UNIT-4

Propositional Logic & Predicate Calculus

Propositional sentence → a sentence which declare true or false but not both is known as propositional sentence.

Ex → it is monday today.
it is 5'o clock.
Please open the door. Not propositional
$x+y = y+x$

1 negation symbol → $\neg, \tilde{x}, \bar{x}$ (used in boolean)
2 conjuction (and)
3 disjunction (or)
4 conditional (if then)
5 biconditional (if and only if)
6 exclusive or (xor)
7 Nand
8 Nor

Connectors

p → today is friday
¬p → today is not friday

① atleast 10 inches of rain fell today in Kanpur.
negation → atmost 10 inches of rain fell today in Kanpur.
atleast 10 inches of rain did not fell today in Kanpur.

p	¬p
T	F
F	T

② Conjunction → symbol → ∧ (and)

$2^2 = 4$

$p \wedge q \wedge r$
↓
$2^3 = 8$

p	q	p∧q
true	true	true
true	false	false
false	true	false
false	false	false

Inclusive or :- The disjunction is true when atleast one of ~~of~~ the two proposition is true.

Ex→1 Roses are red and lotus are pink
2 Jack and Jill went up the hill not conjuction
3 Jack and Jill are ~~cougx~~ cousins ↗

→ let P: Roses are red ~~&P~~
Q: lotus are pink
$P \wedge Q$ → Symbolic form

→ Jack went up the hill and Jill went up the hill.

③ Disjunction → symbol → ∨ (or)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

if exactly 20 or 30 were killed then it will disjunction ↑

Ex→1 Roses are red or violets are blue.
2 Twenty or thirty animals were killed in fire today.
3 You can see the match at home or you can go to stadium.
↓ XOR ⊕ or $\bar{\vee}$
4 There is something wrong with the bulb or with the wiring.
↓ disjunction

④ Exclusive or → XOR symbol → ⊕ or $\bar{\vee}$

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

1 P: I bought a ticket
Q: I won the jackpot

$P \vee Q$ → I bought a ticket or I won the jackpot
$\neg P \wedge \neg Q$ → I did not buy a ticket and I did not won the jackpot
$\neg P \vee Q$ → I did not buy a ticket ~~and~~ or I won the jackpot.

2 P: It is below freezing
Q: It is snowing

It is below freezing but not snowing → $P \wedge \neg Q$

⑤ Conditional → (symbol →) (if then)

A conditional statement is also called an implication.

$$\underset{\text{antecedent}}{\underline{P}} \longrightarrow \underset{\text{consequent}}{\underline{Q}}$$

if P then Q

$$P \rightarrow Q \equiv \neg P \vee Q$$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Ex→1 If $\underbrace{\text{I am elected}}_{P}$ then $\underbrace{\text{I will lower taxes.}}_{Q}$

1 It rains : R
I will be wet : ω
I would stay at home : S
Picnic is cancelled : P

If it rains and the picnic is not cancelled or I don't stay at home then I will wet.

$$((R \land \neg P) \lor \neg S) \rightarrow \omega$$

2 P: swimming is allowed at the shore
Q: Sharks have been spotted near the shore

$P \rightarrow \neg Q$ If swimming is allowed at the shore then sharks have not been spotted near the shore

$\neg Q \rightarrow P$ If sharks have not been spotted near the shore then swimming is allowed at the shore.

$\neg P \rightarrow \neg Q$ If swimming is not allowed at the shore then sharks have not been spotted near the shore

Forms of Conditional →

1 If P then Q If you drive more than 400 miles then you need to buy gasolin
2 If P, Q If you drive more than 400 miles you need to buy gasoline
3 Q if P
4 Q when P
5 Q unless $\neg P$
Same 4,8 6 P is sufficient for Q
7 A necessary condition for P is Q
8 Q whenever P
9 P only if Q

10 P implies Q.

Q whenever P → It rains whenever the wind blows from the east.
↓ Q
consequent
↓ P
antecedent

If P then Q → If the wind blows from the east then it rains.

