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Assignment 4

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Download all python codes from

https://github.com/RaghavJuyal/AI1103/blob/main/ Assignment4/Codes/Assignment4.py

and latex-tikz codes from

https://github.com/RaghavJuyal/AI1103/tree/main/ Assignment4/Assignment4.tex

QUESTION 15, GATE CS 2018

Two people, P and Q decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability that one of them wins on the third trial is?

Solution

Let p represent probability of a tie and q represent probability that one of them wins.

$$p = 1 \times \frac{1}{6} = \frac{1}{6} \tag{0.0.1}$$

$$q = 1 - p = \frac{5}{6} \tag{0.0.2}$$



Markov Diagram where A represents a tie and B represents that one of P and Q wins

The state transition matrix P is shown below:

$$\begin{array}{ccc}
A & B \\
A & \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{5}{6} & 0 \end{bmatrix}
\end{array}$$

Here the initial state, representing the first roll, S_0 is:

$$S_0 = \left[\begin{array}{c} \frac{1}{6} \\ \frac{5}{6} \end{array} \right]$$

For the n^{th} state we have

$$S_n = P^n \times S_0 \tag{0.0.3}$$

To get a win on the third trial we need the $(2,1)^{th}$ position of S_2 .

$$S_3 = P^2 \times S_0$$

$$= \begin{bmatrix} \frac{1}{216} \\ \frac{5}{216} \end{bmatrix} \tag{0.0.4}$$

 \therefore The required probability is $\frac{5}{216}$