Assignment 4

Raghav Juyal - EP20BTECH11018

Download all python codes from

https://github.com/RaghavJuyal/AI1103/blob/main/Assignment4/Codes/Assignment4.py

and latex-tikz codes from

https://github.com/RaghavJuyal/AI1103/tree/main/Assignment4/Assignment4.tex

Question 15, GATE CS 2018

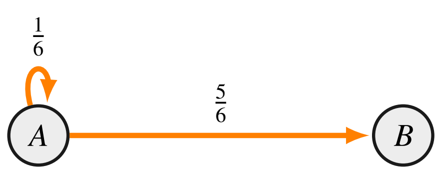
Two people, P and Q decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability that one of them wins on the third trial is?

Solution

Let p represent probability of a tie and q represent probability that one of them wins.

$$p = 1 \times \frac{1}{6} = \frac{1}{6} \tag{0.0.1}$$

$$q = 1 - p = \frac{5}{6} \tag{0.0.2}$$



Markov Diagram where A represents a tie and B represents that one of P and Q wins

The state transition matrix P is shown below :

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} \frac{1}{6} & \frac{5}{6} \\ 0 & 1 \end{bmatrix} \end{array}$$

Here the initial state, representing the first roll, S_0 is:

$$S_0 = \begin{bmatrix} \frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

For the n^{th} state we have

$$S_n = S_0 \times P^n \tag{0.0.3}$$

To get a win on the third trial we need the $(1,2)^{th}$ position of S_2.

$$\begin{aligned} S_2 &= S_0 \times P^2 \\ &= \begin{bmatrix} \frac{1}{216} & \frac{215}{216} \end{bmatrix} \end{aligned} \tag{0.0.4}$$

The $(1,2)^{th}$ position of S_2 includes the probability of getting a win on the third trial if the second trial is also a win. To find the required probability we have to subtract the $(1,2)^{th}$ position of S_1 from $(1,2)^{th}$ position of S_2.

$$\begin{aligned} S_1 &= S_0 \times P \\ &= \begin{bmatrix} \frac{1}{36} & \frac{35}{36} \end{bmatrix} \end{aligned} \tag{0.0.5}$$

$$\tag{0.0.6}$$

$\therefore$ The required probability is $\frac{5}{216}$