
Proof 1.2 – Distance Formula in the Complex Plane $(c_a * c_a) + (c_b * c_b) > 4$

$$c = c_a + i c_b$$

The distance of a complex number from the origin is given by the modulus:

$$|c| = \sqrt{c_a^2 + c_b^2}$$

The Mandelbrot condition requires that if

$$|c| > 2 \text{ (Proof in 1.1)}$$

then c cannot belong to the set.

So:

$$|c| > 2$$

$$\Leftrightarrow \sqrt{c_a^2 + c_b^2} > 2$$

$$\Leftrightarrow c_a^2 + c_b^2 > 4$$

This avoids the square root, making the check faster in code.

Code Equivalent

Distance test: for $c = c_a + i c_b$, $|c| = \sqrt{c_a^2 + c_b^2}$.

If $|c| > 2$ then c cannot be in the Mandelbrot set.

Squaring both sides gives: $(c_a^2 + c_b^2) > 4$
