MATHEMATICS

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Section A

1) 1.If f: $R \longrightarrow R$ be defined by $f(x) = (3x^3)^{\frac{1}{3}}$, then find $f(x)$

2) Write the principal value of $sec^{-}1(-2)$.

3) What positive value of x makes the following pair of determinants equal ?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$$

,

$$\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

4) Evaluate : $\int sec^2(7-4x)dx$

5) Write the adjoint of the following matrix: $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

6) Write the value of the following integral:

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5 x\, dx$$

7) A is a square matrix of order 3 and $|A| = 7$. Write the value of $|adj|$

8) Write the distance of the following plane from the origin: $2xy + 2z + 1 = 0$

9) Write a vector of magnitude 9 units in the direction of vector $-2i + j + 2k$.

10) Find λ if $(2i + 6j + 14k) \times (i - \lambda\hat{J} + 7k) = o$.

11) A family has 2 children. Find the probability that both are boys, if it is known that
(i) the elder child is a boy.
(ii) at least one of the children is a boy

12) Show that the relation S in the set $A = [x\epsilon Z : 0 \le x \le 12)$ given by $S = [(a, b) : a, b\epsilon Z, |a-b|$ is divisible by 4 is an equivalence relation. Find the set of all elements related to 1

13) Prove the following: $\tan^{-1}x + \tan^{-1}x(\frac{2x}{1-x^2}) = \tan^{-1}(\frac{3x-x^3}{1-3x^2})$ (OR) Prove the following: $cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

14) Express the following matrix as the sum of a symmetric and skew symmetric matrix, and verify your result:
$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

15) If $\overrightarrow{a} = i+j+k, \overrightarrow{b} = 4i-2j+3k$ and $\overrightarrow{c} = i-2j+k$ find a vector of magnitude 6 units which is parallel to the vector $2a - b + 3c$.
(OR) Let $\overrightarrow{a} = i + 4j + 2k, \overrightarrow{b} = 3i - 2j + 7k$ and $\overrightarrow{c} = 2i - j + 4k$ find a vector $\overrightarrow{d}$ which is perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$ and $\overrightarrow{c}.\overrightarrow{d} = 18$.

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16) Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3)$.
(OR) Find the distance of the point $P(6, 5, 9)$ from the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$.

17) Solve the following differential equation:
$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}|x|$

OR

$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

18) Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$, is homogeneous and solve it equation

19) Evaluate the following:

$$\int \frac{x+2}{\sqrt{(x-2)(x-3)}}\, dx$$

20) Evaluate the following:

$$\int_1^2 \frac{5x^2}{x^2 + 4x + 3}\, dx$$

21) If $y = e^{a\sin^{-1}x}$, $-1 \leq x \leq 1$ then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

22) If $y = \cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$, find $\frac{dy}{dx}$

23) Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x-y)(y-z)(z-x)$$

(OR)
Find the inverse of the following matrix using elementary operations :

$$\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

24) A bag contains 4 balls. Two balls are drawn at random, and are found to be white. What is the probability that all balls are white?

25) One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150g of flour and 30g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600g of fat, assuming that there is no shortage of the other in gradients used in making the cakes. Make it as an L.P.P. and solve it graphically.

26) Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3, 2, 1) from the plane $2xy + z + 1 = 0$. Find also, the image of the point in the plane

27) Find the area of the circle $4x^2 + 4y^2 = 9$ which the interior to the parabola.$x^2 = 4y$
(OR)
Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

28) If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium, when it is maximum.

29) Find the intervals in which the following function is
(a) strictly increasing,
(b) strictly decreasing.