Discrete time Finite State Markov Chain
(DTMC finite State).

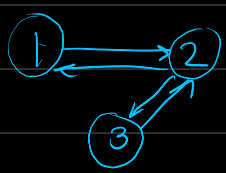
State Space : finite : $S = \{1, 2, \dots M\}$

Time discrete : $T = \{1, 2 \dots n\}$

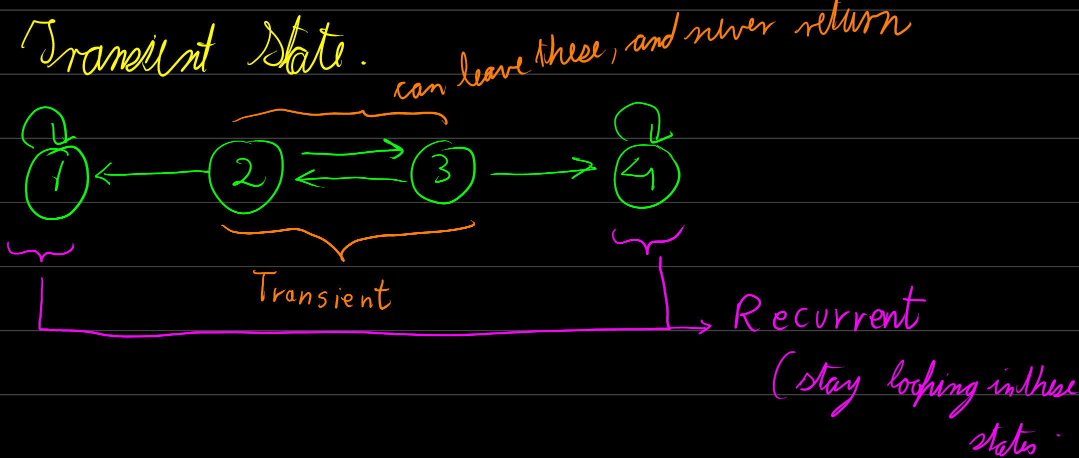
Markov property $P(X_n = j \mid X_{n-1} = i, X_{n-2} = k \dots X_0 = m) = P(X_n = j \mid X_{n-1} = i)$
$= P_{ij}$

(iv) **Irreducible** : If we have only 1 communicating class, then the Markov Chain is irreducible.

eg.

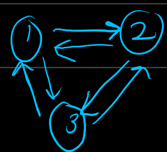


(v) **Recurrent & Transient State.**



If MC enters a state then it will not come out of the loop (class) then we have recurrent states.

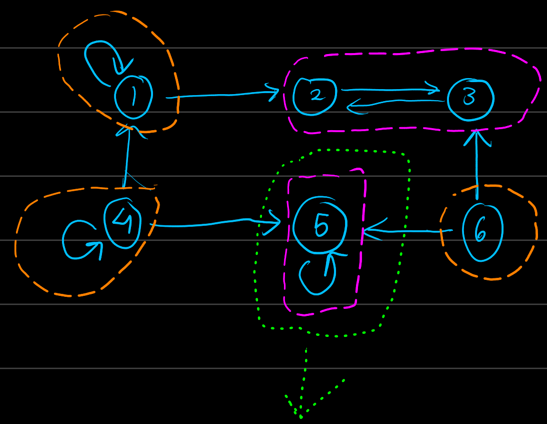
eg :



MC

In transient, the state will leave their class and go to some other class

eg.



Recurring : 2, 3, 5

Transient : 1, 4, 6

Absorption ($P_{ii}^{(1)} = 1$) {when you never exit a state}

Vi Period of a state :-

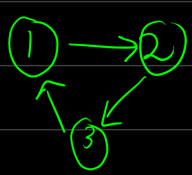
The period of a state i, d(i) is the gcd of those values of n when $P_{ii}^{(n)} > 0$.

