Assignment 4

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EP20BTECH11018

Imports

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as st
from scipy.optimize import curve_fit
from scipy import optimize as opt
from statsmodels.tools import eval_measures as em
```

Question 1

Download the data corresponding to x, y, and σy from
http://www.iith.ac.in/~shantanud/testdata.dat.
Find the best-fit values after fitting the data to linear, quadratic, and cubic polynomials. Find out
which model fits the data best from frequentist model comparison as well as using AIC and BIC. For
frequentist model comparison, using the linear model as the null hypothesis, find out the p value
corresponding to the preferred model. (or if the linear model is the prefered model, then compare it to
the quadratic model). Also show a plot overlaying the data with best fit solutions from linear, quadratic
and cubic functions with different line styles for each of the fits.

In [ ]:
```
testdata = np.loadtxt('testdata.dat')

def linear(x, a, b):
    return a*x + b

def quadratic(x, a, b, c):
    return a*x**2 + b*x + c

def cubic(x, a, b, c, d):
    return a*x**3 + b*x**2 + c*x + d


linearfit_param = curve_fit(linear, xdata = testdata[:,0], ydata = testdata[:,1]
quadraticfit_param = curve_fit(quadratic, testdata[:,0], testdata[:,1], sigma=te
cubicfit_param = curve_fit(cubic, testdata[:,0], testdata[:,1], sigma=testdata[:

print('Linear fit values: ', linearfit_param[0])
```

```
print('Quadratic fit values: ', quadraticfit_param[0])
print('Cubic fit values: ', cubicfit_param[0])
```

```
Linear fit values:  [ 2.79789861 -1.11028082]
Quadratic fit values:  [ 0.50261293  2.38475187 -1.05578915]
Cubic fit values:  [-0.96724992  1.74451332  1.97184055 -1.02910462]
```

Frequentist Comparision

In [ ]:

```
chi2_linear = np.sum(((testdata[:,1] - linear(testdata[:,0], *linearfit_param[0]
chi2_quadratic = np.sum(((testdata[:,1] - quadratic(testdata[:,0], *quadraticfit
chi2_cubic = np.sum(((testdata[:,1] - cubic(testdata[:,0], *cubicfit_param[0]))/


def chi2_gof(x, y, yerr, fit_func, *fit_func_args):
    return np.sum(((y - fit_func(x, *fit_func_args))/yerr)**2)


#Linear fit is considered NULL hypothesis
delta_chi2_quadratic = chi2_linear - chi2_quadratic
delta_chi2_cubic = chi2_linear - chi2_cubic

p_quadratic = 1 - st.chi2.cdf(delta_chi2_quadratic, 1)
p_cubic = 1 - st.chi2.cdf(delta_chi2_cubic, 2)

print('p-value for quadratic fit: ', p_quadratic)
print('p-value for cubic fit: ', p_cubic)
```

```
p-value for quadratic fit:  0.17813275695316744
p-value for cubic fit:  0.32887884419522884
```

Since the p-value of quadratic fit is less than that of cubic fit, cubic fit may be a better fit.

Maximum Likelihood Estimation

In [ ]:

```
def likelihood_estimator(func, data, *args):
    return np.product(np.exp(-0.5 * (((data[:,1] - func(data[:,0], *args))/data[

def max_likelihood(func, data, args):
    return opt.minimize(lambda x: -likelihood_estimator(func, data, *x), args, m


maxlkl = [likelihood_estimator(linear,testdata, *max_likelihood(linear, testdata
likelihood_estimator(quadratic, testdata, *max_likelihood(quadratic, testdata, q
likelihood_estimator(cubic, testdata, *max_likelihood(cubic, testdata, cubicfit_

#Calculate AIC and BIC
bic = -2 * np.log(maxlkl) + np.log(len(testdata)) * np.array([2, 3, 4])

aic = -2 * np.log(maxlkl) + 2 * np.array([2, 3, 4])

aic_c = aic + 2 * np.array([2, 3, 4]) * np.array([3, 4, 5]) / (len(testdata) - r

#Linear fit is considered as NULL hypothesis
delta_aic = aic - aic[0]

delta_bic = bic - bic[0]

print("For linear fit")
print('AIC: ', aic[0])
```

```
print('BIC: ', bic[0])
print()
print("For quadratic fit")
print('AIC: ', aic[1])
print('BIC: ', bic[1])
print()
print("For cubic fit")
print('AIC: ', aic[2])
print('BIC: ', bic[2])


print('\nLinear fit is considered as NULL hypothesis\n')
print("For quadratic fit")
print('Delta AIC: ', delta_aic[1])
print('Delta BIC: ', delta_bic[1])
print()
print("For cubic fit")
print('Delta AIC: ', delta_aic[2])
print('Delta BIC: ', delta_bic[2])
```

