# Cost-Aware Automatic Program Repair

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**Abstract.** We present a formal framework for repairing infinite-state, imperative, sequential programs, with (possibly recursive) procedures and multiple assertions; the framework can generate repaired programs by modifying the original erroneous program in multiple program locations, and can ensure the readability of the repaired program using user-defined expression templates; the framework also generates a set of inductive assertions that serve as a proof of correctness of the repaired program. As a step toward integrating programmer intent and intuition in automated program repair, we present a cost-aware formulation given a cost function associated with permissible statement modifications, the goal is to ensure that the total program modification cost does not exceed a given repair budget. As part of our predicate abstractionbased solution framework, we present a sound and complete algorithm for repair of Boolean programs. We have developed a prototype tool based on SMT solving and used it successfully to repair diverse errors in benchmark C programs.

## 1 Introduction

Program debugging — the process of fault localization and error elimination — is an integral part of ensuring correctness in existing or evolving software. Being essentially manual, program debugging is often a lengthy, expensive part of a program's development cycle. There is an evident need for improved formalization and mechanization of this process. However, program debugging is hard to formalize — there are multiple types of programming mistakes with diverse manifestations, and multiple ways of eliminating a detected error. Moreover, it is particularly challenging to assimilate and mechanize the expert human intuition involved in the choices made in manual program debugging.

In this paper, we present a *cost-aware* formulation of the automated program debugging problem that addresses the above concerns. Our formulation obviates the need for a separate fault localization phase by directly focusing on error elimination, i.e., program repair. We fix a set  $\mathcal{U}$  of

update schemas that may be applied to program statements for modifying them. An update schema is a compact description of a class of updates that may be applied to a program statement in order to repair it. For instance, the update schema assign  $\mapsto$  assign permits replacement of the assignment statement x := y with other assignment statements such as x := x + y or y := x + 1, assign  $\mapsto$  skip permits deletion of an assignment statement, etc. In this paper,  $\mathcal{U}$  includes deletion of statements, replacement of assignment statements with other assignment statements, and replacement of the guards of conditional and loop statements with other guards. We assume we are given a cost function that assigns some user-defined cost to each application of an update schema to a program statement. Given an erroneous program  $\mathcal{P}$ , a cost function c and a repair budget  $\delta$ , the goal of cost-aware automatic program repair is to compute a program  $\widehat{\mathcal{P}}$  such that:  $\widehat{\mathcal{P}}$  is correct,  $\widehat{\mathcal{P}}$  is obtained by modifying  $\mathcal{P}$  using a set of update schemas from  $\mathcal{U}$  and the total modification cost does not exceed  $\delta$ . We postulate that this quantitative formulation [7] is a flexible and convenient way of incorporating user intent and intuition in automatic program debugging. For instance, the user can define appropriate cost functions to search for  $\widehat{\mathcal{P}}$  that differs from  $\mathcal{P}$  in at most  $\delta$  statements, or to penalize any modification within some trusted program fragment, or to favor the application of a particular update schema over another, and so on.

Our approach to cost-aware repair of imperative, sequential programs is based on predicate abstraction [16], which is routinely used by verification tools such as SLAM [6], SLAM2 [3], SATABS [10], etc. for analyzing infinite-state programs. These tools generate Boolean programs which are equivalent in expressive power to pushdown systems and enjoy desirable computational properties such as decidability of reachability [5]. Inevitably, Boolean programs have also been explored for use in automatic repair of sequential programs for partial correctness [17] and total correctness [25]. These papers, however, do not accommodate a quantitative formulation of the repair problem and can only compute repaired programs that differ from the original erroneous program in exactly one expression. Moreover, these papers do not attempt to improve the readability of the concrete program  $\widehat{\mathcal{P}}$ , obtained by concretizing a repaired Boolean program.

Our predicate abstraction-based approach to automatic program repair relaxes the above limitations. Besides erroneous  $\mathcal{P}$ , c, and  $\delta$ , our framework requires a Boolean program  $\mathcal{B}$ , obtained from  $\mathcal{P}$  through iterative predicate abstraction-refinement, such that  $\mathcal{B}$  exhibits a non-spurious

path to an error. We present an algorithm which casts the question of repairability of  $\mathcal{B}$ , given U, c, and  $\delta$ , as an SMT query; if the query is satisfiable, the algorithm extracts a correct Boolean program  $\widehat{\mathcal{B}}$  from the witness to its satisfiability. Along with  $\widehat{\mathcal{B}}$ , we also extract a set of inductive assertions from the witness, that constitute a proof of correctness of  $\widehat{\mathcal{B}}$ . This algorithm for Boolean program repair is sound and complete, relative to  $\mathcal{U}$ , c, and  $\delta$ . A repaired Boolean program  $\widehat{\mathcal{B}}$ , along with its proof, is concretized to obtain a repaired concrete program  $\widehat{\mathcal{P}}$ , along with a proof of correctness. However, the concretized repairs may not be succinct or readable. Hence, our framework can also accept user-supplied templates specifying the desired syntax of the modified expressions in  $\widehat{\mathcal{P}}$  to constrain the concretization.

Alternate approaches to automatic repair and synthesis of sequential programs [20, 28–30] that do not rely on abstract interpretations of concrete programs, also often encode the repair/synthesis problem as a constraint-solving problem whose solution can be extracted using SAT or SMT solvers. Except for [30], these approaches, due to their bounded semantics, are imprecise and cannot handle total correctness<sup>3</sup>. The authors in [20] use SMT reasoning to search for repairs satisfying user-defined templates; the templates are needed not only for ensuring readability of the generated repairs, but also for ensuring tractability of their inherently undecidable repair generation query. They also include a notion of minimal diagnoses, which is subsumed by our more general cost-aware formulation. Given user-defined constraints specifying the space of desired programs and associated proof objects, the scaffold-based program synthesis approach of [30] attempts to synthesizes a program, along with a proof of total correctness consisting of program invariants and ranking functions for loops. In contrast to [30], our framework only interacts with a user for improving the readability of the generated repairs and for the cost function; all predicates involved in the generation of the repaired Boolean program and its proof are discovered automatically. Besides the above, there have been proposals for program repair based on computing repairs as winning strategies in games [18], abstraction interpretation [21], mutations [12], genetic algorithms [2,15], using contracts [31], and focusing on data structure manipulations [27, 32]. There are also customized program repair engines for grading and feedback generation for programming assignments, cf. [26]. Finally, a multitude of algorithms [4, 9, 19, 33] have been proposed for fault localization, based on analyzing error traces.

<sup>&</sup>lt;sup>3</sup> Our framework can be extended to handle total correctness by synthesizing ranking functions along with inductive assertions.

Some of these techniques can be used as a preprocessing step to improve the efficiency of our algorithm, at the cost of giving up on the completeness of the Boolean program repair module.

Summary of contributions: We define a new cost-aware formulation of automatic program repair that can incorporate programmer intuition and intent (Sec. 3). We present a formal solution framework (Sec. 4 and Sec. 5) that can repair infinite-state, imperative, sequential programs with (possibly recursive) procedures and multiple assertions. Our method can modify the original erroneous program in multiple program locations and can ensure the readability of the repaired program using user-defined expression templates. If our method succeeds in generating a repaired program  $\widehat{\mathcal{P}}$ , it generates a proof of  $\widehat{\mathcal{P}}$ 's correctness, consisting of inductive assertions, that guarantees satisfaction of all the assertions in the original program  $\mathcal{P}$ . As part of our predicate abstraction-based solution, we present a sound and complete algorithm for repair of Boolean programs. Finally, we present experimental results for repairing diverse errors in benchmark C programs using a prototype implementation (Sec. 6).

# 2 Background

Review: Predicate Abstraction. Predicate abstraction [5, 16] is an effective approach for model checking infinite-state imperative programs with respect to safety properties. This technique computes a finite-state, conservative abstraction of a concrete program  $\mathcal{P}$  by partitioning  $\mathcal{P}$ 's state space based on the valuation of a finite set  $Pred = \{\phi_1, \dots, \phi_r\}$  of predicates. The resulting abstract program is termed a Boolean program  $\mathcal{B}$  (see Fig. 1a and Fig. 1b): the control-flow of  $\mathcal{B}$  is the same as that of  $\mathcal{P}$  and the set  $V = \{b_1, \ldots, b_r\}$  of variables of  $\mathcal{B}$  are Boolean variables, where for each  $i \in [1, r]$ , the Boolean variable  $b_i$  represents the predicate  $\phi_i$ . Given a concrete program  $\mathcal{P}$ , the overall counterexample-guided abstraction refinement method proceeds as follows. In step one, an initial Boolean program  $\mathcal{B}$  is computed and in step two,  $\mathcal{B}$  is model-checked with respect to its specification. If  $\mathcal{B}$  is found to be correct, the method concludes that  $\mathcal{P}$  is correct. Otherwise, an abstract counterexample path leading to some violated assertion in  $\mathcal{B}$  is computed and examined for feasibility in  $\mathcal{P}$ . If found feasible, the method terminates, reporting an error in  $\mathcal{P}$ . If found infeasible, in step three,  $\mathcal{B}$  is refined into a new Boolean program  $\mathcal{B}'$  that eliminates the spurious counterexample. Thereafter, steps two and three are repeated, as needed. Note that the overall method is

incomplete - it may not always be able to possible to compute a suitable refinement that eliminates a spurious counterexample or to check if an abstract counterexample is indeed spurious.

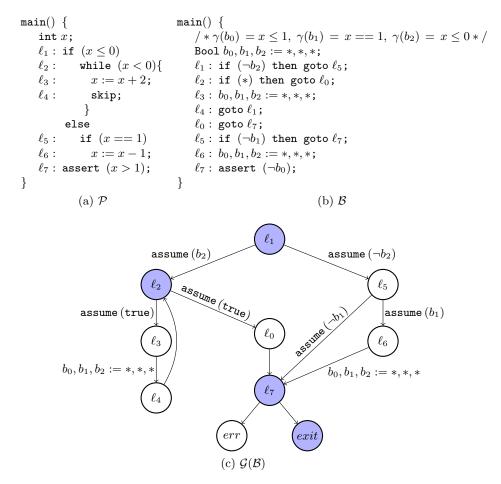


Fig. 1: An example concrete program  $\mathcal{P}$ , a corresponding Boolean program  $\mathcal{B}$  and  $\mathcal{B}$ 's transition graph

In our work, the interesting case is when the method terminates reporting an error. Henceforth, we fix a concrete program  $\mathcal{P}$ , and a corresponding Boolean program  $\mathcal{B}$  that exhibits a non-spurious counterexample path. Let  $\{\phi_1, \ldots, \phi_r\}$  denote the set of predicates used in the abstraction of  $\mathcal{P}$  into  $\mathcal{B}$ , where each predicate is a quantifier-free first

order expression over the variables of  $\mathcal{P}$ . Let  $\{b_1, \ldots, b_r\}$  denote the corresponding Boolean variables of  $\mathcal{B}$ . Let  $\gamma$  denote the mapping of Boolean variables to their respective predicates: for each  $i \in [1, r]$ ,  $\gamma(b_i) = \phi_i$ . The mapping  $\gamma$  can be extended in a standard way to expressions over the Boolean variables in V.

