Combining / Merging of P.P.

2 ind. P.P.

$N_1(t) \sim P(\lambda_1 t)$ $\qquad$ $N(t) = N_1(t) + N_2(t) \sim P(\lambda t)$

$N_2(t) \sim P(\lambda_2 t)$ $\qquad$ $\lambda = \lambda_1 + \lambda_2$

$$M_N(s) = E\left[e^{N(s)}\right] = E\left[e^{N_1(s)+N_2(s)}\right]$$

$$= \prod_{i=1}^{2} E\left[e^{N_i(s)}\right]$$

$$= e^{\lambda_1(e^s-1)} \cdot e^{\lambda_2(e^s-1)} = e^{[\lambda_1+\lambda_2](e^s-1)}$$

$$= P((\lambda_1+\lambda_2)t)$$

(mgf of Poisson)

$N(t) \sim P(\lambda t)$

$\lambda = \lambda_1 + \lambda_2$

$$\begin{array}{ccccc} P(\lambda_1 t) & & P(\lambda_2 t) & \cdots & P(\lambda_k t) \\ \text{SS} & & \text{SS} & & \text{SS} \end{array}$$

$$N(t) = N_1(t) + N_2(t) \cdots N_k(t)$$

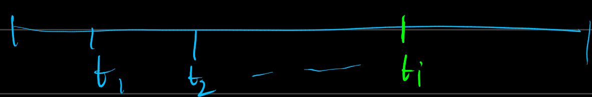
$$P(\lambda t) = P((\lambda_1 + \lambda_2 \cdots \lambda_k)t)$$

$P(\lambda t) \rightarrow$ homo P.P.

$P(\lambda_t t) \rightarrow$ no homo P.P.

Splitting of P.P.

$N(t) \sim P(\lambda t)$



i^{th} arrival was from P_1 with prob p

$\therefore \lambda_1 = p\lambda$

$\lambda_2 = (1-p)\lambda$

Theorem :- For a Poisson process of rate λ & for any $t>0$
The pmf for $N(t)$ is Poisson pmf given by $P_{N(t)}(n) = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$

Proof. $S_n \sim \text{Erlang}(n, \lambda)$

$$P(t < S_{n+1} \leq t+\delta) = P(n \text{ arrivals in } (0,t] \text{ \& } 1 \text{ arrival in } (t, t+\delta])$$
$$= P(N(t)=n) \cdot P(\tilde{N}(t, t+\delta) = 1)$$

$$= P(N(t)=n) \cdot (\lambda\delta + o(\delta)) + o(\delta) \quad (i)$$

$$P(t < S_{n+1} \leq t+\delta) = f_{S_{n+1}}(t)(\delta + o(\delta)) \quad (ii)$$

frm i & ii

$$P(N(t)=n)(\lambda\delta + o(\delta)) + o(\delta) = f_{S_{n+1}}(t)(\delta + o(\delta))$$

div. by δ & $\lim_{\delta \to 0} \dfrac{o(\delta)}{\delta} \to 0$

$$P(N(t)=n)\lambda = f_{S_{n+1}}$$

$$P(N(t)=n) = f_{S_{n+1}}/\lambda = \dfrac{\lambda^{n+1} t^n e^{-\lambda t}}{\dfrac{n!}{\lambda}} = \dfrac{\lambda^n t^n e^{-\lambda t}}{n!}$$