Q
Q if P → The apple trees will bloom if it stays warm for a week.
P

If P then Q → If it stays warm for a week then the apple trees will bloom.

P
P implies Q → That the pistons wins the championship implies that they beat the lakers.
Q

If P then Q → If the pistons win the championship then they beat the lakers.

P
P only if Q → Your gaurantee is good only if you bought your CD player less than 90 days ago.
Q

If P then Q → If your gaurantee is good then you must have bought your CD player less than 90 days ago.

Q is necessary for P → It is necessary to walk 8 miles to get to the top of long's peak.
↓ Q
P ↓

If P then Q → If you get to the top of long's peak then you must have walked 8 miles.

A sufficient condition for Q is P → To get tenure as a professor it is sufficient to be world famous

$\downarrow Q$ $\qquad\qquad$ $\downarrow P$

If P then Q → If you are world famous then you will get tenure as a professor.

Q unless ¬P → John will go swimming unless the water is too cold

$\uparrow Q$ $\qquad\qquad$ $\downarrow \neg P$

If P then Q → If the water is not too cold then John will go swimming.

Ex→1 The crop will be destroyed if there is a flood.

Q $\qquad$ Q if P $\qquad$ P

If P then Q → If there is a flood then the crop will be destroyed

$$\boxed{P \rightarrow Q}$$

? You can access the internet from campus only if you are computer science major or you are not a freshman.

P only if Q

let I: you can access the internet from campus

C: you are a computer science major

F: you are a freshman

P only if Q

If P then Q → If you are accessing the internet from campus then you are computer science major or you are not a freshman.

$$\boxed{I \to (C \lor \neg F)}$$

3 You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

$\neg C$ (over "coaster"), T (under "tall"), O (under "years old")

Q if P
Q unless ¬P

let C: You can ride the roller coaster.
T: You are under 4 feet tall.
O: You are ~~not~~ older than 16 years old.

If P then Q → If you are under 4 feet tall and You are not older than 16 years old then you cannot ride the roller coaster.

$$\boxed{(T \land \neg O) \to \neg C}$$

1 when you buy a new car you get \$2000 cashback or a car loan. Exclusive OR (XOR)

2 To take mathematics you must have taken calculus or a course in computer science. Inclusive OR (disjunction)

3 Dinner for 2 includes, 2 items from column A or 3 items from column B. Exclusive OR (XOR)

1 It is necessary to impress the boss to get promoted.
consequent (Q) antecedent (P)

If P then Q → If you want to get promoted then you have to impress the boss.

2 I will remember to send you the address only if you send me an E-mail message.
P
Q

If P then Q → If I will remember to send you the address then you must have send me an E-mail message.

3 To be a citizen of this country it is sufficient that you were born in US.
P
Q

If P then Q → If you want to be citizen of this country then you must be born in US.

Converse, contrapositive and inverse :-

$P \longrightarrow Q$ original
Converse $Q \to P$
contrapositive $\neg Q \to \neg P$
inverse $\neg P \to \neg Q$

$$\boxed{\begin{array}{l} P \to Q \equiv \neg Q \to \neg P \\ Q \to P \equiv \neg P \to \neg Q \end{array}}$$

P	Q	$P \to Q$	$Q \to P$	$\neg Q \to \neg P$	$\neg P \to \neg Q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Q- The home team wins wherever it is raining.
Q P

$P \longrightarrow Q$ If it is raining then the home team wins.
$Q \to P$ If the home team wins then it is raining.
$\neg Q \to \neg P$ If the home team does not win then it must not be raining.
$\neg P \to \neg Q$ If it is not raining then the home team must not win.

P

Q- A positive integer is a prime only if has no divisor other than 1 or itself.

Q

$P \rightarrow Q$ If positive integer is a prime then it has no divisor other than 1 or itself.

$Q \rightarrow P$ If it has no divisor other than 1 or itself then a positive integer is a prime.

$\neg Q \rightarrow \neg P$ If it has divisor other than 1 or itself then a positive integer is not a prime.

