NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

GATE 2023 BM.48: The function $f(z) = \frac{1}{z-1}$ of a complex variable z on a closed contour in an anti-clockwise direction.For which of the following contours, does this integral have a non-zero value?
$(A)|z-2| = 0.01$
$(B)|z-1| = 0.1$
$(C)|z-3| = 5$
$(D)|z| = 2$

Solution:

Using (**??**)

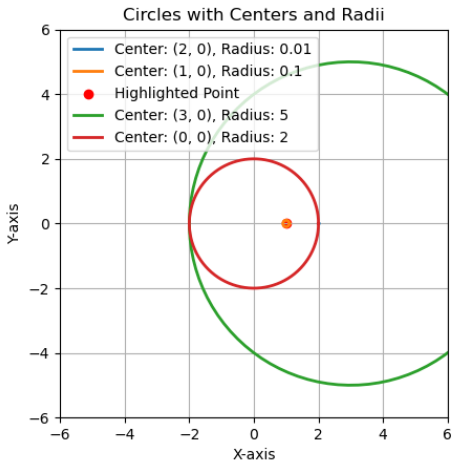

Fig. 0. graphs of all the given contours

$$\oint_c \frac{1}{z-1} dz = 2\pi j Res\left[\frac{1}{z-1}, 1\right] \tag{1}$$

For option A the pole is outside the contour, then Residue is zero.

$$\oint_c \frac{1}{z-1} dz = 2\pi j\,(0) \tag{2}$$

$$= 0 \tag{3}$$

For option B the pole is inside the contour.
Then, using (**??**)

$$Res\left[\frac{1}{z-1}, 1\right] = \lim_{z\to 1} (z-1)\frac{1}{z-1} \tag{4}$$

$$= 1$$

$$\implies \oint_c \frac{1}{z-1} dz = 2\pi j\,(1) \tag{5}$$

$$= 2\pi j \neq 0$$

For option C the pole is inside the contour.
Then, using (**??**)

$$Res\left[\frac{1}{z-1}, 1\right] = \lim_{z\to 1} (z-1)\frac{1}{z-1} \tag{6}$$

$$= 1$$

$$\implies \oint_c \frac{1}{z-1} dz = 2\pi j\,(1) \tag{7}$$

$$= 2\pi j \neq 0$$

For option D the pole is inside the contour.
Then, using (**??**)

$$Res\left[\frac{1}{z-1}, 1\right] = \lim_{z\to 1} (z-1)\frac{1}{z-1} \tag{8}$$

$$= 1$$

$$\implies \oint_c \frac{1}{z-1} dz = 2\pi j\,(1) \tag{9}$$

$$= 2\pi j \neq 0$$

We can conclude that for options B,C,D contours have the non-zero value for this integral.