comparing two independent means: what to report

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birth weight and smoking

2004 survey of 999 births on North Carolina

weight gain during pregnancy and mother's age

2004 survey of 999 births on North Carolina

weight birth weight of baby in pounds

birth weight and smoking

2004 survey of 999 births on North Carolina

weight	birth weight of baby in pounds
habit	status of the mother as a ‘nonsmoker’ or a ‘smoker’

https://pixabay.com/en/cigarette-smoking-tobacco-nicotine-1638135/

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two questions

1. is the average birth weight of babies whose mothers did not smoked different from the average weight gain of younger others?

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two questions

1. is the average birth weight of babies whose mothers did not smoked different from the average weight gain of younger others?
2. if there are differences how large is the effect?

model and priors

- random sample from two populations

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$$p(\mu) = 1 \qquad\qquad p(\sigma^2) = 1/\sigma^2$$

R code

```
library(statsr)
data(nc)
out =bayes_inference(y=weight, x=habit, data=nc,type='ht', null=0,
                     statistic='mean',  alternative='twosided',
                     prior='JZS', r=.5, method='sim', show_summ=FALSE)

## Hypotheses:
## H1: mu_nonsmoker  = mu_smoker
## H2: mu_nonsmoker != mu_smoker
##
## Priors: P(H1) = 0.5  P(H2) = 0.5
##
## Results:
## BF[H2:H1] = 1.4402
## P(H1|data) = 0.4098
## P(H2|data) = 0.5902
##
```

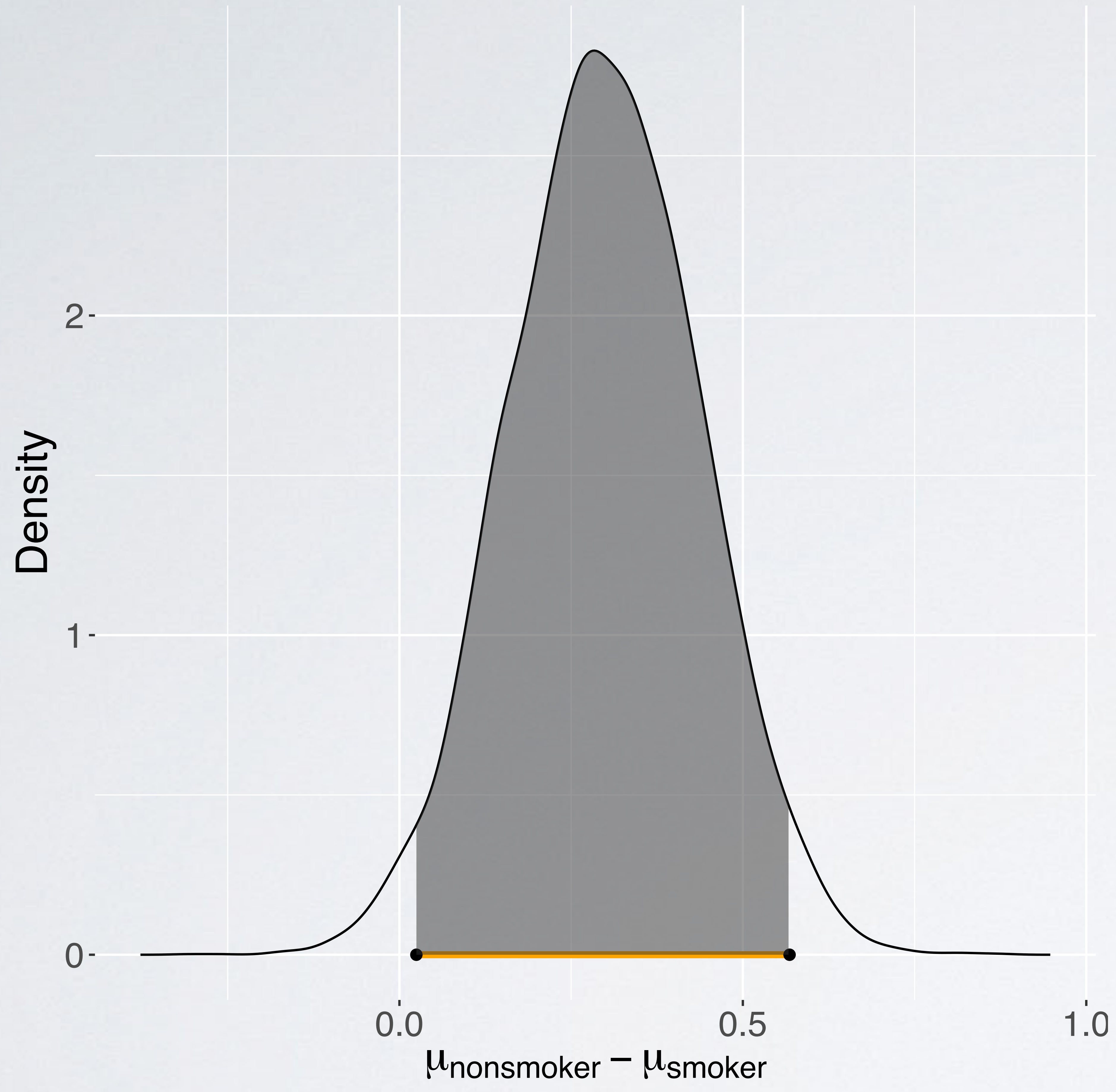
summaries under H_2

R code

```
out.ci = bayes_inference(y=weight, x=habit, data=nc, type='ci',
                         statistic='mean', prior='JZS', mu_0=0,
                         r=.5, method='sim', verbose=FALSE)
print(out.ci$summary, digits=2)

##                              2.5%     25%    50%     75%   97.5%
## overall mean                6.856    6.95    7.0    7.04 7.1e+00
## mu_nonsmoker - mu_smoker    0.023    0.20    0.3    0.39 5.7e-01
## sigma^2                     2.072    2.19    2.3    2.33 2.5e+00
## effect size                 0.015    0.14    0.2    0.26 3.8e-01
## n_0                       184.732 2016.09 4729.1 9594.28 2.6e+04
```

