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## Proof 1.5 — Mandelbrot and Julia Set Iteration Equations

The Mandelbrot and Julia iteration is:

$$z_{n+1} = z_n^2 + c$$

Let  $z_{n+1} = z_{n+1, a} + i z_{n+1, b}$  and  $z_n = z_a + i z_b$  and  $c = c_a + i c_b$ .

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### Step 2: Squaring $z_n$

$$z_n^2 = (z_a + i z_b)^2$$

$$z_n^2 = z_a^2 + (i z_b)^2 + 2 * z_a * z_b i$$

Since  $i^2 = -1$ , this simplifies to:

$$z_n^2 = z_a^2 - z_b^2 + 2 * z_a * z_b i$$

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### Step 3: Adding $c$ and $z_n$ in equation $z_{n+1} = z_n^2 + c$ :

$$z_{n+1} = z_a^2 - z_b^2 + 2 * z_a * z_b i + c_a + i c_b$$

$$z_{n+1} = (z_a^2 - z_b^2 + c_a) + i (2 * z_a * z_b + c_b)$$

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### Step 4: Separating real and imaginary parts:

$$z_{n+1, a} = z_a^2 - z_b^2 + c_a$$

$$z_{n+1, b} = 2 * z_a * z_b + c_b$$

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### Code Equivalent:

```
z_n = z_a + z_b; z_{n+1} = z_n_a + z_n_b; c = c_a + c_b;
```

```
// Mandelbrot iteration step
```

```
double zn_a = (z_a * z_a) - (z_b * z_b) + c_a;
```

```
double zn_b = 2 * (z_a * z_b) + c_b;
```

```
z_a = zn_a;           // Making real part of z_{n+1} = z_n
```

```
z_b = zn_b;           // Making imaginary part of z_{n+1} = z_n
```

Note: In Mandel Brod  $c$  varies in Julia  $z$