**Program Syntax**. For our technical presentation, we fix a common, simplified syntax for sequential concrete and abstract programs. A partial definition of this syntax is shown in Fig. 2. In the syntax, v denotes a variable,  $\langle type \rangle$  denotes the type of a variable, F denotes a procedure,  $\ell$  denotes a statement label or location,  $\langle expr \rangle$  denotes a well-typed expression, and  $\langle bexpr \rangle$  denotes a Boolean-valued expression.

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 \begin{array}{ll} \langle pgm \rangle & ::= \langle vardecl \rangle \langle proclist \rangle \\ \langle vardecl \rangle & ::= \operatorname{decl} v : \langle type \rangle; \mid \langle vardecl \rangle \langle vardecl \rangle \\ \langle proclist \rangle & ::= \langle proc \rangle \langle proclist \rangle \mid \langle proc \rangle \\ \langle proc \rangle & ::= F(v_1, \ldots, v_k) \operatorname{begin} \langle vardecl \rangle \langle stmtseq \rangle \operatorname{end} \\ \langle stmtseq \rangle & ::= \langle labstmt \rangle; \langle stmtseq \rangle \\ \langle labstmt \rangle & ::= \langle stmt \rangle \mid \ell : \langle stmt \rangle \\ \langle stmt \rangle & ::= \operatorname{skip} \mid v_1, \ldots, v_m := \langle expr_1 \rangle, \ldots, \langle expr_m \rangle \\ & \quad \mid \operatorname{if} (\langle bexpr \rangle) \operatorname{then} \langle stmtseq \rangle \operatorname{else} \langle stmtseq \rangle \operatorname{fi} \\ & \quad \mid \operatorname{while} (\langle expr \rangle) \operatorname{do} \langle stmt \rangle \operatorname{od} \mid \operatorname{assume} (\langle bexpr \rangle) \\ & \quad \mid \operatorname{call} F(\langle expr_1 \rangle, \ldots, \langle expr_k \rangle) \mid \operatorname{return} \\ & \quad \mid \operatorname{goto} \ell_1 \operatorname{or} \ldots \operatorname{or} \ell_n \mid \operatorname{assert} (\langle bexpr \rangle) \\ \end{array}
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Fig. 2: Programming language syntax

Thus, a concrete or an abstract (Boolean) program consists of a declaration of global variables, followed by a list of procedure definitions; a procedure definition consists of a declarations of local variables, followed by a sequence of labeled statements; a statement is a skip, (parallel) assignment, conditional, loop, assume, (call-by-value) procedure call, return, goto or assert statement.

We make the following assumptions: (a) there is a distinguished initial procedure main, which is not called from any other procedure, (b) all variable and formal parameter names are globally unique, (c) the number of actual parameters in a procedure call matches the number of formal parameters in the procedure definition, (d) goto statements are not used arbitrarily; they are used only to simulate the flow of control in structured programs, (e) the last statement in the loop body of every while statement is a skip statement, and (f)  $\langle type \rangle$  includes integers and Booleans.

In addition, for Boolean programs, we assume: (a) all variables and formal parameters are of  $\langle type \rangle$  Boolean and (b) all expressions -  $\langle expr \rangle$ ,  $\langle bexpr \rangle$  - are Boolean expressions defined as follows:

where b is a Boolean variable. Thus, a Boolean expression is either a deterministic Boolean expression or the expression \*, which nondeterministically evaluates to true or false<sup>4</sup>. We assume that \* expresses a fair nondeterministic choice, i.e., \* does not permanently evaluate to the same value. We assume that Boolean expressions in assume  $(\langle bexpr \rangle)$  and assert  $(\langle bexpr \rangle)$  statements are always deterministic. Thus, a concrete program contains no nondeterministic expressions, and a Boolean program contains nondeterministic expressions only in the RHS of assignment statements.

Note that the above syntax does not permit return values from procedures. However, return values can be easily modeled using extra global variables. Hence, this syntax simplification does not affect the expressivity of the programming language. Indeed, the above syntax is quite general.

Notation. Let us fix some notation before we proceed. For program  $\mathcal{P}$ , let  $\{F_0,\ldots,F_t\}$  be its set of procedures with  $F_0$  being the main procedure, and let  $GV(\mathcal{P})$  denote the set of global variables. For procedure  $F_i$ , let  $S_i$  and  $\mathcal{L}_i$  denote the sets of statements and locations, respectively, and let  $FV_i$  and  $LV_i$  denote the sets of formal parameters and local variables, respectively, with  $FV_i \subseteq LV_i$ . Let  $V(\mathcal{P}) = GV(\mathcal{P}) \cup \bigcup_{i=1}^t LV_i$  denote the set of variables of  $\mathcal{P}$ , and  $\mathcal{L}(\mathcal{P}) = \bigcup_{i=1}^t \mathcal{L}_i$  denote the set of locations of  $\mathcal{P}$ . For a location  $\ell$  within a procedure  $F_i$ , let  $inscope(\ell) = GV(\mathcal{P}) \cup LV_i$  denote the set of all variables in  $\mathcal{P}$  whose scope includes  $\ell$ . We denote by  $stmt(\ell)$ ,  $formal(\ell)$  and  $local(\ell)$  the statement at  $\ell$  and the sets of formal parameters and local variables of the procedure containing  $\ell$ , respectively. We denote by  $entry_i \in \mathcal{L}_i$  the location of the first statement in  $F_i$ . When

<sup>&</sup>lt;sup>4</sup> In practice, a nondeterministic Boolean expression is any Boolean expression containing \* or the expression  $\mathsf{choose}(e_1, e_2)$ , with  $e_1$ ,  $e_2$  being deterministic Boolean expressions (if  $e_1$  is  $\mathsf{true}$ ,  $\mathsf{choose}(e_1, e_2)$  evaluates to  $\mathsf{true}$ , else if  $e_2$  is  $\mathsf{true}$ ,  $\mathsf{choose}(e_1, e_2)$  evaluates to  $\mathsf{false}$ , else  $\mathsf{choose}(e_1, e_2)$  evaluates to \*). While we handle arbitrary nondeterministic Boolean expressions in our prototype tool (see Sec. 6), we only consider \* expressions in our exposition for simplicity.

the context is clear, we simply use V,  $\mathcal{L}$  instead of  $V(\mathcal{P})$ ,  $\mathcal{L}(\mathcal{P})$  etc.

Transition Graphs. In addition to a textual representation, we will often find it convenient to use a transition graph representation of programs. The transition graph representation of  $\mathcal{P}$ , denoted  $\mathcal{G}(\mathcal{P})$ , comprises a set of labeled, rooted, directed graphs  $\mathcal{G}_0, \ldots, \mathcal{G}_t$ , which have exactly one node, err, in common. Informally, the  $i^{th}$  graph  $\mathcal{G}_i$  captures the flow of control in procedure  $F_i$  with its nodes and edges labeled by locations and corresponding statements of  $F_i$ , respectively. To be more precise,  $\mathcal{G}_i = (N_i, Lab_i, E_i)$ , where the set of nodes  $N_i$ , given by  $\mathcal{L}_i \cup exit_i \cup err$ , includes a unique entry node  $entry_i$ , a unique exit node  $exit_i$  and the error node err, the set of labeled edges  $E_i \subseteq N_i \times Lab_i \times N_i$  is defined as follows: for all  $\ell, \ell' \in N_i$ ,  $(\ell, \varsigma, \ell') \in E_i$  iff:

- $stmt(\ell)$  is an assignment, assume (g) or call  $F(e_1, \ldots, e_k)$  statement,  $\ell'$  is the next sequential location<sup>5</sup> in  $F_i$  after  $\ell$  and  $\varsigma = stmt(\ell)$ , or,
- $stmt(\ell)$  is a skip statement and either (a)  $stmt(\ell)$  is the last statement in the loop body of a statement  $\ell'$ : while (g) and  $\varsigma$  is the empty label, or, (b)  $\ell'$  is the next sequential location in  $F_i$  after  $\ell$  and  $\varsigma$  is the empty label, or,
- $stmt(\ell)$  is if (g), and either (a)  $\ell'$ , denoted  $Tsucc(\ell)$ , is the location of the first statement in the then branch and  $\varsigma = \mathtt{assume}\,(g)$ , or, (b)  $\ell'$ , denoted  $Fsucc(\ell)$ , is the location of the first statement in the else branch and  $\varsigma = \mathtt{assume}\,(\neg g)$ , or,
- $stmt(\ell)$  is while (g), and either (a)  $\ell'$ , denoted  $Tsucc(\ell)$ , is the location of the first statement in the while loop body and  $\varsigma = \mathtt{assume}(g)$ , or, (b)  $\ell'$ , denoted  $Fsucc(\ell)$ , is the next sequential location in  $F_i$  after the end of the while loop body and  $\varsigma = \mathtt{assume}(\neg g)$ , or,
- $stmt(\ell)$  is assert (g), and either  $\ell'$ , denoted  $Tsucc(\ell)$ , is the next sequential location in  $F_i$  after  $\ell$  and  $\varsigma$  is the empty label, or, (b)  $\ell'$ , denoted  $Fsucc(\ell)$ , is the node err and  $\varsigma$  is the empty label, or,
- $stmt(\ell)$  is a goto statement that includes the label  $\ell'$ , and  $\varsigma$  is the empty label, or,
- $stmt(\ell)$  is a return statement,  $\ell' = exit_i$  and  $\varsigma = return$ .

Let  $succ(\ell)$  denote the set  $\{\ell' : (\ell, \varsigma, \ell') \in E_i\}$  for some  $i \in [0, t]$ . A path  $\pi$  in  $\mathcal{G}_i$  is a sequence of labeled connected edges; with some overloading of notation, we denote the sequence of statements labeling the edges in

<sup>&</sup>lt;sup>5</sup> The next sequential location of the last statement in the then or else branch of a conditional statement is the location following the conditional statement. The next sequential location of the last statement in the main procedure is *exit*<sub>0</sub>.

 $\pi$  as  $stmt(\pi)$ . Not that every node in  $\mathcal{G}_i$  is on some path between  $entry_i$  and  $exit_i$ .

The transition graph of Boolean programs can be defined similarly (see Fig. 1c). The main modification is as follows. In defining the set of labeled edges  $E_i$  of graph  $\mathcal{G}_i = (N_i, Lab_i, E_i)$  in the transition graph representation  $\mathcal{G}(\mathcal{B})$  of  $\mathcal{B}$ , for  $\ell \in N_i$  with  $stmt(\ell)$  given by if (\*) or while (\*),  $Tsucc(\ell)$ ,  $Fsucc(\ell)$  are defined as above, but the labels  $\varsigma_1$ ,  $\varsigma_2$  in  $(\ell, \varsigma_1, Tsucc(\ell))$ ,  $(\ell, \varsigma_2, Fsucc(\ell))$  are each set to assume (true).

**Program Semantics and Correctness.** Given a set  $V_s \subseteq V$  of variables, a valuation  $\Omega$  of  $V_s$  is a function that maps each variable in  $V_s$  to an appropriate value of its type.  $\Omega$  can be naturally extended to map well-typed expressions over variables to values.

