binomial distribution

- definition, properties, conditions
- calculating probabilities
- mean and standard deviation

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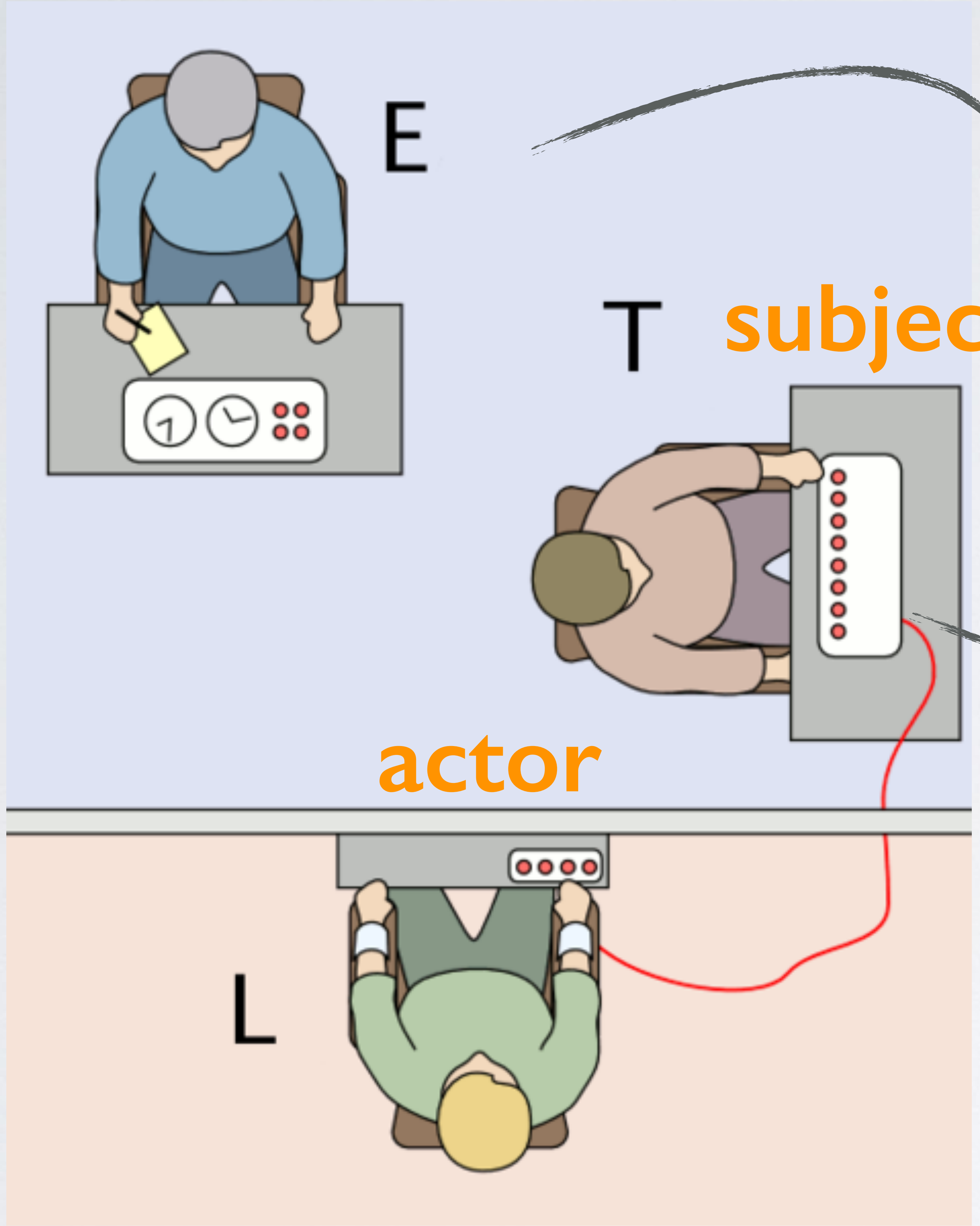
the Milgram experiment



orders the teacher

$P(\text{shock}) = 0.65$

shocks each time the learner answers a question incorrectly

Image source: http://commons.wikimedia.org/wiki/File:Milgram_Experiment_v2.png

Bernouilli random variables

- each person in Milgram's experiment can be thought of as a trial
- a person is labeled a success if she refuses to administer a severe shock, and failure if she administers such shock
- since only 35% of people refused to administer a shock, probability of success is $p = 0.35$.
- when an individual trial has only two possible outcomes, it is called a Bernoulli random variable

