

Proof 1.4 — Distance Formula for z_n in the Complex Plane

Let $z_n = z_a + i z_b$. Its distance from the origin (modulus) is:

$$|z_n| = \sqrt{z_a^2 + z_b^2}$$

From **Proof 1.3**, we see that if $|z_n| > 2$, the point will escape to infinity. This means it **cannot belong to the Mandelbrot set**.

Therefore, only points with $|z_n| \leq 2$ may belong to the Mandelbrot set.

$$\sqrt{z_a^2 + z_b^2} \leq 2$$

Squaring both sides to avoid the square root gives:

$$z_a^2 + z_b^2 \leq 4$$

Code Equivalent:

Distance test: for $z_n = z_a + i z_b$

$$|z_n| = \sqrt{z_a^2 + z_b^2}$$

If $|z_n| > 2$ then z_n will not be in the Mandelbrot set.

$$\sqrt{z_a^2 + z_b^2} > 2$$

Squaring both sides gives: $(z_a^2 + z_b^2) > 4$

So if $((z_a^2 + z_b^2) \leq 4)$ {

// z_n may still belong to the Mandelbrot set

} else {

// z_n has escaped, stop iteration

}