probability trees

$P(A \mid B) \rightarrow P(B \mid A)$

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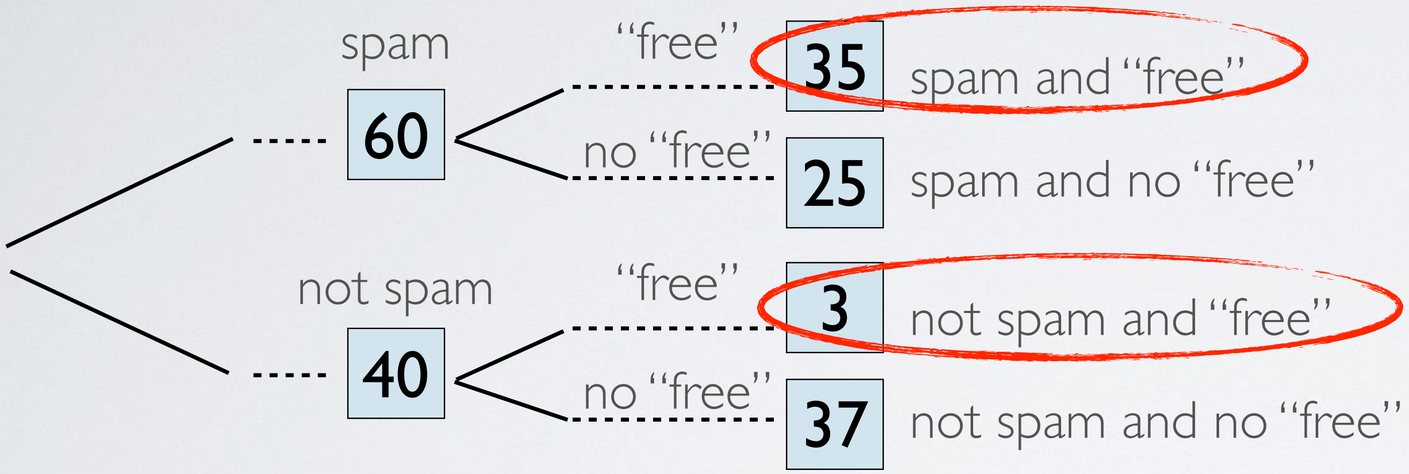
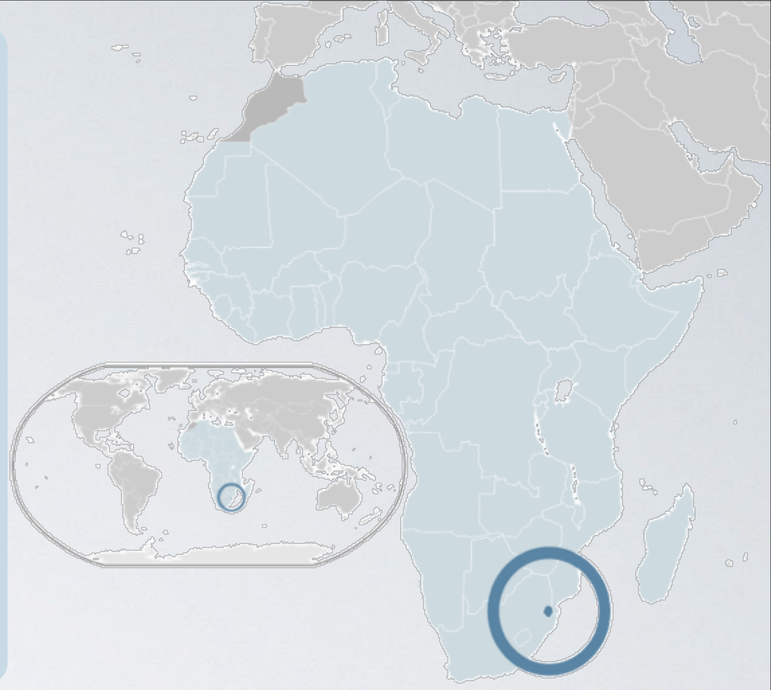
You have 100 emails in your inbox: 60 are spam, 40 are not. Of the 60 spam emails, 35 contain the word "free". Of the rest, 3 contain the word "free". If an email contains the word "free", what is the probability that it is spam?



$$P(\text{spam} \mid \text{"free"}) = \frac{35}{35 + 3}$$

$$= 0.92$$

As of 2009, Swaziland had the highest HIV prevalence in the world. 25.9% of this country's population is infected with HIV. The ELISA test is one of the first and most accurate tests for HIV. For those who carry HIV, the ELISA test is 99.7% accurate. For those who do not carry HIV, the test is 92.6% accurate. If an individual from Swaziland has tested positive, what is the probability that he carries HIV?



