

# NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji\*

**GATE 2023 BM.48:** The function  $f(z) = \frac{1}{z-1}$  of a complex variable  $z$  on a closed contour in an anti-clockwise direction. For which of the following contours, does this integral have a non-zero value?

(A)  $|z - 2| = 0.01$

(B)  $|z - 1| = 0.1$

(C)  $|z - 3| = 5$

(D)  $|z| = 2$

**Solution:**

Using (??)

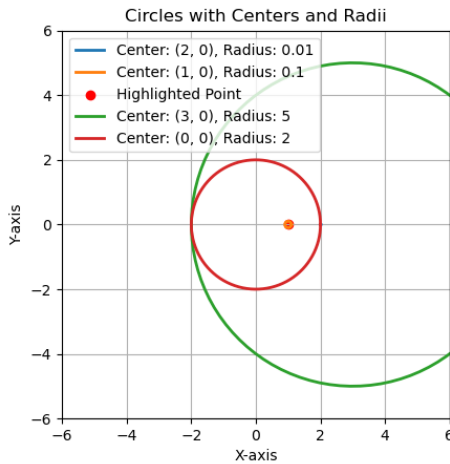


Fig. 0. graphs of all the given contours

$$\oint_c \frac{1}{z-1} dz = 2\pi j \text{Res} \left[ \frac{1}{z-1}, 1 \right] \quad (1)$$

For option A the pole is outside the contour, then

$$\Rightarrow \oint_c \frac{1}{z-1} dz = \int_0^{2\pi} \frac{0.01 j e^{jt}}{1 + 0.01 e^{jt}} dt \quad (2)$$

$$= \int_0^{2\pi} \frac{j e^{jt}}{100 + e^{jt}} dt \quad (3)$$

$$\alpha = e^{jt} \\ = \int_1^1 \frac{1}{100 + \alpha} d\alpha = 0 \quad (4)$$

For option B the pole is inside the contour. Then, using (??)

$$\text{Res} \left[ \frac{1}{z-1}, 1 \right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} = 1 \quad (5)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j (1) = 2\pi j \neq 0 \quad (6)$$

For option C the pole is inside the contour. Then, using (??)

$$\text{Res} \left[ \frac{1}{z-1}, 1 \right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} = 1 \quad (7)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j (1) = 2\pi j \neq 0 \quad (8)$$

For option D the pole is inside the contour. Then, using (??)

$$\text{Res} \left[ \frac{1}{z-1}, 1 \right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} = 1 \quad (9)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j (1) = 2\pi j \neq 0 \quad (10)$$

We can conclude that for options B,C,D contours have the non-zero value for this integral.