, 576$

Q- Show that the sequence $\{ \overset{2+0}{2}, \overset{2+1}{3}, \overset{2+2}{4}, \overset{2+3}{5}, --- 2+n, ---$ for $n \geq 0$ satisfies the Recurrence relation
$$a_k = 2a_{k-1} - a_{k-2}$$
$$k \geq 2$$

$a_n = 2+n$
$a_k = 2+k$
$a_{k-1} = 2+(k-1)$
$a_{k-2} = 2+(k-2)$

$$a_k = 2\left[2+(k-1)\right] - \left[2+(k-2)\right]$$

$$a_k = 4 + 2k - \cancel{2} - 2 - k + \cancel{2}$$

$$\boxed{a_k = 2+k}$$

Hence Proved

Iteration Method →

Ex → $a_n = a_{n-1} + 2, \; n \geq 2, \; a_1 = 3$

Sol^n →

$$a_n = a_{n-1} + 2 \quad \text{— (1)}$$

put $n = n-1$ in eq^n (1)

$$a_{n-1} = a_{n-2} + 2 \quad \text{— (2)}$$

put value of a_{n-1} in eq^n (1)

$$a_n = a_{n-2} + 2 \times 2 \quad \text{— (3)}$$

put $n = n-2$ in eq^n (1)

$$a_{n-2} = a_{n-3} + 2 \quad \text{— (4)}$$

put value of a_{n-2} in eq^n (3)

$$a_n = a_{n-3} + 2 \cdot 3 \quad \text{— (5)}$$

put $n = n-3$ in eq^n (1)

$$a_{n-3} = a_{n-4} + 2 \quad \text{— (6)}$$

put value of a_{n-3} in eq^n (5)

$$a_n = a_{n-4} + 2 \cdot 4 \quad \text{— (7)}$$

$$\vdots \qquad \vdots$$

$$a_n = a_{n-k} + k \cdot 2 \quad \text{— (8)}$$

for $k = n-1$

$$a_n = a_{n-(n-1)} + (n-1) \cdot 2$$

$$a_n = a_1 + (n-1) \cdot 2$$

$$\boxed{a_n = 3 + 2(n-1)}$$ Explicit formula

Ans

Q-1 $a_n = a_{n-1} + 2, \quad a_0 = 1$

$$a_n = a_{n-1} + 2 \quad — (1)$$

put $n = n-1$ in eq^n (1)

$$a_{n-1} = a_{n-2} + 2 \quad — (2)$$

put value of a_{n-1} in eq^n (1)

$$a_n = a_{n-2} + 2 \cdot 2 \quad — (3)$$

put $n = n-2$ in eq^n (1)

$$a_{n-2} = a_{n-3} + 2 \quad — (4)$$

put value of a_{n-2} in eq^n (3)

$$a_n = a_{n-3} + 3 \cdot 2 \quad — (5)$$

put $n = n-3$ in eq^n (1)

$$a_{n-3} = a_{n-4} + 2 \quad — (6)$$

put value of a_{n-3} in eq^n (5)

$$a_n = a_{n-4} + 4 \cdot 2 \quad — (7)$$

$$\vdots \qquad \vdots$$

$$a_n = a_{n-k} + K \cdot 2 \quad — (8)$$

for $K = n$

$$a_n = a_{n-(n)} + (n)2$$

$$a_n = a_0 + 2n$$

$$\boxed{a_n = 1 + 2n}$$

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Q-2

$$a_n = -(a_{n-1}), \quad a_0 = 5$$

$$a_n = (-1)^1 (a_{n-1}) \quad - (1)$$

put $n = n-1$ in eq^n (1)

$$a_{n-1} = -(a_{n-2}) \quad - (2)$$

put value of a_{n-1} in eq^n (1)

$$a_n = (-1)^2 (a_{n-2}) \quad - (3)$$

put $n = n-2$ in eq^n (1)

$$a_{n-2} = -(a_{n-3}) \quad - (4)$$

put value of a_{n-2} in eq^n (3)