$P(HIV) = 0.259$

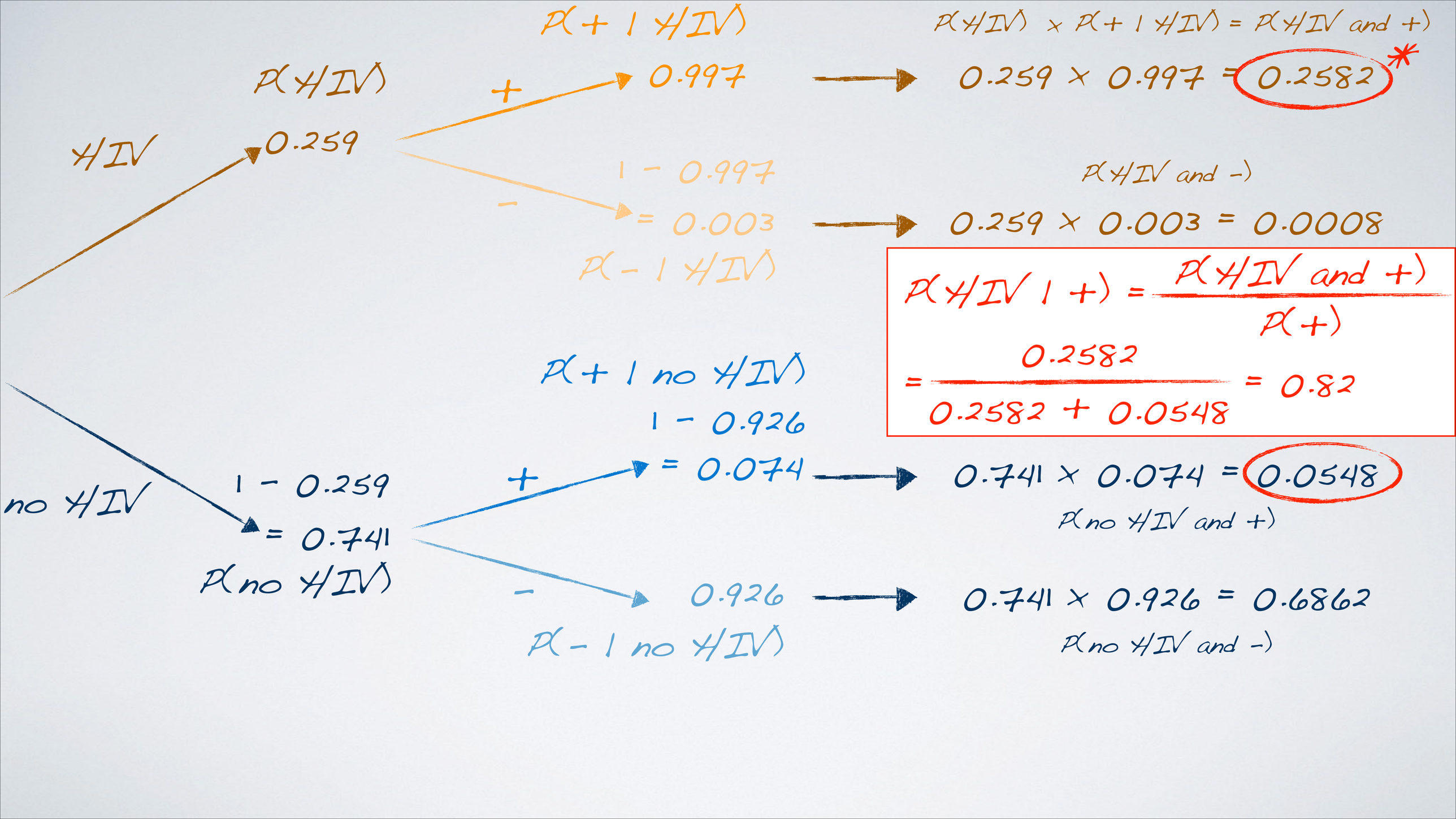
$P(+ \mid HIV) = 0.997$ $P(- \mid no\ HIV) = 0.926$

tree diagram!

$P(HIV \mid +) = ?$

Image source: http://en.wikipedia.org/wiki/File:Location_Swaziland_AU_Africa.svg

Data source: CIA Factbook, Country Comparison: HIV/AIDS - Adult Prevalence Rate
https://www.cia.gov/library/publications/the-world-factbook/rankorder/2155rank.html


P(+ | HIV)
P(HIV) x P(+ | HIV) = P(HIV and +)
P(HIV)
0.997
0.259 x 0.997 = 0.2582 *
+
HIV
0.259
1 - 0.997
= 0.003
P(- | HIV)
P(HIV and -)
0.259 x 0.003 = 0.0008
P(HIV | +) = P(HIV and +) / P(+)
= 0.2582 / (0.2582 + 0.0548) = 0.82
P(+ | no HIV)
1 - 0.926
= 0.074
0.7741 x 0.074 = 0.0548
P(no HIV and +)
no HIV
1 - 0.259
= 0.741
P(no HIV)
+
-
0.926
0.7741 x 0.926 = 0.6862
P(no HIV and -)
P(- | no HIV)

If an individual from Swaziland has tested positive,
what is the probability that he carries HIV?

$$P(\text{HIV} \mid +) = 0.82$$

There is an 82% chance
that an individual from Swaziland
who has tested positive
actually carries HIV.