UNIT - 4

TESTING OF HYPOTHESIS

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Introduction

- Principal Instrument of research
- Function is to suggest experiments and observation.
- Hypothesis testing is often used stratergy for deciding whether sample data offer such support for hypothesis that generalization can be made.

What is hypothesis testing?

- Mere assumption or some supposition to be proved or disproved.
- Defined as a
- “Proposition or a set of proposition set forth as an explaination for the occurance of some specified group of phenomena either asserted merely as a provisional conjecture to guide some investigation or accepted as highly probable in the light of established facts”.

Examples

- “Students who receive counselling will show a greater increase in creativity than students not receiving counselling”
- “The automobile A is performing as well as automobile B”.

Characterstics of Hypothesis

- Should be clear and precise.
- Capable of being tested.
- Should state relationship between variables.
- Hypo should be stated in simple terms and easily understandable.
- Hypo should be consistent with most known facts.
- Hypo should be amenable to testing within reasonable time.

Basic concepts : Null Hypothesis and Alternate Hypothesis

- In context of statistical Analysis:
- ***Null hypothesis*** – If we compare method A and method B and both are equally good(H0).
  - ***Example*** *: “No difference between coke and diet coke”*
- ***Alternate Hypothesis*** – If method A is superior than B(H1).
  - ***Example*** *: “There is difference between coke and diet coke”.*
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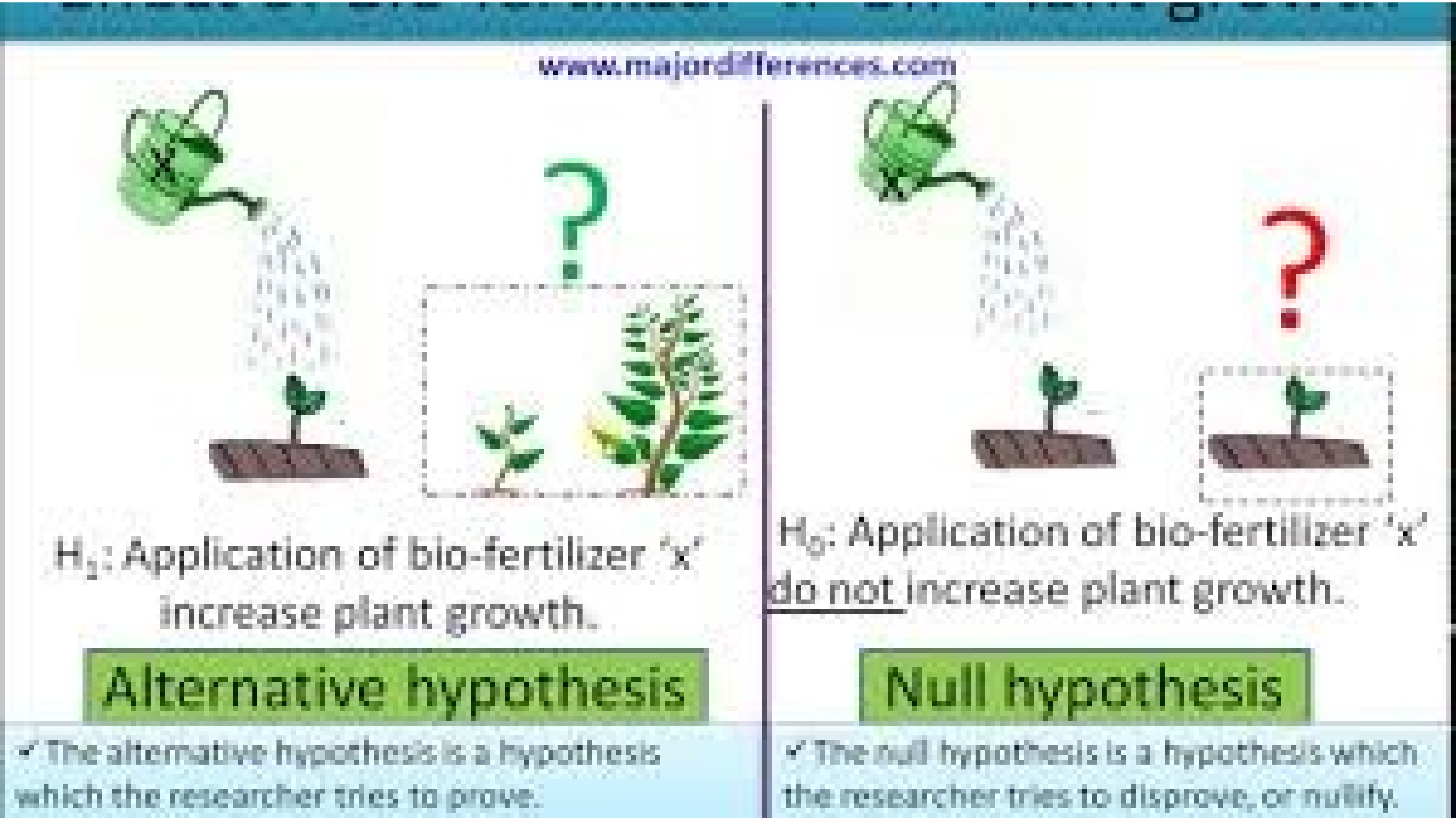
www.majordifferences.com
H1: Application of bio-fertilizer 'x' increase plant growth.
Alternative hypothesis
✓ The alternative hypothesis is a hypothesis which the researcher tries to prove.
H0: Application of bio-fertilizer 'x' do not increase plant growth.
Null hypothesis
✓ The null hypothesis is a hypothesis which the researcher tries to disprove, or nullify.

