NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

Question 11.9.4.3: Find the sum to n terms to the series $3(1)^2 + 5(2)^2 + 7(3)^2 + \ldots$

Solution:

Given series is $3(1)^2 + 5(2)^2 + 7(3)^2 + \ldots$

$x(0)$	3	1st term
$x(n)$	?	$(n+1)$th term
$y(n-1)$	?	sum of n terms

TABLE 0

PARAMETERS

$$x(n) = (2n + 3)(n + 1)^2 \qquad (1)$$

$$y(n) = x(n) * u(n) \qquad (2)$$

$$Y(z) = X(z)U(z) \qquad (3)$$

$$X(z) = \frac{3 + 8z^{-1} + z^{-2}}{\left(1 - z^{-1}\right)^4} \qquad (4)$$

$$U(z) = \frac{1}{1 - z^{-1}} \qquad (5)$$

$$\implies Y(z) = \frac{3 + 8z^{-1} + z^{-2}}{\left(1 - z^{-1}\right)^5} \qquad (6)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \frac{\left(3 + 8z^{-1} + z^{-2}\right)z^{n-1}}{\left(1 - z^{-1}\right)^5} dz \qquad (7)$$

$$y(n) = \lim_{z \to 1} \frac{1}{4!} \frac{d^4}{dz^4} \frac{3z^{n-1} + 8z^{n-2} + z^{n-3}}{\left(1 - z^{-1}\right)^5}\left(1 - z^{-1}\right)^5 \qquad (8)$$

$$\implies y(n) = \frac{(n+1)(n+2)\left(3n^2 + 11n + 9\right)}{6} \qquad (9)$$
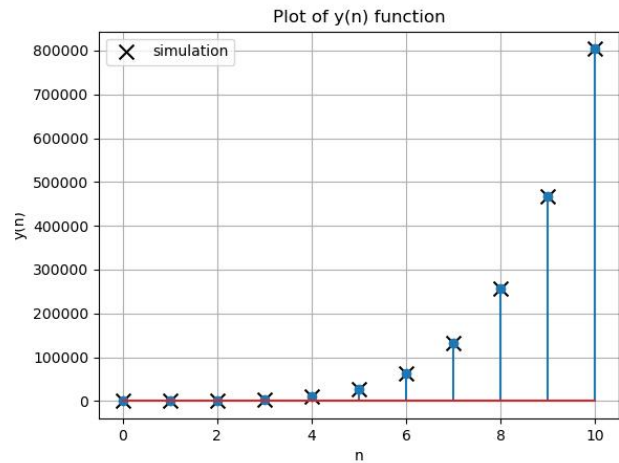
$$\implies y(n-1) = \frac{n(n+1)\left(3n^2 + 5n + 1\right)}{6} \qquad (10)$$

Fig. 0. simulation vs analysis of $y(n)$

The contour integration for the equation (6) is the equation (9).