NCERT DISCRETE

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EE23BTECH11020

Question 11.9.4.3: Find the sum to n terms to the series $3 \times (1)^2 + 5 \times (2)^2 + 7 \times (3)^2 + \dots$

solution:

Given series is $3 \times (1)^2 + 5 \times (2)^2 + 7 \times (3)^2 + \dots$

The nth term of the series can be written as $(2n + 1) \times n^2$.

Let us assume n^{th} term as T_n.

Therefore ,

$$T_n = (2n + 1) \times n^2 \tag{1}$$

Then,

$$\sum T_n = \sum (2n + 1) \times n^2 \tag{2}$$

$$\sum T_n = \sum 2n^3 + \sum n^2 \tag{3}$$

We know that

$$\sum n^3 = \frac{n^2 \times (n + 1)^2}{4}, \sum n^2 = \frac{n \times (n + 1) \times (2n + 1)}{6} \tag{4}$$

Then,

$$\sum T_n = \frac{n^2 \times (n + 1)^2}{2} + \frac{n \times (n + 1) \times (2n + 1)}{6} \tag{5}$$

$$\sum T_n = \frac{n \times (n + 1) \times (3n^2 + 5n + 1)}{6} \tag{6}$$

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Therefore the sum to the n series to the series $3 \times (1)^2 + 5 \times (2)^2 + 7 \times (3)^2 + \dots$
is

$$\sum T_n = \frac{n \times (n+1) \times (3n^2 + 5n + 1)}{6} \tag{6}$$