predictive inference

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the general problem

an inference on random variable X

probability density function $f(x \mid \theta)$

personal probability for θ, denoted by $\pi(\theta)$

to solve this, one needs to integrate:

$$\mathrm{P}[X \leq x] = \int_{-\infty}^{\infty} \mathrm{P}[X \leq x \,|\, \theta] \pi(\theta)\, d\theta$$

a coin example

$\mathbf{P}$[heads] = 0.7

$\mathbf{P}$[heads] = 0.4

the prior

p = probability you have the 0.7 coin

= 0.5

the posterior

$$p^* = \frac{\mathrm{P}[\ 2\ \mathrm{Heads}\,|\,0.7] \times 0.5}{\mathrm{P}\ [\ 2\ \mathrm{Heads}\,|\,0.7] \times 0.5 + \mathrm{P}[\ 2\ \mathrm{Heads}\,|\,0.4] \times 0.5}$$

$$= 0.754$$

the answer

p^* of 0.7 coin = 0.754

p^* of 0.4 coin = 1 − 0.754 = 0.246

$$P[\text{heads}] = P[\text{heads} \mid 0.7] \times 0.754 + P[\text{heads} \mid 0.4] \times 0.246$$

$$= 0.626$$

other applications

what is the probability that a fifth child born in the RU-486 trial will have a mother who received RU-486?

what is the probability that your stock broker's next recommendation will be profitable?

summary

- often the real goal is in **predicting a future outcome**
- for many applications, one needs to use an **integral**
- for some simple cases, one can find a solution **without integration**