Proof 2.1 – Converting Pixel Coordinates to Complex Plane

We want an affine map that takes a pixel coordinate (i , j) where $i \in [0,W]$, $j \in [0,H]$

(where W=SCREEN WIDTH, H=SCREEN HEIGHT)

and returns a complex number

$c = c_a + i\ c_b$ where $c_a \in [x_{min}, x_{max}]$, $c_b \in [y_{min}, y_{max}]$

inside the rectangular complex window.

General Mapping Rule (map $k \in [a, b]$ to $[c, d]$):

Step 1: Normalize to [0,1]

$$\mathbf{k_{ratio} = (k - a) / (b - a)}$$

Step 2: Scale to target range

$$\mathbf{k_{scaled} = k_{ratio} * (d - c)}$$

Step 3: Shift to start at minimum

$$\mathbf{k_{new} = c + K_{scaled}}$$

Final formula:

$$\mathbf{k_{new} = c + ((k - a) / (b - a)) * (d - c)}$$

Applying to Pixel → Complex Mapping:

For $i \in [0, W]$ mapping to $[x_{min}, x_{max}]$:

$$\mathbf{c_a = x_{min} + (i / W) * (x_{max} - x_{min})}$$

For $j \in [0, H]$ mapping to $[y_{min}, y_{max}]$:

$$\mathbf{c_b = y_{min} + (j / H) * (y_{max} - y_{min})}$$

Final Mapping:

$$\mathbf{c = (x_{min} + (i / W) * (x_{max} - x_{min})) + i * (y_{min} + (j / H) * (y_{max} - y_{min}))}$$
