predictive distributions & prior choice

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prior predictive distribution

prior:

$$1/\sigma^2 = \phi \sim \mathsf{Gamma}(v_0/2, s_0^2 v_0/2)$$

$$\mu \mid \sigma^2 \sim \mathsf{N}(m_0, \sigma^2/n_0)$$

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$$Y_i \mid \mu, \sigma^2 \overset{\text{iid}}{\sim} \mathsf{N}(\mu, \sigma^2)$$

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prior predictive distribution for Y:

$$p(Y) = \int\int p(Y \mid \mu, \sigma^2) p(\mu \mid \sigma^2) p(\sigma^2)\, d\mu d\sigma^2$$

$$Y \sim \mathsf{t}(v_0, m_0, s_0^2 + s_0^2/n_0)$$

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set $v_0 = n_0 - 1$ and use the predictive distribution to choose n_0

Monte Carlo sampling from prior predictive distribution

R Code

```
m_0 = (60+10)/2;  s2_0 = ((60-10)/4)^2;
n_0 = 2; v_0 = n_0 - 1
phi = rgamma(1000, v_0/2, s2_0*v_0/2)
sigma = 1/sqrt(phi)
mu =  rnorm(1000, mean=m_0, sd=sigma/(sqrt(n_0)))
y =   rnorm(1000, mu, sigma)
quantile(y, c(.025, .975))

##       2.5%     97.5%
## -135.3916  221.4759
```