If the period of a state 1, it is called aperiodic

If the period of a state ≥ 2, it is called periodic

if no path to come back, periodicity is not defined.

eg: (i)



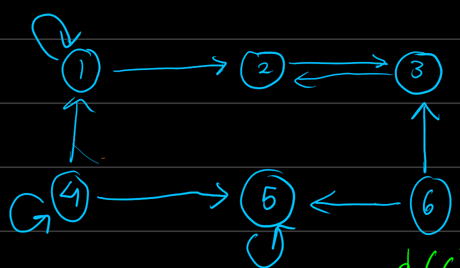
d(1) $P_{11}^{n} > 0$ n = 3, 6, 9 ...

gcd(n) = 3

d(1) = 3 = d(2) = d(3)

(ii)

d(1) = 1



d(4) = 1

d(5) = 1

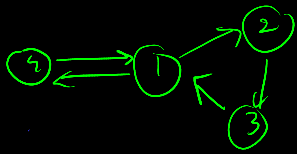
d(6) → ND

$d(2) = P_{22}^{n} > 0$

n = 2, 4, 6 ...

gcd = 2.

d(3) = 2.



$d(1) = P_{11}^{n}$ 2, 3, 4, 6 - - -

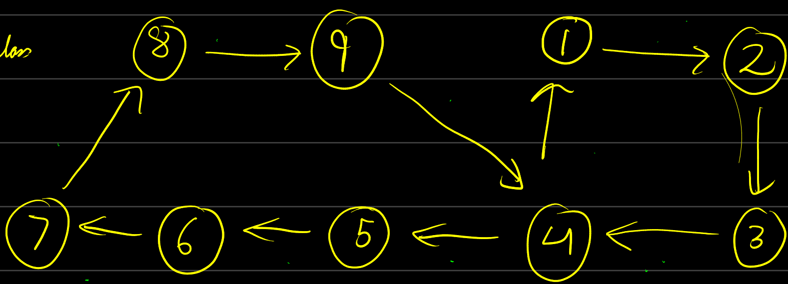
gcd = 1
(aperiodic)

VII Ergodic

In a finite state MC, if all states are recurrent & aperiodic then MC is Ergodic

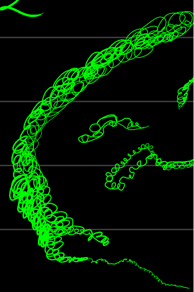
exercise:
i) find recurrent/transient class
ii) $d(i)\ \forall i$

$d(1) = P_{11}^{n} > 0 \rightarrow$ (4, 8, 16 - - -, 10, 16, - -) gcd = 2



$d(2) = 2$ 4, 8, 16, 10, 16 - - -

$d(3) = 2$



TPM

& $\pi^{(0)}$: Initial distribution

$[\ P(X_0=1)\ \ P(X_0=2)\ - - -\ P(X_0=M)]$

$\pi^{(1)}\ [\ P(X_1=1)\ \ P(X_1=2)\ - - -\ P(X_1=M)]$

$\pi^{(2)}\ [\ P(X_2=1)\ - - -\ P(X_2=M)]$

$$P(X_1=j) = \sum_{i\in S} P(X_1=j, X_0=i)$$
$$= \sum_{i\in S} P(X_1=j \mid X_0=i) \cdot P(X_0=i)$$
$$= \sum_{i\in S} P_{ij} \cdot P(X_0=i)$$

$$\underset{(1\times m)}{\pi^{(1)}} = \pi^{(0)} \cdot P$$

$$= \underset{(1\times m)}{[\ P(X_0=1)\ - - -\ P(X_0=M)]} \underset{(m\times m)}{\begin{bmatrix} P_{11} & P_{12} & - - & P_{1M} \\ \vdots & & & \\ P_{M1} & - - & - - & P_{MM} \end{bmatrix}} = [\ P(X_1=1)\ - - -\ P(X_m=1)]$$

$$\sum_{i=1}^{m} P(X_0=i) P_{i1}$$

Similarly

$$\pi^{(2)} = \pi^{(1)} \cdot P = \pi^{(0)} \cdot P^2$$

$$\pi^{(3)} = \pi^{(2)} P$$

$$\vdots$$

$$\pi^{(n)} = \pi^{(n-1)} P = \pi^{(0)} P^n$$