conditional probability

- marginal, joint, conditional probability
- Bayes' theorem
- general product rule

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study

ADOLESCENTS' UNDERSTANDING OF SOCIAL CLASS

study examining teens' beliefs about social class

sample: 48 working class and 50 upper middle class 16-year-olds

study design:

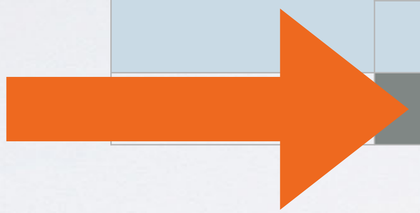
- "objective" assignment to social class based on self-reported measures of both parents' occupation and education, and household income
- "subjective" association based on survey questions

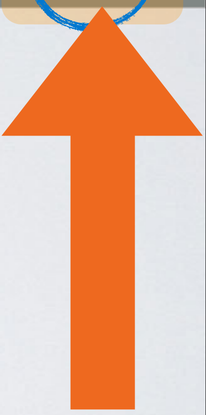
Study reference: Goodman, Elizabeth, et al. "Adolescents' understanding of social class: a comparison of white upper middle class and working class youth." Journal of adolescent health 27.2 (2000): 80-83.

results:		objective social class position		
		working class	upper middle class	Total
subjective social class identity	poor	0	0	0
	working class	8	0	8
	middle class	32	13	45
	upper middle class	8	37	45
	upper class	0	0	0
	Total	48	50	98

marginal

		objective social class position		
		working class	upper middle class	Total
subjective social class identity	poor	0	0	0
	working class	8	0	8
	middle class	32	13	45
	upper middle class	8	37	45
	upper class	0	0	0
	Total	48	50	98

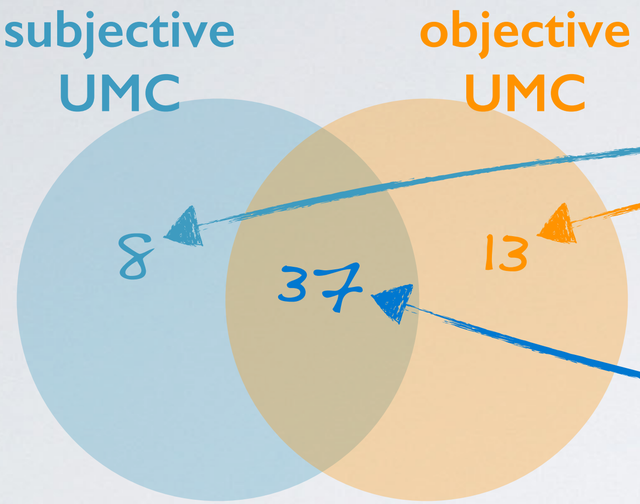




What is the probability that a student's objective social class position is upper middle class?

$P(\text{obj UMC}) = 50 / 98 \approx 0.51$

joint



		objective social class position		
		working class	upper middle class	Total
subjective social class identity	poor	0	0	0
	working class	8	0	8
	middle class	32	13	45
	upper middle	8	37	45
	upper class	0	0	0
	Total	48	50	98

What is the probability that a student's objective position *and* subjective identity are both upper middle class?

$P(\text{obj UMC} \ \& \ \text{subj UMC})$

$= 37 / 98 \approx 0.38$

conditional

		objective social class position		
		working class	upper middle class	Total
subjective social class identity	poor	0	0	0
	working class	8	0	8
	middle class	32	13	45
	upper middle	8	37	45
	upper class	0	0	0
	Total	48	50	98

What is the probability that a student who is objectively in the working class associates with upper middle class?

$P(\text{subj UMC} \mid \text{obj WC})$

$= 8 / 48 \approx 0.17$

Bayes' theorem:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

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	Total	48	50	98

$$P(\text{subj UMC} \mid \text{obj WC}) = \frac{P(\text{subj UMC \& obj WC})}{P(\text{obj WC})} = \frac{8 / 98}{48 / 98} = 8 / 48 \approx 0.17$$

practice

The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services.

The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English at home, and 4.2% fall into both categories.

Based on this information, what percent of Americans live below the poverty line given that they speak a language other than English at home?

$$P(\text{below PL} \mid \text{speak non-Eng}) = ?$$

$$= \frac{P(\text{below PL \& speak non-Eng})}{P(\text{speak non-Eng})} = \frac{0.042}{0.207} \approx 0.2$$

Bayes' theorem:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

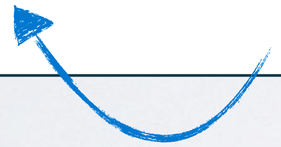
Data source: U.S. Census Bureau, 2010 American Community Survey 1-Year Estimates, Characteristics of People by Language Spoken at Home.

Product rule for independent events:

If A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$

Bayes' theorem:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$



General product rule:

$P(A \text{ and } B) = P(A \mid B) \times P(B)$

independence and conditional probabilities

Generically, if P(A|B) = P(A) then the events A and B are said to be independent.

- Conceptually: Giving B doesn't tell us anything about A.
- Mathematically: If events A and B are independent, P(A and B) = P(A) × P(B). Then,

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

example

		major		
		social science	non-social science	Total
gender	female	30	20	50
	male	30	20	50
	Total	60	40	100

$P(SS) = 60 / 100 = 0.6$

$P(SS \mid F) = 30 / 50 = 0.6$

$P(SS \mid M) = 30 / 50 = 0.6$