multiple comparisons

- which means are different?
- controlling the Type 1 error rate

Duke UNIVERSITY

Dr. Mine Çetinkaya-Rundel
Duke University

which means differ

- two sample t tests for differences in each possible pair of groups
- multiple tests → inflated Type I error rate
- solution: use modified significance level

multiple comparisons

- testing many pairs of groups is called multiple comparisons
- the Bonferroni correction suggests that a more stringent significance level is more appropriate for these tests
  - adjust α by the number of comparisons being considered

Bonferroni correction:

$\alpha^\star = \alpha / K \quad K$: number of comparisons, $K = \frac{k(k-1)}{2}$

practice

The social class variables has 4 levels. If $\alpha = 0.05$ for the original ANOVA, what should the modified significance level be for two sample t tests for determining which pairs of groups have significantly different means?

$k = 4$

$K = \frac{4 \times 3}{2} = 6$

$\alpha^* = 0.05 / 6 \approx 0.0083$

pairwise comparisons

- constant variance → re-think standard error and degrees of freedom:
  - use consistent standard error and degrees of freedom for all tests
- compare the p-values from each test to the modified significance level

Standard error for multiple pairwise comparisons:

$$SE = \sqrt{\frac{MSE}{n_1} + \frac{MSE}{n_2}}$$

indep. groups test:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Degrees of freedom for multiple pairwise comparisons:

$$df = df_E$$

$$df = min(n_1 - 1, n_2 - 1)$$

practice

Is there a difference between the average vocabulary scores between middle and lower class Americans?

	Df	Sum Sq	Mean Sq	F value	Pr(> F)
class	3	236.56	78.855	21.735	<0.0001
Residuals	**791**	2869.80	**3.628**		
Total	794	3106.36			

	n	mean
lower class	41	5.07
middle class	331	6.76

$H_0: \mu_{middle} - \mu_{lower} = 0$

$H_A: \mu_{middle} - \mu_{lower} \neq 0$

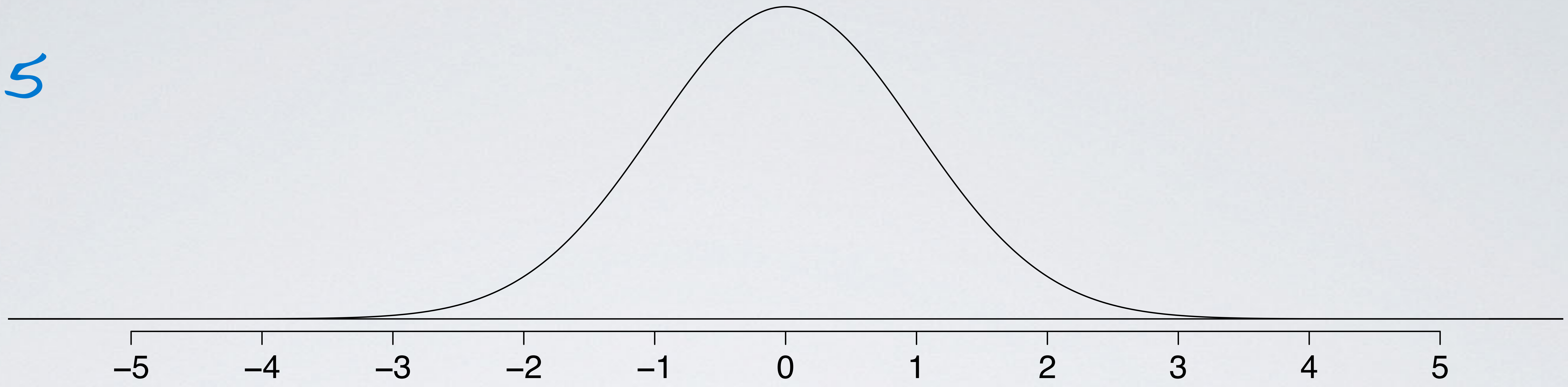
$$T = \frac{(\bar{X}_{middle} - \bar{X}_{lower}) - 0}{\sqrt{\frac{MSE}{n_{middle}} + \frac{MSE}{n_{lower}}}} = \frac{(6.76 - 5.07)}{\sqrt{\frac{3.628}{331} + \frac{3.628}{41}}} = \frac{1.69}{0.315} = 5.365$$

$df = 791$

practice

$T = 5.365$

$df = 791$



R

```
> 2 * pt(5.365, df = 791, lower.tail = FALSE)
[1] 1.063895e-07
```

$\alpha^* = 0.0083$

p-value $< \alpha^* \rightarrow$ Reject H_0