$\neg P \rightarrow \neg Q$ If a positive integer is not a prime then it has ~~no~~ divisor other than 1 or itself.

⑥ Biconditionals →

$$P \longleftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$
$$\equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$$\boxed{P \leftrightarrow Q \equiv \neg(P \oplus Q)}$$

(iff)

You can take the flight if and only if you buy a ticket.

keyword

if P then Q and if Q then P

27/08/2020

Algebra of Propositional →

Logically Equivalent

$A \equiv B$

$A \Leftrightarrow B$.

Rules →

(i) Idempotent Law →

$P \vee P \equiv P \qquad P \wedge P \equiv P$

(ii) Commutative Law →

$P \vee Q \equiv Q \vee P \;|\; P \wedge Q \equiv Q \wedge P$

(iii) Associative Law →

$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

(iv) Distributive Law →

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(v) Absorption Law →

$P \vee (Q \wedge P) \equiv P$

$P \wedge (Q \vee P) \equiv P$

(vi) De Morgan's Law →

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

(vii) Identity law →

① $P \vee \neg P \equiv T$ $\quad$ $P \wedge \neg P \equiv F$

② $P \vee T \equiv T$ $\quad$ $P \wedge F \equiv F$

③ $P \wedge T \equiv P$ $\quad$ $P \vee F \equiv P$

Tautology, Contradiction & Contigency →

Tautology → A propositional formula that is always true no matter what the truth values of the propositional variables that occur in it, is called a tautology.

A compound proposition that is always false is called a contradiction.

Contigency → A compound proposition that is neither a tautology nor a contradiction is called a Contigency.

Q- $\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q$

LHS

$\neg(P \vee (\neg P \wedge Q))$

$\equiv \neg((P \vee \neg P) \wedge (P \vee Q))$ Distributive law

$\equiv \neg(T \wedge (P \vee Q))$ Identity law

$\equiv \neg(P \vee Q)$ Identity law

$\equiv \neg P \wedge \neg Q$ De Morgan's law

RHS

Q- $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology

$\equiv \neg(P \wedge Q) \vee (P \vee Q)$ Conditional

$\equiv (\neg P \vee \neg Q) \vee (P \vee Q)$ De Morgan's law

$\equiv (\neg P \vee P) \vee (\neg Q \vee Q)$ Associative law

$\equiv T \vee T$ Identity law

$\equiv \underline{\underline{T}}$ Identity law

Q- $(\neg P \wedge (P \rightarrow Q)) \rightarrow \neg Q$

$(\neg P \wedge (\neg P \vee Q)) \rightarrow \neg Q$ Conditional

$\neg P \rightarrow \neg Q$ absorption law

$\neg(\neg P) \vee \neg Q$ Conditional

$\underline{P \vee \neg Q}$

Contigency

31/08/2020

Rules of inference (Theory of inference)

Rules
Hypothesis
conclusion

Rules →

1 Modus Pones or MP Rule

$$\begin{array}{l} P \rightarrow Q \\ \underline{P} \\ \underline{\therefore Q} \end{array}$$

statement1
statement2

2 Modus Tollens or MT Rule

$$\begin{array}{l} P \rightarrow Q \\ \underline{\neg Q} \\ \underline{\therefore \neg P} \end{array}$$

3 Hypothetical Syllogism or HS rule

$$\begin{array}{l} P \rightarrow Q \\ \underline{Q \rightarrow R} \\ \underline{\therefore P \rightarrow R} \end{array}$$

4 Disjunctive Syllogism or DS Rule

$$\begin{array}{l} P \lor Q \\ \neg P \\ \hline \therefore Q \end{array}$$

5 Addition when we have lack of hypothesis means only 1 hypothesis is given.

$$\begin{array}{l} P \\ \hline \therefore P \lor Q \end{array}$$

6 Simplification

$$\begin{array}{l} P \land Q \\ \hline \therefore P \text{ or } Q \end{array}$$

7 Conjunction

$$\begin{array}{l} P \\ Q \\ \hline \therefore P \land Q \end{array}$$

8 Resolution

$$\begin{array}{l} P \lor Q \\ \neg P \lor R \\ \hline \therefore Q \lor R \end{array}$$

Q- Show that the premises "It is not sunny and it is colder than yesterday".
"we will go swimming only if it is sunny".
"If we do not go swimming then we will take a trip", and "If we take a trip, then we will be home by Sunset" lead to the conclusion
"we will be home by sunset".

