# Poisson Process.

↳ continuous time.

$N(t_i) = \#$ of arrivals in $(0, t_i]$

0 $t_1$ $t_2$ - - - -

$(0, t_1]$ $(t_1, t_2]$

$\tilde{N}(t_{i-1}, t_i) = \#$ of arrival in $(0, t_i]$
$- \#$ of arrival $(0, t_{i-1}]$
$= N(t_i) - N(t_{i-1})$

→ Arrival Process :- The RVs $\overbrace{S_1, S_2}^{\text{called epochs}}$ --- are called arrival epochs.
The seq. of $\{S_1, S_2 \cdots\}$ forms an arrival process.

Assumption : i Process starts at time = 0

ii Multiple arrivals can't occur simultaneously.

i.e. 0 $t_1$ $t_2$

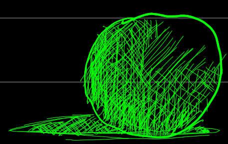
arrivals in $(t_1, t_1 + \delta) = 1$ at most.

→ Counting Process :- $\{N(t), t > 0\}$ or $\{N(t), t = (0, \infty)\}$ a random process is called a counting process if

i $N(0) = 0$

ii $N(t) \in \{0, 1, 2 \cdots\}$

iii if $0 < t \le \tau$ then $N(t) \le N(\tau)$



→ Renewal Process :- A renewal process is an arrival process for when the seq of interarrival time is a seq of IIDs

## I. Property : Indepent Increment Property

A counting process $\{N(t), t > 0\}$ has independent increment Property if for every integer $k > 0$ & every $k$ tuple of times $0 < t_1 < \cdots t_k$ the $k$-tuple of RVs $N(t_1), \tilde{N}(t_1, t_2) \cdots \tilde{N}(t_{k-1}, t_k)$ are statistically independent.

- Non overlapping intervals
- $N(t_i) \perp\!\!\!\perp N(t_i, t_{i+1})$ } → can be any interval
  - non-overlapping

## II. Stationary Increment Property

A counting process $\{N(t), t>0\}$ has the Stationary increment property if $N(t)-N(\tau)$ has the same distribution as $N(t-\tau)$ for every $0<\tau<t$

$r>0$, $\tau$, $t$, $\tau+r$, $t+r$

$\tilde{N}(\tau, t)$
$\tilde{N}(\tau+r, t+r)$ } same distribution

?!

## Def$^n$ of P.P.

I A poisson process is a renewal process in which the interarrival times have an exponential distribution f$^n$ i.e. for some real $\lambda>0$ each $X_i$ has

$$f_{X_i}(x_i)=\lambda e^{-\lambda x_i} \quad x_i>0$$

$\lambda$: rate parameter

i Renewal process
ii $X_i \overset{iid}{\sim} Exp(\lambda)$

II A poisson counting process $\{N(t), t>0\}$ is a counting process that has poisson p.m.f i.e. $P_{N(t)}(n)=\dfrac{e^{-\lambda t}(\lambda t)^n}{n!}$

& has independent & stationary increment property.

i $N(t)\sim Poisson(\lambda t)$
ii ind. & stat. inc. prop.

III A poisson counting process is a counting process that satisfies

$$P(\tilde{N}(t,t+\delta)=0)=e^{-\lambda\delta}\simeq 1-\lambda\delta+$$

$$P(\tilde{N}(t,t+\delta)=1)=\lambda e^{-\lambda\delta}$$

$$P(\tilde{N}(t,t+\delta)\geq 2)=o(\delta)$$

{ still incomplete }