Suppose we randomly select four individuals to participate in this experiment. What is the probability that exactly 1 of them will refuse to administer the shock?

- Four individuals:

(A) Anthony
(B) Brittany
(C) Clara
(D) Dorian

- Multiple scenarios where *"exactly 1 refuses"*

Scenario 1: $\underset{\text{(A) refuse}}{0.35} \times \underset{\text{(B) shock}}{0.65} \times \underset{\text{(C) shock}}{0.65} \times \underset{\text{(D) shock}}{0.65} = 0.0961$

OR

Scenario 2: $\underset{\text{(A) shock}}{0.65} \times \underset{\text{(B) refuse}}{0.35} \times \underset{\text{(C) shock}}{0.65} \times \underset{\text{(D) shock}}{0.65} = 0.0961$

OR

Scenario 3: $\underset{\text{(A) shock}}{0.65} \times \underset{\text{(B) shock}}{0.65} \times \underset{\text{(C) refuse}}{0.35} \times \underset{\text{(D) shock}}{0.65} = 0.0961$

OR

Scenario 4: $\underset{\text{(A) shock}}{0.65} \times \underset{\text{(B) shock}}{0.65} \times \underset{\text{(C) shock}}{0.65} \times \underset{\text{(D) refuse}}{0.35} = 0.0961$

+

$4 \times 0.0961 = 0.3844$

binomial distribution

the binomial distribution describes the probability of having exactly *k* successes in *n* independent Bernouilli trials with probability of success *p*

# of scenarios x P(single scenario)

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n choose k"

$$p^k(1-p)^{(n-k)}$$

probability of success to the power of number of successes

probability of failure to the power of number of failures

How many scenarios yield 1 success in 4 trials?

$n = 4 \qquad k = 1$

$$\binom{4}{1} = \frac{4!}{1! \times (4-1)!}$$

$$= \frac{4 \times \cancel{3 \times 2} \times 1}{1 \times \cancel{3 \times 2} \times 1} = 4$$

SSFFFFFFF

SFSFFFFFF

SFFSFFFFF

...

How many scenarios yield 2 successes in 9 trials?