Example

- Doctors recommend teenagers between 14-18 years to get at least 8 hrs sleep for proper health.
- Statistics class at high school suspects that students at their school are getting less than 8 hours sleep on avg.
- To test we randomly sample of 42 students and ask them how much sleep they get per night. Mean = 7.5 hours.
- Alternate H1: avg amt of sleep student at school is < 8 hrs
- H0 :μ >= 8
- H1: $\mu<8$

- Suppose we want to test the hypothesis that the population mean (μ) is equal to the hypothesised mean (μ H0)=100.
- Then we would say that the null hypothesis is that the population mean is equal to the hypothesised mean 100 and symbolically we can express as:
- H0 : μ = μH0 = 100

Possible alternate hypothesis

Table 9.1

Alternative hypothesis	*To be read as follows*
$H_a : \mu \neq \mu_{H_0}$	(The alternative hypothesis is that the population mean is not equal to 100 i.e., it may be more or less than 100)
$H_a : \mu > \mu_{H_0}$	(The alternative hypothesis is that the population mean is greater than 100)
$H_a : \mu < \mu_{H_0}$	(The alternative hypothesis is that the population mean is less than 100)

Level of Significance

The level of significance:
This is a very important concept in the context of hypothesis testing.
It is always some percentage (usually 5%) which should be chosen with great care, thought and reason.

Type 1 and Type 2 errors

- If Null hypothesis is rejected when it is true – type 1 error.
- If Null hypothesis is accepted when it is not true – Type 2 error.
- In other words Type1 means – rejection of hypothesis when should have been accepted and Type 2 means accepting hypothesis when should have been rejected.

What are these errors?

- These are errors that arise when performing hypothesis testing and decision making

- **Type 1 error** (*false positive* conclusion)
  - Stating difference when there is no difference, alpha
  - Related to p value, ***how?***
  - Set at 1/20 or 0.05 or 5%
  - The probability is distributed at the tails of the normal curve i.e., 0.025 on either tail

- **Type 2 error** (*false negative* conclusion)
  - Stating no difference when there is a difference, beta
  - Occurs when sample size is too small.
  - Conventional values are 0.1 or 0.2
  - Related to power, ***how?***

