

# Assignment 2

Raghav Juyal - EP20BTECH11018

Download all python codes from

<https://github.com/RaghavJuyal/AI1103/blob/main/Assignment2/Codes/Assignment2.py>

and latex-tikz codes from

<https://github.com/RaghavJuyal/AI1103/tree/main/Assignment2/Assignment2.tex>

Let  $X \in \{0, 1\}$  be a random variable where 0 represents reflexive relation chosen from  $\mathcal{R}$  and 1 represents non-reflexive relation chosen from  $\mathcal{R}$ . In this case,  $n=3$ .

$$\begin{aligned} \Pr(X = 0) &= \frac{2^{n^2-n}}{2^{n^2}} \\ &= \frac{2^6}{2^9} \end{aligned} \quad (0.0.4)$$

$$\therefore \text{Answer} = \frac{1}{8} \quad (0.0.5)$$

## QUESTION 17, GATE CS 2020

Let  $\mathcal{R}$  be the set of all binary relations on the set  $\{1, 2, 3\}$ . Suppose a relation is chosen from  $\mathcal{R}$  at random. The probability that the chosen relation is reflexive is?

### SOLUTION

Let  $A$  be a set of  $n$  numbers. No. of pairs formed from elements of  $A$ :

$${}^nC_1 \times {}^nC_1 = n^2 \quad (0.0.1)$$

For each pair we have 2 choices, whether to include it in the relation or not.

$\therefore$  Number of binary relations on  $A$ :

$$2 \times 2 \times \dots n^2 \text{ times} = 2^{n^2} \quad (0.0.2)$$

**Definition 1.** A reflexive relation is one in which every element maps to itself, i.e., a relation  $R$  on set  $A$  is reflexive if  $(a, a) \in R \forall a \in A$ .

In a reflexive relation, out of the  $n^2$  pairs (0.0.1),  $n$  have to be included ( $n$  pairs of the form  $(a, a)$ ) which means there is only 1 way to include them. For the remaining  $n^2 - n$  pairs we have 2 choices, whether to include it in the relation or not.

$\therefore$  Number of reflexive relations are:

$$1 \times 2^{n^2-n} = 2^{n^2-n} \quad (0.0.3)$$