$$a_n = (-1)^3 (a_{n-3}) \quad - (5)$$

$$\vdots \qquad \vdots$$

$$a_n = (-1)^K (a_{n-K}) \quad - (6)$$

for $K = n$

$$a_n = (-1)^n a_0$$

$$\boxed{a_n = (-1)^n \cdot 5}$$

Q-3 $a_n = 3a_{n-1}$, $a_0 = 2$

$$a_n = 3a_{n-1} \quad — (1)$$

put $n = n-1$ in eq^n (1)

$$a_{n-1} = 3a_{n-2} \quad - (2)$$

put value of a_{n-1} in eq^n (1)

$$a_n = (3)^2 a_{n-2} \quad - (3)$$

put $n = n-2$ in eq^n (1)

$$a_{n-2} = 3a_{n-3} \quad — (4)$$

put value of a_{n-2} in eq^n (3)

$$a_n = (3)^3 a_{n-3} \quad - (5)$$

$$\vdots \qquad \vdots$$

$$a_n = (3)^K a_{n-K} \quad — (6)$$

for $K = n$

$$a_n = (3)^n a_0$$

$$\boxed{a_n = (3)^n \cdot 2}$$

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Q-4 $a_n = 2n\, a_{n-1}$, $a_0 = 1$

$a_n = 2n\, a_{n-1}$ — (1)

put $n = n-1$ in eq^n (1)

$a_{n-1} = 2(n-1)\, a_{n-2}$ — (2)

put value of a_{n-1} in eq^n (1)

$a_n = 2^2 n(n-1)\, a_{n-2}$ — (3)

put $n = n-2$ in eq^n (1)

$a_{n-2} = 2(n-2)\, a_{n-3}$ — (4)

put value of a_{n-2} in eq^n (3)

$a_n = 2^3 n(n-1)(n-2)\, a_{n-3}$ — (5)

$\vdots$

$a_n = 2^K n(n-1)(n-2) \cdots (n-(K-1))\, a_{n-K}$ — (6)

for $K = n$

$a_n = 2^n a_0\, n(n-1)(n-2) \cdots (n-n+1)$

$a_n = 2^n a_0\, n!$

$$\boxed{a_n = 2^n \times n!}$$

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Characteristic Root Method →

Linear recurrence relation

$$a_n = C_1 a_{n-1} + C_2 a_{n-2} + C_3 a_{n-3} + \dots \dots + C_K a_{n-K} + f(n)$$

if $f(n) = 0$ then eq^n is said to be as homogeneous recurrence relation.
There are only recurrence terms.

if there are both recurrence & non recurrence terms then it is said to be as nonhomogeneous recurrence relation.

Homogeneous Part →

Ex →

$$a_n = a_{n-1} + 2a_{n-2} \quad \text{— (1)}$$

$$n \geq 2, \; a_0 = 0$$

$$a_1 = 1$$

let $a_n = r^n$

$$r^n = r^{n-1} + 2r^{n-2} \quad \text{— (2)}$$

$$r^n - r^{n-1} - 2r^{n-2} = 0$$

$$r^{n-2}[r^2 - r - 2] = 0$$

$$(r-2)(r+1) = 0$$

$$r = 2, -1 \quad \text{characteristic roots}$$

☆ Roots must be real.

