Master Theorem

If $f(n) \in \Theta(n^d)$ or $f(n) = c * n^d$ where $d \geq 0$

in recurrence $T(n) = aT(n/b) + f(n)$ then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n \log_b a) & \text{if } a > b^d \end{cases}$$

1) $T(n) = 8T(n/2) + 1000n^2$

$T(n) = aT(n/b) + f(n)$

$a = 8 \quad b = 2 \quad f(n) = cn^d$

$= 1000n^2$

$c = 1000, \quad d = 2$

Since

$a > b^d$

i.e $8 > 2^2$

master theorem case (3)

$T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 8})$

$T(n) = \Theta(n^3)$

2) $T(n) = 2T(n/2) + n^2$

$a = 2 \quad b = 2 \quad c = 1 \quad d = 2$

Since

$a < b^d$

$2 < 2^2$

Case (1) $T(n) = \Theta(n^d)$

$\boxed{T(n) = \Theta(n^2)}$

5) $T(n) = 2T(n/2) + 10n$

$a = 2 \quad b = 2 \quad c = 10 \quad d = 1$

since $a = b^d$

$2 = 2^1$

Case (2)

i.e $T(n) = O(n^d \log n)$

$\boxed{T(n) = O(n \log n)}$