
Proof: Why c lies between -2 and 2 on the real axis, and -2i and 2i on the imaginary axis

We study the iteration:

$Z_{n+1}=Z_n^2+c$, starting with $Z_0 = 0$.

At the first step, $Z_1 = c$. This means the sequence begins directly at the value of c itself.

If c is very large in the size, then Z_1 is already large. The next term is $Z_2 = c^2 + c$, which becomes even larger, since the c^2 part dominates. From here the sequence will keep increasing and diverge.

How large can c be without diverging immediately?

If $|c| > 2$, then already at $Z_1 = c$ we are outside radius 2 from the origin. From this point the values will only grow larger and escape to infinity.

Therefore, any c with $|c| > 2$ cannot belong to the Mandelbrot set.

More generally, think of the quadratic expression: when $|c| > 2$, the c^2 term dominates and guarantees that values will grow larger and larger. When $|c| \le 2$, the c^2 term is not strong enough to always dominate, so the value may go up or may go down depending on c. This means for $|c| \le 2$ we must actually check by iteration to know whether it stays bounded or escapes.

The condition $|c| \le 2$ means all possible c values worth testing lie inside the circle of radius 2 centred at the origin. On the real axis this circle runs from -2 to 2 on the imaginary axis it runs from -2i to 2i. Thus, the Mandelbrot set is contained in the circle region bounded by -2, 2, -2i, and 2i.