An operational semantics can be defined for our programs by formalizing the effect of each type of program statement on a program configuration. A configuration  $\eta$  of a program  $\mathcal{P}$  is a tuple of the form  $(\ell, \Omega, \zeta)$ , where where  $\ell \in \bigcup_{i=0}^t N_i$ ,  $\Omega$  is a valuation of the variables in  $inscope(\ell)^6$ , and  $\zeta$  is a stack of elements. Each element of  $\zeta$  is of the form  $(\widetilde{\ell}, \widetilde{\Omega})$ , where  $\widetilde{\ell} \in \mathcal{L}_i$  for some i and  $\widetilde{\Omega}$  is a valuation of the variables in  $local(\widetilde{\ell})$ . A program state is a pair of the form  $(\ell, \Omega)$ , where  $\ell$  and  $\Omega$  are as defined above; thus a program state excludes the stack contents. A configuration  $(\ell, \Omega, \zeta)$  of  $\mathcal{P}$  is called an initial configuration if  $\ell = entry_0$  is the entry node of the main procedure and  $\zeta$  is the empty stack. We use  $\eta \leadsto \eta'$  to denote that  $\mathcal{P}$  can transition from configuration  $\eta = (\ell, \Omega, \zeta)$  to configuration  $\eta' = (\ell', \Omega', \zeta')$ ; the transitions rules for each type of program statement at  $\ell$  and for exit nodes of procedures are presented in Fig. 3.

Let us take a closer look at the last two transition rules in Fig. 3 - the only transition rules that affect the stack contents. Upon execution of the statement call  $F_j(e_1, \ldots, e_k)$  in program configuration  $(\ell, \Omega, \zeta)$ , control moves to the entry node of the called procedure  $F_j$ ; the new valuation  $\Omega'$  of program variables is constrained to agree with  $\Omega$  on the values of all global variables, and maps the formal parameters of  $F_j$  to the values of the actual arguments according to  $\Omega$ ; finally, the element  $(succ(\ell), \Delta)$  is pushed onto the stack, where  $succ(\ell)$  is the location to which control returns after  $F_j$  completes execution and  $\Delta$  is a valuation of all local variables of the calling procedure, as recorded in  $\Omega$ . The last transition rule in Fig. 3 captures the return of control to the calling procedure, say  $F_i$ , after completion of execution of a called procedure, say  $F_j$ ; the top of the stack element  $(\ell_{ret}, \Delta)$  is removed and is used to retrieve the location

<sup>&</sup>lt;sup>6</sup> For  $\ell = exit_i$ ,  $inscope(\ell) = GV \cup LV_i$ , and for  $\ell = err$ ,  $inscope(\ell)$  is undefined.

Cases		$\ (\ell,\Omega,\zeta)\leadsto(\ell',\Omega',\zeta')$ if:							
	skip	$\ell' = succ(\ell), \ \Omega' = \Omega \text{ and } \zeta' = \zeta$							
$stmt(\ell)$ :	return	$  \epsilon - succ(\epsilon),   \epsilon - s  $ and $\zeta - \zeta$							
	$ extstyle \operatorname{ extstyle goto} \ell_1 \operatorname{ extstyle or} \ldots \operatorname{ extstyle or} \ell_n$	$ \begin{aligned} & \ell' \in succ(\ell), \ \Omega' = \Omega \ \text{and} \ \zeta' = \zeta \\ & \Omega(g) = true, \ \ell' = succ(\ell), \ \Omega' = \Omega \ \text{and} \ \zeta' = \zeta \end{aligned} $							
	${\tt assume}g$								
	if g	either $\Omega(g) = \text{true}, \ \ell' = Tsucc(\ell), \ \Omega' = \Omega \text{ and } \zeta' = \zeta, \text{ or,}$							
	t while g	$\Omega(g) = \mathtt{false}, \ \ell' = Fsucc(\ell), \ \Omega' = \Omega \ \mathrm{and} \ \zeta' = \zeta$							
	$\mathtt{assert}g$	either $\Omega(g) = \text{true}, \ \ell' = Tsucc(\ell), \ \Omega' = \Omega \text{ and } \zeta' = \zeta, \text{ or,}$							
	$\Omega(g) = \mathtt{false} \ \mathrm{and} \ \ell' = Fsucc(\ell) = err$								
	$v_1,\ldots,v_m:=$	$\ell' = succ(\ell),$							
	$ e_1,\ldots,e_m   \forall i \in [1,m] : \Omega'(v_i) = \Omega(e_i),$								
		$\forall v \notin \{v_1, \dots, v_m\} : \Omega'(v) = \Omega(v) \text{ and }$							
		$ \zeta' = \zeta $							
	$call F_j(e_1,\ldots,e_k)$	$\ell' = entry_j,$							
		$\forall v_i \in formal(\ell') : \Omega'(v_i) = \Omega(e_i),$							
		$\forall v \in GV(\mathcal{P}) : \Omega'(v) = \Omega(v) \text{ and }$							
		$ \zeta' = (succ(\ell), \Delta).\zeta$ , where $\forall v \in local(\ell) : \Delta(v) = \Omega(v)$							
$\ell$ :	$exit_j$	$ \ell' = \ell_{ret},$							
		$\forall v \in local(\ell') : \Omega'(v) = \Delta(v),$							
		$\forall v \in GV(\mathcal{P}) : \Omega'(v) = \Omega(v) \text{ and}$							
		$\left  \zeta = (\ell_{ret}, \Delta).\zeta' \right $							

Fig. 3: Transition rules for  $(\ell, \Omega, \zeta) \rightsquigarrow (\ell', \Omega', \zeta')$ .

 $\ell_{ret}$  of  $F_i$  to which control must return as well the valuation  $\Delta$  of the local variables of  $F_i$ ; the new valuation  $\Omega'$  of program variables is constrained to agree with  $\Omega$  on the values of all global variables, and to agree with  $\Delta$  on the values of all local variables of  $F_i$ .

An execution path of program  $\mathcal{P}$  is a sequence of configurations,  $\eta \leadsto \eta' \leadsto \eta'' \leadsto \ldots$ , obtained by repeated application of the transition rules from Fig. 3, starting from an initial configuration  $\eta$ . Note that an execution path may be finite or infinite. The last configuration  $(\ell, \Omega, \zeta)$  of a finite execution path may either be a terminating configuration with  $\ell = exit_0$ , or an error configuration with  $\ell = err$ , or a stuck configuration with  $\ell \neq exit_0$ . An execution path ends in a stuck configuration  $\eta$  if none of the transition rules from Fig. 3 are applicable to  $\eta$ . In particular, notice that notice that a transition from configuration  $(\ell, \Omega, \zeta)$  with  $stmt(\ell)$  being assume (g) is defined only when  $\Omega(g) = \texttt{true}$ .

The operational semantics of Boolean programs can be defined similarly. The main modifications are as follows. For  $stmt(\ell)$  given by if(\*) or while (\*), we say  $(\ell, \Omega, \zeta) \leadsto (\ell', \Omega', \zeta')$  if  $\ell' \in succ(\ell)$ ,  $\Omega' = \Omega$  and  $\zeta' = \zeta$ . For  $stmt(\ell)$  given by the assignment statement  $b_1, \ldots, b_j, \ldots, b_m := e_1, \ldots, *, \ldots, e_m$ , we say  $(\ell, \Omega, \zeta) \leadsto (\ell', \Omega', \zeta')$  if  $\ell' = succ(\ell), \zeta' = \zeta$ ,  $\forall i \in \{1, \ldots, j-1, j+1, \ldots, m] : \Omega'(b_i) = \Omega(e_i), \forall v \notin \{b_1, \ldots, b_m\}$ :

 $\Omega'(v) = \Omega(v)$ , and either  $\Omega'(b_j) = \text{true}$  or  $\Omega'(b_j) = \text{false}$ . This transition rule can be extended to handle other scenarios such as assignment statements with multiple \* expressions in the RHS, and call statements with \* expressions in the actual arguments.

An assertion in program  $\mathcal{P}$ , is a statement of the form  $\ell$ : assert (g), with g being a quantifier-free, first order expression representing the expected values of the program variables in  $inscope(\ell)$  at  $\ell$ . We will use the term assertion to denote both the statement  $\ell$ : assert (g) as well as the expression g. We say a program configuration  $(\ell, \Omega, \zeta)$  satisfies an assertion, if the embedded variable valuation  $\Omega$  satisfies the same.

Given a program  $\mathcal{P}$  annotated with a set of assertions,  $\mathcal{P}$  is partially correct iff every finite execution path of  $\mathcal{P}$  ends in a terminating configuration. We say  $\mathcal{P}$  is totally correct iff every execution path is finite and ends in a terminating configuration. In what follows, we assume that all programs are annotated with a set of assertions.

In specifying correctness for Boolean programs, we interpret the nondeterminism in them as Dijkstra's demonic nondeterminism [13]. Given a program  $\mathcal{B}$  annotated with a set of assertions,  $\mathcal{B}$  is partially correct iff every finite execution path of  $\mathcal{B}$  ends in a terminating configuration for all nondeterministic choices that  $\mathcal{B}$  might make.  $\mathcal{B}$  is totally correct iff every execution path is finite and ends in a terminating configuration, for all nondeterministic choices that  $\mathcal{B}$  might make.

Unless otherwise specified, an *incorrect* program is one that is not partially correct.

Remark: While we found it convenient to define Boolean programs as we did above, it is worth noting that formalisms such as pushdown systems [8] and recursive state machines [1] are equivalent to Boolean programs.

## 3 Cost-aware Program Repair

#### 3.1 The Problem

Let  $\Sigma$  denote the set of statement types in program  $\mathcal{P}$ . As can be seen from Fig. 1c, it suffices to consider the set of statement types given by  $\Sigma = \{ \mathtt{skip}, \mathtt{assign}, \mathtt{assume}, \mathtt{assert}, \mathtt{call}, \mathtt{return}, \mathtt{goto} \}$ . Given a statement  $\varsigma$ , let  $\tau(\varsigma)$  be an element of  $\Sigma$  denoting the statement type of  $\varsigma$ . Let  $\mathcal{U} = \{u_0, u_1, \ldots, u_d\}$  be a set of permissible, statement-level update schemas:  $u_0 = id$  is the identity update schema that maps every statement to itself, and  $u_i, i \in [1, m]$ , is a function  $\sigma \mapsto \widehat{\sigma}, \sigma, \widehat{\sigma} \in \Sigma \setminus \{\mathtt{assert}\}$ , that maps a statement type to a statement type. For each update schema u, given by  $\sigma \mapsto \widehat{\sigma}$ , we say u can be applied to statement  $\varsigma$  to get statement  $\widehat{\varsigma}$  if

 $\tau(\varsigma) = \sigma; \ \tau(\widehat{\varsigma})$  is then given by  $\widehat{\sigma}$ . For example, u, given by assign  $\mapsto$  assign, can be applied to the assignment statement  $\ell: x := y$  to get other assignment statements such  $\ell: x := x + y, \ \ell: y := x + 1$  etc. Notice that update schemas in  $\mathcal{U}$  do not affect the label of a statement, and that we do not permit any modification of an assert statement. In this paper, we fix the following set of permissible update schemas for programs:

$$\mathcal{U} = \{id, \mathtt{assign} \mapsto \mathtt{assign}, \mathtt{assign} \mapsto \mathtt{skip}, \mathtt{assume} \mapsto \mathtt{assume}, \quad (1)$$

$$\mathtt{call} \mapsto \mathtt{call}, \mathtt{call} \mapsto \mathtt{skip}\}.$$

We extend the notion of a statement-level update to a program-level update as follows. For programs  $\mathcal{P}$ ,  $\widehat{\mathcal{P}}$ , let the respective sets of locations be  $\mathcal{L}$ ,  $\widehat{\mathcal{L}}$  and let  $stmt(\ell)$ ,  $\widehat{stmt}(\ell)$  denote the respective statements at location  $\ell$ . Let  $\mathbb{R}_{\mathcal{U},\mathcal{L}}: \mathcal{L} \mapsto \mathcal{U}$  be a function that maps each location of  $\mathcal{P}$  to an update schema in  $\mathcal{U}$ . We say  $\widehat{\mathcal{P}}$  is a  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$ -update of  $\mathcal{P}$  iff  $\mathcal{L} = \widehat{\mathcal{L}}$  and for each  $\ell \in \mathcal{L}$ ,  $\widehat{stmt}(\ell)$  is obtained by applying  $\mathbb{R}_{\mathcal{U},\mathcal{L}}(\ell)$  on  $stmt(\ell)$ .

