Discrete time Finite State Markov Chain
(DTMC finite State).

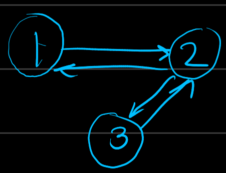
State Space : finite : $S = \{1, 2, \dots M\}$

Time discrete : $T = \{1, 2 \dots n\}$

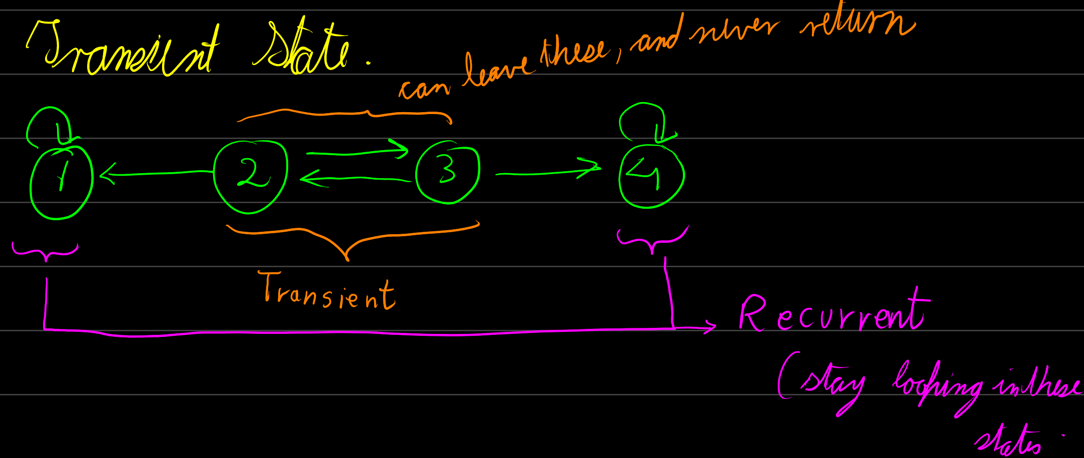
Markov property $P(X_n = j \mid X_{n-1} = i, X_{n-2} = k \dots X_0 = m) = P(X_n = j \mid X_{n-1} = i)$
$= P_{ij}$

(iv) **Irreducible :** If we have only 1 communicating class, then the Markov Chain is irreducible.

eg.

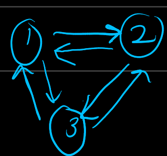


(v) **Recurrent & Transient State.**



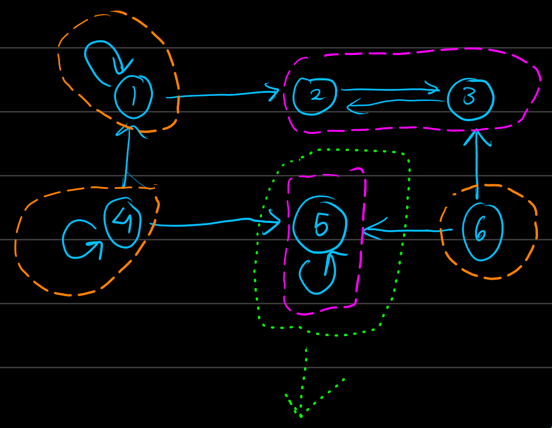
If MC enters a state then it will not come out of the loop (class) then we have recurrent states.

eg :



In transient MC, the state will leave their class and go to some other class

eg.



Recurring : 2, 3, 5

Transient : 1, 4, 6

Absorption ($P_{ii}^{(1)}=1$) {when you never exit a state}

Vi Period of a state :-

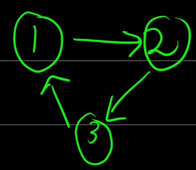
The period of a state i, d(i) is the gcd of those values when $p_{ii}^{(n)}>0$.

