

# Assignment 3

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Download latex-tikz codes from

<https://github.com/RaghavJuyal/AI1103/tree/main/Assignment3/Assignment3.tex>

QUESTION 80, GATE MA 2003

$E_1, E_2$  are independent events such that,

$$\Pr(E_1) = \frac{1}{4}, \Pr(E_2|E_1) = \frac{1}{2} \text{ and } \Pr(E_1|E_2) = \frac{1}{4}$$

Define random variables  $X$  and  $Y$  by

$$X = \begin{cases} 1, & \text{if } E_1 \text{ occurs} \\ 0, & \text{if } E_1 \text{ does not occur} \end{cases}$$

$$Y = \begin{cases} 1, & \text{if } E_2 \text{ occurs} \\ 0, & \text{if } E_2 \text{ does not occur} \end{cases}$$

Consider the following statements

$\alpha$  :  $X$  is uniformly distributed on the set  $\{0, 1\}$

$\beta$  :  $X$  and  $Y$  are identically distributed

$\gamma$  :  $\Pr(X^2 + Y^2 = 1) = \frac{1}{2}$

$\delta$  :  $\Pr(XY = X^2Y^2) = 1$

Choose the correct combination

(a)  $(\alpha, \beta)$  (c)  $(\beta, \gamma)$

(b)  $(\alpha, \gamma)$  (d)  $(\gamma, \delta)$

SOLUTION

Since events  $E_1$  and  $E_2$  are independent,

$$\Pr(E_1E_2) = \Pr(E_1) \times \Pr(E_2)$$

$$\Pr(E_2|E_1) = \frac{\Pr(E_1E_2)}{\Pr(E_1)} = \Pr(E_2)$$

$$\therefore \Pr(E_2) = \frac{1}{2} \quad (0.0.1)$$

From the given information we get,

	0	1
$\Pr(X)$	$\frac{3}{4}$	$\frac{1}{4}$
$\Pr(Y)$	$\frac{1}{2}$	$\frac{1}{2}$

TABLE 4: Probability of  $X \in \{0, 1\}$  and  $Y \in \{0, 1\}$

$$F_X(x) = \begin{cases} 1, & x \geq 1 \\ \frac{3}{4}, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases}$$

$$F_Y(y) = \begin{cases} 1, & y \geq 1 \\ \frac{1}{2}, & 0 \leq y \leq 1 \\ 0, & y < 0 \end{cases}$$

(1)  $X$  is not uniformly distributed on the set  $\{0, 1\}$  as it is not continuous in  $\{0, 1\}$  (both  $X$  and  $Y$  are Bernoulli Distributions).  
 $\therefore$  Statement  $\alpha$  is incorrect.

(2) Since  $F_X(x) \neq F_Y(y)$ ,  $X$  and  $Y$  are not identically distributed.  
 $\therefore$  Statement  $\beta$  is incorrect.

$$(3) \Pr(X^2 + Y^2 = 1)$$

$$= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0)$$

$$= \frac{1}{2} \quad (0.0.2)$$

$\therefore$  Statement  $\gamma$  is correct.

$$(4) \Pr(XY = X^2Y^2)$$

$$= \sum_{i=0}^1 \sum_{j=0}^1 \Pr(X = i, Y = j)$$

$$= 1 \quad (0.0.3)$$

$\therefore$  Statement  $\delta$  is correct.

(a) This option is incorrect as statement  $\alpha$  is incorrect (1) and statement  $\beta$  is incorrect (2).

- (b) This option is incorrect as statement  $\gamma$  is correct (3) but statement  $\alpha$  is incorrect (1).
- (c) This option is incorrect as statement  $\gamma$  is correct (3) but statement  $\beta$  is incorrect (2).
- (d) This option is correct as statement  $\gamma$  is correct (3) and statement  $\delta$  is correct (4).

$\therefore$  Option (d),  $(\gamma, \delta)$ , is the answer.