Let  $c_{\mathcal{U},\mathcal{L}}: \mathcal{U} \times \mathcal{L} \to \mathbb{N}$  be a cost function that maps a tuple, consisting of a statement-level update schema u and a location  $\ell$  of  $\mathcal{P}$ , to a certain cost. Thus,  $c_{\mathcal{U},\mathcal{L}}(u,\ell)$  is the cost of applying update schema u to the  $stmt(\ell)$ . We impose an obvious restriction on  $c_{\mathcal{U},\mathcal{L}}: \forall \ell \in \mathcal{L}: c_{\mathcal{U},\mathcal{L}}(id,\ell) = 0$ . Since we have already fixed the set  $\mathcal{U}$  and the set  $\mathcal{L}$  of locations of program  $\mathcal{P}$  (or equivalently, of Boolean program  $\mathcal{B}$ ), we henceforth use c,  $\mathbb{R}$  instead of  $c_{\mathcal{U},\mathcal{L}}$ ,  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$ , respectively, The total cost,  $Cost_c(\mathbb{R})$ , of performing a  $\mathbb{R}$ -update of  $\mathcal{P}$  is given by  $\sum_{\ell \in \mathcal{L}} c(\mathbb{R}(\ell), \ell)$ .

Given an incorrect concrete program  $\mathcal{P}$  annotated with assertions, a cost function c and a repair budget  $\delta$ , the goal of cost-aware program repair is to compute  $\widehat{\mathcal{P}}$  such that:

- 1.  $\widehat{\mathcal{P}}$  is totally correct, and,
- 2. there exists  $\mathbb{R}$ :
  - (a)  $\widehat{\mathcal{P}}$  is some  $\mathbb{R}$ -update of  $\mathcal{P}$ , and
  - (b)  $Cost_c(\mathbb{R}) \leq \delta$ .

If there exists such a  $\widehat{\mathcal{P}}$ , we say  $\widehat{\mathcal{P}}$  is a  $(\mathcal{U}, c, \delta)$ -repair of  $\mathcal{P}$ .

In addition to the above problem, we propose another problem as follows. Let  $\mathcal{T} = \{\mathcal{T}_1, \dots, \mathcal{T}_h\}$  be a set of templates or grammars, each representing a syntactical restriction for the modified expressions in  $\widehat{\mathcal{P}}$ . The syntax of an example template, say  $\mathcal{T}_1$ , defining Boolean-valued linear arithmetic expressions over the program variables, denoted  $\langle blaexpr \rangle$ , is shown below:

In the above, const and var denote integer-valued or real-valued constants and program variables, respectively. Expressions such as  $v_1 + 2 \times v_2 \leq v_3$ ,  $(v_1 < v_2) \wedge (v_3 = 3)$  etc., that satisfy the syntactical requirements of the template  $\mathcal{T}_1$ , are said to belong to the language of the template, denoted  $L(\mathcal{T}_1)$ .

Let  $\mathbb{E}_{\mathcal{T},\mathcal{L}}: \mathcal{L} \to \mathcal{T}$ , be a function that maps each location of  $\mathcal{P}$  to a template in  $\mathcal{T}$ . Let  $\mathcal{E}(stmt(\ell))$  denote a set that includes all expressions in certain statement types and be defined as follows: if  $stmt(\ell)$  is  $v_1, \ldots, v_m := e_1, \ldots, e_m$ ,  $\mathcal{E}(stmt(\ell)) = \{e_1, \ldots, e_m\}$ , else if  $stmt(\ell)$  is  $call F_j(e_1, \ldots, e_k), \mathcal{E}(stmt(\ell)) = \{e_1, \ldots, e_k\}$ , else if  $stmt(\ell)$  is assume  $(g), \mathcal{E}(stmt(\ell)) = \{g\}$  else,  $\mathcal{E}(stmt(\ell))$  is the empty set.

Given  $\mathbb{E}_{\mathcal{T},\mathcal{L}}$ , along with (incorrect)  $\mathcal{P}$ , c and  $\delta$ , the goal of template-based, cost-aware program repair is to compute  $\widehat{\mathcal{P}}$  such that:

- 1.  $\widehat{\mathcal{P}}$  is correct, and,
- 2. there exists  $\mathbb{R}$ :
  - (a)  $\widehat{\mathcal{P}}$  is some  $\mathbb{R}$ -update of  $\mathcal{P}$ ,
  - (b)  $Cost_c(\mathbb{R}) \leq \delta$ , and
  - (c) for each location  $\ell$ :  $\mathbb{R}(\ell) \neq id \Rightarrow \forall e \in \mathcal{E}(\widehat{stmt}(\ell)) : e \in L(\mathbb{E}_{\mathcal{T},\mathcal{L}}(\ell)).$

We conjecture that an insightful choice for the cost function c can help prune the search space for repaired programs and help incorporate expert user intuition and intent in automatic program repair. Exploration of suitable cost-functions is beyond the scope of this dissertation. For now, we would only like to emphasize that our cost-function is quite flexible, and can be used to constrain the computation of  $\widehat{\mathcal{P}}$  in diverse ways. For example, the user can choose to search for  $\widehat{\mathcal{P}}$  that differs from  $\mathcal{P}$  in at most  $\delta$  statements by defining c as:

$$\forall \ell \in \mathcal{L}, u \in \mathcal{U} : u \neq id \Rightarrow c_{\mathcal{U},\mathcal{L}}((id,\ell)) = 1.$$

Or, the user can choose to search for  $\widehat{\mathcal{P}}$  that does not modify any statement within a *trusted* procedure  $\mathcal{P}_i$  by defining c as:

$$\forall \ell \in \mathcal{L}, u \in \mathcal{U} : u \neq id \land \ell \in \mathcal{L}_i \implies c_{\mathcal{U}, \mathcal{L}}((u, \ell)) = N \text{ and}$$
$$u \neq id \land \ell \notin \mathcal{L}_i \implies c_{\mathcal{U}, \mathcal{L}}((u, \ell)) = 1,$$

where N is some prohibitively large number. Or, the user can choose to favor the application of a particular update schema, say  $u_1$ , over others by defining c as:

$$\forall \ell \in \mathcal{L}, u \in \mathcal{U} : u \neq id \text{ and } u \neq u_1 \Rightarrow c_{\mathcal{U},\mathcal{L}}((u,\ell)) = N \text{ and } u = u_1 \Rightarrow c_{\mathcal{U},\mathcal{L}}((u,\ell)) = 1,$$

where N is some prohibitively large number, and so on. Similarly, insightful templates choices can help guide the search for repairs based on user input.

#### 3.2 Solution Overview

We present a predicate abstraction-based framework for cost-aware program repair. Recall that we had fixed a Boolean program  $\mathcal{B}$  in Sec. 2 such that  $\mathcal{B}$  is obtained from  $\mathcal{P}$  via iterative predicate abstraction-refinement and  $\mathcal{B}$  exhibits a non-spurious counterexample path. In addition to  $\mathcal{P}$ ,  $c_{\mathcal{U},\mathcal{L}}$  and  $\delta$ , our framework requires: the Boolean program  $\mathcal{B}$  and the corresponding function  $\gamma$  that maps Boolean variables to their respective predicates. The computation of a suitable repaired program  $\widehat{\mathcal{P}}$  involves two main steps:

- 1. Cost-aware repair of  $\mathcal{B}$  to obtain  $\widehat{\mathcal{B}}$ , and
- 2. Concretization of  $\widehat{\mathcal{B}}$  to obtain  $\widehat{\mathcal{P}}$ .

The problem of cost-aware repair of a Boolean program  $\mathcal{B}$  can be defined in a manner identical to cost-aware repair of concrete program  $\mathcal{P}$ . Concretization of  $\widehat{\mathcal{B}}$  involves mapping each statement of  $\widehat{\mathcal{B}}$  that has been modified by  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$  into a corresponding statement of  $\mathcal{P}$ , using the function  $\gamma$ . For template-based repair of  $\mathcal{P}$ , the concretization needs to ensure that the modified expressions of  $\mathcal{P}$  meet the syntactic requirements of the corresponding templates. In the following sections, we describe these two steps in detail.

## 4 Cost-aware Repair of Boolean Programs

Our solution to cost-aware repair of a Boolean program  $\mathcal{B}$  relies on automatically computing *inductive assertions*, along with a suitable  $\widehat{\mathcal{B}}$ , that together certify the partial correctness of  $\widehat{\mathcal{B}}$ . In what follows, we explain our adaptation of the method of inductive assertions [14,22] for cost-aware program repair.

Cut-set. Let  $N = N_0 \cup ... \cup N_t$  be the set of nodes in  $\mathcal{G}(\mathcal{B})$ , the transition graph representation of  $\mathcal{B}$ . We define a cut-set  $\Lambda \subseteq N$  as a set of nodes, called *cut-points*, such that for every  $i \in [0,t]$ : (a)  $entry_i, exit_i \in \Lambda$ , (b) for every edge  $(\ell, \varsigma, \ell') \in E_i$  where  $\varsigma$  is a procedure call,  $\ell, \ell' \in \Lambda$ , (c) for every edge  $(\ell, \varsigma, \ell') \in E_i$  where  $\varsigma$  is an assert statement,  $\ell, \ell' \in \Lambda$ , and (d) every cycle in  $\mathcal{G}_i$  contains at least one node in  $\Lambda$ . A pair of cut-points  $\ell, \ell'$  in some  $\mathcal{G}_i$  is said to be *adjacent* if every path from  $\ell$  to  $\ell'$  in  $\mathcal{G}_i$  contains no other cut-point. A *verification path* is any path from a cut-point to an adjacent cut-point; note that there can be more than one verification path between two adjacent cut-points.

