An Under-Approximate Relational Logic Heralding Logics of Insecurity, Incorrect Implementation & More

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Abstract. Recently, authors have proposed *under-approximate* logics for reasoning about programs. So far, all such logics have been confined to reasoning about individual program behaviours. Yet there exist many over-approximate *relational* logics for reasoning about pairs of programs and relating their behaviours.

We present the first under-approximate relational logic, for the simple imperative language IMP. We prove our logic is both sound and complete. Additionally, we show how reasoning in this logic can be decomposed into non-relational reasoning in an under-approximate Hoare logic, mirroring Beringer's result for over-approximate relational logics. We illustrate the application of our logic on some small examples in which we provably demonstrate the presence of insecurity.

1 Introduction

Almost all program logics are *over-approximate*, in that they reason about an over-approximation of program behaviour. Recently, authors have proposed *under-approximate* logics [16,8], for their promise to increase the scalability of reasoning methods, avoid false positives, and *provably* detect bugs.

So far, all such logics have been confined to reasoning about individual program executions. Yet, many properties of interest talk about pairs of program behaviours. Examples are security properties like noninterference [11], but also function sensitivity, determinism, correct implementation (refinement) and program equivalence. Such properties can be reasoned about using *relational* logics [5], in which pre- and post-conditions are relations on program states. Overapproximate relational logics for doing so have been well-studied [5,4,10,12].

Here, we present (Section 2) the first under-approximate relational logic, for the simple imperative language IMP [14]. We prove our logic is both sound and complete (Section 3). Additionally (Section 4), we show how reasoning in this logic can be decomposed into non-relational reasoning in an under-approximate Hoare logic, mirroring Beringer's result for over-approximate relational logics [6]. Under-approximate logics are powerful for provably demonstrating the presence of bugs. We illustrate the application of our logic on some small examples (Section 5) for provably demonstrating violations of noninterference security. In this

sense, our logic can be seen as the first *insecurity logic*. However, being a general relational logic, it should be equally applicable for provably demonstrating violations of the other aforementioned relational properties. All results in this paper have been mechanised in Isabelle/HOL [15].

2 The Logic

2.1 The Language IMP

Our logic is defined for the simple imperative language IMP [14]. Commands c in IMP include variable assignment, sequencing, conditionals (if-then-else statements), iteration (while-loops), and the no-op **skip**.

$$c ::= x := e \mid c; \ c \mid \mathbf{if} \ b \ \mathbf{then} \ c \ \mathbf{else} \ c \mid \mathbf{while} \ b \ \mathbf{do} \ c \mid \mathbf{skip}$$

Here, e and b denote respectively (integer) arithmetic and boolean expressions. The big-step semantics of IMP is entirely standard (omitted for brevity) and defines its semantics over states s that map variables x to (integer) values. It is characterised by the judgement $(c, s) \Rightarrow s'$ which denotes that command c when execute in initial state s terminates in state s'.

2.2 Under-Approximate Relational Validity

Relations R are binary relations on states. We write R(s, s') when states s and s' are related by R.

Given pre- and post-relations R and S and commands c and c', under-approximate relational validity is denoted $\vDash \langle R \rangle c$, $c' \langle S \rangle$ and is defined analogously to its Hoare logic counterpart [8,16].

Definition 1 (Under-Approximate Relational Validity).

$$\models \langle R \rangle \ c, \ c' \ \langle S \rangle \equiv (\forall \ t \ t'. \ S(t,t') \implies (\exists \ s \ s'. \ R(s,s') \land (c,s) \Rightarrow t \land (c',s') \Rightarrow t'))$$

It requires that for all final states t and t' in S, there exists pre-states s and s' in R for which executing c and c' from each respectively terminates in t and t'.

