

NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji*

Question 11.9.4.3: Find the sum to n terms to the series $3(1)^2 + 5(2)^2 + 7(3)^2 + \dots$

Solution:

Given series is $3(1)^2 + 5(2)^2 + 7(3)^2 + \dots$

$x(0)$	3	1st term
$x(n)$?	$(n+1)$ th term
$y(n-1)$?	sum of n terms

TABLE 0
PARAMETERS

$$x(n) = (2n + 3)(n + 1)^2 \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

$$X(z) = \frac{3 + 8z^{-1} + z^{-2}}{(1 - z^{-1})^4} \quad (4)$$

$$U(z) = \frac{1}{1 - z^{-1}} \quad (5)$$

$$\Rightarrow Y(z) = \frac{3 + 8z^{-1} + z^{-2}}{(1 - z^{-1})^5} \quad (6)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \frac{(3 + 8z^{-1} + z^{-2})z^{n-1}}{(1 - z^{-1})^5} dz \quad (7)$$

(8)

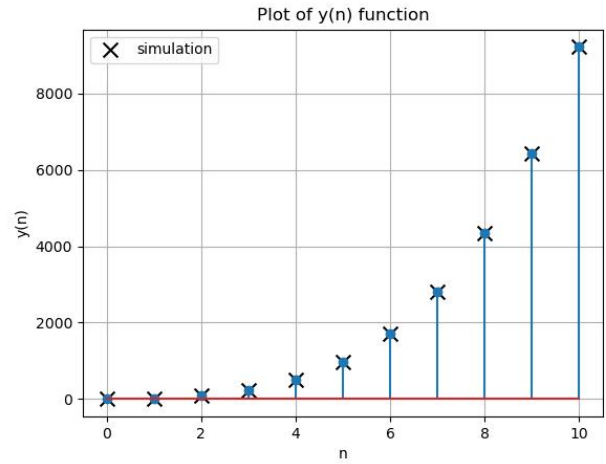


Fig. 0. simulation vs analysis of $y(n)$

Using (??).

$$y(n) = \lim_{z \rightarrow 1} \frac{1}{4!} \frac{d^4}{dz^4} \frac{3z^{n-1} + 8z^{n-2} + z^{n-3}}{(1 - z^{-1})^5} (1 - z^{-1})^5 \quad (9)$$

$$\Rightarrow y(n) = \frac{(n+1)(n+2)(3n^2 + 11n + 9)}{6} \quad (10)$$

$$\Rightarrow y(n-1) = \frac{n(n+1)(3n^2 + 5n + 1)}{6} \quad (11)$$