estimates of effect under H_2



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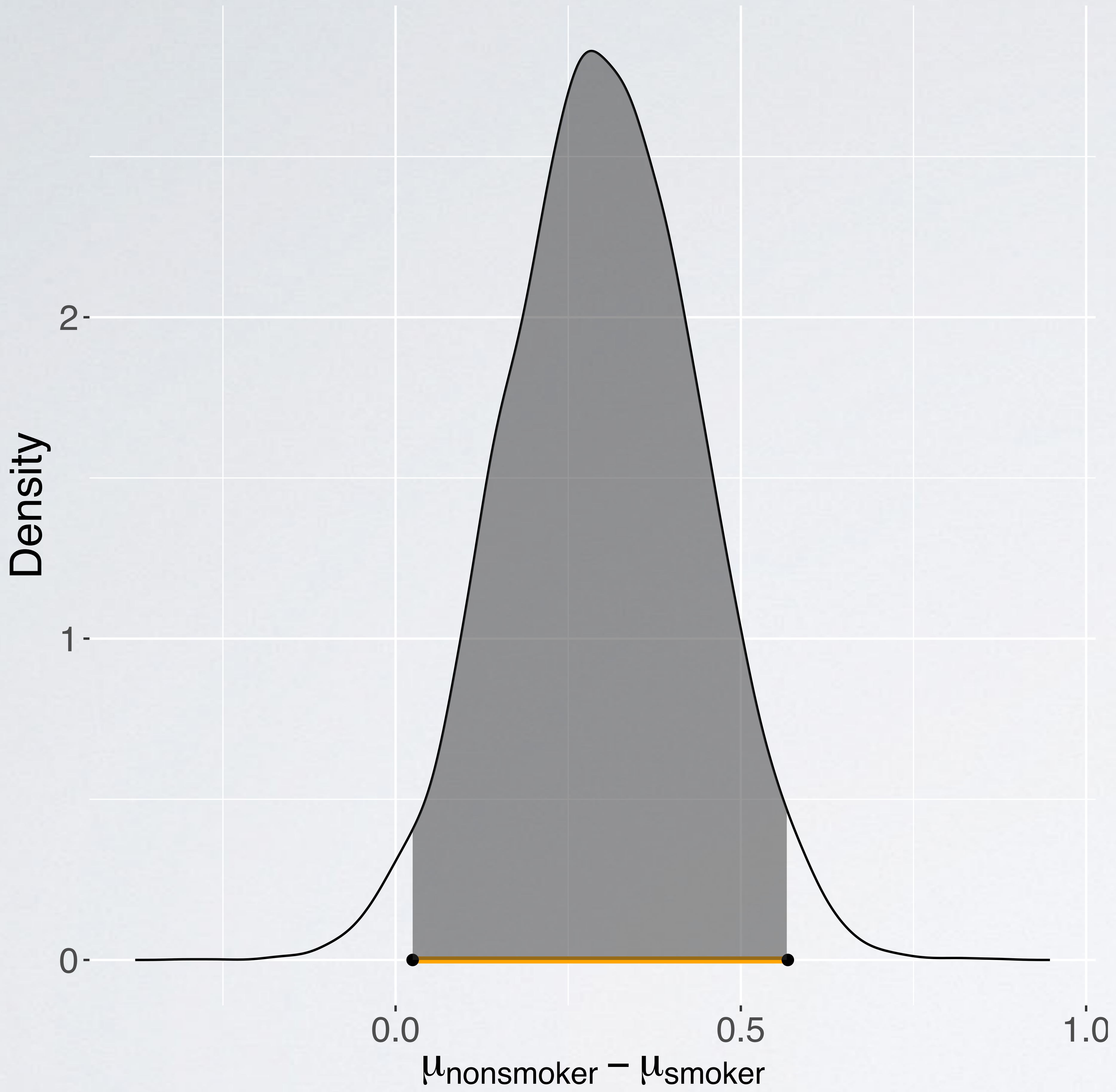

under H_2, 95% chance the average birth weight of babies born to nonsmokers is 0.02 to 0.57 pounds higher than that of babies born to smokers

overall coverage

- conditional probability $P(0.02 < \alpha < 0.57 \mid \mathrm{data}, H_2) = 0.95$

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$$P(0.02 < \alpha < 0.57 \mid \text{data})$$

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$$
\begin{aligned}
&P(0.02 < \alpha < 0.57 \mid \text{data}) \\
&= P(0.02 < \alpha < 0.57 \mid \text{data}, H_1) P(H_1 \mid \text{data}) +
\end{aligned}
$$

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&= I(0 \text{ in CI }) P(H_1 \mid \text{data}) + 0.95 \times P(H_2 \mid \text{data})
\end{aligned}$$

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the average birth weight of babies born to nonsmokers is 0.02 to 0.57 pounds higher than that of babies born to smokers with probability 0.56

summary

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next: regression models to adjust for continuous explanatory variables