Assignment 2

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Download all python codes from

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https://github.com/RaghavJuyal/AI1103/blob/main/
    Assignment2/Codes/Assignment2.py
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and latex-tikz codes from

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https://github.com/RaghavJuyal/AI1103/tree/main/
    Assignment2/Assignment2.tex
```

Question 17, GATE CS 2020

Let $\mathcal{R}$ be the set of all binary relations on the set $\{1,2,3\}$. Suppose a relation is chosen from $\mathcal{R}$ at random. The probability that the chosen relation is reflexive is?

Solution

Let A be a set of n numbers. No. of pairs formed from elements of A:

$$^{n}C_1 \times {}^{n}C_1 = n^2 \tag{0.0.1}$$

For each pair we have 2 choices, whether to include it in the relation or not.
$\therefore$ Number of binary relations on A:

$$2 \times 2 \times ... \; n^2 \text{ times } = 2^{n^2} \tag{0.0.2}$$

Definition 1. *A reflexive relation is one in which every element maps to itself, i.e., a relation R on set A is reflexive if $(a,a) \in R \; \forall \; a \in A$.*

For example, consider the set $A = \{1,2,3\}$. A possible reflexive relation on A is $R_1 = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ as every element in A is related to itself in R_1 while in relation $R_2 = \{(1,1),(2,2),(1,2)\}$ is not a reflexive relation on A as $3 \in A$ but $(3,3) \notin R_2$.

In a reflexive relation, out of the n^2 pairs (0.0.1), n have to be included (n pairs of the form (a,a)) which means there is only 1 way to include them. For the remaining $n^2 - n$ pairs we have 2 choices, whether to include it in the relation or not.
$\therefore$ Number of reflexive relations are:

$$1 \times 2^{n^2-n} = 2^{n^2-n} \tag{0.0.3}$$

Let $X \in \{0,1\}$ be a random variable where 0 represents reflexive relation chosen from $\mathcal{R}$ and 1 represents non-reflexive relation chosen from $\mathcal{R}$. In this case, n=3.

$$\Pr(X=0) = \frac{2^{n^2-n}}{2^{n^2}} = \frac{2^6}{2^9} \tag{0.0.4}$$

$$\therefore \; \text{Answer } = \frac{1}{8} \tag{0.0.5}$$