Type I Error

Type II Error

Example 1

Null Hypothesis	Type I Error / False Positive	Type II Error / False Negative
Person is not guilty of the crime	Person is judged as **guilty** when the person actually **did not** commit the crime (convicting an innocent person)	Person is judged **not guilty** when they actually **did** commit the crime (letting a guilty person go free)
Cost Assessment	Social costs of sending an innocent person to prison and denying them their personal freedoms (which in our society, is considered an almost unbearable cost)	Risks of letting a guilty criminal roam the streets and committing future crimes

Example 2

Null Hypothesis	Type I Error / False Positive	Type II Error / False Negative
Wolf is not present	Shepherd thinks wolf is present (shepherd cries wolf) when no wolf is actually present	Shepherd thinks wolf is NOT present (shepherd does nothing) when a wolf is actually present
Cost Assessment	Costs (actual costs plus shepherd credibility) associated with scrambling the townsfolk to kill the non-existing wolf	Replacement cost for the sheep eaten by the wolf, and replacement cost for hiring a new shepherd

Example 3

Null Hypothesis	Type I Error / False Positive	Type II Error / False Negative
Medicine A cures Disease B	(H_0 **true**, but rejected as **false**)Medicine A **cures** Disease B, but is **rejected as false**	(H_0 **false**, but accepted as **true**)Medicine A **does not cure** Disease B, but is **accepted as true**
