Assignment 3

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Download latex-tikz codes from

https://github.com/RaghavJuyal/AI1103/tree/main/Assignment3/Assignment3.tex

Question 80, GATE MA 2003

E_1, E_2 are independent events such that,

$$\Pr(E_1) = \frac{1}{4}, \Pr(E_2|E_1) = \frac{1}{2} \text{ and } \Pr(E_1|E_2) = \frac{1}{4}$$

Define random variables X and Y by

$$X = \begin{cases} 1, & \text{if } E_1 \text{ occurs} \\ 0, & \text{if } E_1 \text{ does not occur} \end{cases}$$

$$Y = \begin{cases} 1, & \text{if } E_2 \text{ occurs} \\ 0, & \text{if } E_2 \text{ does not occur} \end{cases}$$

Consider the following statements

α : X is uniformly distributed on the set $\{0, 1\}$
β : X and Y are identically distributed
γ : $\Pr\left(X^2 + Y^2 = 1\right) = \frac{1}{2}$
δ : $\Pr\left(XY = X^2Y^2\right) = 1$

Choose the correct combination

(a) (α, β)

(b) (α, γ)

(c) (β, γ)

(d) (γ, δ)

Solution

Since events E_1 and E_2 are independent,

$$\Pr(E_1E_2) = \Pr(E_1) \times \Pr(E_2)$$
$$\Pr(E_2|E_1) = \frac{\Pr(E_1E_2)}{\Pr(E_1)} = \Pr(E_2)$$
$$\therefore \Pr(E_2) = \frac{1}{2} \tag{0.0.1}$$

From the given information we get,

$$F_X(x) = \begin{cases} 1, & x \geq 1 \\ \frac{3}{4}, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases}$$

	0	1
$\Pr(X)$	$\frac{3}{4}$	$\frac{1}{4}$
$\Pr(Y)$	$\frac{1}{2}$	$\frac{1}{2}$

TABLE 4: Probability of $X \in \{0, 1\}$ and $Y \in \{0, 1\}$

$$F_Y(y) = \begin{cases} 1, & y \geq 1 \\ \frac{1}{2}, & 0 \leq y \leq 1 \\ 0, & y < 0 \end{cases}$$

(1) X is not uniformly distributed on the set $\{0, 1\}$ as it is not continuous in $\{0, 1\}$ (both X and Y are Bernoulli Distributions).
$\therefore$ Statement α is incorrect.

(2) Since $F_X(x) \neq F_Y(y)$, X and Y are not identically distributed.
$\therefore$ Statement β is incorrect.

(3) $\Pr\left(X^2 + Y^2 = 1\right)$

$$= \Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0)$$
$$= \frac{1}{2} \tag{0.0.2}$$

$\therefore$ Statement γ is correct.

(4) $\Pr\left(XY = X^2Y^2\right)$

$$= \sum_{i=0}^{1} \sum_{j=0}^{1} \Pr(X = i, Y = j)$$
$$= 1 \tag{0.0.3}$$

$\therefore$ Statement δ is correct.

(a) This option is incorrect as statement α is incorrect (1) and statement β is incorrect (2).

(b) This option is incorrect as statement γ is correct (3) but statement α is incorrect (1).

(c) This option is incorrect as statement γ is correct (3) but statement β is incorrect (2).

(d) This option is correct as statement γ is correct (3) and statement δ is correct (4).

$\therefore$ Option (d), (γ, δ), is the answer.