*Example*: The set  $\{\ell_1, \ell_2, \ell_7, exit\}$  is a valid cut-set for Boolean program  $\mathcal{B}$  in Fig. 1. The verification paths in  $\mathcal{G}(\mathcal{B})$  corresponding to this cut-set are as follows:

$$\begin{array}{l} 1. \ \ \ell_1 \xrightarrow{\mathtt{assume} \ (b_2)} \ \ell_2 \\ 2. \ \ \ell_2 \xrightarrow{\mathtt{assume} \ (\mathtt{T})} \ \ell_3 \xrightarrow{b_0,b_1,b_2:=\,*,*,*} \ell_4 \to \ell_2 \\ 3. \ \ \ell_2 \xrightarrow{\mathtt{assume} \ (\mathtt{T})} \ell_0 \to \ell_7 \\ 4. \ \ \ell_1 \xrightarrow{\mathtt{assume} \ (\neg b_2)} \ell_5 \xrightarrow{\mathtt{assume} \ (\neg b_1)} \ell_7 \\ 5. \ \ \ell_1 \xrightarrow{\mathtt{assume} \ (\neg b_2)} \ell_5 \xrightarrow{\mathtt{assume} \ (b_1)} \ell_6 \xrightarrow{b_1,b_1,b_2:=\,*,*,*} \ell_7 \\ 6. \ \ \ell_7 \xrightarrow{\mathtt{assert} \ (\neg b_0)} exit^7 \\ \end{array}$$

Inductive assertions. We denote an inductive assertion associated with cut-point  $\ell$  in  $\Lambda$  by  $\mathcal{I}_{\ell}$ . Informally, an inductive assertion  $\mathcal{I}_{\ell}$  has the property that whenever control reaches  $\ell$  in any program execution,  $\mathcal{I}_{\ell}$  must be true for the current values of the variables in scope. Thus, for a Boolean program, an inductive assertion  $\mathcal{I}_{\ell}$  is in general a Boolean formula over the variables whose scope includes  $\ell$ . To be precise,  $\mathcal{I}_{\ell}$  is a Boolean formula over  $V_s[\ell]$ , where  $V_s[\ell]$  denotes an  $\ell^{th}$  copy of the subset  $V_s$  of the program variables, with  $V_s = GV \cup formal(\ell)$  if  $\ell \in \{exit_1, \ldots, exit_t\}$ , and  $V_s = inscope(\ell)$  otherwise. Thus, except for the main procedure, the inductive assertions at the exit nodes of all procedures exclude the local variables declared in the procedure. Let  $\mathcal{I}_{\Lambda}$  denote the set of inductive assertions associated with all the cut-points in  $\Lambda$ .

<sup>&</sup>lt;sup>7</sup> Labeling this edge with assert  $(\neg b_0)$  is a slight abuse of the semantics of an assert statement. Our justification is that the constraints formulated later in this section require that the assertion is **true** whenever control reaches location  $\ell_7$  in an execution path.

Verification conditions. A popular approach to verification of sequential, imperative programs is to compute  $\mathcal{I}_{\Lambda}$  such that  $\mathcal{I}_{\Lambda}$  satisfies a set of constraints called *verification conditions*. Let  $\pi$  be a verification path in  $\mathcal{G}_i$ , from cut-point  $\ell$  to adjacent cut-point  $\ell'$ . The verification condition corresponding to  $\pi$ , denoted  $VC(\pi)$ , is essentially the Hoare triple  $\langle \mathcal{I}_{\ell} \rangle stmt(\pi) \langle \mathcal{I}_{\ell'} \rangle$ , where  $stmt(\pi)$  is the sequence of statements labeling  $\pi$ . When  $\mathcal{I}_{\ell}$ ,  $\mathcal{I}_{\ell'}$  are unknown,  $VC(\pi)$  can be seen as a constraint encoding all possible solutions for  $\mathcal{I}_{\ell}$ ,  $\mathcal{I}_{\ell'}$  such that: every program execution along path  $\pi$ , starting from a set of variable valuations satisfying  $\mathcal{I}_{\ell'}$ . Note that the definitions of cut-sets and adjacent cut-points ensure that we do not have to worry about non-termination along verification paths.

The Hoare triple  $\langle \mathcal{I}_{\ell} \rangle stmt(\pi) \langle \mathcal{I}_{\ell'} \rangle$  can be defined using weakest preconditions or strongest postconditions. In this paper, as we will see shortly, we find it convenient to use strongest postconditions.

Program verification using the inductive assertions method. Given a program  $\mathcal{B}$  annotated with assertions, and a set  $\Lambda$  of cut-points,  $\mathcal{B}$  is partially correct if one can compute a set  $\mathcal{I}_{\Lambda}$  of inductive assertions such that: for every verification path  $\pi$  between every pair  $\ell, \ell'$  of adjacent cut-points in  $\mathcal{G}$ ,  $VC(\pi)$  is valid.

Cost-aware repairability conditions for partial correctness. Let  $\mathcal{C}: \bigcup_{i=0}^t N_i \to \mathbb{N}$  be a function mapping locations to costs. We find it convenient to use  $\mathcal{C}_{\ell}$  to denote the value  $\mathcal{C}(\ell)$  at location  $\ell$ . We set  $\mathcal{I}_{entry_0} = \texttt{true} \text{ and } \mathcal{C}_{\ell} = 0 \text{ if } \ell \in \{entry_0, \dots, entry_t\}. \text{ Informally, } \mathcal{C}_{\ell} \text{ with }$  $\ell \in N_i$  can be seen as recording the cumulative cost of applying a sequence of update schemas to the statements in procedure  $F_i$  from location  $entry_i$ to  $\ell$ . Thus, for a specific update function  $\mathbb{R}$  with cost function c,  $\mathcal{C}_{exit_0}$ records the total cost  $Cost_c(\mathbb{R})$  of performing an  $\mathbb{R}$ -update of the program. Given a verification path  $\pi$  in  $\mathcal{G}_i$ , from cut-point  $\ell$  to adjacent cut-point  $\ell'$ , we extend the definition of  $VC(\pi)$  to define the cost-aware repairability condition corresponding to  $\pi$ , denoted  $CRC(\pi)$ .  $CRC(\pi)$  can be seen as a constraint encoding all possible solutions for inductive assertions  $\mathcal{I}_{\ell}$ ,  $\mathcal{I}_{\ell'}$  and update functions  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$ , along with associated functions  $\mathcal{C}$ , such that: every program execution that proceeds along path  $\pi$  via statements modified by applying the update schemas in  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$ , starting from a set of variable valuations satisfying  $\mathcal{I}_{\ell}$ , terminates in a set of variable valuations satisfying  $\mathcal{I}_{\ell'}$ , for all nondeterministic choices that the program might make along  $\pi$ .

Before we proceed, recall that  $\mathcal{I}_{\ell}$  is a Boolean formula over  $V_s[\ell]$ , with  $V_s = GV \cup formal(\ell)$  if  $\ell \in \{exit_1, \ldots, exit_t\}$ , and  $V_s = inscope(\ell)$  otherwise. Thus, for all locations  $\lambda \neq \ell'$  in verification path  $\pi$  from  $\ell$  to  $\ell'$ ,  $V_s = inscope(\lambda)$ . In what follows, the notation  $\llbracket u \rrbracket (stmt(\lambda))$  represents the class of statements that may be obtained by applying update schema u on  $stmt(\lambda)$ , and is defined for our permissible update schemas in Fig. 4. Here,  $f, f_1, f_2$  etc. denote unknown Boolean expressions<sup>8</sup>, over the variables in  $inscope(\lambda)$ . Note that the update schema  $assign \mapsto assign$ , modifies any assignment statement, to one that assigns unknown Boolean expressions to all variables in  $V_s$ .

u	$\llbracket u \rrbracket (stmt(\lambda))$
id	$ stmt(\lambda) $
$\mathtt{assign} \mapsto \mathtt{skip}$	skip
$\mathtt{assume} \mapsto \mathtt{skip}$	skip
$\mathtt{call} \mapsto \mathtt{skip}$	skip
$ ext{assign} \mapsto  ext{assign}$	$ b_1, \dots, b_{ V_s } := f_1, \dots, f_{ V_s }$
	$\mathtt{assume} f$
$\mathtt{call} \mapsto \mathtt{call}$	call $F_j(f_1,\ldots,f_k)$ , where $stmt(\lambda)$ : call $F_j(e_1,\ldots,e_k)$

Fig. 4: Definition of  $[\![u]\!](stmt(\lambda))$ 

We now define  $CRC(\pi)$ . There are three cases to consider.

1.  $stmt(\pi)$  does not contain a procedure call or assert statement: Let  $\mathcal{A}_{\lambda}$  denote an assertion associated with location  $\lambda$  in  $\pi$ .  $CRC(\pi)$  is given by the (conjunction of the) following set of constraints:

$$\mathcal{A}_{\ell} = \mathcal{I}_{\ell}$$

$$\mathcal{A}_{\ell'} \Rightarrow \mathcal{I}_{\ell'}$$

$$\bigwedge_{\ell \leq \lambda \prec \ell'} \bigwedge_{u \in \mathcal{U}_{stmt(\lambda)}} \mathbb{R}(\lambda) = u \Rightarrow \mathcal{C}_{\lambda'} = \mathcal{C}_{\lambda} + c(u, \lambda) \wedge$$

$$\mathcal{A}_{\lambda'} = sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda}).$$
(2)

In the above,  $\prec$  denotes the natural ordering over the sequence of locations in  $\pi$  with  $\lambda$ ,  $\lambda'$  being consecutive locations, i.e.,  $\lambda' \in succ(\lambda)$ .

<sup>&</sup>lt;sup>8</sup> To keep our exposition simple, we assume that these unknown Boolean expressions are deterministic. However, in our prototype tool (see Sec. 6), we also have the ability to compute modified statements with nondeterministic expressions such as \* or  $choose(f_1, f_2)$ .

The notation  $\mathcal{U}_{stmt(\lambda)} \subseteq \mathcal{U}$  denotes the set of all update schemas in  $\mathcal{U}$  which may be applied to  $stmt(\lambda)$ . The notation  $sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$  denotes the strongest postcondition of the assertion  $\mathcal{A}_{\lambda}$  over the class of statements  $\llbracket u \rrbracket(stmt(\lambda))$ . We define the strongest postcondition using multiple variable copies - a copy  $V_s[\lambda]$  for each location  $\lambda$  in  $\pi$ . Let us assume that  $\mathcal{A}_{\lambda}$  is a Boolean formula of the form<sup>9</sup>:

$$\mathcal{A}_{\lambda} = \rho[\ell, \dot{\lambda}] \wedge \bigwedge_{b \in V_s} b[\lambda] = \xi[\dot{\lambda}], \tag{3}$$

where  $\lambda$ ,  $\lambda$  are consecutive locations in  $\pi$  with  $\lambda \in succ(\lambda)$ ,  $\rho[\ell, \lambda]$  is a Boolean expression over all copies  $V_s[\mu]$ ,  $\ell \leq \mu \leq \lambda$ , representing the path condition imposed by the program control-flow, and  $\xi[\ell]$  is a Boolean expression over  $V_s[\lambda]$  representing the  $\lambda^{th}$  copy of each variable b in terms of the  $\lambda^{th}$  copy of the program variables. Note that  $\mathcal{A}_{\ell} = \mathcal{I}_{\ell}$  is of the form  $\rho[\ell]$ .