Cost Assessment	Lost opportunity cost for rejecting an effective drug that could cure Disease B	Unexpected side effects (maybe even death) for using a drug that is not effective

Type 1 and Type 2 Hypothesis.

Possible Errors in Hypothesis Test Decision Making

(continued)

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability 1 - α	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Probability 1 - β

Level of Significance

- The significance level, also denoted as α, is the probability of rejecting the null hypothesis when it is true
- Ex : a significance level of 0.05 indicates a 5% risk of concluding that a difference exists when there is no actual difference

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One tailed and two tailed test

- We test 3 types of Hypotheses given by:
  - Ho:μ =μ0 Against h1:μ not-equal μ0
  - Ho:μ =μ0 Against ha:μ > μ0 or Ho:μ <=μ0 Against h1:μ > μ0
  - Ho:μ =μ0 Against ha:μ < μ0 or Ho:μ >=μ0 Against h1:μ < μ0l
  - If we have not equal sign in alternate hypotheses – Two tailed
  - If we have > sign in alternate hypotheses – **right tailed**
  - If we have < sign in alternate hypotheses – left **tailed**
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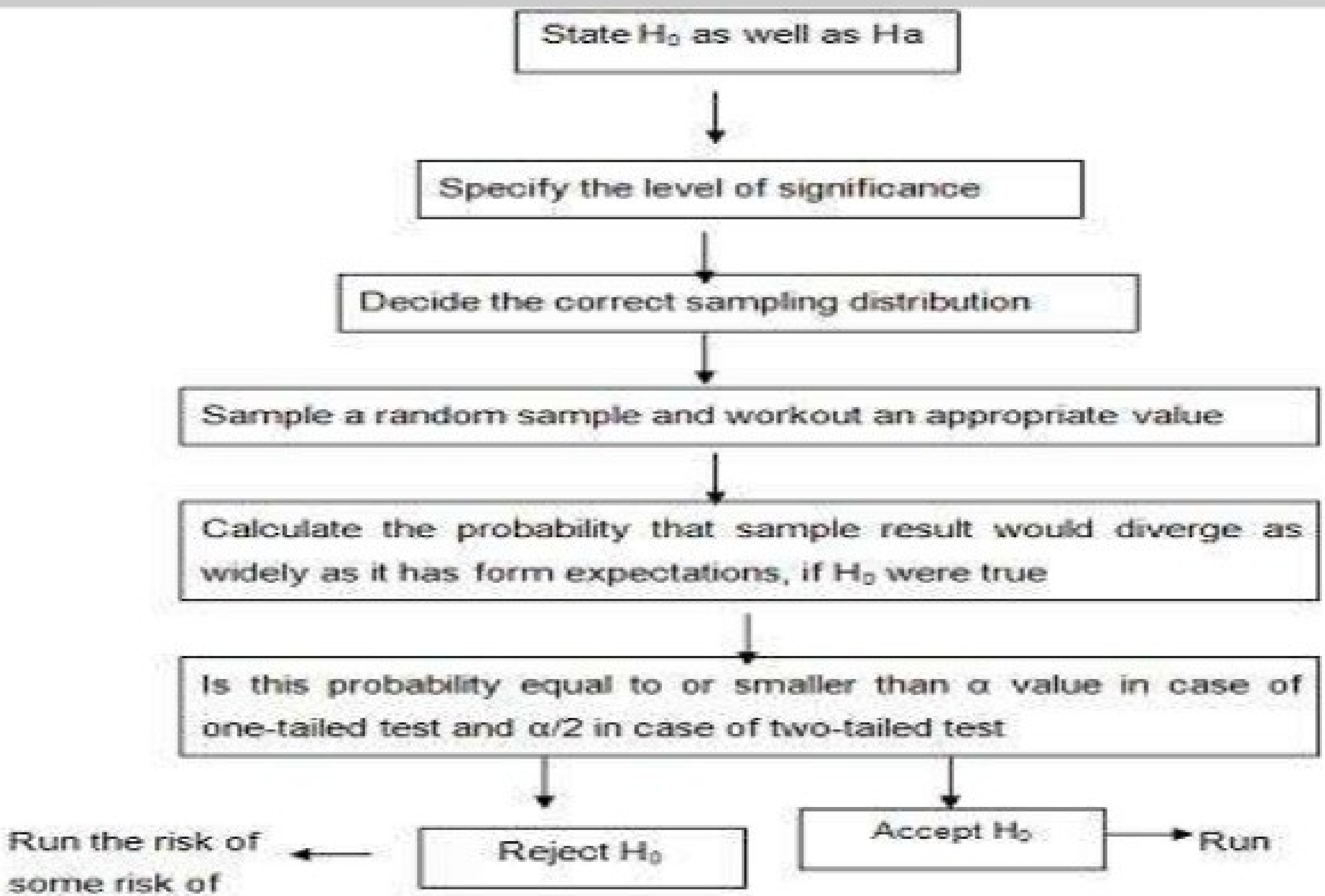
Steps in Hypothesis Testing