If the period of a state 1, it is called aperiodic

If the period of a state ≥ 2, it is called periodic

if no path to come back, periodicity is not defined.

eg: (i)



d(1) $\quad P_{11}^{n}>0$

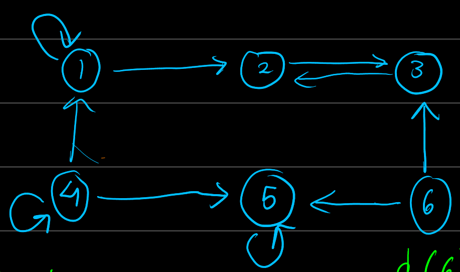
n = 3, 6, 9 ---

gcd(n) = 3

d(1) = 3 = d(2) = d(3)

(ii)

d(1) = 1



d(4) = 1

d(5) = 1

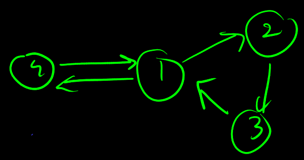
d(6) → ND

d(2) = $P_{22}^{n}>0$

n = 2, 4, 6 ---

gcd = 2

d(3) = 2.



$d(1) = P_{11}^n$ $\quad 2, 3, 4, 6 \ldots$

$gcd = 1$ (aperiodic)

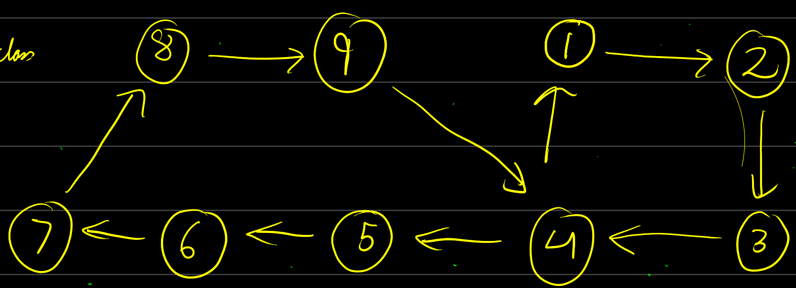
VII Ergodic

In a finite state MC, if all states are recurrent & aperiodic then MC is Ergodic

exercise:

i) find recurrent/transient class

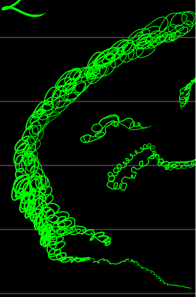
ii) $d(i) \; \forall i$



$d(1) = P_{11}^n > 0 \rightarrow \begin{pmatrix} 4, 8, 16, \ldots \\ 10, 16, \ldots \end{pmatrix}$ $gcd = 2$

$d(2) = 2$ $\quad 4, 8, 16$ $\quad 10, 16 \ldots$

$d(3) = 2$



TPM

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

& $\pi^{(0)}$: Initial distribution.

$[P(X_0=1) \quad P(X_0=2) \quad \cdots \quad P(X_0=M)]$

$\pi^{(1)} \quad [P(X_1=1) \quad P(X_1=2) \quad \cdots \quad P(X_1=M)]$

$\pi^{(2)} \quad [P(X_2=1) \quad \cdots \quad P(X_2=M)]$

$$P(X_1=j) = \sum_{i \in S} P(X_1=j, X_0=i)$$

$$= \sum_{i \in S} P(X_1=j \mid X_0=i) \cdot P(X_0=i)$$

$$= \sum_{i \in S} P_{ij} \cdot P(X_0=i)$$

$$\pi^{(1)} = \pi^{(0)} \cdot P$$

$$\underset{(1\times m)}{\pi^{(1)}} = \underset{(1\times m)}{[P(X_0=1) \; \cdots \; P(X_0=M)]} \underset{(m\times m)}{\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1M} \\ \vdots & & & \\ P_{M1} & \cdots & \cdots & P_{MM} \end{bmatrix}} = [P(X_1=1) \; \cdots \; P(X_M=1)]$$

$\sum_{i=1}^{m} P(X_0=i) P_{i1}$

Similarly

$$\pi^{(2)} = \pi^{(1)} \cdot P = \pi^{(0)} \cdot P^2$$

$$\pi^{(3)} = \pi^{(2)} P$$

$$\vdots$$

$$\pi^{(n)} = \pi^{(n-1)} P = \pi^{(0)} P^n$$