Formulae List

1] Laplace/Inverse

1) $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

2) $L[e^{-at}] = \frac{1}{s-a} \Rightarrow L^{-1}\left[\frac{1}{s-a}\right] = e^{-at}$

3) $L[\sin at] = \frac{a}{s^2+a^2} \Rightarrow L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$

4) $L[\cos at] = \frac{s}{s^2+a^2} \Rightarrow L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$

5) $L[\sinh at] = \frac{a}{s^2-a^2} \Rightarrow L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{\sinh at}{a}$

6) $L[\cosh at] = \frac{s}{s^2-a^2} \Rightarrow L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$

7) $L[t^n] = \frac{n!}{s^{n+1}} \Rightarrow L^{-1}\left[\frac{1}{s^{n+1}}\right] = \frac{t^n}{n!}$

8) **Linearity Property**

$L[af(t) + bg(t)] = aL[f(t)] + bL[g(t)]$

$L^{-1}[a\phi_1(s) + b\phi_2(s)] = aL^{-1}[\phi_1(s)] + bL^{-1}[\phi_2(s)]$

9) **Change of Scale**

$L[f(at)] = \frac{1}{a}\phi(s/a)$

10) **First Shifting**

$L[e^{at}f(t)] = \phi(s-a) \Rightarrow L^{-1}[\phi(s-a)] = e^{at}f(t)$

$L[e^{-at}f(t)] = \phi(s+a) \Rightarrow L^{-1}[\phi(s+a)] = e^{-at}f(t)$

4)

Date
Page

11) Second shifting

$$g(t) = \begin{cases} f(t-a) & \text{for } t > a \\ 0 & \text{for } t < a \end{cases}$$

$$L[g(t)] = e^{-as}\phi(s)$$

$$L^{-1}[e^{-as}\phi(s)] = g(t) = \begin{cases} f(t-a) & \because t > a \\ 0 & t < a \end{cases}$$

12) Multiplication by t

$$L[t^n f(t)] = \cancel{(-1)^n \frac{d^n}{dt^n} f(} \; (-1)^n \frac{d^n}{dt^n}\phi(s)$$

$$L^{-1}[\phi^n(s)] = (-1)^n t^n L^{-1}[\phi(s)]$$

13) Division by t

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \phi(s)\,ds$$

14) Derivative Property

$$L[f^n(t)] = s^n\phi(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots$$

15) ~~Laplace of~~ Integration Property

$$L\left[\int_0^t f(u)\,du\right] = \frac{\phi(s)}{s}$$

Note: $erf\sqrt{t} = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$

16) Convolution theorm

$$L^{-1}[\phi_1(s) \times \phi_2(s)] = \int_0^t f_1(u) f_2(t-u)\,du$$

Date
Page

17) Periodic function

$$L[f(t)] = \frac{1}{1-e^{-as}} \int_0^a e^{-st} f(t)\, dt$$

18) Heaviside's unit step function

i) $L[H(t-a)] = \frac{e^{-as}}{s} \Rightarrow L^{-1}\left[\frac{e^{-as}}{s}\right] = H(t-a)$

ii) $L[f(t-a)\, H(t-a)] = e^{-as} L[f(t)]$

iii) $L[f(t)\, H(t-a)] = e^{-as} L[f(t+a)]$

iv) If $f(t) = \begin{cases} f_1(t) & 0 \le t < a \\ f_2(t) & a < t \le b \\ f_3(t) & t > b \end{cases}$

$$f(t) = f_1(t)[H(t) - H(t-a)] + f_2(t)[H(t-a) - H(t-b)] + f_3(t)[H(t-b)]$$

19) Dirac Delta function

i) $L[\delta(t-a)] = e^{-as}$

ii) $L^{-1}[1] = \delta(t)$

iii) $L^{-1}[f(t)\,\delta(t-a)] = e^{-as} f(a)$

2] Fourier

1) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

where, $a_0 = \frac{1}{L}\int_a^{a+2L} f(x)\, dx$, $a_n = \frac{1}{L}\int_a^{a+2L} f(x) \cos\frac{n\pi x}{L}\, dx$

$b_n = \frac{1}{L}\int_a^{a+2L} f(x) \sin\frac{n\pi x}{L}\, dx$

2) For $(-L, L)$ & $(-\pi, \pi)$

Odd $\Rightarrow b_n = 0$

Even $\Rightarrow a_n = a_0 = 0$

$$\& \int_{-L}^{L} \text{ or } \int_{-\pi}^{\pi} = 2\int_{0}^{\pi \text{ or } L} \text{ for evem}$$

$$= 0 \text{ for odd}$$

3) Parseval's

$$\frac{1}{L}\int_{0}^{a+2L} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

4) HRFS of cosine

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$\frac{2}{L}\int_{0}^{L} f^2(x)\,dx = \sum_{n=1}^{\infty} (a_n^2) + \frac{a_0^2}{2}$$