$[\![u]\!](stmt(\lambda))$	$sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$					
skip	$\rho[\ell, \dot{\lambda}] \wedge \bigwedge_{b \in V_a} b[\lambda'] = b[\lambda]$					
goto	$[\rho_{[c}, \lambda] \land \lambda_{b \in V_s} \circ [\lambda] = \sigma[\lambda]$					
$\mathtt{assume}g$	$g[\lambda] \wedge \rho[\ell, \dot{\lambda}] \wedge \bigwedge_{b \in V_s} b[\lambda'] = b[\lambda]$					
$\mathtt{assume} f$	$f[\lambda] \wedge \rho[\ell, \dot{\lambda}] \wedge \bigwedge_{b \in V_s} b[\lambda'] = b[\lambda]$					
$b_1,\ldots,b_m:=e_1,\ldots,e_m$	$\rho[\ell, \grave{\lambda}] \wedge \bigwedge_{b_i \in V_s, i \in [1, m]} b_i[\lambda'] = e_i[\lambda] \wedge \\ \bigwedge_{b_i \in V_s, i \notin [1, m]} b_i[\lambda'] = b_i[\lambda]$					
$b_1,\ldots,b_{ V_s }:=f_1,\ldots,f_{ V_s }$	$\rho[\ell, \dot{\lambda}] \wedge \bigwedge_{b_i \in V_s} b_i[\lambda'] = f_i[\lambda]$					

Fig. 5: Definition of  $sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$ 

Given  $\mathcal{A}_{\lambda}$  of the form in (3),  $sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$  is defined in Fig. 5. Observe that  $sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$  is a Boolean formula of the same form as (3), over variable copies from  $V_s[\ell]$  to  $V_s[\lambda']$ . For the entries assume g and  $b_1, \ldots, b_m := e_1, \ldots, e_m$ , the expressions  $g, e_1, \ldots, e_m$ 

<sup>&</sup>lt;sup>9</sup> In general,  $\mathcal{A}_{\lambda}$  is a disjunction over Boolean formulas of this form;  $sp(\llbracket u \rrbracket(stmt(\lambda)), \mathcal{A}_{\lambda})$  can then be obtained by computing a disjunction over the strongest postconditions obtained by propagating each such Boolean formula through  $\llbracket u \rrbracket(stmt(\lambda))$  using the rules in Fig. 5.

are known beforehand (these entries correspond to u=id). For the entries assume f and  $b_1,\ldots,b_{|V_s|}:=f_1,\ldots,f_{|V_s|}$ , the expressions f,  $f_1,\ldots,f_{|V_s|}$  are unknown (these entries correspond to u= assume  $\mapsto$  assume and u= assign  $\mapsto$  assign, respectively). Notation such as  $f[\lambda]$  denotes that f is an unknown Boolean expression over  $V_s[\lambda]$ . For nondeterministic expressions in the RHS of an assignment statement  $b_1,\ldots,b_m:=e_1,\ldots,e_m$ , the strongest postcondition is computed as the disjunction of the strongest postconditions over all possible assignment statements obtained by substituting each \* expression with either false or true.

Thus, to summarize, the set of constraints in (2) encodes all  $\mathcal{I}_{\ell}$ ,  $\mathcal{C}_{\ell'}$ ,  $\mathcal{C}_{\ell'}$  and  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$  such that: if  $\mathbb{R}_{\mathcal{U},\mathcal{L}}$  is applied to the sequence of statements  $stmt(\pi)$  to get some modified sequence of statements, say  $\widehat{stmt}(\pi)$ , and program execution proceeds along  $\widehat{stmt}(\pi)$ , then  $\mathcal{I}_{\ell'}$  is the strongest postcondition  $sp(\widehat{stmt}(\pi), \mathcal{I}_{\ell})$ , and  $\mathcal{C}_{\ell'}$  equals the cumulative modification cost, counting up from  $\mathcal{C}_{\ell}$ .

2.  $stmt(\pi)$  contains a procedure call, say call  $F_j(e_1, \ldots, e_k)$ : The path  $\pi$ , given by  $(\ell, \text{call } F_j(e_1, \ldots, e_k), \ell')$ , is a verification path of length 1. Suppose the formal parameters of  $F_j$  are  $b_1, \ldots, b_k$ .  $CRC(\pi)$  is then given by the following set of constraints:

$$\mathbb{R} = id \quad \Rightarrow \mathcal{C}_{\ell'} = \mathcal{C}_{\ell} + \mathcal{C}_{exit_{j}} \wedge \\ \mathcal{I}_{\ell} \Rightarrow \mathcal{I}_{entry_{j}} [\bigwedge_{i \in [1,k]} b_{i}[entry_{j}]/e_{i}[\ell]] \wedge \\ \mathcal{I}_{exit_{j}} [\bigwedge_{i \in [1,k]} b_{i}[exit_{j}]/e_{i}[\ell']] \Rightarrow \mathcal{I}_{\ell'} \\ \mathbb{R} = \text{call} \mapsto \text{skip} \quad \Rightarrow \mathcal{C}_{\ell'} = \mathcal{C}_{\ell} + c(\text{call} \mapsto \text{skip}, \ell) \wedge \\ \mathcal{I}_{\ell'} = \mathcal{I}_{\ell} [\bigwedge_{i \in [1,k]} b_{i}[\ell]/b_{i}[\ell']] \\ \mathbb{R} = \text{call} \mapsto \text{call} \quad \Rightarrow \mathcal{C}_{\ell'} = \mathcal{C}_{\ell} + \mathcal{C}_{exit_{j}} + c(\text{call} \mapsto \text{call}, \ell) \wedge \\ \mathcal{I}_{\ell} \Rightarrow \mathcal{I}_{entry_{j}} [\bigwedge_{i \in [1,k]} b_{i}[entry_{j}]/f_{i}[\ell]] \wedge \\ \mathcal{I}_{exit_{j}} [\bigwedge_{i \in [1,k]} b_{i}[exit_{j}]/f_{i}[\ell']] \Rightarrow \mathcal{I}_{\ell'}$$

For  $\mathbb{R} = id$ , the constraints involve replacing the  $entry_j^{th}$ ,  $exit_j^{th}$  copies of the formal parameters in  $\mathcal{I}_{entry_j}$ ,  $\mathcal{I}_{exit_j}$  with the corresponding actual parameters  $e_1, \ldots, e_k$  expressed over the  $\ell^{th}$ ,  $\ell^{th}$  copies of the

program variables, respectively. For  $\mathbb{R} = \mathtt{call} \mapsto \mathtt{call}$ , a similar substitution is performed, except the actual parameters are unknown expressions  $f_1, \ldots, f_k$ . Finally, for  $\mathbb{R} = \mathtt{call} \mapsto \mathtt{skip}$ , the inductive assertion essentially stays the same, with variable copies appropriately adjusted.  $\mathcal{C}_{\ell'}$  is in general the sum of  $\mathcal{C}_{\ell}$ , the cumulative modification cost  $\mathcal{C}_{exit_j}$  of procedure  $F_j$ , and the cost of applying the update schema in question.

3.  $stmt(\pi)$  contains an assert statement, say assert g: Again,  $\pi$ , given by  $(\ell, \texttt{assert}\, g, \ell')$ , is a verification path of length 1, and  $CRC(\pi)$  is given by the following set of constraints:

$$\mathcal{I}_{\ell} \left[ \bigwedge_{i \in [1, |V_s|]} b_i[\ell] / b_i[tmp] \right] \Rightarrow g \left[ \bigwedge_{i \in [1, |V_s|]} b_i / b_i[tmp] \right] 
\mathcal{I}_{\ell} \left[ \bigwedge_{i \in [1, |V_s|]} b_i[\ell] / b_i[tmp] \right] \Rightarrow \mathcal{I}_{\ell'} \left[ \bigwedge_{i \in [1, |V_s|]} b_i[\ell'] / b_i[tmp] \right] 
\mathcal{C}_{\ell'} = \mathcal{C}_{\ell}.$$

In the above, we uniformly convert the expressions  $\mathcal{I}_{\ell}$ , g and  $\mathcal{I}_{\ell'}$  into expressions over some temporary copy of the program variables to enable checking the implications (informally, these implications are  $\mathcal{I}_{\ell} \Rightarrow g$  and  $\mathcal{I}_{\ell} \Rightarrow \mathcal{I}_{\ell'}$ ).

Cost-aware Boolean program repair. Given a cut-set  $\Lambda$  of  $\mathcal{G}(\mathcal{B})$ , let  $\Pi_{\Lambda}$  be the set of all verification paths between every pair of adjacent cut-points in  $\Lambda$ . Given incorrect program  $\mathcal{B}$  annotated with assertions, the set  $\mathcal{U}$ , cost function c and repair budget  $\delta$ , we say  $\mathcal{B}$  is repairable within budget  $\delta$  if given a cut-set  $\Lambda$  in  $\mathcal{G}$ , one can compute a set  $\mathcal{I}_{\Lambda}$  of inductive assertions, an update function  $\mathbb{R}$ , along with models for all unknown expressions associated with applications of update schemas in  $\mathbb{R}$ , and the valuations of a cumulative-cost-recording function  $\mathcal{C}$  such that:  $\mathcal{C}_{exit_0} \leq \delta$ , for every verification path  $\pi \in \Pi_{\Lambda}$ ,  $CRC(\pi)$  is valid and some other constraints are met. Mathematically,  $\mathcal{B}$  is repairable within budget  $\delta$  if the following formula is true:

$$\exists Unknown \ \forall Var: \ \mathcal{C}_{exit_0} \leq \delta \ \land \ \bigwedge_{\pi \in \Pi_A} CRC(\pi) \ \land \ Assume Constraints,$$

$$\tag{5}$$

where Unknown is the set of all unknowns and Var is the set of all Boolean program variables and their copies used in encoding each constraint  $CRC(\pi)$ . The set of unknowns includes the inductive assertions

in  $\mathcal{I}_A$ , update function  $\mathbb{R}$ , unknown expressions  $f, f_1$  etc. associated with applying the update schemas in  $\mathbb{R}$  and valuations at each program location of the function  $\mathcal{C}$ . Finally, AssumeConstraints ensures that any modifications to the guards of assume statements corresponding to the same conditional statement are consistent. Thus, for every pair of updated assume  $(f_1)$ , assume  $(f_2)$  statements labeling edges starting from the same node in the transition graph, the uninterpreted functions  $f_1, f_2$  are constrained to satisfy  $f_1 = \neg f_2$ .

If the above formula is **true**, then we can extract models for all the unknowns from the witness to the satisfiability of the formula:  $\forall Var$ :  $C_{exit_0} \leq \delta \wedge \bigwedge_{\pi \in \Pi_A} CRC(\pi) \wedge AssumeConstraints$ . In particular, we can extract an  $\mathbb{R}$  and the corresponding modified statements to yield a correct Boolean program  $\widehat{\mathcal{B}}$ . The following theorem states the correctness and completeness of the above algorithm for repairing Boolean programs for partial correctness.