→ Roots are real and distinct

General solution

$$a_n = b_1(r_1)^n + b_2(r_2)^n + \dots + b_K(r_K)^n$$

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→ Roots are real and same

general solution

$$a_n = (b_1 + b_2 n + b_3 n^2 + \cdots + b_k n^{k-1}) r_k^n$$

roots are 3, 3, 2

$$a_n = (b_1 + b_2 n) 3^n + b_3 2^n$$

roots are 1, 2, 3

$$a_n = b_1 (1)^n + b_2 (2)^n + b_3 (3)^n.$$

General sol^n → $a_n = b_1 (2)^n + b_2 (-1)^n$

for $n = 0$

$$0 = b_1 (2)^0 + b_2 (-1)^0$$

$$0 = b_1 + b_2$$

for $n = 1$

$$1 = b_1 (2)^1 + b_2 (-1)^1$$

$$1 = 2b_1 - b_2$$

$$0 = b_1 + \cancel{b_2}$$

$$1 = 2b_1 - \cancel{b_2}$$

$$1 = 3b_1$$

$$b_1 = \frac{1}{3}, \quad b_2 = -\frac{1}{3}$$

$$\boxed{a_n = \frac{1}{3}(2)^n - \frac{1}{3}(-1)^n}$$

13-01-2020 sheet on official group

Q-9

a) $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$

$$a_n = 2a_{n-1} \quad \text{— (1)}$$

let $a_n = r^n$

$$r^n = 2r^{n-1}$$

$$r^n - 2r^{n-1} = 0$$

$$r^{n-1}[r-2] = 0$$

$$r - 2 = 0$$

$\underline{r = 2}$ characteristic root

General solⁿ → $a_n = b_1(2)^n$

for $n = 0$

$$a_0 = b_1(2)^0$$

$$3 = b_1$$

$$\boxed{a_n = 3(2)^n}$$

b) $a_n = a_{n-1}$ for $n \geq 1$, $a_0 = 2$

$$a_n = a_{n-1} \quad \text{— (1)}$$

let $a_n = r^n$

$$r^n = r^{n-1}$$

$$r^n - r^{n-1} = 0$$

$$r^{n-1}[r-1] = 0$$

$r - 1 = 0$

$\underline{r = 1}$ characterstic root

general sol^n $\rightarrow$ $a_n = b_1 (1)^n$

for $n = 0$

$$a_0 = b_1 (1)^0$$
$$2 = b_1$$

$$\boxed{a_n = 2(1)^n}$$

c) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

$$a_n = 5a_{n-1} - 6a_{n-2} \quad — (1)$$

let $a_n = r^n$

$$r^n = 5r^{n-1} - 6r^{n-2}$$
$$r^n - 5r^{n-1} + 6r^{n-2} = 0$$
$$r^{n-2}\left[r^2 - 5r + 6 \right] = 0$$
$$r^2 - 5r + 6 = 0$$
$$r^2 - 2r + 3r + 6 = 0$$
$$r(r - 2) - 3(r - 2) = 0$$
$$(r - 3)(r - 2) = 0$$

$\underline{r = +3, 2}$ characterstic root

General sol^n $\rightarrow$ $a_n = b_1 (+3)^n + b_2 (2)^n$

for $n = 0$, $1 = b_1 + b_2$

for $n = 1$, $0 = +3b_1 + 2b_2$

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$b_1 = -2$

$b_2 = 3$

$$\boxed{a_n = -2(3)^n - 3(2)^n}$$

$\cancel{1 = b_1 + b_2}$

$\cancel{0 = b_1 + 6b_2}$

$\cancel{1 = 7b_2}$

$\cancel{b_2 = \frac{1}{7}, \quad b_1 = \frac{6}{7}}$

$$\boxed{\cancel{a_n = \frac{6}{7}(-1)^n} + \frac{1}{7}(6)^n}$$

d) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$

$a_n = 4a_{n-1} - 4a_{n-2}$ — (1)

let $a_n = r^n$

$r^n = 4r^{n-1} - 4r^{n-2}$

$r^n - 4r^{n-1} + 4r^{n-2} = 0$

$r^{n-2}[r^2 - 4r + 4] = 0$

$r^2 - 4r + 4 = 0$

$\underline{r = 2, 2}$ characteristic root

General soln $\rightarrow$ $a_n = (b_1 + b_2 n)2^n$

for $n = 0$, $a_0 = (b_1 + b_2 \times 0)2^0$

$6 = b_1$

for $n = 1$, $a_1 = (b_1 + b_2)2^1$

$8 = (b_1 + b_2)2$

$8 = (6 + b_2)2$

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$$4 = 6 + b_2$$
$$-2 = b_2$$

$$\boxed{a_n = (6 - 2n)2^n}$$

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Non-homogeneous Part →

$$\boxed{a_n = C_1 a_{n-1} + C_2 a_{n-2} + C_3 a_{n-3} - - - - C_k a_{n-k}} + f(n)$$

$\underbrace{C_1 a_{n-1} + \dots + C_k a_{n-k}}_{\text{homogeneous part}}$ $\qquad$ $\underbrace{f(n)}_{\text{nonhomogeneous part.}}$

value of $f(n)$ can be
① $2^n, 3^n, 4^n$
② n^3, n^2+1, n
③ $n^3 4^n$
$(n+1)5^n$
④ $2, 3, 4.$

