Assignment 3

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Download latex-tikz codes from

https://github.com/RaghavJuyal/AI1103/tree/main/Assignment3/Assignment3.tex

Question 80, GATE MA 2003

E_1, E_2 are independent events such that,

$$\Pr(E_1) = \frac{1}{4},\ \Pr(E_2|E_1) = \frac{1}{2} \text{ and } \Pr(E_1|E_2) = \frac{1}{4}$$

Define random variables X and Y by

$$X = \begin{cases} 1, & \text{if } E_1 \text{ occurs} \\ 0, & \text{if } E_1 \text{ does not occur} \end{cases}$$

$$Y = \begin{cases} 1, & \text{if } E_2 \text{ occurs} \\ 0, & \text{if } E_2 \text{ does not occur} \end{cases}$$

Consider the following statements

α : X is uniformly distributed on the set $\{0, 1\}$
β : X and Y are identically distributed
γ : $\Pr\left(X^2 + Y^2 = 1\right) = \frac{1}{2}$
δ : $\Pr\left(XY = X^2Y^2\right) = 1$

Choose the correct combination

(a) (α, β)
(b) (α, γ)
(c) (β, γ)
(d) (γ, δ)

Solution

Since events E_1 and E_2 are independent,

$$\Pr(E_1E_2) = \Pr(E_1) \times \Pr(E_2)$$

$$\Pr(E_2|E_1) = \frac{\Pr(E_1E_2)}{\Pr(E_1)} = \Pr(E_2)$$

$$\therefore \Pr(E_2) = \frac{1}{2} \tag{0.0.1}$$

From the given information we get,

$$F_X(x) = \begin{cases} 1, & x \geq 1 \\ \frac{3}{4}, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases}$$

$$F_Y(y) = \begin{cases} 1, & y \geq 1 \\ \frac{1}{2}, & 0 \leq y \leq 1 \\ 0, & y < 0 \end{cases}$$

- We can see that both X and Y are Bernoulli distributed.
  $\therefore$ Statement α is incorrect.

- Since $F_X(x) \neq F_Y(y)$, X and Y are not identically distributed.
  $\therefore$ Statement β is incorrect.

- $\Pr\left(X^2 + Y^2 = 1\right)$

  $$= \Pr((X = 0)(Y = 1)) + \Pr((X = 1)(Y = 0))$$
  $$= \frac{1}{2} \tag{0.0.2}$$

  $\therefore$ Statement γ is correct.

- $\Pr\left(XY = X^2Y^2\right)$

  $$= \sum_{X=0}^{1} \sum_{Y=0}^{1} \Pr(XY)$$
  $$= 1 \tag{0.0.3}$$

  $\therefore$ Statement δ is correct.

$\therefore$ Option (d) is correct.