NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

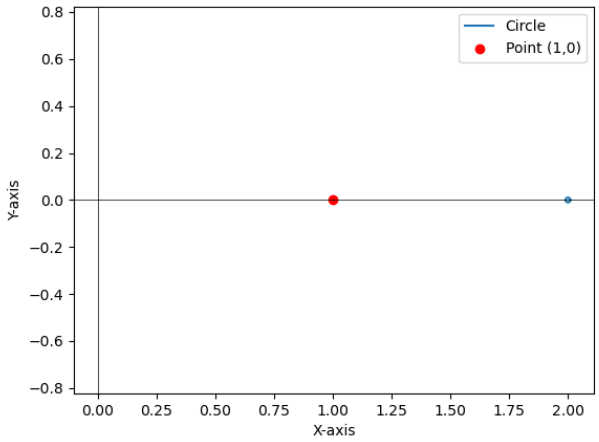

Fig. 0. graph of option A

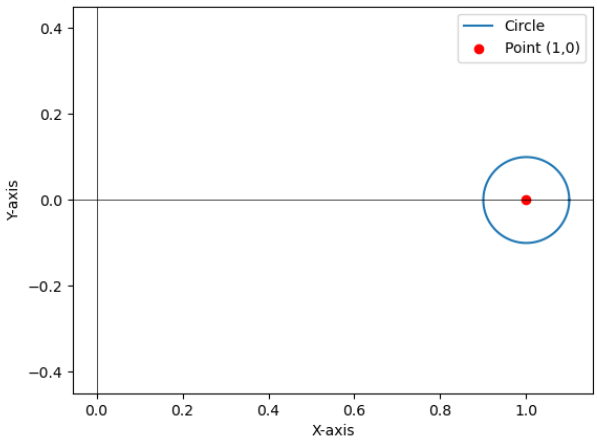

Fig. 0. graph of option B

GATE 2023 BM.48: The function $f(z) = \frac{1}{z-1}$ of a complex variable z on a closed contour in an anti-clockwise direction.For which of the following contours, does this integral have a non-zero value?
$(A)|z-2| = 0.01$
$(B)|z-1| = 0.1$
$(C)|z-3| = 5$
$(D)|z| = 2$

Solution:

Using (**??**)

$$\oint_c \frac{1}{z-1} dz = 2\pi i Res\left[\frac{1}{z-1}, 1\right] \tag{1}$$

For option A the pole is outside the contour, then Res=0.

$$\implies \oint_c \frac{1}{z-1} dz = 2\pi i\,(0) \tag{2}$$
$$= 0$$

For all other options the pole is inside the contours.

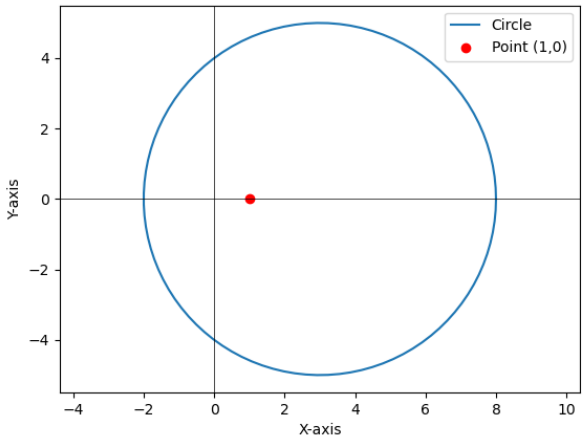

Fig. 0. graph of option C

Then, using (**??**)

$$Res\left[\frac{1}{z-1}, 1\right] = \lim_{z\to 1} (z-1)\frac{1}{z-1} \tag{3}$$
$$= 1$$
$$\implies \oint_c \frac{1}{z-1} dz = 2\pi i\,(1) \tag{4}$$
$$= 2\pi i$$

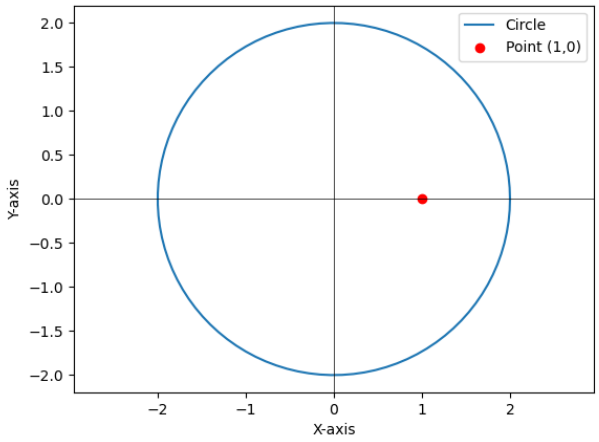

Fig. 0. graph of option D

We can conclude that for options B,C,D contours have the non-zero value for this integral.