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Department of Mathematics

Subject: Mathematics Foundations for Computing, Probability and Statistics **Date: 24-06-2023**

Subject Code : 21MATCS41 **Semester : IV**

Assignment No. 1

Module-1

Answer the following Questions.

1. Define the following with an example for each
   i) Proposition iii) Contradiction
   ii) Tautology iv) Dual of statement
2. Let p,q and r be propositions having truth values 0,0 and 1 respectively. Find the truth values of the following compound proposition
   i) $(p\wedge q) \to r$ ii) $p \to (q\wedge r)$ iii) $p\wedge(r \to q)$ iv) $p \to (q \to (\neg r))$.
3. Define tautology. Prove that for any propositions p,q,r the compound proposition $[(p\vee q)\wedge\{(p \longrightarrow r)\wedge(q \to r)\}] \to r$ is a tautology
4. Establish the validity of the following argument $p \longrightarrow q$

$$p \to (q\wedge r)$$
$$\neg r\vee(\neg t\vee u)$$
$$\underline{p\wedge t}$$
$$\therefore u$$

5. Prove that for any propositions p, q ,r $[p \to (q\wedge r)] \Leftrightarrow [p \longrightarrow q)\wedge(p \to r]$ using the truth table.
6. Prove that for any propositions p, q ,r $[p \to (q \longrightarrow r)] \longrightarrow [p \longrightarrow q) \to (p \to r]$ is a tautology
7. Prove the following logical equivalence using the laws of logic:

$$(p \to q)\wedge[\neg q\wedge(r\vee \neg q)] \Leftrightarrow \neg (q\vee p)$$

8. Prove the following logical equivalence using the laws of logic:

$$[\neg p\wedge(\neg q\wedge \mathrm{r})]\vee(q\wedge \mathrm{r})\vee(p\wedge \mathrm{r}) \Leftrightarrow r$$

9. Prove the validity of the arguments using the rule of inference

$$(\neg p\vee \neg q) \to (r\wedge s)$$
$$r \to t$$
$$\underline{\neg t}$$
$$\therefore p$$

10. Establish the validity of the following argument

$$\forall x, [p(x)\vee q(x)]$$
$$\exists x, \neg p(x)$$
$$\forall x, [\neg q(x)\vee r(x)]$$
$$\underline{\forall x\ [s(x) \to \neg r(x)]}$$
$$\therefore \exists x, \neg s(x)$$

11. Find whether the following argument is valid:

No Engineering student of 1^{st} or 2^{nd} semester studies logic
Anil is an Engineering student who studies logic

∴ Anil is not in second semester.

12. Give i) Direct proof and ii) Proof by contradiction for the following statement. " If 'n' is an odd integer ,then n+9 is an even integer" .
13. Give a direct proof for the following:
    i)For all integers k and l , if k and l are both even , then k+l is even
    ii)For all integers k and l , if k and l are both even , then k×l is even
14. Give i) Direct proof and ii) Indirect Proof iii) Proof by contradiction for the following statement. " If 'm' is an even integer, then m+7 is an odd".
15. Prove that for all integers 'k' and 'l', if 'k' and 'l' are both odd ,then k+l is even and kl is odd by direct proof
16. Give a direct proof of the statement "The square of an odd integer is an odd integer".
17. Find the possible truth values for p,q and r
    i) $p \rightarrow (q \vee r)$ – False
    ii) $p \wedge (q \rightarrow r)$ - True
18. Establish the validity of the following argument using rules of inference. "If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made, therefore the band could play rock music".
19. Find whether the following arguments are valid or not for which the universe is set of all triangles . In triangle XYZ ,there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it is isosceles. If the triangle is isosceles ,then it has two angles of equal measure. Therefore triangle XYZ has no two sides of equal length
20. Determine the truth value of the following statements If the universe comprises all nonzero integers
    i) $\exists x, \exists y\ [xy = 2]$
    ii) $\exists x, \forall y\ [xy = 2]$
    iii) $\forall x, \exists y\ [xy = 2]$
    iv) $\exists x, \exists y\ [(3x + y = 8) \wedge (2x - y = 7)]$
    v) $\exists x, \exists y\ [(4x + 2y = 3) \wedge (x - y = 1)]$
21. Define i) Open sentence ii) Quantifiers. For the following statements, the universe comprises all nonzero integers i) $\exists x, \exists y\ [xy = 1]$ ii) $\exists x, \forall y\ [xy = 1]$ iii) $\forall x, \exists y\ [xy = 1]$
22. Define dual of a logical statement. Verify the principle of duality for the following logical equivalence
    $[\neg(p \wedge q) \rightarrow \neg p \vee (\neg p \vee q))] \Leftrightarrow (\neg p \vee q)$.
23. Prove the following by using laws of logic
    i)) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
    ii) $[\neg p \wedge (\neg q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$
24. Establish the validity of the following argument using the rules of inference
    $$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$$
25. Negate and simplify each of the following: i) $\exists x, [p(x) \vee q(x)]$
    ii) $\forall x, [p(x) \wedge \neg q(x)]$
    iii) $\forall x\ [p(x) \rightarrow q(x)]$

iv) $\exists x, [\{p(x) \vee q(x)\} \to r(x)]$

26. Show that $R \vee S$ follows logically from the premises $C \vee D, (C \vee D) \to \neg H, \to \neg H \to (A \wedge \neg B),\ (A \wedge \neg B) \to R \vee S$
27. Establish the validity of the following argument using the rules of inference

$$\forall x\ [p(x) \to q(x)]$$
$$\forall x\ [q(x) \to r(x)])$$
$$\overline{\quad\forall x\ [p(x) \to r(x)]\quad}$$

