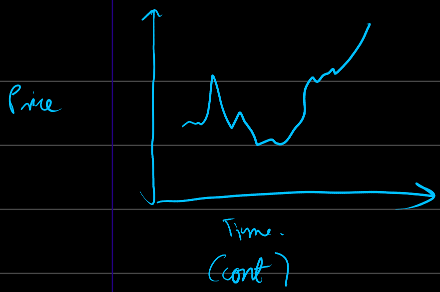
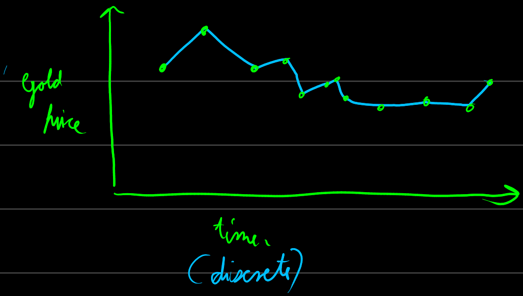
Random Processes. → inf. collectⁿ of R.V.s indexed on time
↳ integer or real no.





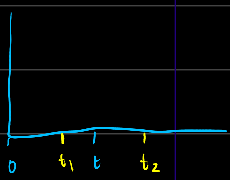
X_1: Price of gold on 1^{st} day
X_2: Price of " " 2^{nd} day
different RVs

eg: playing a game, → win / lose

t: Discrete
or t: continuous

$$X_t = \begin{cases} 1 & \text{if win / arrival} \\ 0 & \text{if lose / no-arrival} \end{cases}$$

$$\{ Z_t : t = \{0, 1, 2 \cdots\} \}$$



from 0 to t ←

$$\{ Z(t),\ t > 0 \} \text{ or } \{ Z(t),\ t \in R \}$$

$Z(t_1, t_2)$

Classification of RPs

Time
- Discrete time RP
- Continuous time RP

value that Z_t can take ← State Space
- Discrete time RP
- Continuous time RP

time & State
- discrete time & discrete state (Bernoulli) Z_t → Bernoulli
- time discrete & state conti. (Gaussian with Discrete time)
- time cont. state discr. (Poisson)
- time cont & state cont (Gaussian with cont. time)

Bernoulli eg: win/lose

$$Z_t = \begin{cases} 1 & \text{if win / arrival} \\ 0 & \text{if lose / no-arrival.} \end{cases}$$

$$\{ Z_t : f = \{0, 1, 2 \dots\} \}$$

eg: failure / no-failure of a machine

$$Z_t = \begin{cases} 1 & \text{if no failure / arrival} \\ 0 & \text{if failure / no-arrival.} \end{cases}$$

Bernoulli :-
- 2 outcomes
- constant $p \ \forall Z_t$.
- $P(\text{arrival}) = p$

$$Z_t \overset{iid}{\sim} \text{Bernoulli}(p)$$

A Bernoulli process is a seq $\{Z_1, Z_2 \dots Z_t \dots\}$ of IID (RVs) Bernoulli(p) where $P(Z_t = 1) = p$

$P(Z_t = 0) = 1 - p = q$

$\forall f = \{0, 1, 2 \dots\}$

Assumption:

i Time is discrete

ii IID RVs ie $Z_t \overset{iid}{\sim} \text{Bernoulli}(p)$

iii Only one arrival happens at time $t = p$

iv the process starts at $t = 0$ but arrival can happen only $t \geq 1$

$$\boxed{\begin{aligned} &E(X) = \Sigma x P(X = x) \\ &E(g(x)) = \Sigma g(x) P(X = x) \\ &V(X) = E(X^2) - (E(X))^2 \end{aligned}}$$

$Z_t \sim \text{Bernoulli}(p)$

$P(Z_t = 1) = p$

$P(Z_t = 0) = q = 1 - p$

$E(Z_t) = p, \quad V(Z_t) = pq$

Yalagar
Sheldon

mgf: moment generating function

$$M_X(t) = \sum_{x \in X} e^{xt} P(X = x)$$

$$\left. \frac{d^k M_x(t)}{dt^k} \right|_{t=0} = E(X^k)$$