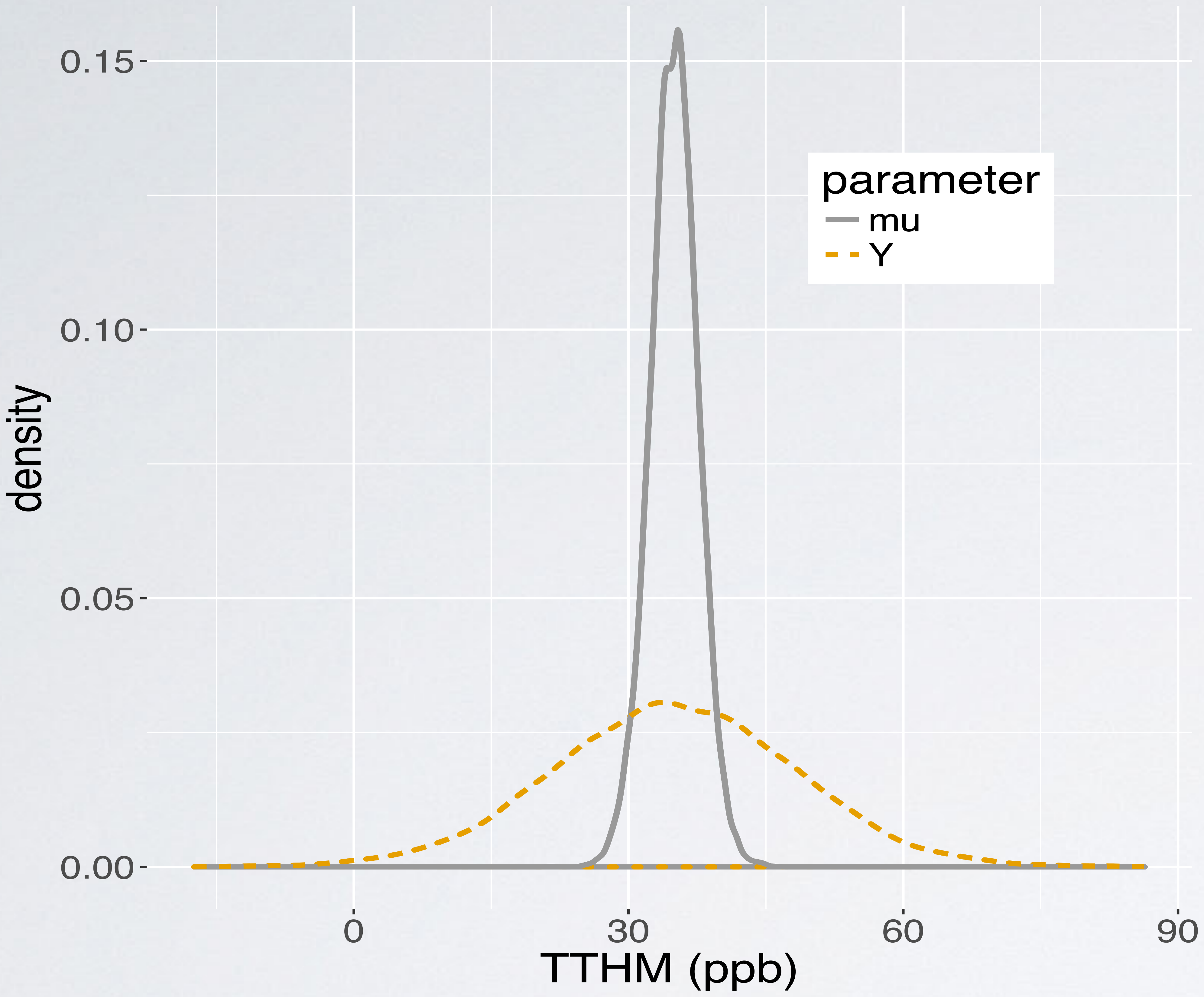
adjust prior sample size

R Code

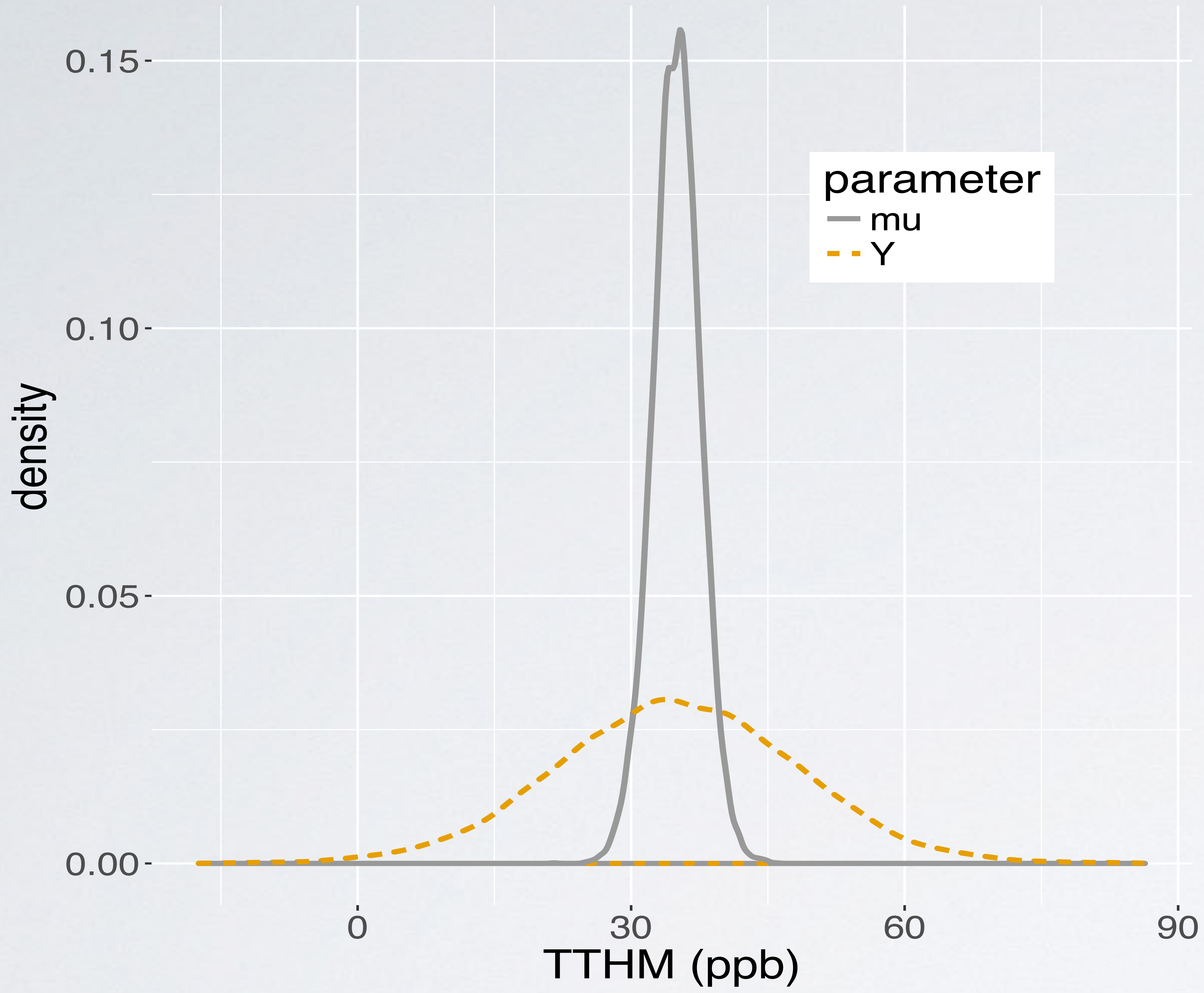
```
m_0 = (60+10)/2;  s2_0 = ((60-10)/4)^2;
n_0 = 25; v_0 = n_0 - 1
phi = rgamma(10000, v_0/2, s2_0*v_0/2)
sigma = 1/sqrt(phi)
mu =  rnorm(10000, mean=m_0, sd=sigma/(sqrt(n_0)))
y =   rnorm(10000, mu, sigma)
quantile(y, c(.025, .975))

##       2.5%     97.5%
##  8.522367 61.126357
```