2.3 Proof Rules

Judgements of the logic are written $\vdash \langle R \rangle c$, $c' \langle S \rangle$. Fig. 1 depicts the rules of the logic. For $\vdash \langle R \rangle c$, $c' \langle S \rangle$ we define a set of rules for each command c. Almost all of these rules has a counterpart in under-approximate Hoare logic [16,8]. There are also three additional rules (Conseq, Disj, and Sym). Of these, only the final one Sym does not have a counterpart in prior under-approximate logics.

The SKIP rule simply encodes the meaning of $\vDash \langle R \rangle$ skip, $c' \langle S \rangle$ in the premise. The SEQ rule essentially encodes the intuition that $\vDash \langle R \rangle$ (c; d), $c' \langle S \rangle$ holds precisely when $\vDash \langle R \rangle$ (c; d), $(c'; \mathbf{skip}) \langle S \rangle$ does. However we can also view this rule differently: once c_1 and c' have both been executed, the left-hand side

still has c_2 to execute; however the right-hand side is done, represented by **skip**. The relation S then serves as a witness for linking the two steps together.

This same pattern is seen in the other rules too: given the pre-relation R the Assign rule essentially computes the witness relation R[x := e] that holds following the assignment x := e, which (like the corresponding rule in [16]) simply applies the strongest-postcondition predicate transformer to R.

The rules for if-statements, IFTRUE and IFFALSE, are the relational analogues of the corresponding rules from under-approximate Hoare logic. The rules WhileTrue and WhileFalse encode the branching structure of unfolding a while-loop. The rule Back-Var is the analogue of the "backwards variant" iteration rule from [16,8]. Here R is a family of relations indexed by natural number n (which can be thought of as an iteration count) such that the corresponding relation R(k) must hold after the kth iteration. Unlike the corresponding rule in [16], this rule explicitly encodes that the loop executes for some number of times (possibly 0) to reach a frontier, at which point $\exists n. R(n)$ holds and the remainder of the loop is related to command c'.

The Conseq and Disj rules are the relational analogues of their counterparts in [16]. Finally, the SYM rule allows to flip the order of c and c' when reasoning about $\vdash \langle R \rangle c$, $c' \langle S \rangle$ so long as the relations are flipped accordingly, denoted R etc. Naturally, this rule is necessary for the logic to be complete.

3 Soundness and Completeness

Theorem 1 (Soundness). For relations R and S and commands c and c', if $\vdash \langle R \rangle c$, $c' \langle S \rangle$ then $\models \langle R \rangle c$, $c' \langle S \rangle$.

Proof. By structural induction on $\vdash \langle R \rangle$ c, $c' \langle S \rangle$. Almost all cases are straightforward. The only exception is the BACK-VAR rule, which, similarly to [16], requires an induction on the argument n to R(n) and generalising the result from R(0) to some R(k) to get a sufficiently strong induction hypothesis.

Theorem 2 (Completeness). For relations R and S and commands c and c', $if \models \langle R \rangle \ c, \ c' \ \langle S \rangle \ then \models \langle R \rangle \ c, \ c' \ \langle S \rangle$.

Proof. By induction on c. This proof follows a similar structure to the completeness proof of [16].

4 Under-Approximate Relational Decomposition

For over-approximate relational logic, Beringer [6] showed how one can decompose relational reasoning into over-approximate Hoare logic reasoning. We show that the same is true for under-approximate relational logic, in which relational reasoning can be decomposed into *under-approximate* Hoare logic reasoning.

The essence of the approach is to decompose $\vDash \langle R \rangle c$, $c' \langle S \rangle$ into two statements: one about c and the other about c'. $\vDash \langle R \rangle c$, $c' \langle S \rangle$ says that for every

$$\frac{\forall t \ t'. \ S(t,t') \implies \exists s'. \ R(t,s') \land (c',s') \Rightarrow t'}{\vdash \langle R \rangle \ \text{skip}, \ c' \ \langle S \rangle} \text{Skip}$$

