NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

Question 11.9.4.3: Find the sum to n terms to the series $3\,(1)^2 + 5\,(2)^2 + 7\,(3)^2 + \dots$

Solution:

Given series is $3\,(1)^2 + 5\,(2)^2 + 7\,(3)^2 + \dots$

$x\,(0)$	3	1st term
$x\,(n)$	?	$(n+1)$th term
$y\,(n-1)$	?	sum of n terms

TABLE 0

PARAMETERS

$$x\,(n) = (2n + 3)\,(n + 1)^2 \qquad (1)$$

$$y\,(n) = x\,(n) * u\,(n) \qquad (2)$$

$$Y\,(z) = X\,(z)\,U\,(z) \qquad (3)$$

$$X\,(z) = \frac{3 + 8z^{-1} + z^{-2}}{\left(1 - z^{-1}\right)^4} \qquad (4)$$

$$U\,(z) = \frac{1}{1 - z^{-1}} \qquad (5)$$

$$\implies Y\,(z) = \frac{3 + 8z^{-1} + z^{-2}}{\left(1 - z^{-1}\right)^5} \qquad (6)$$

$$y\,(n) = \frac{1}{2\pi j} \oint_C Y\,(z)\,z^{n-1} dz \qquad (7)$$

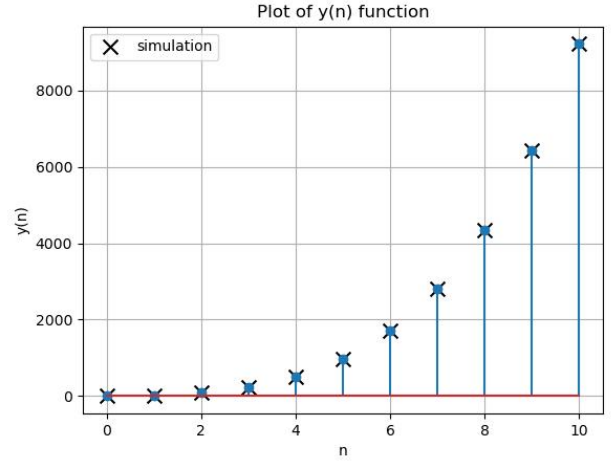$$= \frac{1}{2\pi j} \oint_C \frac{\left(3 + 8z^{-1} + z^{-2}\right) z^{n-1}}{\left(1 - z^{-1}\right)^5} dz \qquad (8)$$

Using (**??**)

$$y\,(n) = \lim_{z \to 1} \frac{1}{4!} \frac{d^4}{dz^4} \frac{3z^{n-1} + 8z^{n-2} + z^{n-3}}{\left(1 - z^{-1}\right)^5} \left(1 - z^{-1}\right)^5 \qquad (9)$$

$$\implies y\,(n) = \frac{(n + 1)\,(n + 2)\left(3n^2 + 11n + 9\right)}{6} \qquad (10)$$

$$\implies y\,(n - 1) = \frac{n\,(n + 1)\left(3n^2 + 5n + 1\right)}{6} \qquad (11)$$

Fig. 0. simulation vs analysis of y(n)