	$f(n)$	Assumed Particular Solution
1	b^n (if b is not the root of characteristic eq^n)	$\underbrace{A}_{\text{constant}} b^n$
2	$P(n)$ i.e. polynomial of degree m	$A_0 + A_1 n + A_2 n^2 + A_3 n^3 - - - - + A_m n^m$
3	$b^n P(n)$	$(A_0 + A_1 n + A_2 n^2 + - - - - + A_m n^m) b^n$
4	b^n (if b is the root of characteristic eq^n of multiplicity s).	$A b^n \cdot n^s$
5	$b^n P(n)$ (if b is the - - - - - - - - - - multiplicity s)	$(A_0 + A_1 n + A_2 n^2 + - - - - + A_m n^m) b^n n^s$
6	b	A

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Q- $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$

for homogeneous solⁿ

$$a_{n+2} - 4a_{n+1} + 4a_n = 0$$

let $a_n = r^n$

$$r^{n+2} + 4r^{n+1} + 4r^n = 0$$
$$r^n [r^n - 4r + 4] = 0$$
$$(r-2)^2 = 0$$
$$r = 2, 2$$
$$a_n^{(h)} = (c_1 + c_2 n) 2^n$$

for Particular solⁿ

$$a_n^{(p)} = A 2^n n^2 .$$

$$a_{n+1} = A 2^{n+1} (n+1)^2$$
$$a_{n+2} = A 2^{n+2} (n+2)^2$$

$$A 2^{n+2} (n+2)^2 - 4 A 2^{n+1} (n+1)^2 + 4 A 2^n n^2 = 2^n$$

$$[A 2^2 (n+2)^2 - 4A\, 2 (n+1)^2 + 4 A n^2] = 1$$

$$[4A(n+2)^2 - 8A(n+1)^2 + 4An^2] = 1$$

$$[4A(n^2+4+4n) - 8A(n^2+1+2n) + 4An^2] = 1$$

$$\cancel{4An^2} + 16A + \cancel{16An} - \cancel{8An^2} - 8A - \cancel{16An} + \cancel{4An^2} = 1$$

$$8A = 1 \Rightarrow A = \frac{1}{8}$$

$$a_n^{(p)} = \frac{1}{8} 2^n n^2$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$\boxed{a_n = (C_1 + C_2 n) 2^n + \frac{1}{8} 2^n n^2}$$

Q- $y_{n+2} - y_{n+1} - 2y_n = n^2$

<u>for homogeneous sol^n</u>

$$y_{n+2} - y_{n+1} - 2y_n = 0$$

let $y_n = r^n$

$$r^{n+2} - r^{n+1} - 2r^n = 0$$

$$r^n [r^2 - r - 2] = 0$$

$$r = -1, 2$$

$$y_n^{(h)} = C_1 (-1)^n + C_2 (2)^n$$

<u>for Particular sol^n →</u>

$$y_n^{(p)} = A_0 + A_1 n + A_2 n^2$$

$$y_{n+1} = A_0 + A_1 (n+1) + A_2 (n+1)^2$$

$$y_{n+2} = A_0 + A_1 (n+2) + A_2 (n+2)^2$$

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Q- What is the general form of a particular solⁿ guarented to exists of the linear non-homogeneous recurrance relation

$$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + f(n)$$

if a) $f(n) = n^2$ case 2 → $A_0 + A_1 n + A_2 n^2$

b) $f(n) = 2^n$ case 4 → $A 2^n n^3$

c) $f(n) = n.2^n$ case 5 → $(A_0 + A_1 n) 2^n n^3$

d) $f(n) = (-2)^n$ case 1 → $A(-2)^n$

e) $f(n) = n^2 2^n$ case 5 → $(A_0 + A_1 n + A_2 n^2) 2^n n^3$

f) $f(n) = n^3(-2)^n$ case 3 → $(A_0 + A_1 n + A_2 n^2 + A_3 n^3)(-2)^n$

g) $f(n) = 3$ case 6 → A.

