Assignment 6

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EP20BTECH11018

Imports

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy import stats, ndimage, optimize
import emcee
```

Question 1

In 1919, two expeditions sailed from Britain to test if the light deflection from stars agrees
with Einstein's General Theory of Relativity. Einstein's theory predicts a value of 1.74 arc-seconds,
whereas Newtonian gravity predicts a value exactly half of that. The team by Eddington obtained a
value of 1.61 ± 0.40 arc-seconds, while the team by Crommelin reported a value of 1.98 ± 0.16 arc-seconds.
Calculate the Bayes factor between General Relativity and Newtonian gravity from those data, assuming Gaussian likelihoods.

In [ ]:
```
# data from the given question
time_einstein = 1.74
time_newton = 0.87
time_eddington = 1.61
error_eddington = 0.40
time_crommelin = 1.98
error_crommelin = 0.16

# pdf of Einstein-Eddington distribution
einstein_eddington_pdf = stats.norm(time_einstein,error_eddington).pdf(time_eddi
# pdf of Einstein-Cromelin distribution
einstein_crom_pdf = stats.norm(time_einstein,error_crommelin).pdf(time_crommelir
# pdf of Newton-Eddington distribution
newton_eddington_pdf = stats.norm(time_newton,error_eddington).pdf(time_eddingtc
# pdf of Newton-Cromelin distribution
newton_crom_pdf = stats.norm(time_newton,error_crommelin).pdf(time_crommelin)

bayes_factor = (newton_crom_pdf*newton_eddington_pdf)/(einstein_crom_pdf*einstei

print(f"Bayes factor between General Relativity and Newtonian gravity from the g
```

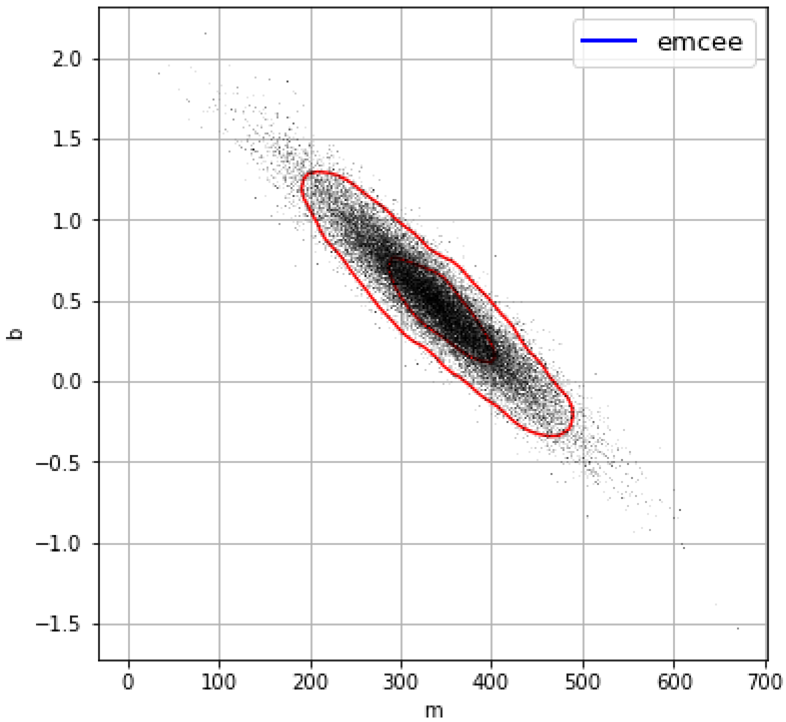
```
Bayes factor between General Relativity and Newtonian gravity from the given da
ta (assuming Gaussian Likelihoods) = 2.0762126610332088e-11
```

Question 2

For exercise 1 in arXiv:1008.4686, calculate the 68% and 95% joint confidence intervals on b and m.

In [ ]:
```
# utility function for log prior
def log_prior(theta):
    alpha, beta, sigma = theta
    if sigma < 0:
        return -np.inf
    else:
        return -1.5 * np.log(1 + beta ** 2) - np.log(sigma)

# utility function for log likelihood
def log_llihood(theta, x, y):
    a, b, s = theta
    y_model = a + b * x
    return -0.5 * np.sum(np.log(2 * np.pi * s ** 2) + (y - y_model) ** 2 / s **

# function for log posterior
def log_posterior(theta, x, y):
    return log_prior(theta) + log_llihood(theta, x, y)

# function for calculating sigma level
def sigma_level(t1, t2, nbins=20, smoothing=3):
    L, xbins, ybins = np.histogram2d(t1, t2, nbins)
    L[L == 0] = 1E-16
    logL = np.log(L)

    shape = L.shape
    L = L.ravel()

    i_sort = np.argsort(L)[::-1]
    i_unsort = np.argsort(i_sort)
    # cumulative sum
    L_cumsum = L[i_sort].cumsum()
    L_cumsum /= L_cumsum[-1]

    sigma = L_cumsum[i_unsort].reshape(shape)

    if smoothing > 1:
        sigma = ndimage.zoom(sigma, smoothing)
        xbins = np.linspace(xbins[0], xbins[-1], sigma.shape[0] + 1)
        ybins = np.linspace(ybins[0], ybins[-1], sigma.shape[1] + 1)

    xbins = 0.5 * (xbins[1:] + xbins[:-1])
    ybins = 0.5 * (ybins[1:] + ybins[:-1])
    return xbins, ybins, sigma