Chi-Square tests

A chi-square goodness of fit test determines if a sample data matches a population.

- Used to obtain confidence interval estimate of unknown population variance.
- Non-parametric test and as such no rigid assumptions are necessary in respect of type of population.
- chi-square can be used (i) as a test of goodness of fit and (ii) as a test of independence.
- ***As a test of goodness of fit,*** X^2 test enables us to see how well does the assumed theoretical distribution (such as Binomial distribution, Poisson distribution or Normal distribution) fit to the observed data.
- ***As a test of independence,*** X^2 test enables us to explain whether or not two attributes are associated.

$$\chi^2 = \sum \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

- Degree of Freedom – Number of Independent value which are assigned to statistical distribution
- (n-1) or (r-1)(c-1)
- Oij -observed frequency in ith row and jth column
- Eij – Expected frequency in ith row and jith column.
- = {Corresponding row total * corresponding column total} / grand total

Problem - 1

- A die is thrown 132 times with following results:
- Is the die biased?
-

Number turned up	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Is the die unbiased?

Table 3: Critical Values of χ^2

Degrees of freedom	Probability under H_0 that of χ^2 > Chi square						
	.99	.95	.50	.10	.05	.02	.01
1	.000157	.00393	.455	2.706	3.841	5.412	6.635
2	.0201	.103	1.386	4.605	5.991	7.824	9.210
3	.115	.352	2.366	6.251	7.815	9.837	11.341
4	.297	.711	3.357	7.779	9.488	11.668	13.277
5	.554	.1145	4.351	9.236	11.070	13.388	15.086
6	.872	1.635	5.348	10.645	12.592	15.033	16.812
7	1.239	2.167	6.346	12.017	14.067	16.622	18.475
8	1.646	2.733	7.344	13.362	15.507	18.168	20.090
9	2.088	3.325	8.343	14.684	16.919	19.679	21.666
10	2.558	3.940	9.342	15.987	18.307	21.161	23.209
11	3.053	4.575	10.341	17.275	19.675	22.618	24.725
12	3.571	5.226	11.340	18.549	21.026	24.054	26.217
13	4.107	5.892	12.340	19.812	22.362	25.472	72.688
14	4.660	6.571	13.339	21.064	23.685	26.873	29.141
15	4.229	7.261	14.339	22.307	24.996	28.259	30.578
16	5.812	7.962	15.338	23.542	26.296	29.633	32.000
17	6.408	8.672	16.338	24.769	27.587	30.995	33.409
18	7.015	9.390	17.338	25.989	28.869	32.346	34.805
19	7.633	10.117	18.338	27.204	30.144	33.687	36.191
20	8.260	10.851	19.337	28.412	31.410	35.020	37.566
21	8.897	11.591	20.337	29.615	32.671	36.343	38.932
22	9.542	12.338	21.337	30.813	33.924	37.659	40.289

A die is thrown 132 times

# turned up	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Is die biased?

Sol^n Die is unbiased.

Prob of obtaining any one of 6 #s is $1/6$

expected freq $= 132 \times \frac{1}{6} = 22$

To compute χ^2.

# of turn up	obtained freq O_i	exp freq. E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	16	22	-6	36	36/22
2	20	22	-2	4	4/22
3	25	22	3	9	9/22
4	14	22	-8	64	64/22
5	29	22	7	49	49/22
6	28	22	6	36	36/22

$$\Sigma \frac{(O_i - E_i)^2}{E_i} = 9$$

$\therefore \chi^2 = 9$

Degree of freedom $= n-1 = 6-1 = 5$.

- For a degree of freedom @ 5% level of sig $= \underline{11.071}$
- comparing calculated value & table value, χ^2, we say calculated value < table value

$\therefore$ Die is unbiased.

Problem - 2

2. Find the value of X^2 for the following information

Class	A	B	C	D	E
Observed frequency	8	29	44	15	4
Theoretical (or expected) frequency	7	24	38	24	7

Find value of χ^2 for following info:

Class	A	B	C	D	E
Obs freq.	8	29	44	15	4
Exp freq	7	24	38	24	7

Soln: Since some of freq is less than than 10, we shall regroup & then work on χ^2

Class	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
A & B	37	31	6	36	36/31
C	44	38	6	36	36/31
D & E	19	31	-12	144	144/31

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 6.76$$

- **Problem - 3**
- Genetic theory states that children having one parent of blood type *A* and the other of blood type *B* will always be of one of three types, *A*, *AB*, *B* and that the proportion of three types will on an average be as 1 : 2 : 1.
- A report states that out of 300 children having one *A* parent and *B* parent, 30 per cent were found to be types *A*, 45 per cent per cent type *AB* and remainder type *B*.
- Test the hypothesis by chi^2 test

χ^2 test.

Soln. Observed freq of type A - 90 = 300 × 30% - 90

AB - 135 = 300 × 45% - 135

B - 75 = 300 × 25% - 75.

300

Total students = 300.

1:2:1 = 1+2+1 = (4).

$\frac{1}{4} \times 300 = 75.$

$\frac{2}{4} \times 300 = 150.$

$\frac{1}{4} \times 300 = 75$

Expected freq.

Type.	O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
A	90	75	15	225	225/75 = 3
AB	135	150	15	225	225/150 = 1.5
B	75.	75	0.	0.	0/75 = 0.

$\therefore \chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 3 + 1.5 = 4.5.$

$\chi^2 = 4.5.$

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$\boxed{\text{Degree of freedom } (n-1) = (3-1) = 2.}$

χ^2 for 2, D F at 5%,

$\boxed{\text{Level of significance} = 5.991}$

Calulated value of χ^2 is 4.5 < table value

$\therefore$ This supports theoritical hypothesis of genetic theory.

