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Date

Page

# PARAMETER EVALUATION

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#1 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{x-y}{2\sigma^2}\right)^2}$

Let $x_1, x_2, x_3 \ldots\ldots x_n$ be the sample of size n

$L(x_1, x_2, x_3 \ldots x_n) = f(x_1).f(x_2) \ldots\ldots f(x_n)$

$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-y)^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-y)^2}{2\sigma^2}}\right)$

taking ln on both sides

$\ln(L) = \frac{-n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-y)^2}{2\sigma^2}\right) - ①$

Taking partial derivatives w.r.t y on above equation

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i-\mu)'}{2\sigma^2}\right) = 0$

$= \sum_{i=1}^{n}(x_i-\mu) = 0$

$\Rightarrow n\bar{x} - n\mu = 0 \Rightarrow \bar{x} = \mu$

$\theta_1 = \bar{x}$ is therefore the sample mean

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Taking derivation w.r.t $\sigma^2$ (of eq(1))

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$= -n + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$\Rightarrow n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)$$

hence $\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$

2. Binomial distribution

$\Rightarrow {}^nC_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n}\left(\log({}^nC_{x_i}) + \log \theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

differenciate w.r.t $\theta$

Date
Page

$$\frac{\partial \log(L)}{\partial \theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{1}{1-\theta} \Sigma(n - x_i) = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\Rightarrow \frac{\Sigma 1}{\theta(1-\theta)} \Sigma x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\Sigma x_i}{n^2}}$$