# utility function for plotting MCMC
def plot_MCMC(ax, xdata, ydata, t, scatter=False, nbins=20, smoothing=3, **kwarg
    xbins, ybins, sigma = sigma_level(t[0], t[1], nbins, smoothing)
    ax.contour(xbins, ybins, sigma.T, levels=[0.68 ** 2, 0.95 ** 2], **kwargs)
```

```
    if scatter:
        ax.plot(t[0], t[1], ',k', alpha=0.1)
    ax.set_xlabel('m')
    ax.set_ylabel('b')


# given data
x = np.array([201, 244, 47, 287, 203, 58, 210, 202, 198, 158, 165, 201, 157, 131
y = np.array([592,401, 583, 402, 495,173, 479, 504, 510, 416, 393, 442, 317, 311
sigmay = np.array([61,25,38, 15 , 21 , 15, 27 , 14 , 30 , 16 , 14 , 25 , 52 , 16

# number of dimensions
ndim = 3
# MCMC walkers
nwalkers = 50
# burn period
nburn = 1000
# number of MCMC steps
nsteps = 2000

# defining intial guess
guesses = np.random.random((nwalkers, ndim))

# sampler value
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_posterior,args=[x, y])
sampler.run_mcmc(guesses, nsteps)
emcee_ = sampler.chain[:, nburn:, :].reshape(-1, ndim).T

# plotting the required things
fig, ax = plt.subplots(figsize=(6,6))
plot_MCMC(ax, x, y, emcee_, True, colors='red', linewidths=1.5)
ax.plot([0, 0], [0, 0], 'blue', lw=2)
ax.legend(ax.lines[-1:] + ax.collections[::2], ['emcee'], fontsize=13)
plt.grid()
plt.show()
```



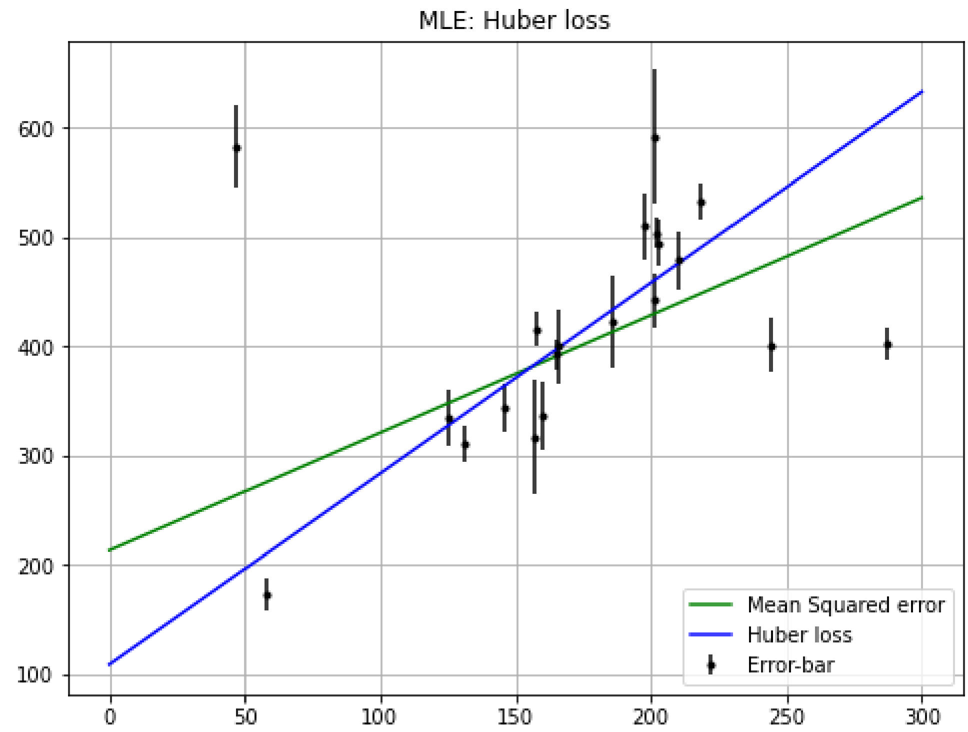
Question 3

Fit the data in Table 1 of arXiv:1008.4686 to a straight line, after including all the data points, (after ignoring σx and ρxy) using both maximum likelihood analysis and using
a Bayesian analysis to identify the outliers, using the same procedure as in the second of Jake VanDerPlas blog article. Show graphically the best fit line using both maximum likelihood analysis and also using Bayesian analysis, including the outlier points.

In [ ]:
```
# mean squared error loss
def mse(theta, x=x, y=y, sigmay=sigmay):
    dy = y - theta[0] - theta[1] * x
    return np.sum(0.5 * (dy / sigmay) ** 2)

