Introduction to Dynamic Analysis & Simulation

BA Economics

L1: Economic dynamics

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- An economic model is an abstract description of a particular economic reality.
- The subjective nature of scientific exploration in economics arises from our *conception* of the particular economic reality, which is underpinned by socio-cultural-political factors.
- As a result there are models from multiple and differing perspectives.

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- The fundamental purpose of *theoretical models* is to elicit features of the underlying, or generative mechanisms that generate those particular stylised facts.

- In contrast, *empirical models* aim to verify those theoretical propositions about the generative mechanisms of the phenomenon under investigation.

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- This is one of the reasons why there are several *schools of thought* in Economics.

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- The static mode is one where the notion of time is suspended, as if one is analysing a snapshot of the system, and the analysis focuses on the relationships between variables in that timeless context.

- If the analysis, on the other hand, is concerned with the evolution of the system then the notion of *time* needs to be explicitly introduced.

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- But such an analysis is underpinned the assumption that the system under investigation is **stable**.

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Foundational concepts

- Existence
- Uniqueness
- Stability

An Illustration

- To illustrate the use of dynamical systems in the study of stability of markets, let us consider this simple model of Demand and Supply.

$$
\begin{aligned}
D_t &= a + bp_t, \qquad b < 0, && (1)\\
S_t &= a_1 + b_1 p_{t-1}, \quad b_1 > 0. && (2)
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- Where D_t equation represents the demand at time t as a function of price at time t and S_t equation represents the supply at time t as a function of price at time $t-1$.

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- The economic intuition behind such a supply function is that suppliers (for instance, farmers) would supply more in the present period t, if they had received a favourable price in the last time period $t-1$.

Equilibrium

- The equilibrium condition, where neither the demander nor the supplier are unsatisfied in the exchange, is

$$D_t = S_t,$$

i.e. it is the point where both consumer and the supplier agree to exchange their goods.

- Substituting equations (1) and (2) in the equilibrium condition would yield

$$bp_t - b_1 p_{t-1} = a_1 - a, \tag{3}$$

which is a first-order difference equation.

Analysis

- The general solution of the homogenous part of the equation is

$$A\left(\frac{b_1}{b}\right)^t.$$

- The homogenous part of the equation is

$$bp_t - b_1 p_{t-1} = 0 \quad \Rightarrow \quad bp_t = b_1 p_{t-1} \quad \Rightarrow \quad p_t = \frac{b_1}{b} p_{t-1}.$$

- To see the solution of the homogenous part, let us iterate the above solution

$$p_1 = \frac{b_1}{b} p_0, \quad p_2 = \left(\frac{b_1}{b}\right)^2 p_0, \quad \ldots \quad p_t = \left(\frac{b_1}{b}\right)^t p_0.$$

By letting $p_0 = A$, we have

$$p_t = \left(\frac{b_1}{b}\right)^t A,$$

which is the general solution of the homogenous part of equation (3).

- The particular solution of the equation is

$$p_e = \frac{a_1 - a}{b - b_1}.$$

- The particular solution is obtained by substituting

$$p_t = p_{t-1} = p_e$$

in equation (3), since the right hand side of the equation is constant.[1]

[1] Refer to Chapter 3 in Gandolfo for solving the first-order difference equations for the case where the RHS is a function of time or exponential function or some other generic function of time

- Now combining both the solutions, the general solution of equation (3) is

$$p(t) = \frac{a_1 - a}{b - b_1} + A\left(\frac{b_1}{b}\right)^t \quad \Rightarrow \quad p(t) = p_e + A\left(\frac{b_1}{b}\right)^t.$$

- Finally, we have to find the value of A. To do that, let $t = 0$ in the above general solution

$$\Rightarrow p(0) = p_e + A \quad \Rightarrow \quad A = p(0) - p_e.$$

- And hence, the general solution of equation (3) is

$$p(t) = (p(0) - p_e)\left(\frac{b_1}{b}\right)^t + p_e. \tag{4}$$

Stability Analysis

- The stability of the Demand-Supply model can be understood from equation (4). In the equation

$$\frac{b_1}{b}$$

represent the ratio of the slope of the supply curve and the demand curve and p_e is the equilibrium price.
- Note that the demand curve has a negative slope $b < 0$ and the supply curve has a positive slope $b_1 > 0$. Then $\frac{b_1}{b} < 0$, so the price will have an oscillatory movement around its equilibirum value p_e
- Three cases arise depending upon the value of the ratio of the slope.

- **The case when** $|\frac{b_1}{b}| < 1$

  This implies the slope of the supply curve is less than that of the demand curve, i.e. $|b_1| < |b|$. In this case, you can very well infer from the equation (4) that for any initial condition the distance $(p_0 - p_e)$ will vanish and the succession of prices over time will converge towards the equilibrium price p_e. This is the case of **Oscillatory Convergence**.

