conjugacy

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prior beliefs:

- binomial with known n and unknown p
- beta probability density function (α,β)

observe:

- x successes in n trials

new belief:

- beta($\alpha + x, \beta + n - x$)

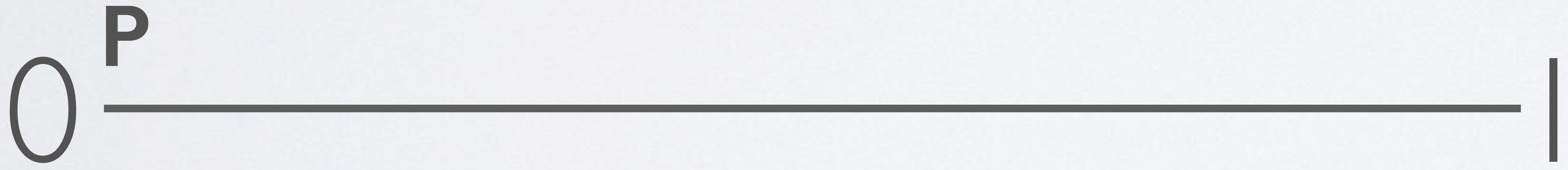
conjugacy

when the **posterior distribution** is in the **same family** as your prior belief but with **new parameter values**

Bayes' rule

$$\mathrm{P}[A_i|B] = \frac{\mathrm{P}[B|A_i]*\mathrm{P}[A_i]}{\sum_{j=1}^{n} \mathrm{P}[B|A_j]*\mathrm{P}[A_j]}$$

cannot apply to continuous random variables



integral form of Bayes' rule

$$\pi^*(p \mid x) = \frac{\boxed{\mathrm{P}(x \mid p)}\boxed{\pi(p)}}{\boxed{\int_0^1 \mathrm{P}(x \mid p)\pi(p)\, dp}}$$

beta-binomial case

$$= \frac{\left[\binom{n}{x} p^{x}(1-p)^{n-x}\right] * \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}\right]}{\text{some number}}$$

$$= c * p^{x+\alpha-1}(1-p)^{\beta+n-x-1}$$

$$c = \Gamma(\alpha^* + \beta^*)/[\Gamma(\alpha^*)\Gamma(\beta^*)]$$

$$\text{where } \alpha^* = \alpha + x$$

$$\beta^* = \beta + n - x$$

summary

some pairs of distributions are conjugate. If your prior is in one and your data comes from the other, then your posterior is in the same family as the prior, but with new parameters.

summary

we looked at the beta-binomial and saw that the integral form of Bayes' rule implied that the posterior on p had to be beta.