NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

Question 11.9.4.3: Find the sum to n terms to the series $3(1)^2 + 5(2)^2 + 7(3)^2 + \dots$

Solution:

Given series is $3(1)^2 + 5(2)^2 + 7(3)^2 + \dots$

$x(0)$	3	1st term
$x(n)$	?	$(n+1)$th term
$y(n-1)$	?	sum of n terms

TABLE 0
PARAMETERS

$$x(n) = (2n+3)(n+1)^2 \tag{1}$$

$$y(n) = x(n) * u(n) \tag{2}$$

$$Y(z) = X(z)\,U(z) \tag{3}$$

$$X(z) = \frac{3 + 8z^{-1} + z^{-2}}{(1-z^{-1})^4} \tag{4}$$

$$U(z) = \frac{1}{1-z^{-1}} \tag{5}$$

$$\implies Y(z) = \frac{3 + 8z^{-1} + z^{-2}}{(1-z^{-1})^5} \tag{6}$$

$$Y(z) = \frac{23z^{-1}}{1-z^{-1}} + \frac{63z^{-2}}{(1-z^{-1})^2} + \frac{81z^{-3}}{(1-z^{-1})^3} + \frac{50z^{-4}}{(1-z^{-1})^4} + \frac{12z^{-5}}{(1-z^{-1})^5} + 3 \tag{7}$$

$$\delta(n) \overset{z}{\longleftrightarrow} 1 \tag{8}$$

$$u(n-1) \overset{z}{\longleftrightarrow} \frac{z^{-1}}{1-z^{-1}} \tag{9}$$

$$(n-1)\,u(n-1) \overset{z}{\longleftrightarrow} \frac{z^{-2}}{(1-z^{-1})^2} \tag{10}$$

$$\frac{(n-1)(n-2)\,u(n-1)}{2} \overset{z}{\longleftrightarrow} \frac{z^{-3}}{(1-z^{-1})^3} \tag{11}$$

$$\frac{(n-3)(n-2)(n-1)\,u(n-1)}{6} \overset{z}{\longleftrightarrow} \frac{z^{-4}}{(1-z^{-1})^4} \tag{12}$$

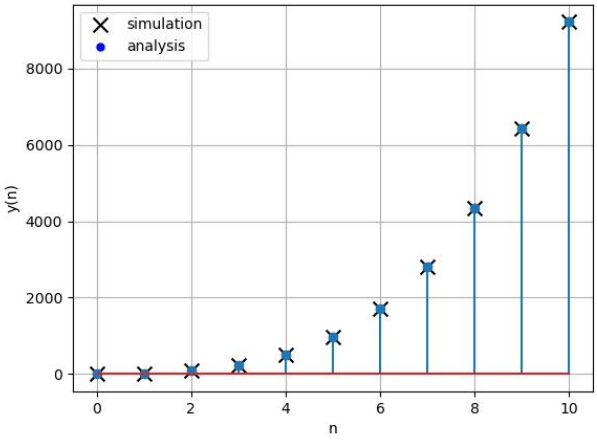

Fig. 0. simulation vs analysis of y(n)

$$\frac{(n-4)(n-3)(n-2)(n-1)\,u(n-1)}{24} \overset{z}{\longleftrightarrow} \frac{z^{-5}}{(1-z^{-1})^5} \tag{13}$$

By using above 6 equations, we get

$$y(n) = 3\delta n + 23u(n-1) + 63(n-1)\,u(n-1) + \frac{81(n-1)(n-2)\,u(n-1)}{2} + \frac{50(n-3)(n-2)(n-1)\,u(n-1)}{6} + \frac{(n-4)(n-3)(n-2)(n-1)\,u(n-1)}{24} \tag{14}$$