# utility function for finding Huber Loss
def huber_loss(t, c=3):
    return ((abs(t) < c) * 0.5 * t ** 2 + (abs(t) >= c) * -c * (0.5 * c - abs(t)

# computes huber loss
def total_huber_loss(theta, x=x, y=y, sigmay=sigmay, c=3):
    return huber_loss((y - theta[0] - theta[1] * x) / sigmay, c).sum()

def log_prior(theta):
    if 1 > all(theta[2:]) > 0:
        return 0
    else:
        return -np.inf

# function for log likelihood
def log_llhood(theta, x, y, sigmay, sigmaB):
    dy = y - theta[0] - x*theta[1]
    temp = np.clip(theta[2:], 0, 1)
    lL1 = np.log(temp) - 0.5 * np.log(2 * np.pi * sigmay ** 2)
    lL1=lL1- 0.5 * (dy / sigmay) ** 2
    lL2 = np.log(1 - temp) - 0.5 * np.log(2 * np.pi * sigmaB ** 2)
    lL2+=- 0.5 * ((dy / sigmaB) ** 2)
    su =  np.sum(np.logaddexp(lL1, lL2))
    return su

def log_posterior(theta, x, y, sigmay, sigmaB):
    return log_prior(theta) + log_llhood(theta, x, y, sigmay, sigmaB)
```

In [ ]:
```
# optimized parameters from mean squared loss
theta1 = optimize.fmin(mse, [0, 0], disp=False)
# optimized parameters from huber loss
theta2 = optimize.fmin(total_huber_loss, [0, 0], disp=False)

# data
x = np.array([201,244,47,287,203,58,210,202,198,158,165,201,157,131,166,160,186,
y = np.array([592,401,583,402,495,173,479,504,510,416,393,442,317,311,400,337,42
sigmay= np.array([61,25,38,15,21,15,27,14,30,16,14,25,52,16,34,31,42,26,16,22])

# defining points on x-axis
x_1 = np.linspace(0, 300, 1000)
```

```
# plotting the required things
plt.figure(figsize=(8,6))
plt.errorbar(x, y, sigmay, fmt='.k', label='Error-bar')
plt.plot(x_1, theta1[0] + x_1*theta1[1] , color='g', label='Mean Squared error')
plt.plot(x_1, theta2[0] + x_1*theta2[1], color='b', label='Huber loss')
plt.title('MLE: Huber loss')
plt.legend()
plt.grid()
plt.show()
```



In [ ]:

```
# dimensions in the model
ndim = 2 + len(x)
# number of MCMC walkers
nwalkers = 50
# burn rate
nburn = 10000
# number of steps
nsteps = 15000

# for initial guess
starting_guesses = np.zeros((nwalkers, ndim))
starting_guesses[:, :2] = np.random.normal(theta2, 1, (nwalkers, 2))
starting_guesses[:, 2:] = np.random.normal(0.5, 0.1, (nwalkers, ndim - 2))

# computes sampler value
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_posterior, args=[x, y, sigma
sampler.run_mcmc(starting_guesses, nsteps)

sample = sampler.chain
sample = sampler.chain[:, nburn:, :].reshape(-1, ndim)

# bayesian marginalisation
theta3 = np.mean(sample[:, :2], 0)
g = np.mean(sample[:, 2:], 0)
# outliers
outliers = (g < 0.38)
```

```
# plotting the required things
plt.figure(figsize=(8,6))
plt.errorbar(x, y, sigmay, fmt='.k')
plt.plot(x_1, theta1[0] + x_1*theta1[1], color='g', label='Mean Squared error')
plt.plot(x_1, theta2[0] + x_1*theta2[1] ,color='b', label='Huber loss')
plt.plot(x_1, theta3[0] +  x_1* theta3[1] , color='black', label='bayesian margi
plt.grid()
plt.legend()
plt.title('MLE: Bayesian Marginalization');
plt.show()
```

```
C:\Users\Admin\AppData\Local\Temp\ipykernel_13080\1873603062.py:24: RuntimeWarn
ing: divide by zero encountered in log
  lL1 = np.log(temp) - 0.5 * np.log(2 * np.pi * sigmay ** 2)
c:\Users\Admin\anaconda3\lib\site-packages\emcee\moves\red_blue.py:99: RuntimeW
arning: invalid value encountered in double_scalars
  lnpdiff = f + nlp - state.log_prob[j]
C:\Users\Admin\AppData\Local\Temp\ipykernel_13080\1873603062.py:26: RuntimeWarn
ing: divide by zero encountered in log
  lL2 = np.log(1 - temp) - 0.5 * np.log(2 * np.pi * sigmaB ** 2)
```