for homogeneous solⁿ

$$a_n - 6a_{n-1} + 12a_{n-2} - 8a_{n-3} = 0$$

let $a_n = r^n$

$$r^n - 6r^{n-1} + 12r^{n-2} - 8r^{n-3} = 0$$

$$r^{n-3}[r^3 - 6r^2 + 12r - 8] = 0$$

$$r = 2, 2, 2$$

$$a_n^{(h)} = (C_1 + C_2 n + C_3 n^2) 2^n$$

a) for Particular solⁿ →

$$y_n^{(P)} = A_0 + A_1 n + A_2 n^2$$

$$y_{n-1}^{(P)} = A_0 + A_1(n-1) + A_2(n-1)^2$$

$$y_{n-2}^{(P)} = A_0 + A_1(n-2) + A_2(n-2)^2$$

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$$y_{n-3}^{(p)} = A_0 + A_1(n-3) + A_2(n-3)^2$$

$$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + f(n)$$

$$a_n - 6a_{n-1} + 12a_{n-2} - 8a_{n-3} = f(n)$$

$$\Rightarrow A_0 + A_1 n + A_2 n^2 - 6\left[A_0 + A_1(n-1) + A_2(n-1)^2\right] + 12\left[A_0 + A_1(n-2) + A_2(n-2)^2\right] - 8\left[A_0 + A_1(n-3) + A_2(n-3)^2\right] = n^2$$

Generating function Method →

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_k x^k + \cdots$$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n$$

1 $(1, 1, 1, 1 \cdots, 1, \cdots)$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots$$

$$= \sum_{n=0}^{\infty} x^n$$

2 $(1, 1, 1, 1, \cdots 1)$

$$\frac{1-x^{n+1}}{1-x} = 1x^0 + 1x^1 + 1x^2 \cdots + 1x^n$$

$$= \sum_{k=0}^{n} x^k$$

3 $(a^0, a^1, a^2, a^3, \cdots, a^k, \cdots)$

$$\cancel{\frac{1}{a x^0}} \quad \frac{1}{1-ax} = a^0 x^0 + a^1 x^1 + a^2 x^2 \cdots + a^k x^k + \cdots$$

$$= \sum_{n=0}^{\infty} a^n x^n$$

4 $$\frac{1}{1-x^r} = 1 + x^r + x^{2r} + x^{3r} + \cdots x^{kr} + \cdots$$

$$= \sum_{n=0}^{\infty} x^{nr}$$

5 $(1, 2, 3, 4, 5, \cdots)$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

$$= \sum_{n=0}^{\infty} (n+1) x^n.$$

Shifting Properties of Generating functions:-

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_k x^k + \cdots$$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n$$

$(a_0, a_1, a_2, a_3 \cdots a_k \cdots)$

→ Multiplication

$$x\,G(x) = 0.x^0 + a_0 x + a_1 x^2 + a_2 x^3 + a_3 x^4 + \cdots + a_k x^{k+1} \cdots$$

$\cdots$ $(0, a_0, a_1, a_2, a_3 \cdots a_k \cdots)$

$$x^2 G(x) = 0x^0 + 0x^1 + a_0 x^2 + a_1 x^3 + a_2 x^4 + a_3 x^5 + \cdots a_k x^{k+2} + \cdots$$

$(0, 0, a_0, a_1, a_2, a_3 \cdots)$

→ division

$$G(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ----$$
$$G(x) - a_0 = a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + ----$$

$$\frac{G(x) - a_0}{x} = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + ----$$

$$(a_1, a_2, a_3, a_4 ----)$$

$$G(x) - a_0 - a_1 x = a_2 x^2 + a_3 x^3 + a_4 x^4 ----$$

$$\frac{G(x) - a_0 - a_1 x}{x^2} = a_2 + a_3 x + a_4 x^2 ----$$

$$(a_2, a_3, a_4, a_5 ----)$$

Q- $(1, 1, 1, 1, 1, 1)$

$$1x^0 + 1x^1 + 1x^2 + 1x^3 + 1x^4 + 1x^5$$
$$= 1 + x + x^2 + x^3 + x^4 + x^5$$
$$= \frac{1 - x^{5+1}}{1-x} = \frac{1-x^6}{1-x}.$$