5) HRFS of sine

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L b_n \sin \frac{n\pi x}{L} \, dx$$

$$\frac{2}{L} \int_0^L f^2(x) \, dx = \sum_{n=1}^{\infty} b_n^2$$

6) Complex form

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{-\frac{in\pi x}{L}}$$

$$C_n = \frac{1}{2L} \int_a^{a+2L} f(x) e^{-\frac{in\pi x}{L}} \, dx$$

$$C_0 = \frac{a_0}{2}$$

$$C_n = \frac{1}{2} (a_n - ib_n)$$

$$C_{-n} = \frac{1}{2} [a_n + ib_n]$$

7) Fourier Integral

i) $$f(x) = \frac{1}{\pi} \int_{w=0}^{\infty} \int_{s=-\infty}^{\infty} f(s) \cos w(s-x) \, dw \, ds$$

ii) Cosine

$$f(x) = \frac{2}{\pi} \int_{w=0}^{\infty} \cos wx \left[\int_{s=0}^{\infty} f(s) \cos ws \, ds \right] dw \rightarrow f \text{ is even}$$

iii) Sine

$$f(x) = \frac{2}{\pi} \int_{\omega=0}^{\infty} \sin \omega x \left[\int_{s=0}^{\infty} f(s) \sin \omega s \, ds \right] d\omega$$

f is odd

8) Fourier Transform

1) $F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{is\cancel{t}} \, d\cancel{t}$

$$f(\cancel{t}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-is\cancel{t}} \, d\cancel{s}$$

2) $F_s[f(t)] = \int_0^{\infty} f(t) \sin st \, dt$

$$f(\cancel{t}) = \frac{2}{\pi} \int_0^{\infty} F_s[f(t)] \sin s\cancel{t} \, ds$$

3) $F_c[f(t)] = \int_0^{\infty} f(t) \cos st \, dt$

$$f(\cancel{t}) = \frac{2}{\pi} \int_0^{\infty} F_c[f(t)] \cos s\cancel{t} \, ds$$

i) $F[f(x) \cos a x] = \frac{1}{2}[F(s+a) + F(s-a)]$

ii) $F_s[f(x) \cancel{\cos} \, a t] = \frac{1}{2}[F_s(s+a) + F_s(s-a)]$

iii) $F_c[f(x) \sin at] = \frac{1}{2}[F_c(s+a) - F_c(s-a)]$

iv) $F_s[f(x) \sin at] = \frac{1}{2}[F_c(s-a) - F_c(s+a)]$

v) $F_c[f(x) \cos at] = \frac{1}{2}[F_c(s+a) + F_c(s-a)]$

3] Vector [algebra]

1) $(\bar{a} \times \bar{b}) \cdot \bar{c} = [\bar{a}\ \bar{b}\ \bar{c}]$

2) $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

3) $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{a}) = \begin{vmatrix} \bar{a} \cdot \bar{c} & \bar{b} \cdot \bar{c} \\ \bar{a} \cdot \bar{d} & \bar{b} \cdot \bar{d} \end{vmatrix}$

4) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{b}\ \bar{d}]\bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$

~~4] Differentiation~~

4) Vector (differentiation)

1) $\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

2) $\nabla \cdot \bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

3) $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$

4) Directional Derivative

$D\hat{u} f(a) = \nabla f(a) \cdot \hat{u}$

$= |\nabla f(a)| \cos\theta$

i) $\theta = 0 \Rightarrow max$

ii) $\theta = 180 \Rightarrow min$

5) Total differentiation

$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$

6) $\nabla f(r) = f'(r) \frac{\bar{r}}{r}$

7) $\nabla \cdot (\phi \bar{J}) = \phi(\nabla \cdot \bar{J}) + \bar{J} \cdot (\nabla \phi)$

5] Vector (Integration)

1) Theorm

i) $\int_C \bar{F} \cdot d\bar{r}$ is independent of path joining the endpoint of curve 'c' but only dependents only on the endpoint of curve 'c'

ii) $\bar{F}$ is conservative that is there exist ϕ such that $\bar{F} = \nabla \phi$

iii) For any closed curve 'c' $\oint_C \bar{F} \cdot d\bar{r} = 0$

2) $\nabla \times \bar{F} = \bar{0}$ ⇒ irrotational (iff conservative)

3) Green's Theorm

$$\int_C P\,dx + Q\,dy = \iint_R \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dx\,dy$$

4) Stoke's Theorm

$$\int_C \bar{F} \cdot d\bar{r} = \iint_S \underbrace{\hat{N}}_{\text{Unit outward Normal}} \cdot (\nabla \times \bar{F})\, ds$$

$ds = \dfrac{dx\,dy}{|\hat{N} \cdot \hat{k}|}$ → Projection xy plane

$dx = \dfrac{dy\,dz}{|\hat{N} \cdot \hat{i}|}$ → Projection yz plane

$ds = \dfrac{dx\,dz}{|\hat{N} \cdot \hat{j}|}$ → Projection xz plane

5) Gauss ~~Ther~~ Divergence Thm

$$\iint_S \hat{N} \cdot \bar{F}\, ds = \iiint_V \nabla \cdot \bar{F}\, dv$$

6] Z - transform

1) $Z[f(k)] = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$

2) Change of scale

$Z[a^k f(k)] = F\left(\frac{z}{a}\right)$

$Z[a^{-k} f(k)] = F(az)$

3) Shifting Prop

$Z[f(k \pm n)] = z^{\pm n} F(z)$

4) Linearity

$Z[a f(k) + b g(k)] = a Z[f(k)] + b[g(k)]$

5) Multiplication by K

$Z[k^n f(k)] = \left(-z \frac{d}{dz}\right)^n F(z)$

6) Convolution

$h(k) = g(k) * f(k)$

$h(k) = \sum_{n=-\infty}^{\infty} f(n) g(k-n) = \sum_{n=-\infty}^{\infty} g(n) f(k-n)$

Then

$Z[h(k)] = Z[g(k)] \cdot Z[f(k)]$

7) Inverse Z - Transform

1) $Z^{-1}\{F(z)\} = f(k)$

2) Methods

2) Direct Division method

$$F(z) = \frac{P(z)}{Q(z)}$$

3) Binomial Expansion Method

4) Partial Fraction Method