$$\frac{\vdash \langle R \rangle \ c, \ c' \ \langle S \rangle \qquad \vdash \langle S \rangle \ d, \ \text{skip} \ \langle T \rangle}{\vdash \langle R \rangle \ (c; \ d), \ c' \ \langle T \rangle} \text{SEQ1} \qquad \frac{\vdash \langle R[x := e] \rangle \ \text{skip}, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ x := e, \ c' \ \langle S \rangle} \text{Assign}$$

$$\frac{\vdash \langle R \land b^1 \rangle \ c_1, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{if} \ b \ \text{then} \ c_1 \ \text{else} \ c_2, \ c' \ \langle S \rangle} \text{If} \text{True}$$

$$\frac{\vdash \langle R \land \neg b^1 \rangle \ c_2, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{if} \ b \ \text{then} \ c_1 \ \text{else} \ c_2, \ c' \ \langle S \rangle} \text{If} \text{False}$$

$$\frac{\vdash \langle R \land \neg b^1 \rangle \ \text{skip}, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle} \text{WhileFalse}$$

$$\frac{\vdash \langle R \land b^1 \rangle \ (c; \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle} \text{WhileTrue}$$

$$\frac{\vdash \langle R \land b^1 \rangle \ (c; \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle} \text{WhileTrue}$$

$$\frac{\vdash \langle R \land b^1 \rangle \ (c; \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle}{\vdash \langle R \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle} \text{Back-Var}$$

$$\frac{\vdash \langle R(n) \land b^1 \rangle \ c, \ \text{skip} \ \langle R(n+1) \rangle \qquad \vdash \langle \exists \ n. \ R(n) \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle}}{\vdash \langle R(0) \rangle \ \text{while} \ b \ \text{do} \ c, \ c' \ \langle S \rangle} \text{Conseq}$$

$$\frac{R_1 \implies R_2 \qquad \vdash \langle R_1 \rangle \ c, \ c' \ \langle S_1 \rangle \qquad S_2 \implies S_1 \text{Conseq}}{\vdash \langle R_2 \rangle \ c, \ c' \ \langle S_2 \rangle} \text{Disj} \qquad \frac{\vdash \langle R \rangle \ c', \ c \ \langle S \rangle}{\vdash \langle R \rangle \ c, \ c' \ \langle S \rangle} \text{Sym}$$

Fig. 1. Proof rules. R[x:=e] denotes the transformer on relation R following the assignment x:=e. It is defined as: $R[x:=e] \equiv \lambda \ s \ s'. \ \exists \ v. \ R(s(x:=v),s') \land s(x) = [e]_{s(x:=v)}$. Logical operators \land and \lor are lifted to relations, e.g.: $R \land S \equiv \lambda \ s \ s'. \ R(s,s') \land S(s,s')$. For boolean expression $b, \ b^1 \equiv \lambda \ s \ s'. \ [b]_s$. Also, $\exists \ n. \ R(n) \equiv \lambda \ s \ s'. \ \exists \ n. \ R(n)(s,s')$. $R \Longrightarrow S$ denotes when relation R is a subset of relation S. Finally, $R \equiv \lambda \ s \ s'. \ R(s',s)$.

S-pair of final states t and t', we can find an R-pair of states s and s' from which t and t' can be reached respectively by executing c and c'. To decompose this we think about what it would mean to first execute c on its own. At this point we would have reached a state t which has a c-predecessor s for which R(s,s'), and the execution of c' would not yet have begun and would still be in the state s' which has a c'-successor t' for which S(t,t'). We write decomp(R,c,c',S) that defines this relation that holds between t and s':

$$\mathsf{decomp}(R,c,c',S)(t,s') \equiv \exists \ s \ t'. \ P(s,s') \land (c,s) \Rightarrow t \land (c',s') \Rightarrow t' \land Q(t,t')$$

This allows us to decompose $\models \langle R \rangle$ c, $c' \langle S \rangle$ into two separate relational statements about c and c'. Here we use **skip** to indicate when one command executes but the other does not make a corresponding step, to simulate the idea that first c executes followed by c'.