- **The case when** $|\frac{b_1}{b}| > 1$

  This case implies that $|b_1| > |b|$, i.e. the slope of the supply curve is greater than that of the demand curve. From the solution one can infer that for any intital condition p_0 the distance $(p_0 - p_e)$ will become greater and the succession of prices over time will diverge from the equilibrium price p_e. This is the case of **Oscillatory Divergence**.

- **The case when** $|\frac{b_1}{b}| = 1$
  In this case the succession of prices over time will have a constant amplitude oscillations (**Limit Cycle**)

A General Demand-Supply Model

- Let us consider the previous model in a more general setting. This would generalization would help you to see the role of **behaviour** of human agents in the simple model that we discussed in the last section.
- The simple model of demand-supply analyzed in the last section can be modified in the following way:

$$D_t = a + bp_t, \qquad b < 0$$
$$S_t = a_1 + b_1 p_t^e, \quad b_1 > 0$$

- Note that in this model supply is a function of expected price (p_t^e). With b_1, the slope of the function being positive this means that the supply of a good is positively related to the expected price, i.e. if the supplier expectes that he might get a higher price for his product in the next period $(t+1)$, he will increase his supply in this period (t).
- The equilibrium condition again is the same, i.e

Specifying Expectation Rules: Naive Expectation

- Now in order to complete the model, we need to specify the rule for expectations i.e. we need to specify how do the supplier form his expectations about the future price, p_t^e.
- This is one of the most challenging problems in economics and in fact it is this behavioural aspect that distinguishes economics from other scientific disciplines. As you would agree that forming expectations of about the future can vary anything from tossing a coin to studying the past time series using the frontier techniques, and articulating a theory of expectations has been one of the most toughest challenges for economics.
- Nonetheless, we can introduce simple rules of expectations and study the dynamics of the model.
- In the fist instance, let us specify that the supplier has naive expectations,

$$p_t^e = p_{t-1},$$

i.e. he expects the next period's price to be the same as the current

The Normal Price expectation case

- The second way to model expectations is to assume that the suppliers have some notion of a **normal price** or a **long-run price** to which the current market price will tend to over time.
- The simplest way to formalize this expectations rule is

$$p_t^e = p_{t-1} + c(p_N - p_{t-1}), \qquad 0 < c < 1. \tag{5}$$

- The intuition behind the rule is if the current price is lower than the normal price (p_N), the suppliers would expect an increase in the next period's price.
- And the fact that the positive constant c, which is called as the **speed of adjustment**, is assumed to be less than unity is equivalent to the assumption that the suppliers think that the convergence of the last time period's price to the normal price is not immediate and the process will take some time (measured by the reciprocal of c).

- However, we need to specify the **normal price** without which this specification would be a tautology! For this case, we assume that the normal price p_N is nothing but the equilibrium price p_e that we dervied in the earlier model, i.e

$$p_N = p_e = \frac{a_1 - a}{b - b_1}.$$

- Such an assumption could be justified by the reasoning that the suppliers have perfect information, but know that due to market frictions and other institutional rigidities the current price cannot immediately get back to the normal price, which is p_e.

- Now let us substitute these assumptions in the general model and study the stability properties

$$b_1(1-c)p_{t-1} - bp_t = a - a_1 - b_1 c p_e,$$

whose solution is

$$p(t) = (p_0 - p_e)\left(\frac{b_1(1-c)}{b}\right)^t + p_e. \tag{6}$$

The stability condition is

$$\left|\frac{b_1(1-c)}{b}\right| < 1 \qquad \text{or} \qquad |b_1(1-c)| < |b|.$$

- Analyze the solution
- When $\left|\frac{b_1(1-c)}{b}\right| < 1$, starting from any initial condition the succession of prices would converge to the equilibrium value.
- When $\left|\frac{b_1(1-c)}{b}\right| > 1$, starting from any initial condition the succession of prices would diverge away from the equilibirum value.
- When $\left|\frac{b_1(1-c)}{b}\right| = 1$, we should have constant-amplitude oscillations.

- How does this model compare with the earlier model?
- In the case $\left|\frac{b_1(1-c)}{b}\right| < 1$, the convergence is faster because

$$\left|\frac{b_1(1-c)}{b}\right| < \left|\frac{b_1}{b}\right| < 1$$

and hence the absolute value of $\left(\frac{b_1(1-c)}{b}\right)^t$ tends to zero more rapidly than the absolute value of $(\frac{b_1}{b})^t$.
- The repeated oscillation can also becomes damped in this model. This is so because, if $|b_1| = |b|$, then $|b_1(1-c)| < |b|$.

- And the divergent movement is slower because the absolute value of $\left(\frac{b_1(1-c)}{b}\right)^t$ increases at a slower rate than the absolute value of $\left(\frac{b_1}{b}\right)^t$.
- However, for values of the parameter c is sufficiently colse to unity the divergent movement may become convergent because when c is greater $(1-c)$ is smaller so that it is more likely that $|b_1(1-c)| \leq |b|$ even if $|b_1| > |b|$. This makes economic sense becasue the larger the value of c means that the suppliers expect a faster approach to the current price towards its equilibrium price.
- To conclude, the introduction of expectations based on the normal price makes the model more stable.