$n = 9 \qquad k = 2$

$$\binom{9}{2} = \frac{9!}{2! \times 7!}$$

$$= \frac{9 \times 8 \times \cancel{7!}}{2 \times 1 \times \cancel{7!}} = 36$$

R

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> choose(9,2)
[1] 36
```

Binomial distribution:

If *p* represents probability of success, *(1-p)* represents probability of failure, *n* represents number of independent trials, and *k* represents number of successes

$$P(\text{k successes in n trials}) = \binom{n}{k} p^k \ (1-p)^{(n-k)}$$

where $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

binomial conditions

1. the trials must be independent
2. the number of trials, n, must be fixed
3. each trial outcome must be classified as a success or a failure
4. the probability of success, p, must be the same for each trial

practice

According to a 2013 Gallup poll, worldwide only 13% of employees are engaged at work (psychologically committed to their jobs and likely to be making positive contributions to their organizations). Among a random sample of 10 employees, what is the probability that 8 of them are engaged at work?

$n = 10$

$p = 0.13$

$1 - p = 0.87$

$k = 8$

$$P(K = 8) = \binom{10}{8} 0.13^8 \times 0.87^2$$

$$= \frac{10!}{8! \times 2!} \times 0.13^8 \times 0.87^2$$

$$= \frac{10 \times 9 \times \cancel{8!}}{\cancel{8!} \times 2 \times 1} \times 0.13^8 \times 0.87^2$$

$$= 45 \times 0.13^8 \times 0.87^2$$

$$= 0.0000278$$

Poll: http://www.gallup.com/poll/165269/worldwide-employees-engaged-work.aspx practice

R

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> dbinom(8, size = 10, p = 0.13)
[1] 2.77842e-06
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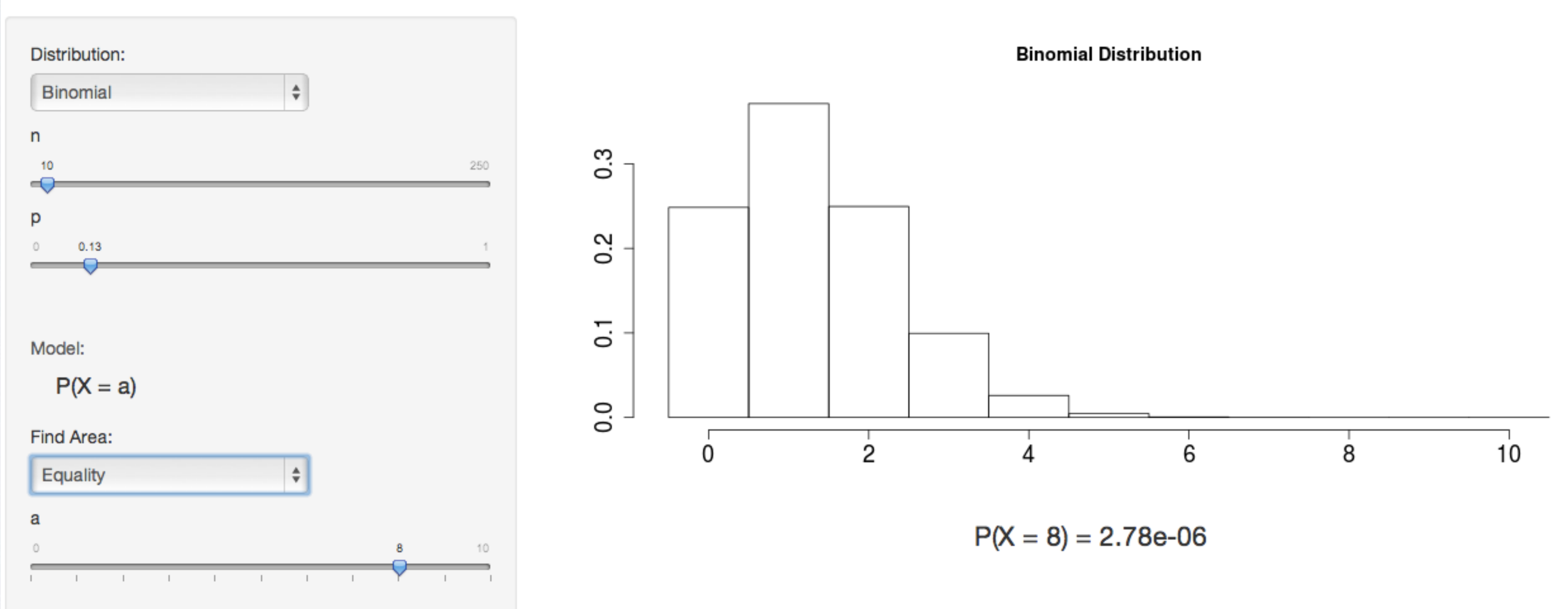
http://bit.ly/dist_calc



Among a random sample of 100 employees, how many would you expect to be engaged at work? Remember: $p = 0.13$.

$$\mu = 100 \times 0.13 = 13$$

Expected value (mean) of binomial distribution: $\mu = np$

Standard deviation of binomial distribution: $\sigma = \sqrt{np(1-p)}$

$$\sigma = \sqrt{100 \times 0.13 \times 0.87} = 3.36$$