Lemma 1 (Relational Validity Decomposition). For relations R and S and commands c and c',

$$\models \langle R \rangle \ c, \ c' \ \langle S \rangle \iff \begin{pmatrix} \models \langle R \rangle \ c, \ \textit{skip} \ \langle \mathsf{decomp}(R, c, c', S) \rangle \ \land \\ \models \langle \mathsf{decomp}(R, c, c', S) \rangle \ \textit{skip}, \ c' \ \langle S \rangle \end{pmatrix}$$

Proof. From the definitions of $\vDash \langle R \rangle c$, $c' \langle S \rangle$ and $\mathsf{decomp}(R, c, c', S)$.

Now $\vDash \langle R \rangle$ c, skip $\langle S \rangle$ talks about only one execution (since the second command skip doesn't execute by definition). And indeed it can be expressed equivalently as a statement in under-approximate Hoare logic.

For Hoare logic pre- and post-conditions P and Q and command c, we write $\langle P \rangle c \langle Q \rangle$ to mean that Q under-approximates the post-states t that c can reach beginning from a P-pre-state s [8,16].

Definition 2 (Under-approximate Hoare logic validity).

$$\langle P \rangle \ c \ \langle Q \rangle \equiv (\forall \ t. \ Q(t) \implies (\exists \ s. \ P(s) \land (c, s) \Rightarrow t))$$

Then the following result is a trivial consequence:

Lemma 2.

$$\vDash \langle R \rangle \ skip, \ c' \ \langle Q \rangle \iff (\forall \ t. \ \langle R(t) \rangle \ c' \ \langle S(t) \rangle)$$

Theorem 3 below follows straightforwardly from Lemma 1 and Lemma 2. It states how every under-approximate relational statement $\vDash \langle R \rangle$ c, $c' \langle S \rangle$ can be decomposed into two under-approximate Hoare logic statements: one about c and the other about c'.

Theorem 3. Under-Approximate Relational Decomposition $\models \langle R \rangle$ c, c' $\langle S \rangle$ if and only if

$$\left(\begin{array}{l} \forall \ t'. \ \langle (\lambda \ t. \ R(t,t')) \rangle \ c \ \langle (\lambda \ t. \ \mathsf{decomp}(R,c,c',S)(t,t')) \rangle \ \land \\ \forall \ t. \ \langle \mathsf{decomp}(R,c,c',S)(t) \rangle \ c' \ \langle Q(t) \rangle \end{array} \right)$$

This theorem implies that, just as over-approximate Hoare logic can be used to prove over-approximate relational properties [6], all valid under-approximate relational properties can be proved using (sound and complete) under-approximate Hoare logic reasoning. This theorem therefore provides strong evidence for the applicability of under-approximate reasoning methods (such as under-approximate symbolic execution) for proving under-approximate relational properties, including e.g. via product program constructions [3,1,2,9].

5 Using the Logic

While the logic is complete, its rules on their own are inconvenient for reasoning about common relational properties. Indeed, often when reasoning in relational logics one is reasoning about two very similar programs c and c'. For instance, when proving noninterference [11] c and c' might be identical, differing only when the program branches on secret data [13]. When proving refinement from an abstract program c to a concrete program c', the two may often have similar structure [7]. Hence for the logic to be usable one can derive familiar-looking proof rules that talk about similar programs.

5.1 Deriving Proof Rules via Soundness and Completeness

For instance, the following rule is typical of relational logics (both over- and under-approximate) for decomposing matched sequential composition.

$$\frac{\vdash \langle R \rangle \ c, \ c' \ \langle S \rangle \qquad \vdash \langle S \rangle \ d, \ d' \ \langle T \rangle}{\vdash \langle R \rangle \ (c; \ d), \ (c'; \ d') \ \langle T \rangle} \text{Seq-Matched}$$

Rather than appealing to the rules of Fig. 1, this rule is simpler to derive from the corresponding property on under-approximate relational validity. In particular, it is straightforward to prove from Definition 1 and the big-step semantics of IMP that when $\vDash \langle R \rangle$ c, c' $\langle S \rangle$ and $\vDash \langle S \rangle$ d, d' $\langle T \rangle$ hold, then $\vDash \langle R \rangle$ (c; d), (c'; d') $\langle T \rangle$ follows. The proof rule above is then a trivial consequence of soundness and completeness.