prior densities



prior densities




```r
sum(y < 0)/length(y)  # P(Y < 0) a priori

## [1] 0.0064
```

posterior predictive distribution

predictive distribution of $Y_{n+1} \mid Y_1, \ldots, Y_n \sim \mathsf{t}(v_n, m_n, s_n^2(1 + 1/n_n))$

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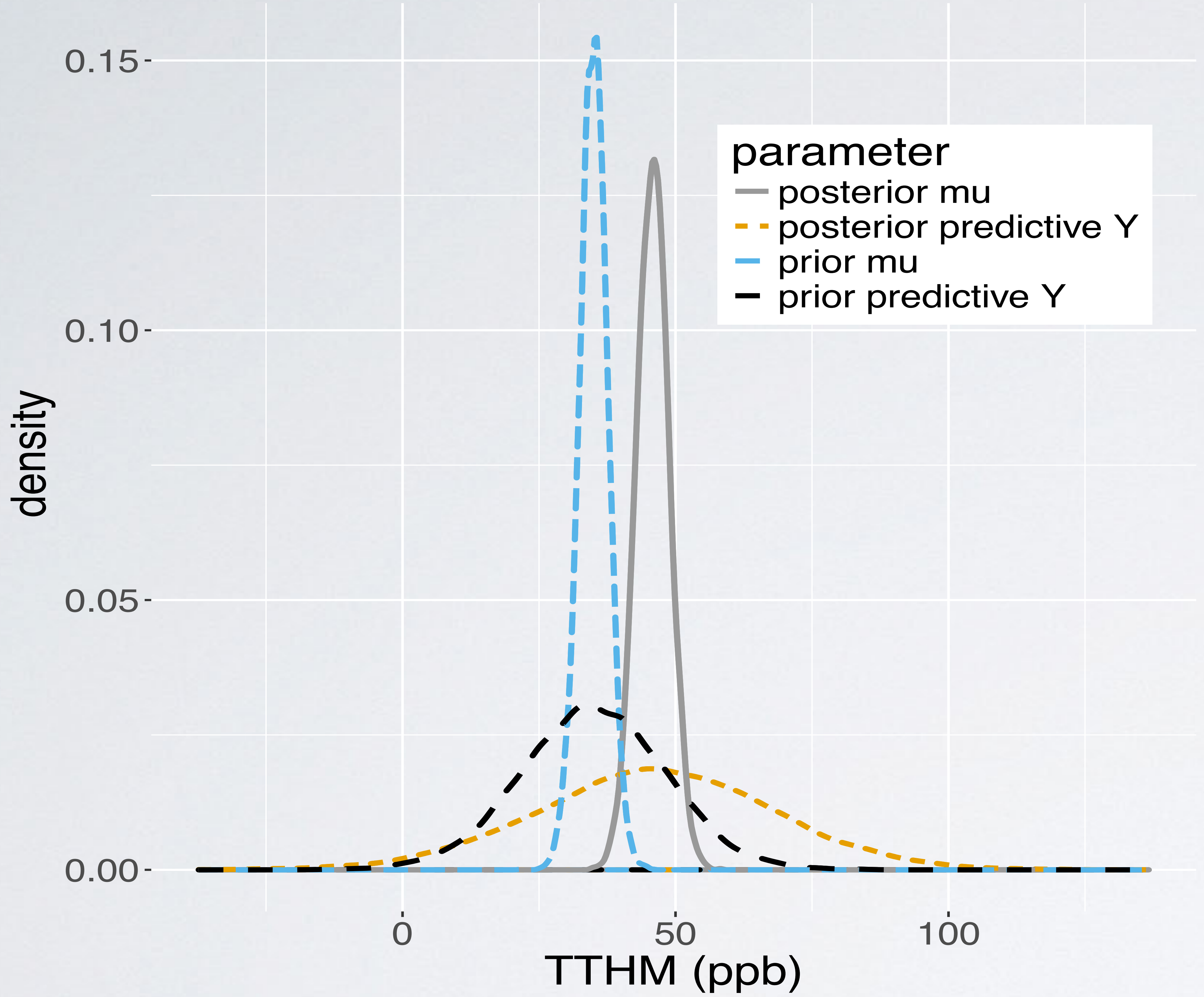
R Code

