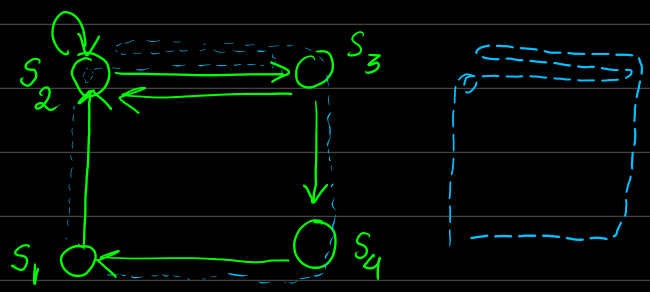
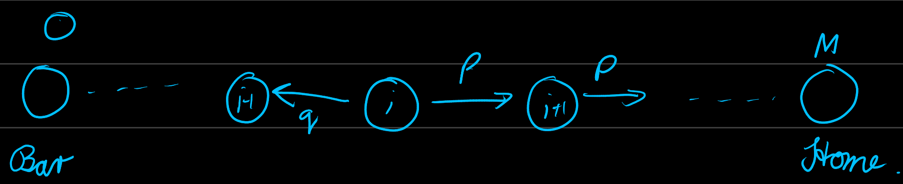
Markov Chains.

· Discrete Time Finite State



$T = \{0, 1, 2, 3, 4, 5, 6, 7\}$
$S = \{1\ 2\ 2\ 3\ 2\ 3\ 4\ 1\}$



Markov Property

At n^{th} step, the MC is in j-th state.

$$P(X_n = j \mid X_{n-1} = i, X_{n-2} = k \cdots\cdots X_0 = m) = P(X_n = j \mid X_{n-1} = i)$$

$$= \underbrace{p_{ij}^{(1)}}_{\text{from } i \text{ to } j \text{ in 1 step}} = p_{ij}$$

$$P(X_{n+1} = j \mid X_n = i, X_{n-1} = k \cdots\cdots X_0 = m) = P(X_{n+1} = j \mid X_n = i)$$

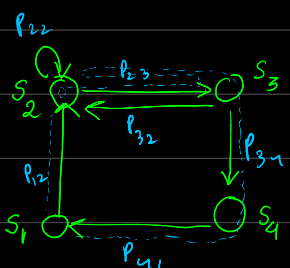
$$= p_{ij}$$

Formally, MC are Stochastic Process defined at discrete time and discrete finite statespace. They are represented by $\{X_n ; n \geq 0\}$.

$\{X_n = i\}$ represents at time n, the process is in the i^{th} state.

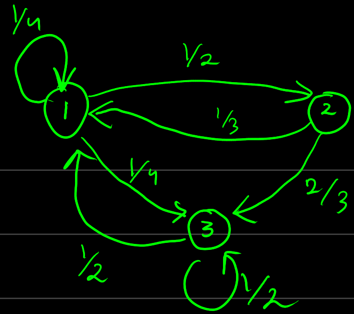
$$P(X_n = j \mid X_{n-1} = i, X_{n-2} = k \ldots X_0 = m) = P(X_n = j \mid X_{n-1} = i) = p_{ij}$$

TPM (Transition Probability Matrix)



$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \ (j) \end{array} \\ (i) \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & P_{12} & 0 & 0 \\ 0 & P_{22} & P_{23} & 0 \\ 0 & P_{32} & 0 & P_{34} \\ P_{41} & 0 & 0 & 0 \end{bmatrix} \end{array} = P$$

eg:

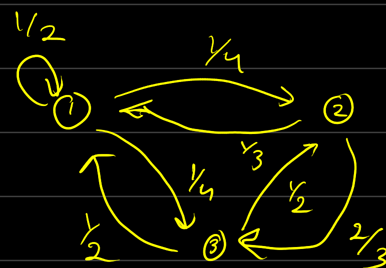


$S = \{1, 2, 3\}$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1/4 & 1/2 & 1/4 \\ 2 & 1/3 & 0 & 2/3 \\ 3 & 1/2 & 0 & 1/2 \end{array}$$

eg

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1/2 & 1/4 & 1/4 \\ 2 & 1/3 & 0 & 2/3 \\ 3 & 1/2 & 1/2 & 0 \end{array} \equiv$$



- walk, path, cycle.

- Accessible.

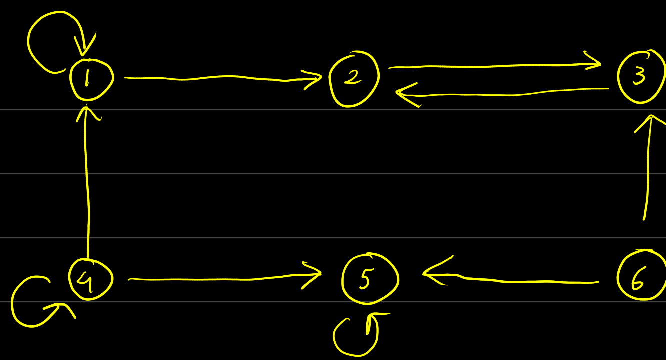
  reach j for i

  $i \to j$

- Communication: both are accessible.

  $i \leftrightarrow j$

Result: if $i \leftrightarrow j$ & $j \leftrightarrow m$: then $i \leftrightarrow m$.



$2 \leftrightarrow 3$

$S = \{1, 2, 3 \cdots 6\}$

$S = \bigcup_{i=1}^{\infty} C_i$ classes :

$C_1 \{2\ 3\}$

$C_2 \{1\}$

$C_3 \{4\}$

$C_4 \{5\}$

$C_5 \{6\}$