

Assignment 3

Raghav Juyal - EP20BTECH11018

Download all python codes from

<https://github.com/RaghavJuyal/AI1103/blob/main/Assignment3/Codes/Assignment3.py>

and latex-tikz codes from

<https://github.com/RaghavJuyal/AI1103/tree/main/Assignment3/Assignment3.tex>

	0	1
Pr(X)	$\frac{3}{4}$	$\frac{1}{4}$
Pr(Y)	$\frac{1}{2}$	$\frac{1}{2}$

TABLE 4: Probability of $X \in \{0, 1\}$ and $Y \in \{0, 1\}$

$$F_X(x) = \begin{cases} 1, & x \geq 1 \\ \frac{3}{4}, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases}$$

QUESTION 80, GATE MA 2003

E_1, E_2 are independent events such that,

$$\Pr(E_1) = \frac{1}{4}, \Pr(E_2|E_1) = \frac{1}{2} \text{ and } \Pr(E_1|E_2) = \frac{1}{4}$$

Define random variables X and Y by

$$X = \begin{cases} 1, & \text{if } E_1 \text{ occurs} \\ 0, & \text{if } E_1 \text{ does not occur} \end{cases}$$

$$Y = \begin{cases} 1, & \text{if } E_2 \text{ occurs} \\ 0, & \text{if } E_2 \text{ does not occur} \end{cases}$$

Consider the following statements

α : X is uniformly distributed on the set $\{0, 1\}$

β : X and Y are identically distributed

γ : $\Pr(X^2 + Y^2 = 1) = \frac{1}{2}$

δ : $\Pr(XY = X^2Y^2) = 1$

Choose the correct combination

(a) (α, β) (c) (β, γ)

(b) (α, γ) (d) (γ, δ)

SOLUTION

Since events E_1 and E_2 are independent,

$$\Pr(E_1 E_2) = \Pr(E_1) \times \Pr(E_2)$$

$$\Pr(E_2|E_1) = \frac{\Pr(E_1 E_2)}{\Pr(E_1)} = \Pr(E_2)$$

$$\therefore \Pr(E_2) = \frac{1}{2} \quad (0.0.1)$$

From the given information we get,

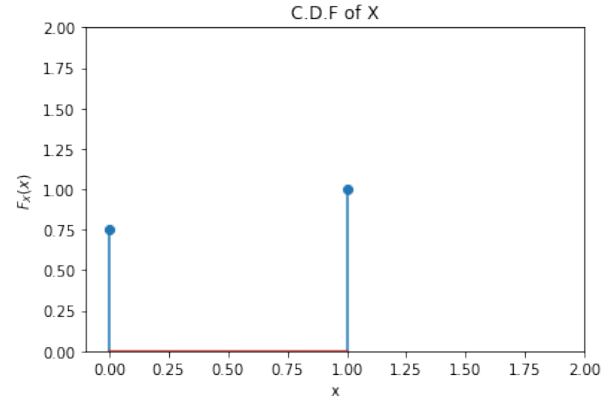


Fig. 4: CDF of X

$$F_Y(y) = \begin{cases} 1, & y \geq 1 \\ \frac{1}{2}, & 0 \leq y \leq 1 \\ 0, & y < 0 \end{cases}$$

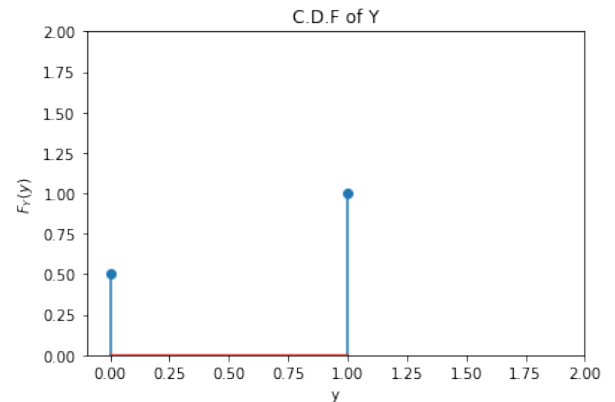


Fig. 4: CDF of Y

- We can see that both X and Y are Bernoulli distributed.
 \therefore Statement α is incorrect.

- Since $F_X(x) \neq F_Y(y)$, X and Y are not identically distributed.
 \therefore Statement β is incorrect.

- $\Pr(X^2 + Y^2 = 1)$
 $= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0)$
 $= \frac{1}{2} \quad (0.0.2)$

\therefore Statement γ is correct.

- $\Pr(XY = X^2Y^2)$
 $= \sum_{i=0}^1 \sum_{j=0}^1 \Pr(X = i, Y = j)$
 $= 1 \quad (0.0.3)$

\therefore Statement δ is correct.

\therefore Option (d) is correct.