```
For linear fit
AIC:  15.309175575502223
BIC:  17.300640122610204

For quadratic fit
AIC:  15.496041767569295
BIC:  18.483238588231266

For cubic fit
AIC:  17.085043873972353
BIC:  21.067972968188315

Linear fit is considered as NULL hypothesis

For quadratic fit
Delta AIC:  0.18686619206707178
Delta BIC:  1.1825984656210622

For cubic fit
Delta AIC:  1.7758682984701295
Delta BIC:  3.7673328455781103
```

delta AIC is in range 0-2 for quadratic and cubic fit so supports both models

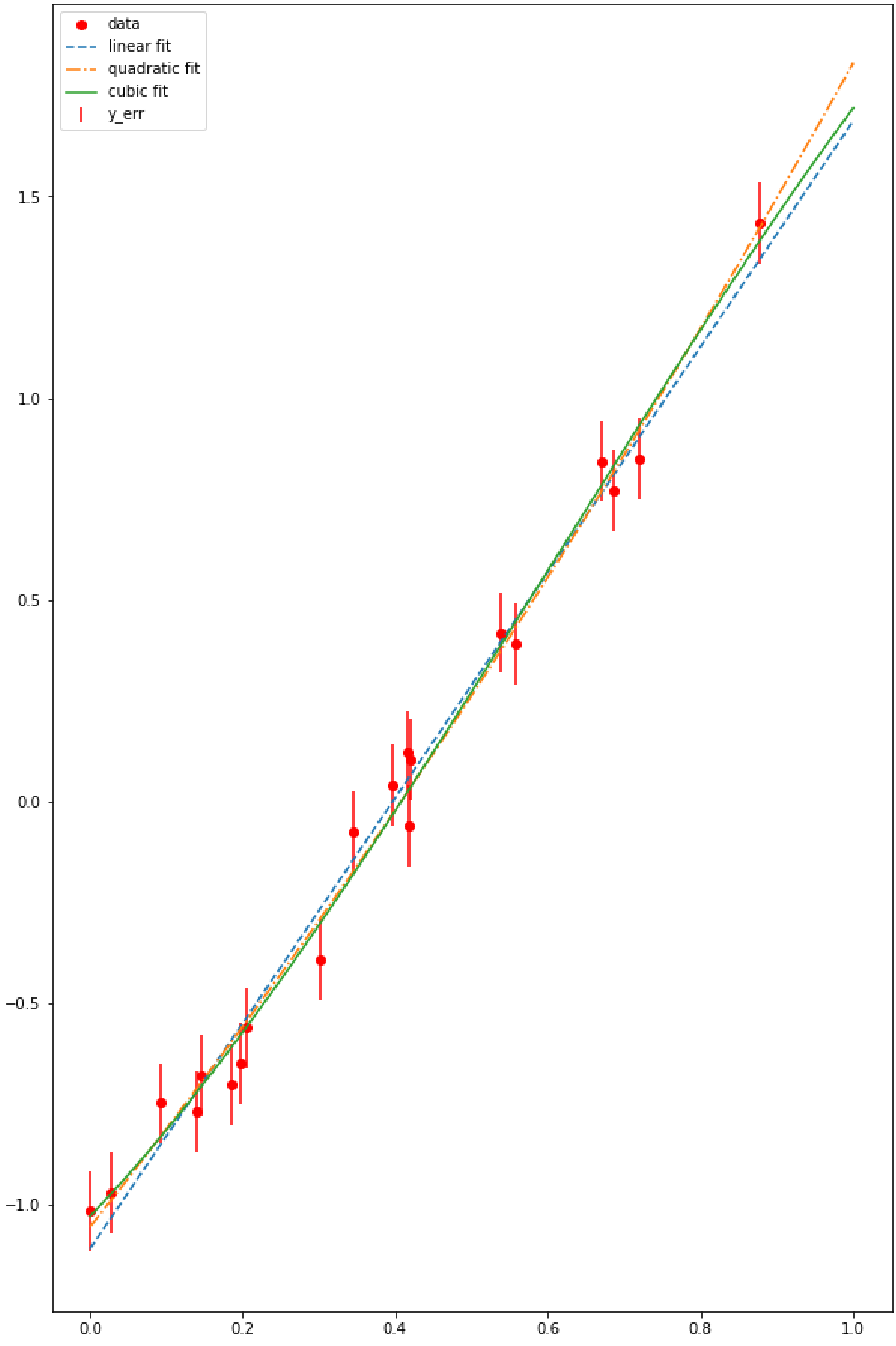
delta BIC is < 2 for quadratic and > 2 for cubic(positive evidence against cubic)

Therefore, quadratic model is preferred

In [ ]:

```
#Plotting the data and the fits

trange = np.linspace(0, 1, 1000)
plt.figure(figsize=(10,16))
plt.scatter(testdata[:,0], testdata[:,1], label='data', c='r')
plt.errorbar(testdata[:,0], testdata[:,1], yerr=testdata[:,2], fmt='none', label
plt.plot(trange, linear(trange, *linearfit_param[0]), label='linear fit', ls = '
plt.plot(trange, quadratic(trange, *quadraticfit_param[0]), label='quadratic fit
plt.plot(trange, cubic(trange, *cubicfit_param[0]), label='cubic fit')
plt.legend()
plt.show()
```



Question 2

For the model comparison problem shown in class on JVDP's blog, calculate AIC and BIC for the linear and quadratic models. Do these results agree with the frequentist model comparison results shown on the blog? Also mention the qualitative significance using strengt of evidence rules.

```
In [ ]: data = np.array([[ 0.42,  0.72,  1e-7  ,  0.3 ,  0.15,
                           0.09,  0.19,  0.35,  0.4 ,  0.54,
                           0.42,  0.69,  0.2 ,  0.88,  0.03,
                           0.67,  0.42,  0.56,  0.14,  0.2  ],
                         [ 0.33,  0.41, -0.22,  0.01, -0.05,
                          -0.05, -0.12,  0.26,  0.29,  0.39,
                           0.31,  0.42, -0.01,  0.58, -0.2 ,
                           0.52,  0.15,  0.32, -0.13, -0.09 ],
                         [ 0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                           0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                           0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                           0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1  ]])

        x, y, sigma_y = data
        linearfit_param = curve_fit(linear, xdata = x, ydata = y, sigma=sigma_y)
        quadraticfit_param = curve_fit(quadratic, xdata = x, ydata = y, sigma=sigma_y)


        maxlkl = [likelihood_estimator(linear, data.transpose(), *max_likelihood(linear,
        likelihood_estimator(quadratic, data.transpose(), *max_likelihood(quadratic, dat
```

```
In [ ]: bic = -2 * np.log(maxlkl) + np.log(len(x)) * np.array([2, 3])

        aic = -2 * np.log(maxlkl) + 2 * np.array([2, 3])

        aic_c = aic + 2 * np.array([2, 3]) * np.array([3, 4]) / (len(testdata) - np.arra

        #Linear fit is considered as NULL hypothesis
        delta_aic = aic - aic[0]

        delta_bic = bic - bic[0]
        print("For linear fit")
        print('AIC: ', aic[0])
        print('BIC: ', bic[0])
        print()
        print("For quadratic fit")
        print('AIC: ', aic[1])
        print('BIC: ', bic[1])
        print()
```

```
For linear fit
AIC:  15.324128704090546
BIC:  17.31559325119853

For quadratic fit
AIC:  15.46283414153923
BIC:  18.450030962201204
```

Taking the linear fit as null hypothesis we see that there is negligible difference in
the AIC of both models and the BIC of linear fit is slightly lesser that that of quadratic fit.
This implies there is only slight evidence against the quadratic model which is in
agreement with
the results shown in the blog.

Question 3

Find out one paper in research literature which uses the Kolmogorov-Smirnov test and explain briefly how it was used in that paper. Is K-S test used incorrectly (in this paper) as per the warnings on the Penn State website discussed in class?

PAPER: EWF : simulating exact paths of the Wright--Fisher diffusion(https://arxiv.org/abs/2301.05459)

Here the output was validated by generating 10,000 samples for a wide variety of cases and subsequently
comparing this to a truncation of the transition density by means of Kolmogorov–Smirnov test
as well as QQ-plots. They point out that they present only neutral output here as the non-neutral
output is generated using the same rejection procedure as used in Jenkins and Span`o.

Question 4

Calculate the significance in terms of no of sigmas of the Higgs boson discovery claim from the p value given in the abstract of the ATLAS discovery paper, arXiv:1207.7214.

Do the same for the LIGO discovery of GW150914, for which the pvalue = 2 × 10−7. (Hint : look up norm.isf)

From the Super-K discovery paper for neutrino oscillations (hep-ex/9807003), calculate the χ^2 GOF using the best-fit
$\nu_\mu \leftrightarrow \nu_\tau$ oscillation solution. (Hint : check page 4 of the paper, second column,last paragraph)

In [ ]:
```
print(f'Significance of Higgs Boson discovery claim with p value of 1.7e-9 is: {
print(f'Significance of LIGO discovery of GW150914 with p value of 2e-7 is := {s
print(f'chi2 GOF of Super-K discovery of neutrino oscillations = {st.chi2(67).sf
```

```
Significance of Higgs Boson discovery claim with p value of 1.7e-9 is: 5.911017
938341624
Significance of LIGO discovery of GW150914 with p value of 2e-7 is := 5.0689577
49717791
chi2 GOF of Super-K discovery of neutrino oscillations = 0.5394901931099036
```