Let S: It is sunny
C: It is colder than yesterday
Sw: we will go swimming

T: we will take a trip
H: we will be home by sunset

1. $\neg S \wedge C$
2. $S_\omega \rightarrow S$
3. $\neg S_\omega \rightarrow T$
4. $T \rightarrow H$ $\quad / \therefore H$

~~/∴H~~

5	$\neg S_\omega \rightarrow H$	3,4	HS
6	$\neg S$	1	Simplification
7	$\neg S_\omega$	2,6	MT
8	T	3,7	MP
9	$\underline{H}$	4,8	MP

Q- Show that the hypothesis "If you send me an e-mail message then I will finish writing the program." "If you do not send me an e-mail message, then I will go to sleep early" and "If I go to sleep early, then I will wake up feeling refreshed", lead to the conclusion "If I do not finish writting the program then I will wake up feeling refreshed".

Let P: You send me an e-mail message
Q: I will finish writing the program
R: I will go to sleep early
S: I will wake up feeling refreshed

Terrible Mansahor

1. $P \rightarrow Q$
2. $\neg P \rightarrow R$
3. $R \rightarrow S$ $/ \therefore \neg Q \rightarrow S$

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4. $\neg P \rightarrow S$ 2, 3 HS
5. $\neg Q \rightarrow \neg P$ 1 Contrapositive
6. $\neg Q \rightarrow S$ 5, 4 HS

Q-

1. $P \vee (Q \rightarrow S)$
2. $\neg R \rightarrow (S \rightarrow T)$
3. $P \rightarrow R$
4. $\neg R$ $/ \therefore Q \rightarrow T$

---

5 $\neg P$ 3, 4 MT
6 $Q \rightarrow S$ 1, 5 DS
7 $R \vee (S \rightarrow T)$ ~~2,4~~2 Conditional
8 $S \rightarrow T$ 7, 4 DS
9 $Q \rightarrow T$ 6, 8 HS

Q-

1. $(P \vee Q) \rightarrow (R \wedge S)$
2. $\neg R$ $/ \therefore \neg Q$

---

3. $\neg R \vee \neg S$ 2 addition
4. $\neg (R \wedge S)$ 3 De Morgon's
5. $\neg (P \vee Q)$ 1, 4 MT
6. $\neg P \wedge \neg Q$ De Morgon's
7. $\neg Q$ Simplification

Some advanced rules →

1. Constructive Dilemma CD Rule

$(P \to Q) \land (R \to S)$

$\underline{P \lor R}$

$\therefore Q \lor S$

2. Destructive Dilemma DD Rule

$(P \to Q) \land (R \to S)$

$\underline{\neg Q \lor \neg S}$

$\therefore \neg P \lor \neg R$

Q-

1. $(P \to Q) \land (R \to S)$
2. $(Q \land S) \to T$
3. $\neg T \quad /\therefore \neg (P \land R)$

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4. $\neg (Q \land S)$ 2,3 MT
5. $\neg Q \lor \neg S$ 4 De Morgan's
6. $\neg P \lor \neg R$ 1,5 DD Rule
7. $\neg (P \land R)$ 6 De Morgan's

3. Rule of Conditional Proof (CP Rule)

1
2
3 $/\therefore (\_\_\_) \to (\_\_\_)$

4 ↙ $/\therefore$ ↙

we can add the antecedent part of conclusion to the list of statements and left the consequent part which is ~~to~~ next become the new conclusion.

