Assignment 2

Raghav Juyal - EP20BTECH11018

Download all python codes from

```
https://github.com/RaghavJuyal/AI1103/blob/main/
    Assignment2/Codes/Assignment2.py
```

and latex-tikz codes from

```
https://github.com/RaghavJuyal/AI1103/tree/main/
    Assignment2/Assignment2.tex
```

Question 17, GATE CS 2020

Let $\mathcal{R}$ be the set of all binary relations on the set $\{1, 2, 3\}$. Suppose a relation is chosen from $\mathcal{R}$ at random. The probability that the chosen relation is reflexive is?

Solution

Let A be a set of n numbers. No. of pairs formed from elements of A:

$$ {}^{n}C_1 \times {}^{n}C_1 = n^2 \tag{0.0.1} $$

For each pair we have 2 choices, whether to include it in the relation or not.
$\therefore$ Number of binary relations on A:

$$ 2 \times 2 \times ... \ n^2 \text{ times } = 2^{n^2} \tag{0.0.2} $$

In a reflexive relation, out of the n^2 pairs (0.0.1) n have to be included which means there is only 1 way to include them. For the remaining $n^2 - n$ pairs we have 2 choices, whether to include it in the relation or not.
$\therefore$ Number of reflexive relations are:

$$ 1 \times 2^{n^2-n} = 2^{n^2-n} \tag{0.0.3} $$

Let $X \in \{0, 1\}$ be a random variable where 0 represents reflexive relation chosen from $\mathcal{R}$ and 1 represents non-reflexive relation chosen from $\mathcal{R}$. In this case, n=3.

$$ \begin{aligned} \Pr(X = 0) &= \frac{2^{n^2-n}}{2^{n^2}} \\ &= \frac{2^6}{2^9} \qquad (0.0.4) \\ \therefore \text{ Answer } &= \frac{1}{8} \qquad (0.0.5) \end{aligned} $$