inference via Monte Carlo sampling

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- inference about σ or other transformations of the parameters?

Monte Carlo simulation

https://pixabay.com/en/gambling-roulette-game-bank-2001079/

Monte Carlo inference for the precision

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- improved approximation as S increases

$$\frac{\sum_{i=1}^{S} g(\phi^{(i)})}{S} \to \mathsf{E}[g(\phi) \mid \text{data}]$$

tap water example

R code

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set.seed(8675309)
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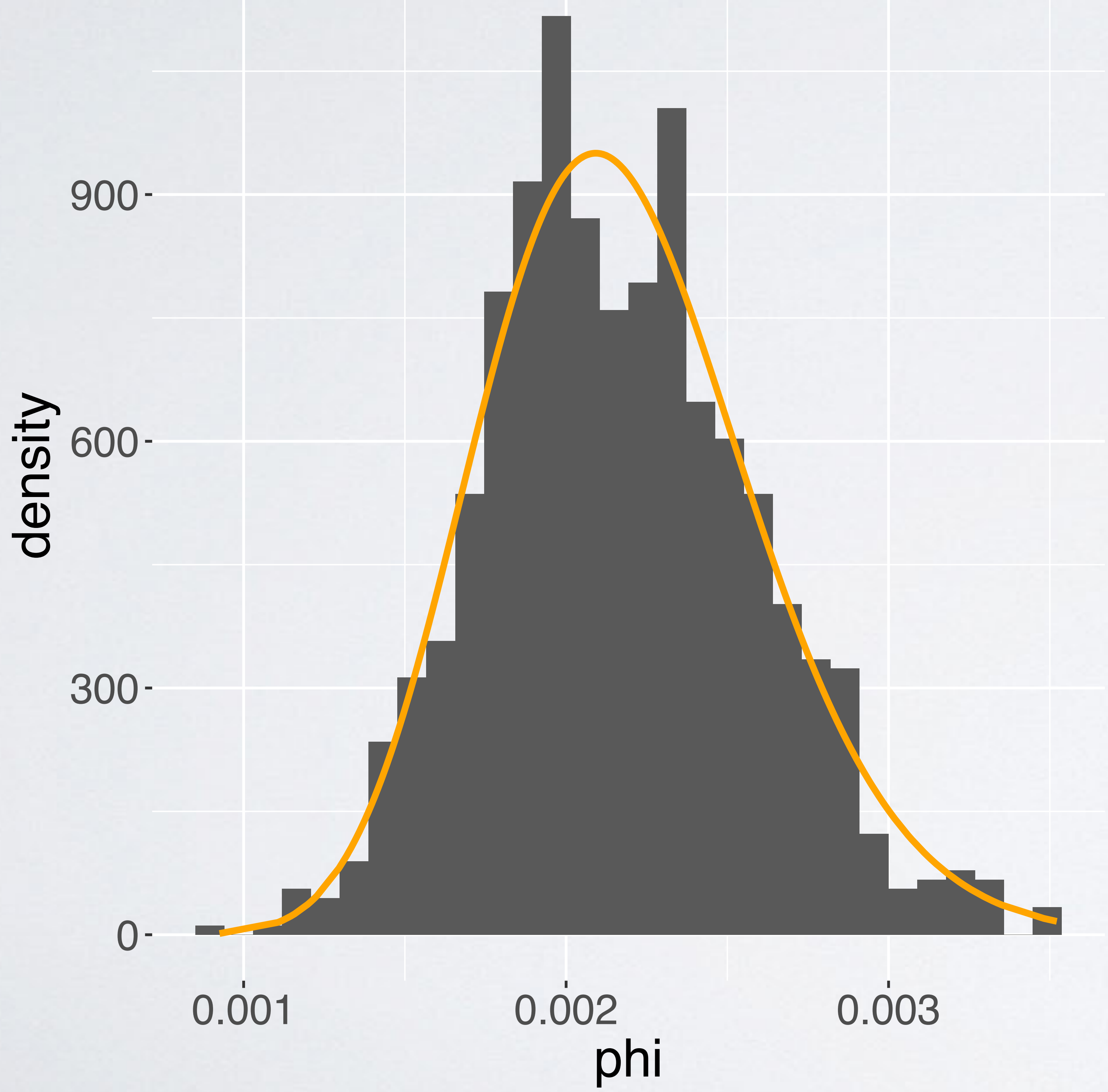

inference about σ

R code

```
phi = rgamma(1000, shape = v_n/2, rate=s2_n*v_n/2)
sigma = 1/sqrt(phi)
mean(sigma)   # posterior mean of sigma

## [1] 21.79296

quantile(sigma, c(0.025, .975))

##      2.5%    97.5%
## 18.13778 26.60354
```

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next: predictive distributions and choice of hyper-parameters