Q- 1. $P \to Q$ $\quad /\therefore P \to (P \land Q)$

2. P $\quad /\therefore P \land Q$ 1 CP Rule
3 Q $\quad$ 1, 2 MP Rule
4 $P \land Q$ $\quad$ 2, 3 Conjunction

Q- 1 $(P \lor Q) \to ((R \lor S) \to T)$ $\quad /\therefore P \to (R \land S) \to T$

2 P $\quad /\therefore (R \land S) \to T$ 1 CP rule
3 $R \land S$ $\quad /\therefore T$ 2 CP rule
4 $P \lor Q$ $\quad$ 2 addition
5 $(R \lor S) \to T$ $\quad$ 1, 4 MP
6 R $\quad$ 3 simplification
7 $R \lor S$ $\quad$ 6 addition
8 T $\quad$ 5, 7 MP rule

03/09/2020 Rule of Indirect Proof or Proof by Contradiction →

In this method we first assume that the negation of conclusion is true i.e the negation of the conclusion become new premise then we start deduction from new premise including new premise then we reach our contradiction it means that the negation of the conclusion is true, was a wrong assumption.

{conclusion: In negation}

Q- Show that the conclusion $\neg(P \vee R)$ follows logically from $(P \to Q) \wedge (R \to S)$, $(Q \vee S) \to T$ & $\neg T$

1 $(P \to Q) \wedge (R \to S)$
2 $(Q \vee S) \to T$
3 $\neg T$ $\quad / \therefore \neg(P \vee R)$

---

4 $\neg\neg(P \vee R)$ 1 IP rule
5 $(P \vee R)$ 4 double negation
6 $(Q \vee S)$ 1,5 CD rule
7 T 2,6 MP rule
8 $T \wedge \neg T$ 3,7 conjunction
9 false. $\therefore$ our assumption is wrong so $\neg(P \vee R)$ is true.

Q- Show that the formula $(\neg Q \wedge (P \to Q)) \to \neg Q$ is a tautology.

$/ \therefore (\neg Q \wedge (P \to Q)) \to \neg Q$

---

1. $(\neg Q \wedge (P \to Q))$ $\quad / \therefore \neg Q$ CP Rule

---

2. $\neg\neg Q$ 1 IP rule
3. Q 2 double negation
4. $\neg Q$ 1 simplification
5. $Q \wedge \neg Q$ 3,4 conjunction
6. false

$\therefore$ our assumption is wrong
so $\neg Q$ is true

Fellacies →

Q- Show that the following premises are inconsistant.

(i) If Jack misses many class through illness, then he fails high school.
(ii) If Jack fails high school then he is uneducated.
(iii) If Jack reads a lot of books then he is not uneducated.
(iv) Jack misses the class through illness and reads a lot of books.

let P: ~~If~~ Jack misses many class through illness.
Q: he fails high school
R: he is uneducated
S: Jack reads a lot of books.

1. $P \rightarrow Q$
2. $Q \rightarrow R$
3. $S \rightarrow \neg R \quad / \therefore P \wedge S$

4	$P \rightarrow R$	1,2 HS rule
5	$\neg P \vee R$	4 conditional
6	$P \wedge \neg R$	5 De Morgan's
7	P	6 simplification
8	$\neg R$	6 simplification
9	R	4,7 MP rule
10	$R \wedge \neg R$	9,8 conjunction
11	F	10 Identity law.

07/09/2020

Quantifiers →

1 Universal Quantifier ($\forall$)
~~Extential~~

2 Existential Quantifier ($\exists$)

1 Universal Quantifier → $\underbrace{\overset{\nearrow \text{subject}}{\forall x}\ P(x)}_{\downarrow \text{predicate}}$ ~~P~~ → for all x P of x.

keywords → all of, for arbitary, for any, for all, for every, for each

Ex → $\underset{\forall}{\underline{\text{All}}}$ men are mortal.

let domain of dis course: All men in the world

m: man
M: are mortal

$\forall m\, M(m)$ → for every man in this world men are mortal

1 Q-
1 Every bird can fly $\forall b. F(b)$
2 Every Koala can climb ~~$\forall K C(K)$~~ $\forall k\, C(k)$
3 Every one ~~student in this~~ in your class is friendly $\forall s\, F(s)$

1 let domain of dis course: all the bird in the world

b: bird
F: fly

Symbolic form: $\forall b\, F(b)$

let:
2 ^domain of dis course: All the Koala in the world.
k: Koala
C: can climb
Symbolic form: $\forall k\, C(k)$.