28. Check whether the following is a valid argument

If I study ,then I will not fail in the examination. If I don't watch TV in the evening, then I will study.
I failed in the examination.∴

∴ I must watch TV in the evening

MODULE – 2

1. For any non- empty sets A,B,C prove that,

   i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ ii) $\big(A \times (B - C)\big) = (A \times B) - (A \times C)$

2. Let $f: R \to R$ be defined by f(x) $= \begin{cases} 3x - 5\ for\ x > 0 \\ -3x + 1\ for\ x \le 0 \end{cases}$

   i)Determine $f(0), f(\frac{5}{3})$ ii)Find $f'([-5,5])$.

3. Let f,g,h be functions form Z to Z defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0\ if\ x\ is\ even \\ 1\ if\ x\ is\ odd \end{cases}$ .verify that $(fog)oh(x) = fo(goh)(x)$.
4. Let $A = \{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b". Represent the relation R as a matrix and draw its diagraph.
5. Draw the Hasse diagraph representing the positive divisors of 36.
6. Let $A = \{1,2,3,4,5\}$, define a relation R on $A \times A$, by (X1,Y1)$R(X2,Y2)$ if and only if $X1 + Y1 = X2 + Y2$

   i) verify that R is an equivalence relation.

   ii) Find the partition of $A \times A$ induced by R.

7. If $A = \{1,2,3,4,5\}$ and there are 6720 injective functions $f: A \to B$, What is $|B|$?
8. Six books each of Physics , Chemistry, Mathematics and four books of Biology totally contains 12225 pages. Find the least number of pages contained in book.
9. The set $A = \{1,3,4,7,9\}$ and $B = \{2,4,6,7,8\}$ and $f: R \to R$ is given by $f(x) = 2x + 5$.Verify the following results for
   i)$f(A \cup B) = f(A) \cup f(B)$
   ii)$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$
   iii)$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$
10. Consider poset whose Hasse diagram is given below . Consider $B = \{3,4,5\}$. Find the upper and lower bounds of B , least upper bound and greatest lower bound of B.

11. Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & for\ x > 0 \\ -3x + 1 & for\ x \leq 0 \end{cases}$
    i) Determine $f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$
    ii) Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1\,)$ $f^{-1}(3), f^{-1}(-3)$
    iii) what are $f^{-1}([-5,5])$, $f^{-1}([-6,5])$
12. Let $f, g, h: R \to R$ where $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$,Show that $\big(fo(goh)\big) = \big((fog)oh\big)$
13. Let $A = \{1,2,3,4\}$ and let R be the relation on A defined by xRy if and only if " x divides y",
    a) Write down R as a set of ordered pairs
    b) Draw the digraph of R
    c) Write matrix of R.
14. Let $A = \{1,2,3,4,5,6\}$, $B = \{6,7,8,9,10\}$ and f be a function from A to B defined by $f = \{(1,7)(2,7)(3,8)(4,6)(5,9)(6,9)\}$. Then find $f^{-1}(9), f^{-1}(6)$. If $B_1 = \{7,8\}$, $B_2 = \{8,9,10\}$ and find $f^{-1}(B_1)$, $f^{-1}(B_2)$.
15. Prove that if 30 dictionaries in a library contain a total of 61,327 pages ,then at least one of the dictionaries must have at least 2045 pages.
16. Define the following with an example to each.
    (i) Simple graph (ii) Complete graph (iii) Regular graph (iv) Spanning sub graph (v) Induced sub graph
    (vi) Complete Bipartite graph (vii) Bipartite graph (viii) Complement of a graph
17. Merge sort the list -1,7,4,11,5,-8,15,-3,-2,6,10,3
18. Merge sort the list 7,3,8,4,5,10,6,2,9
19. Prove that in a graph .The sum of degrees of all vertices is an even number and is equal to twice the number of edges
20. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur with respective frequencies 78,16,30,35,125,31,20,50,80,3
21. Define optimal prefix code. Obtain the optimal prefix code for the message “ROAD IS GOOD”. Indicate the code.
22. Obtain the optimal prefix code for the message “LETTER RECEIVED”. Indicate the code.
23. Obtain the optimal prefix code for the message “ENGINEERING”. Indicate the code.
24. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with the frequencies 20,28,4,17,12,7 respectively
25. A tree with ‘n’ vertices having ‘n-1’ edges. Prove the given statement.
26. Define isomorphism. Verify the following graphs are isomorphism.
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