Other such rules that we have also proved by appealing to corresponding properties of under-approximate relational validity with soundness and completeness include the following one for reasoning about matched while-loops.

$$\frac{\forall \ n. \vdash \langle R(n) \land b^1 \land b'^2 \rangle \ c, \ c' \ \langle R(n+1) \rangle}{\vdash \langle \exists \ n. \ R(n) \rangle \ \textbf{while} \ b \ \textbf{do} \ c, \ \textbf{while} \ b' \ \textbf{do} \ c' \ \langle S \rangle}{\vdash \langle R(0) \rangle \ \textbf{while} \ b \ \textbf{do} \ c, \ \textbf{while} \ b' \ \textbf{do} \ c' \ \langle S \rangle} \text{Back-Var-Matched}$$

5.2 Derived Proof Rules

Other rules for matched programs are more easily derived by direct appeal to existing proof rules. For instance, the following rule for matched assignments can be derived straightforwardly from Assign and Sym.

$$\vdash \langle R \rangle \ x := e, \ x' := e' \ \langle R[x := e][x' := e'] \rangle$$
ASSIGN-MATCHED

Similarly the following rule follows from Back-Var-Matched above, with While-False and Sym.

$$\frac{\forall \ n. \vdash \langle R(n) \wedge b^1 \wedge b'^2 \rangle \ c, \ c' \ \langle R(n+1) \rangle}{\vdash \langle R(0) \rangle \ \textbf{while} \ b \ \textbf{do} \ c, \ \textbf{while} \ b' \ \textbf{do} \ c' \ \langle \exists \ n. \ R(n) \wedge \neg b^1 \wedge \neg b'^2 \rangle} \\ \text{Back-Var-Matched 2}$$

It is useful for reasoning past two matched loops (rather than for detecting violations of relational validity within their bodies) and is an almost direct relational analogue of the backwards variant rule from [16].

5.3 Examples

We illustrate the logic on a few small examples. Each of these examples concerns the relational property of noninterference [11]. Under-approximate logics are aimed to prove the existence of bugs rather than to prove their absence [16]. Each example has a security bug that violates noninterference that we provably demonstrate via our logic.

In a programming language-based setting [18], termination-insensitive non-interference holds for a program c if, when we execute it from two initial states s and s' that agree on the value of all public (aka "low") variables, then whenever both executions terminate the resulting states t and t' must also agree on the values of all public variables.

Taking the variable low to be the only public variable, we can define when two states agree on the value of all public variables, written L, as follows:

$$L(s, s') \equiv (s(low) = s'(low))$$

Then a program c is insecure if it has two executions beginning in states s and s' for which L(s, s') that terminate in states t and t' for which $\neg L(t, t')$. This holds precisely when there exists some non-empty relation S, for which

$$S \implies \neg \mathsf{L} \text{ and } \vDash \langle \mathsf{L} \rangle \ c, \ c \ \langle S \rangle$$

In fact, since our logic is sound, if we can prove $\vdash \langle \mathsf{L} \rangle \ c, \ c \ \langle S \rangle$ for some satisfiable S such that $S \implies \neg \mathsf{L}$, then the program must be insecure.

The left program in Fig. 2 is trivially insecure and this is demonstrated straightforwardly by using a derived proof rule (omitted for brevity) for matching if-statements that follows the then-branch of the first and the else-branch of the second, and then by applying the matched assignment rule ASSIGN-MATCHED.

```
1 \times := 0;
                                                           1 \times := 0;
                             2 while n > 0 do
                                                           2 while x < 4000000 do
                                  x := x + n;
1 if x > 0
                                                                x := x + 1;
     \mathsf{low} := 1
                                  n := y;
                                                                if x = 2000000
                                if x = 2000000
3 else
                                                                  low := high
                                  low := high
     low := 0
                                                           6
                                                                else
                                else
                                                                  skip
                                  skip
```

Fig. 2. Examples (some inspired from [16]).