**Theorem 41** Given the set  $\mathcal{U}$  specified in (1), and given an incorrect Boolean program  $\mathcal{B}$  annotated with assertions, cost function c and repair budget  $\delta$ ,

- 1. if there exists a  $(\mathcal{U}, c, \delta)$ -repair of  $\mathcal{B}$ , the above method finds a  $(\mathcal{U}, c, \delta)$ -repair of  $\mathcal{B}$ ,
- 2. if the above method finds a  $\widehat{\mathcal{B}}$ , then  $\widehat{\mathcal{B}}$  is a  $(\mathcal{U}, c, \delta)$ -repair of  $\mathcal{B}$ .

*Proof.* Note that the formula (5) is a  $\exists \forall$  formula over Boolean variables (Boolean program variables and their copies), unknown Boolean expressions over these Boolean variables (inductive assertions and expressions in modified program statements), sequences of update schemas (update functions) and corresponding sequences of integer costs (valuations of  $\mathcal{C}$ ). The number of Boolean variables is finite and hence, the number of unknown Boolean expressions over them is finite. There are a finite number of update functions drawn from finite sequences of update schemas in the finite set  $\mathcal{U}$ , and a corresponding finite number of  $\mathcal{C}$  functions, with  $\mathcal{C}_{entry_0}$  set to 0. Besides these (5) includes Boolean operators, the + operator and a finite number of integer constants (corresponding to the cost function c). Clearly, the truth of the formula in (5) is decidable. In particular, the formula has a finite number of models.

Given the set  $\mathcal{U}$  specified in (1), the completeness of our method follows from the completeness of Floyd's inductive assertions method and the decidability of the formula in (5).

The soundness of our method follows from the soundness of Floyd's inductive assertions method.

Example: For the Boolean program in Fig. 1b, our tool modifies two statements: (1) the guard for  $stmt(\ell_1)$  is changed from b2 to  $b0 \lor b1 \lor \neg b2$  and (2) the guard for  $stmt(\ell_2)$  is changed from \* to  $b0 \lor b1 \lor b2$ .

# 5 Concretization

We now present the second step in our framework for computing a concrete repaired program  $\widehat{\mathcal{P}}$ . In what follows, we assume that we have already extracted models for  $\widehat{\mathcal{B}}$  and  $\mathcal{I}_A$ . Recall that  $\gamma$  denotes the mapping of Boolean variables to their respective predicates: for each  $i \in [1, |V(\mathcal{B})|]$ ,  $\gamma(b_i) = \phi_i$ . The mapping  $\gamma$  can be extended in a standard way to map expressions over the Boolean variables in  $V(\mathcal{B})$  to expressions over the concrete program variables in  $V(\mathcal{P})$ .

Concretization of  $\widehat{\mathcal{B}}$ . The goal of concretization of a repaired Boolean program  $\widehat{\mathcal{B}}$  is to compute a corresponding repaired concrete program  $\widehat{\mathcal{P}}$ . This involves computing a mapping, denoted  $\Gamma$ , from each modified statement of  $\widehat{\mathcal{B}}$  into a corresponding modified statement in the concrete program. In what follows, we define  $\Gamma$  for each type of modified statement in  $\widehat{\mathcal{B}}$ . Let us fix our attention on a statement at location  $\ell$ , with  $V_s(\mathcal{B})$ ,  $V_s(\mathcal{P})$  denoting the set of concrete, abstract program variables, respectively, whose scope includes  $\ell$ . Let  $r = |V_s(\mathcal{B})|$  and  $q = |V_s(\mathcal{P})|$ .

- 1.  $\Gamma(\text{skip}) = \text{skip}$
- 2.  $\Gamma(\operatorname{assume}(g)) = \operatorname{assume}(\gamma(g))$
- 3.  $\Gamma(\operatorname{call} F_i(e_1,\ldots,e_k) = \operatorname{call} F_i(\gamma(e)_1,\ldots,\gamma(e)_k)$
- 4. The definition of  $\Gamma$  for an assignment statement is non-trivial. In fact, in this case,  $\Gamma$  may be the empty set, or may contain multiple concrete assignment statements.

We say that an assignment statement  $b_1, \ldots, b_r := e_1, \ldots, e_r$  in  $\mathcal{B}$  is concretizable if one can compute expressions  $f_1, \ldots, f_q$  over  $V_s(\mathcal{P})$ , of the same type as the concrete program variables  $v_1, \ldots, v_q$  in  $V_s(\mathcal{P})$ , respectively, such that a certain set of constraints is valid. To be precise,  $b_1, \ldots, b_r := e_1, \ldots, e_r$  in  $\mathcal{B}$  is concretizable if the following formula is true:

$$\exists f_1, \dots, f_q \ \forall v_1, \dots, v_q : \ \bigwedge_{i=1}^r \ \gamma(b_i)[v_1/f_1, \dots, v_q/f_q] = \ \gamma(e_i)$$
 (6)

Each quantifier-free constraint  $\gamma(b_i)[v_1/f_1,\ldots,v_q/f_q]=\gamma(e_i)$  above essentially expresses the concretization of the abstract assignment  $b_i=0$ 

 $e_i$ . The substitutions  $v_1/f_1, \ldots, v_q/f_q$  reflect the *new* values of the concrete program variables after the concrete assignment  $v_1, \ldots, v_q$  :=  $f_1, \ldots, f_q$ .

If the above formula is **true**, we can extract models  $expr_1, \ldots, expr_q$  for  $f_1, \ldots, f_q$ , respectively, from the witness to the satisfiability of the inner  $\forall$ -formula. We then say:

$$v_1, \ldots, v_q := expr_1, \ldots, expr_q \in \Gamma(b_1, \ldots, b_r := e_1, \ldots, e_r).$$

Note that, in practice, for some  $i \in [1, q]$ ,  $expr_i$  may be equivalent to  $v_i$ , thereby generating a redundant assignment  $v_i := v_i$ . The parallel assignment can then be compressed by eliminating each redundant assignment. In fact, it may be possible to infer some such  $v_i$  without using (6) by analyzing the dependencies of concrete program variables on the predicates in  $\{\phi_1, \ldots, \phi_r\}$  that are actually affected by the Boolean assignment in question; this exercise is beyond the current scope of this work.

Template-based concretization of  $\widehat{\mathcal{B}}$ . Recall that  $\mathbb{E}_{\mathcal{T},\mathcal{L}}(\ell)$ , associated with location  $\ell$ , denotes a user-supplied template from  $\mathcal{T}$ , specifying the desired syntax of the expressions in any concrete modified statement at  $\ell$ . Henceforth, we use the shorthand  $\mathbb{E}(\ell)$  for  $\mathbb{E}_{\mathcal{T},\mathcal{L}}(\ell)$ . We find it helpful to illustrate template-based concretization using an example template. Let us assume that for each concrete program variable  $v \in V(\mathcal{P}), v \in$  $\mathbb{N} \cup \mathbb{R}$ . We fix  $\mathbb{E}(\ell)$  to (Boolean-valued) linear arithmetic expressions over the program variables, of the form  $c_0 + \sum_{p=1}^q c_p * v_p \leq 0$ , for assume and call statements, and (integer or real-valued) linear arithmetic terms over the program variables, of the form  $c_0 + \sum_{p=1}^q c_p * v_p$ , for assignment statements. Let us assume that the parameters  $c_0, c_1, \ldots, c_q \in \mathbb{R}$ . Given  $\mathbb{E}(\ell)$ , let  $\Gamma_{\mathbb{E}(\ell)}$  denote the mapping of abstract statements into concrete statements compatible with  $\mathbb{E}(\ell)$ . We can define  $\Gamma_{\mathbb{E}(\ell)}$  for each type of modified statement in  $\widehat{\mathcal{B}}$  as shown below. The basic idea is to compute suitable values for the template parameters  $c_0, \ldots, c_q$  that satisfy certain constraints. Note that, in general,  $\Gamma_{\mathbb{E}(\ell)}$  may be the empty set, or may contain multiple concrete statements.

- 1.  $\Gamma_{\mathbb{E}(\ell)}(\mathtt{skip}) = \mathtt{skip}$
- 2. The statement assume(g) is concretizable if the following formula is true:

$$\exists c, \dots, c_q \ \forall v_1, \dots, v_q : \ (c_0 + \sum_{p=1}^q c_p * v_p \le 0) = \gamma(g).$$
 (7)

If the above formula is **true**, we extract values from the witness to the satisfiability of the inner  $\forall$ -formula, and say,

$$c_0 + \sum_{p=1}^q c_p * v_p \le 0 \in \Gamma_{\mathbb{E}(\ell)}(\operatorname{assume}(g)).$$

3. Similarly, the statement call  $F_j(e_1, \ldots, e_k)$  is concretizable if the following formula is true:

$$\exists c_{1,0}, \dots, c_{k,q} \ \forall v_1, \dots, v_q : \ \bigwedge_{i=1}^k \left( (c_{i,0} + \sum_{p=1}^q c_{i,p} * v_p \le 0 \right) = \gamma(e_i) \right).$$

If the above formula is true, we can extract values from the witness to the satisfiability of the inner  $\forall$ -formula to generate a concrete call statement in  $\Gamma_{\mathbb{E}(\ell)}(\text{call }\mathcal{P}_j(e_1,\ldots,e_k))$ .

4. The statement  $b_1, \ldots, b_r := e_1, \ldots, e_r$  is concretizable if the formula in (8) is true. For convenience, let  $h_j = c_{j,0} + \sum_{p=1}^q c_{j,p} * v_p$ , for  $j \in [1,q]$ .

$$\exists c_{1,0}, \dots, c_{r,q} \, \forall v_1, \dots, v_q : \, \bigwedge_{i=1}^r \, \gamma(b_i)[v_1/h_1, \dots, v_q/h_q] = \gamma(e_i). \quad (8)$$

If the above formula is **true**, we can extract values from the witness to the satisfiability of the inner  $\forall$ -formula to generate a concrete assignment statement in  $\Gamma_{\mathbb{E}(\ell)}(b_1,\ldots,b_r:=e_1,\ldots,e_r)$ .

Example: For our example in Fig. 1, the modified guards,  $b0 \lor b1 \lor \neg b2$  and  $b0 \lor b1 \lor b2$ , in  $stmt(\ell_1)$  and  $stmt(\ell_2)$  of  $\widehat{\mathcal{B}}$ , respectively are concretized into true and  $x \le 1$ , respectively using  $\gamma$ .

Concretization of inductive assertions. The concretization of each inductive assertion  $\mathcal{I}_{\ell} \in \mathcal{I}_{\Lambda}$  is simply  $\gamma(\mathcal{I}_{\ell})$ .