3 let: domain of dis course: Every student in the class
s: student
F: is friendly
Symbolic form: $\forall s\, F(s)$

2 Existential Quantifier → $\exists x\, P(x)$

Keywords → for some, for at least one, there is

Ex→ Some men are mortal.

let domain of discourse: all men in this world
Sub:
m: man
M: are mortal
Symbolic form: $\exists m\, M(m)$
↓
There is a man in this world such that the man is mortal.

Q- 1 Some of your classmates are perfect
2 There is a student in this class who owns a PC

1 let: domain of dis course: all ~~classmates of~~ classmate in the class
C: classmates

P: are perfect

Symbolic form: $\exists C\, P(C) \rightarrow$ There is a student in your class such that that student is perfect.

2 let: domain of discourse: all the students in this class

S: student   P: who owns a PC

Symbolic form: $\exists S\, P(S)$ There is a student in this class such that that student who owns a PC.

Negating Quantified Expression:

① Everyone like sweets

$\forall x\, L(x) \rightarrow$ for all personx in this world, x like sweets

There is no one who does not like sweets.

$$\neg \exists x \neg L(x)$$

$$\boxed{\forall x\, L(x) \equiv \neg[\exists x \neg L(x)]}$$

09/09/2020

② Not everyone like sweets

$\neg[\forall x\, L(x)] \rightarrow$ There is someone who does not like sweets

$$\downarrow$$

$$\exists x \neg L(x)$$

$$\boxed{\neg[\forall x\, L(x)] \equiv \exists x \neg L(x)}$$

③ Someone like sweets

$\exists x L(x)$

Not everyone dislike sweets

$\neg[\forall x \neg L(x)]$

$$\boxed{\exists x L(x) \equiv \neg[\forall x \neg L(x)]}$$

④ Not someone like sweets

$\neg[\exists x L(x)]$

Everyone does not like sweets

$\forall x \neg L(x)$

$$\boxed{\neg(\exists x L(x)) \equiv \forall x \neg L(x)}$$

① No one is perfect
② Everyone is perfect
③ All ignorant people are vain
④ There is a person who cannot speak hindi
⑤ All birds cannot fly.

⑤ Step 1 → all birds cannot fly
Step 2 → domain: all birds in this world
subject: a bird: b
Predicate: canfly : F

$\forall b \neg F(b)$

Step 3 → $\neg[\forall b \neg F(b)]$
Step 4 → $\exists b\, F(b)$

Step 5 → There is a bird in this world such that that bird can fly.

Some birds can fly

Inference theory of Predicate Calculus→

biggest possible domain
? predicate

<u>Ex→</u> all birds can fly

domain: all birds in this world
New domain: all creatures in this world

$$\forall x\left[B(x) \longrightarrow F(x)\right]$$

For every creature in this world if that creature is a bird then that creature can fly.

Rules of TOI (Predicate Calculus)

	Name	Rule
1	Universal Instantiation	$\forall x P(x)$ $\therefore P(c)$
2	Universal Generalization	$P(a)$ for any arbitary a $\therefore \forall x P(x)$
3	Existential Instantiation	$\exists x P(x)$ $\therefore P(c)$ for some element
4	Existential Generalization	$P(c)$ for some element $\therefore \exists x P(x)$

Instantiation:
Predicate → Propositional   c → specific

Generalization:
Propositional → Predicate

Q- "Everyone in this Discrete Maths class has taken a course in CS" and "Marla is the student in this discrete mathematics class."

"Marla has taken a course in CS" → Conclusion

Domain :- All students in this world.