Q-1 0, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, ----

2 0, 0, 0, 1, 1, 1, 1, 1, ----

3 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1 ----

4 2, 4, 8, 16, 32, 64, 128, 256 ----

5 2, -2, 2, -2, 2, -2, 2, -2 -----

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6 $1, 1, 0, 1, 1, 1, 1, 1, 1 \text{ ----}$

7 $0, 0, 0, 1, 2, 3, 4 \text{ ----}$

1 $0 \cdot x^0 + 2x^1 + 2x^2 + 2x^3 + 2x^4 + 2x^5 + 2x^6 + 0x^7 + 0 \cdot x^8 + 0 \cdot x^9 \text{ -----}$

$$2x^1 + 2x^2 + 2x^3 + 2x^4 + 2x^5 + 2x^6$$
$$2x(1 + x + x^2 + x^3 + x^4 + x^5)$$

$$= 2x\left(\frac{1-x^6}{1-x}\right)$$

2 $0x^0 + 0x^1 + 0x^2 + 1x^3 + 1x^4 + 1x^5 \text{ -----}$

$$1x^3 + 1x^4 + 1x^5 + 1x^6 + \text{ -----}$$
$$x^3(1 + x + x^2 + x^3 + \text{ ----})$$
$$= x^3\left(\frac{1}{1-x}\right)$$

3 $0x^0 + 1x^1 + 0x^2 + 0x^3 + 1x^4 + 0x^5 + 0x^6 + 1x^7 + \text{---}$

$$1x^1 + 1x^4 + 1x^7 + 1x^{10} \text{ -----}$$
$$x[1 + x^3 + x^6 + x^9 \text{ ----}]$$
$$x[1 + x^3 + x^{2\times3} + x^{3\times3} + \text{ ----}]$$
$$= x\left[\frac{1}{1-x^3}\right]$$

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4 $(2, 4, 8, 16, 32, 64, 128, 256 \ldots) \ldots$

$2^1 \cdot x^0 + 2^2 x^1 + 2^3 x^2 + 2^4 x^3 + 2^5 x^4 + 2^6 x^7 + 2^7 x^8 + 2^8 x^9 \ldots$

$2[x^0 + 2x^1 + 2^2 x^2 + 2^3 x^3 + 2^4 x^4 + 2^5 x^5 + 2^6 x^6 + 2^7 x^7 + 2^8 x^8 \ldots]$

$$= 2 \sum_{n=0}^{\infty} 2^n x^n$$

$$= 2 \frac{1}{(1-2x)}$$

5 $2, -2, 2, -2, 2, -2, \ldots$

$2x^0 + (-2x^1) + 2x^2 + (-2x^3) \ldots$

$2[1 + (-1x^1) + 1x^2 + (-1x^3) \ldots]$

$2[(-1)^0 + (-1^1 x^1) + (-1)^2 x^2 + \ldots]$

$$2\left[\frac{1}{1-(-x)}\right] = \frac{2}{1+x}.$$

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op
add or sub x^2

6 $1, 1, 0, 1, 1, 1, 1 \ldots$

$$x^0 + x^1 + 0x^2 + x^3 + x^4 + x^5 \ldots$$

$$1 + x + x^3[1 + x + x^2 + x^3 \ldots]$$

$$1 + x + x^3\left(\frac{1}{1-x}\right)$$

$$\frac{1 - x + x - x^2 + x^3}{1-x}$$

$$= \frac{1 - x^2 + x^3}{1-x}.$$

7 $0, 0, 0, 1, 2, 3, 4 \ldots$

$$0x^0 + 0x^1 + 0x^2 + 1x^3 + 2x^4 + 3x^5 + \ldots$$

$$x^3(1 + 2x + 3x^2 + \ldots)$$

$$x^3\left(\frac{1}{1-x^2}\right) = \frac{x^3}{1-x^2}.$$

Q- a) $a_n = 5 \quad \forall n = 0, 1, 2, \ldots$

$a_1 = 5$

$a_2 = 5$

$\vdots$

$$5x^0 + 5x^1 + 5x^2 + \ldots$$

$$5[1 + x + x^2 + \ldots]$$

$$5\left(\frac{1}{1-x}\right) = \frac{5}{1-x}.$$