# 6 Experiments with a Prototype Tool

We have built a prototype tool for repairing Boolean programs. The tool accepts Boolean programs generated by the predicate abstraction tool SATABS (version 3.2) [10] from sequential C programs. In our experience, we found that for C programs with multiple procedures, SATABS generates (single procedure) Boolean programs with all procedure calls inlined within the calling procedure. Hence, we only perform intraprocedural analysis in this version of our tool. The set of update schemas handled currently is  $\{id, assign \rightarrow assign, assume \rightarrow assume\}$ ; we do

```
\begin{array}{ll} \operatorname{handmade1}: \\ & \operatorname{int} \ x; \\ & \ell_1: \ \ \operatorname{while} \ (x < 0) \\ & \ell_2: \ \ x := x + 1; \\ & \ell_3: \ \operatorname{assert} \ (x > 0); \\ \\ \hline \\ & \operatorname{Boolean} \ \operatorname{program} \ \operatorname{vars/predicates:} \\ & 1. \ \gamma(b0) = x \leq 0 \\ \hline \\ & \operatorname{Boolean} \ \operatorname{program} \ \operatorname{repair:} \\ & 1. \ \operatorname{Change} \ \operatorname{guard} \ \operatorname{for} \ \operatorname{stmt}(\ell_1) \ \operatorname{from} \ * \ \operatorname{to} \ b0 \\ \hline \\ & \operatorname{Concrete} \ \operatorname{program} \ \operatorname{repair:} \\ & 1. \ \operatorname{Change} \ \operatorname{guard} \ \operatorname{for} \ \operatorname{stmt}(\ell_1) \ \operatorname{to} \ x \leq 0 \\ \hline \end{array}
```

Fig. 6: Repairing program handmade1

not permit statement deletions. We set the costs  $c(\mathtt{assign} \to \mathtt{assign}, \ell)$  and  $c(\mathtt{assume} \to \mathtt{assume}, \ell)$  to some large number for every location  $\ell$  where we wish to disallow statement modifications, and to 1 for all other locations. We initialize the tool with a repair budget of 1. We also provide the tool with a cut-set of locations for its Boolean program input.

Given the above, the tool automatically generates an SMT query corresponding to the inner  $\forall$ -formula in (5). When generating this repairability query, for update schemas involving expression modifications, we stipulate every deterministic Boolean expression g be modified into an unknown deterministic Boolean expression f (as described in Fig. 4), and every nondeterministic Boolean expression be modified into an unknown nondeterministic expression of the form  $choose(f_1, f_2)$ . The SMT query is then fed to the SMT-solver Z3 (version 4.3.1) [23]. The solver either declares the formula to be satisfiable, and provides models for all the unknowns, or declares the formula to be unsatisfiable. In the latter case, we can choose to increase the repair budget by 1, and repeat the process.

Once the solver provides models for all the unknowns, we can extract a repaired Boolean program. Currently, the next step — concretization — is only partly automated. For assignment statements, we manually formulate SMT queries corresponding to the inner ∀-formula in (6), and

```
handmade2:
int main() {
   int x;
   \ell_1: if (x \leq 0)
           while (x < 0)
              x := x + 2;
   \ell_4:
              skip;
         else
           if (x == 1)
   \ell_6:
              x := x - 1;
   \ell_7: assert (x>1);
Boolean program vars/predicates:
 1. \gamma(b0) = x \le 1
 2. \gamma(b1) = x == 1
 3. \gamma(b2) = x \le 0
Boolean program repair:
 1. Change guard for stmt(\ell_1) from b2 to b0 \lor b1 \lor \neg b2
 2. Change guard for stmt(\ell_2) from * to b0 \vee b1 \vee b2
Concrete program repair:
 1. Change guard for stmt(\ell_1) to true
 2. Change guard for stmt(\ell_2) to x \leq 1
```

Fig. 7: Repairing program handmade2

feed these queries to Z3. If the relevant queries are found to be satisfiable, we can obtain a repaired C program. If the queries are unsatisfiable, we attempt template-based concretization using linear-arithmetic templates. We manually formulate SMT queries corresponding to the inner  $\forall$ -formulas in (7) and (8), and call Z3. In some of our experiments, we allowed ourselves a degree of flexibility in guiding the solver to choose the right template parameters.

In Fig. 6, Fig. 7, Fig. 8 and Fig. 9, we present some of the details of repairing four C programs. The first two programs are handmade, with the second one being the same as the one shown in Fig. 1. The next two programs are mutations of two programs drawn from the NEC Laboratories Static Analysis Benchmarks [24].

```
necex6:
int x, y;
int foo(int *ptr) {
   \ell_4: if (ptr == \&x)
   \ell_5: *ptr := 0;
   \ell_6: if (ptr == \& y)
   \ell_7: *ptr := 1;
   return 1;
int main() {
   \ell_1: foo (&x);
   \ell_2: foo (&y);
   \ell_3: assert (x > y);
Boolean program vars/predicates:
1. \gamma(b0) = y < x
2. \ \gamma(b1) = \&y == ptr
3. \gamma(b2) = \&x == ptr
Boolean program repair:
1. Change stmt(\ell_7) from b0 := * to b0 := b0 \lor b1 \lor b2
Concrete program repair:
1. Change stmt(\ell_7) to *ptr := -1
```

Fig. 8: Repairing program necex6

We emphasize that the repairs for the respective Boolean programs (not shown here due to lack of space) are obtained automatically. The concretization of the repaired Boolean program in Fig. 6 was trivial – it only involved concretizing the guard b0 corresponding to the statement at location  $\ell_1$ . Concretization of the repaired Boolean program in Fig. 7 involved concretizing two different guards,  $b0 \lor b1 \lor \neg b2$  and  $b0 \lor b1 \lor b2$ , corresponding to the statements at locations  $\ell_1$  and  $\ell_2$ , respectively. We manually simplified the concretized guards to obtain the concrete guards true and  $x \le 1$ , respectively. Concretization of the repaired Boolean program in Fig. 8 involved concretizing the assignment statement at location  $\ell_7$ . We manually formulated an SMT query corresponding to the

```
necex14:
int main() {
   int x, y;
   int a[10];
   \ell_1: x := 1U;
   \ell_2: while (x \leq 10U) {
           y := 11 - x;
           assert (y \ge 0 \land y < 10);
           a[y] := -1;
   \ell_6:
           x := x + 1;
       }
Boolean program vars/predicates:
 1. \gamma(b0) = y < 0
 2. \gamma(b1) = y < 10
Boolean program repair:
 1. Change stmt(\ell_3) from b0, b1 := *, * to b0, b1 := F, T
Concrete program repair:
 1. Change stmt(\ell_3) to y := 10 - x
```

Fig. 9: Repairing program necex14

formula in (6), after simplifying  $\gamma(b_0 \vee b_1 \vee b_2)$  to y < x and restricting the LHS of  $stmt(\ell_7)$  in the concrete program to remain unchanged. The query was found to be satisfiable, and yielded -1 as the RHS of the assignment statement in the concrete program. We repeated the above exercise to concretize the assignment statement at location  $\ell_3$  in Fig. 9, and obtained y := 0 as the repair for the concrete program. Unsatisfied by this repair, we formulated another SMT query corresponding to the formula in (8), restricting the RHS of  $stmt(\ell_3)$  to the template -x + c, where c is unknown. The query was found to be satisfiable, and yielded c = 10.

In Table 1, we present the results of repairing the above four programs and some benchmark programs from the 2014 Competition on Software Verification [11]. The complexity of the programs from [11] stems from nondeterministic assignments and function invocations within loops. All

experiments were run on the same machine, an Intel Dual Core 2.13GHz Unix desktop with 4 GB of RAM.

We enumerate the time taken for each individual step involved in generating a repaired Boolean program. The columns labeled  $LoC(\mathcal{P})$  and  $LoC(\mathcal{B})$  enumerate the number of lines of code in the original C program and the Boolean program generated by SATABS, respectively. The column labeled  $V(\mathcal{B})$  enumerates the number of variables in each Boolean program. The column  $\mathcal{B}$ -time enumerates the time taken by SATABS to generate each Boolean program, the column Que-time enumerates the time taken by our tool to generate each repairability query and the column Sol-time enumerates the time taken by Z3 to solve the query. The columns # Asg and # Asm count the number of assign  $\to$  assign and assume  $\to$  assume update schemas applied, respectively, to obtain the final correct program.

Notice that our implementation either produces a repaired program very quickly, or fails to do so in reasonable time whenever there is a significant increase in the number of Boolean variables, as was the case for example, in <code>veris.c\_NetBSD-libc\_loop\_true</code>. This is because the SMT solver might need to search over simultaneous non-deterministic assignments to all the Boolean variables for every assignment statement in  $\mathcal B$  in order to solve the repairability query. For the last two programs, SATABS was the main bottleneck, with SATABS failing to generate a Boolean program with a non-spurious counterexample after 10 minutes. In particular, we experienced issues while using SATABS on programs that relied heavily on character manipulation.

We emphasize that when successful, our tool can repair a diverse set of errors in programs containing loops, multiple procedures and pointer and array variables. In our benchmarks, we were able to repair operators (e.g., an incorrect conditional statement x < 0 was repaired to x > 0) and array indices (e.g., an incorrect assignment x:=a[0] was repaired to x:=a[j]), and modify constants into program variables (e.g. an incorrect assignment x:=0 was repaired to x:=d, where d was a program variable). Also, note that for many benchmarks, the repaired programs required multiple statement modifications.

#### 7 Discussion

While the algorithm presented in this paper separates the computation of a repaired Boolean program  $\widehat{\mathcal{B}}$  from its concretization to obtain  $\widehat{\mathcal{P}}$ , this separation is not necessary. In fact, the separation may be sub-optimal - it

Table 1: Experimental results

Name	LoC(P)	$LoC(\mathcal{B})$	$V(\mathcal{B})$	$\mathcal{B}$ -time	Que-time	Sol-time	# Asg	# Asm
handmade1	6	58	1	0.180s	0.009s	0.012s	0	1
handmade2	16	53	3	0.304s	0.040s	0.076s	0	2
necex6	24	66	3	0.288s	0.004s	0.148s	1	0
necex14	13	60	2	0.212s	0.004s	0.032s	1	0
while_infinite_loop_1_true	5	33	1	0.196s	0.002s	0.008s	0	1
array_true	23	57	4	0.384s	0.004s	0.116s	1	1
n.c11_true	27	50	2	0.204s	0.002s	0.024s	1	0
terminator_03_true	22	38	2	0.224s	0.004s	0.036s	1	1
trex03_true	23	58	3	0.224s	0.036s	0.540s	1	1
trex04_true	29	36	1	0.200s	0.004s	0.004s	2	0
$veris.c_NetBSD-libc_loop_true$	30	144	23	3.856s	-	-	-	-
vogal_true	41	-	-	> 10m	-	-	-	-
count_up_down_true	18	-	-	> 10m	-	-	-	-

may not be possible to concretize all modified statements of a computed  $\widehat{\mathcal{B}}$ , while there may indeed exist some other concretizable  $\widehat{\mathcal{B}}$ . The solution is to directly search for  $\widehat{\mathcal{B}}$  such that all modified statements of  $\widehat{\mathcal{B}}$  are concretizable. This can be done by combining the constraints presented in Sec. 5 with the one in (5). In particular, the set Unknown in (5) can be modified to include unknown expressions/template parameters needed in the formulas in Sec. 5, and  $CRC(\pi)$  can be modified to include the inner quantifier-free constraints in the formulas in Sec. 5.

As noted in Sec. 1, we can target total correctness of the repaired programs by associating ranking functions along with inductive assertions with each cut-point in  $\Lambda$ , and including termination conditions as part of the constraints.

Finally, we wish to explore ways to ensure that the repaired program does not unnecessarily restrict correct behaviors of the original program. We conjecture that this can be done by computing the weakest possible set of inductive assertions and a least restrictive  $\widehat{\mathcal{B}}$ .

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