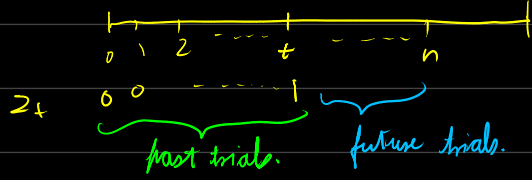
Bernoulli property.

(i) Fresh start property

Starting from any given pt in time, the future is also modelled as Bernoulli Process. which is independent of the past.



(ii) Memoryless property.

time taken for 1st arrival

$$P(X_2 > n+t \mid X_2 > t) = \frac{P(X_2 > n+t \cap X_2 > t)}{P(X_2 > t)}$$

$$= \frac{P(X_2 > n+t)}{P(X_2 > t)} = \frac{1 - P(X_2 < n+t)}{1 - P(X_2 < t)}$$

$$= \frac{q^{n+t}}{q^t} = q^n$$

↑ independent of t

$$= P(X_1 > n)$$

$X_k \overset{iid}{\sim}$ Geometric (p). $\forall k = 1, 2 \ldots$

$$S_k = \sum_{i=1}^{k} X_i$$

$$M_{S_k}(s) = E[e^{S_k s}]$$

$$= E[e^{s(X_1 + X_2 \ldots X_k)}]$$

$$= \prod_{i=1}^{k} E[e^{sX_i}]$$

$$= \prod_{i=1}^{k} M_{X_i}(s) = \left(\frac{pe^s}{1 - qe^s}\right)^k$$

$S_k \sim$ Pascal $(k, p) \approx$ negative Binomial.

derivation
&
Probs.

$$E(S_k) = E\left[\sum X_i\right] = k \times \left(\frac{1}{p}\right)$$

$$V(S_k) = V\left(\sum X_i\right) = \sum X_i + 2\cdot\cancel{\mathrm{Cov}} \; 0$$

$$= k \times \frac{q}{p^2}$$