Subject :- A student 'x'

Predicate :- DM(x) : x is a student in discrete mathematics class

for every student x in this world, if x is a student in DM class then x has taken a course in CS

CS(x) : x has taken a course in CS

$$\begin{array}{ll|l} 1) & \forall x\,[DM(x) \rightarrow CS(x)] & \\ 2) & DM(Marla) & \therefore CS(Marla) \end{array}$$

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3)	$DM(Marla) \rightarrow CS(Marla)$	1) UI
4)	$CS(Marla)$	3,2 MP

Q- "A student in this class has not read the book"
"Everyone in this class passed the first exam"
"Someone who passed the first exam has not read the book" → conclusion

Domain :- All students in this world
Subject :- A student 'x'
Predicate :- C(x) : x is a student in this class
R(x) : x has read the book.
P(x) : x has passed the first exam

$P(a) \land \neg R(a)$

$$\begin{array}{ll|l} 1) & \exists x\,(C(x) \land \neg R(x)) & \\ 2) & \forall x\,[C(x) \rightarrow P(x)] & \therefore \exists x\,(P(x) \land \neg R(x)) \end{array}$$

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3)	$C(a) \land \neg R(a)$	1) EI
4)	$C(a) \rightarrow P(a)$	2) UI
5)	$C(a)$	3) simplification
6)	$P(a)$	4,5 MP
7)	$\neg R(a)$	3) simplification

8) $P(a) \land \neg R(a)$ 6, 7 conjunction
9) $\exists x (P(x) \land \neg R(x))$ 8 EG

Nested Quantifiers→

$\forall x \forall y P(x, y)$

$\forall x \exists y P(x, y)$

$\exists x \forall y P(x, y)$

$\exists x \exists y P(x, y)$

Ex→ $\forall x \forall y (x + y = y + x)$

domain: Real number

for every real number x and for every real number y $x + y = y + x$.

$\forall x \forall y (((x > 0) \land (y < 0)) \rightarrow (xy < 0))$

The product of a positive and a negative real number is allways negative.

↑ convert into simpleat

for every real no x and for every real no y if $x > 0$ and $y < 0$ then $xy < 0$

Q-1 The sum of two positive integers is always positive. $\forall x \forall y ((x>0) \wedge (y \wedge 0) \rightarrow (x+y>0))$

domain: integers

Q-2 Every real no except zero has a multiplicative inverse. domain: real no

$\forall x \forall y ((x \neq 0) \rightarrow (xy=1))$ $\forall x ((x \neq 0) \longrightarrow \exists y (xy=1))$

Q-3 Every student in your school has a computer or has a friend who has a computer.

domain: every student in school

$\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$

C(x): x has a computer
C(y): y has a computer
F(x,y): x is friend of y

If a person is a female and is a parent, then this person is someone's mother.

domain: all the person in this world

$\forall x (F(x) \wedge P(x)) \rightarrow \exists y M(x,y)$

Everyone has exactly one best friend.

$\forall x \exists y (B(x,y) \wedge \forall z ((z \neq y) \rightarrow \neg B(x,z)))$

domain: every person x in this world

Sheet 6 c) The disjunction of two contigencies can be a tautology.

domain: all propositional formula

C(x) : x is a contigency
C(y) : y is a contigency

T(x v y) : disjunction of x & y is a tautology

$\exists x\, \exists y ((C(x) \wedge C(y)) \rightarrow T(x \vee y))$

for some propositional formula x & for some propositional formula y if x is a contigency and y is a contigency then disjunction of x & y is a tautology

Satisfiability → [for Propositionals]

→ make a truthtable of given Propositional formula
→ find all values and if we get one or more true value then formula is satisfiable at that values of truth table.

9 b) whenever there is an active alert, all queued messages are transmitted.

Ⓟ Ⓠ

$$\exists x (A(x)) \longrightarrow \overset{\text{all messages}}{\forall m} (Q(m) \rightarrow T(m))$$

for some alert x if there is an active alert x then for all messages if there are queued messages then messages will be transmitted

→ queued messages

$$\exists x (A(x)) \longrightarrow \forall m \, T(m)$$

↓

for some alert x if there is an active alert x then for every queued messages ~~there is a transmitted~~ messages are transmitted.