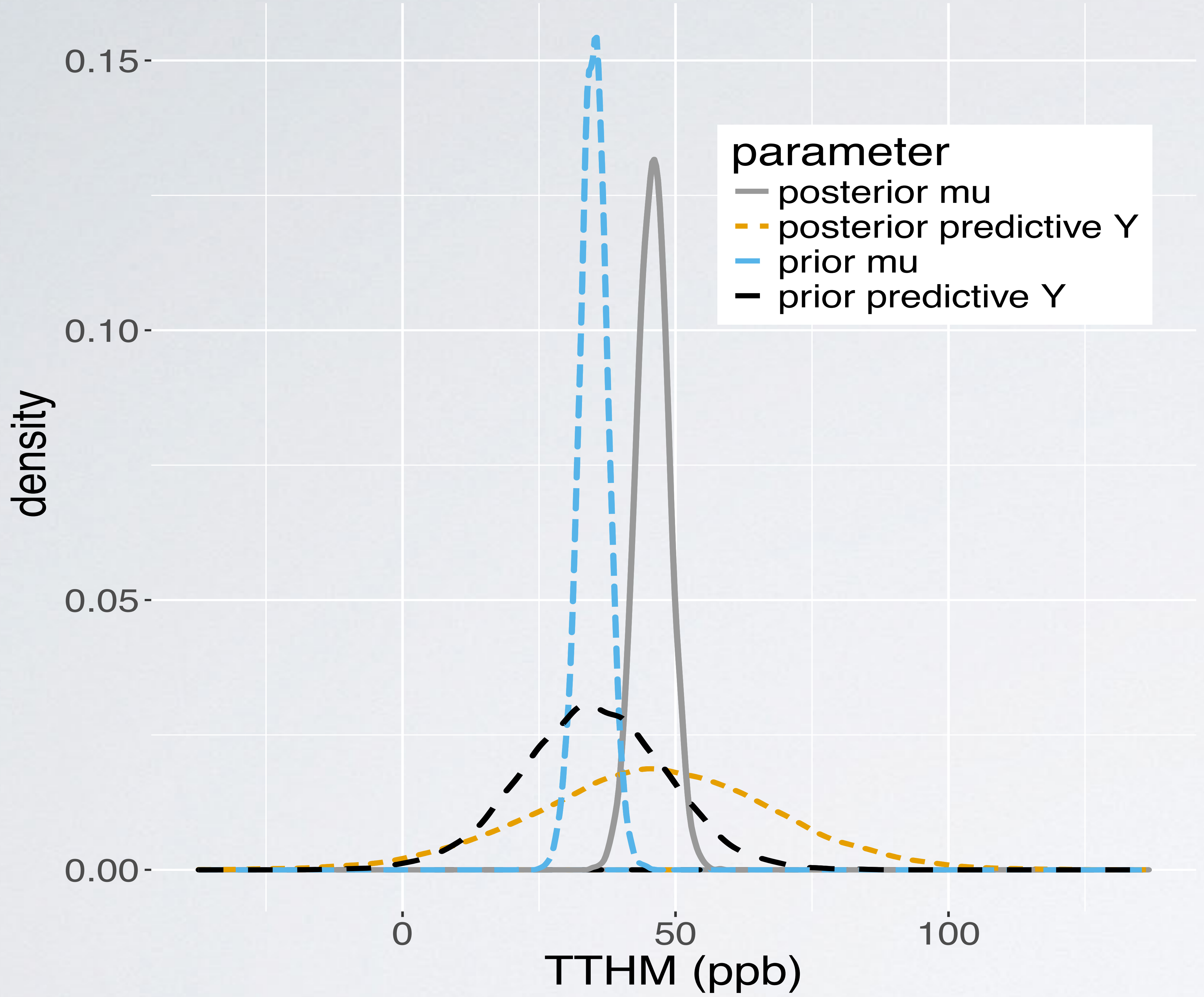
```
phi = rgamma(10000, v_n/2, s2_n*v_n/2)
sigma = 1/sqrt(phi)
post_mu = rnorm(10000, mean=m_n, sd=sigma/(sqrt(n_n)))
pred_y =  rnorm(10000,post_mu, sigma)
quantile(pred_y, c(.025, .975))

##     2.5%    97.5%
##  1.75404 89.30412
```

posterior densities



posterior densities




```
sum(pred_y > 80)/length(pred_y)   # P(Y > 80 | data)

## [1] 0.0613
```

summary

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next: reference prior distributions and sensitivity