Problem - 4

Eight coins were tossed 256 times and the following results were obtained:

Numbers of heads	0	1	2	3	4	5	6	7	8
Frequency	2	6	30	52	67	56	32	10	1

Are the coins biased? Use χ^2 test.

Soln. Hypothesis - Coins not biased

- Probability of 1 coin falling with head upward is 1/2 & tail upward is 1/2. & it remains same.

- expected value of getting 0,1,2.... heads in a single throw in 256 throws of 8 eight coins is.

# of head	expected frequency	
0	${}^8C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^8 \cdot 256.$	1
1	${}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 \cdot 256.$	8
2	${}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 \cdot 256$	26
3	${}^8C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^5 \cdot 256$	56
4	${}^8C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^4 \cdot 256$	70
5	${}^8C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3 \cdot 256$	56
6	${}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 \cdot 256$	28
7	${}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 \cdot 256$	8
8	${}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0 \cdot 256.$	1.

Value of χ^2 can be computed as

# of heads	O_i	E_i	$O_i - E_i$	$(O_i-E_i)^2/E_i$
0	2	1	1	$1/1 = 1.00$
1	6	8	-2	$4/8 = 0.50$
2	30	28	2	$4/28 = 0.14$
3	52	56	-4	$16/56 = 0.29$
4	67	70	-3	$9/70 = 0.13$
5	56	56	0	$0/56 = 0.00$
6	32	28	4	$16/28 = 0.56$
7	10	8	2	$4/8 = 0.50$
8	1	1	0	$0/1 = 0.00$

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 3.13.$$

Degree of fredom = 7.

Table value of χ^2 for D.F @ 5% L of S = 15.507

$\therefore$ $3.13 < 15.507$ ie coins are not Biased

Problem - 5

③.

④ Table given below shows data obtained during out break of small pox.

	Attacked	Not attacked	Total
Vaccinated	31	469	550
Not Vaccinated	185	1315	1500
Total	216	1784	2000

Test the effectiveness of vaccination in preventing attack from small pox. Test your result with help of χ^2 @ 5% level of significance.

Soln. Hypo: Vaccine is not effective in preventing small pox. On this basis, expected. freq corresponding to # of person Vaccinated (A) & attacked (B) } independent.

expectation of $AB = \frac{A \cdot B}{N}$.

$\therefore$ (A) = 500, (B) = 216, N = 200

$AB = \frac{500 \times 216}{2000} = 54.$

	Attacked (B)	Nt attacked (b)	Total
Vaccinated (A)	AB - 54	Ab - 446	500
Not Vaccinated (a)	aB - 162	ab - 1338	1500
Total	216	1784	2000

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(A) (B)

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Group	Given O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
A B	31	54			
A b	469	446			
a B	158	162			
a b	1315	1338			

$$\chi^2 = 14.642$$

$$DF = (r-1)(c-1) = (2-1)(2-1) = 1$$

table value of χ^2 for 1 degree of freedom @ 5% LOF

$$= 3.841$$

$14.642 > 3.841$ doesnot support hypothesis.

Conclusion- Vaccine is effective in preventing attack from Smal pox.

Home work -1

Two research workers classified some people in income groups on the basis of sampling studies. Their results are as follows:

Investigators	*Income groups*			*Total*
	Poor	*Middle*	*Rich*	
A	160	30	10	200
B	140	120	40	300
Total	300	150	50	500

Home work 2

3. An experiment was conducted to test the efficacy of chloromycetin in checking typhoid. In a certain hospital chloromycetin was given to 285 out of the 392 patients suffering from typhoid. The number of typhoid cases were as follows:

	Typhoid	*No Typhoid*	*Total*
Chloromycetin	35	250	285
No chloromycetin	50	57	107
Total	85	307	392

With the help of χ^2, test the effectiveness of chloromycetin in checking typhoid.

(The χ^2 value at 5 per cent level of significance for one degree of freedom is 3.841).

- Refer text book and solve worked example problems :- 11.2 , 11.3,11.7 to 11.14.
- Also solve exercise problems :- 3, 4, 5, 6, 7.


Thank you!