The middle program of Fig. 2 is also insecure, and is inspired by the client0 example of [16]. To prove it insecure, we first use the consequence rule to strengthen the pre-relation to one that ensures that the values of the high variable in both states are distinct. We use the matched assignment and sequencing rules, ASSIGN-MATCHED and SEQ-MATCHED. Then we apply some derived proof rules (omitted) for matched while-loops that (via the disjunction rule DISJ) consider both the case in which both loops terminate immediately (ensuring x=0) and the case in which both execute just once (ensuring x>0). Taking the disjunction gives the post-relation for the loops that $x\geq 0$ in both states, which is of course the pre-relation for the following if-statement. The rule of consequence allows this pre-relation to be strengthened to x=2000000 in both states, from which a derived rule (omitted) for matching if-statements is applied that explores both then-branches, observing the insecurity.

The rightmost example of Fig. 2 makes full use of the rule Back-Var-Matched. This rule is applied first with the backwards-variant relation R(n) instantiated to require that in both states x=n and $0 \le x < 2000000$. This effectively reasons over both loops up to the critical iteration in which x will be incremented to become 2000000. At this point, we apply a derived rule for matching while loops similar to WhileTrue that unfolds both for one iteration. This allows witnessing the insecurity. We then need to reason over the remainder of the loops to reach the termination of both executions. This is done using the Back-Var-Matched rule, with R(n) instantiated this time to require that in both states x=2000000+n and $20000000 \le x \le 4000000$ and, critically, that low = high in both states (ensuring that the security violation is not undone during these subsequent iterations).

6 Related Work

Over-approximate relational logics have been well-studied. Indeed recent work purports to generalise many previous and forthcoming relational logics [12]. Our logic is different from these prior relational logics in that those were all over-approximate. Ours, on the other hand, is an under-approximate logic [16,8]. Thus, while traditional logics aim to provably establish when relational properties hold, ours aims instead for provably demonstrating *violations* of relational properties.

The power of under-approximate logics for provably detecting bugs was recently explained by O'Hearn [16]. He extended the previous Reverse Hoare Logic of de Vries and Koutavas [8], so that the semantics explicitly tracks erroneous executions, allowing post-conditions to distinguish between normal and erroneous execution, producing an Incorrectness Logic.

The semantics of our language, IMP, like that of de Vries and Koutavas does not explicitly track erroneous execution. Doing so offers less benefit for the purposes of relational reasoning, where erroneous behaviour cannot be characterised by individual executions but rather only by *comparing* two executions.

Just as our relational logic can be seen as the relational analogue of prior under-approximate Hoare logics, our decomposition result (Section 4) can also be seen as the relational analogue of Beringer's decomposition result for over-approximate relational logic [6].

7 Conclusion

We presented the first under-approximate relational logic, for the simple imperative language IMP. Our logic is sound and complete. We also showed a decomposition principle allowing under-approximate relational logic assertions to be proved via under-approximate Hoare logic reasoning. We briefly discussed our logic's application to some small examples, for provably demonstrating the presence of insecurity. Being a general relational logic, it should be equally applicable for provably demonstrating violations of other relational properties too.

O'Hearn's Incorrectness Logic [16] is an under-approximate Hoare logic that was very recently extended to produce the first Incorrectness Separation Logic [17] Over-approximate relational separation logics have been studied [19,10] and an interesting direction for future research would include investigation of underapproximate relational separation logics. Others naturally include the use of under-approximate Hoare logic reasoning tools for under-approximate relational verification. We hope that this paper serves as a step towards automatic, provable detection of relational incorrectness.

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