the normal-normal conjugate families

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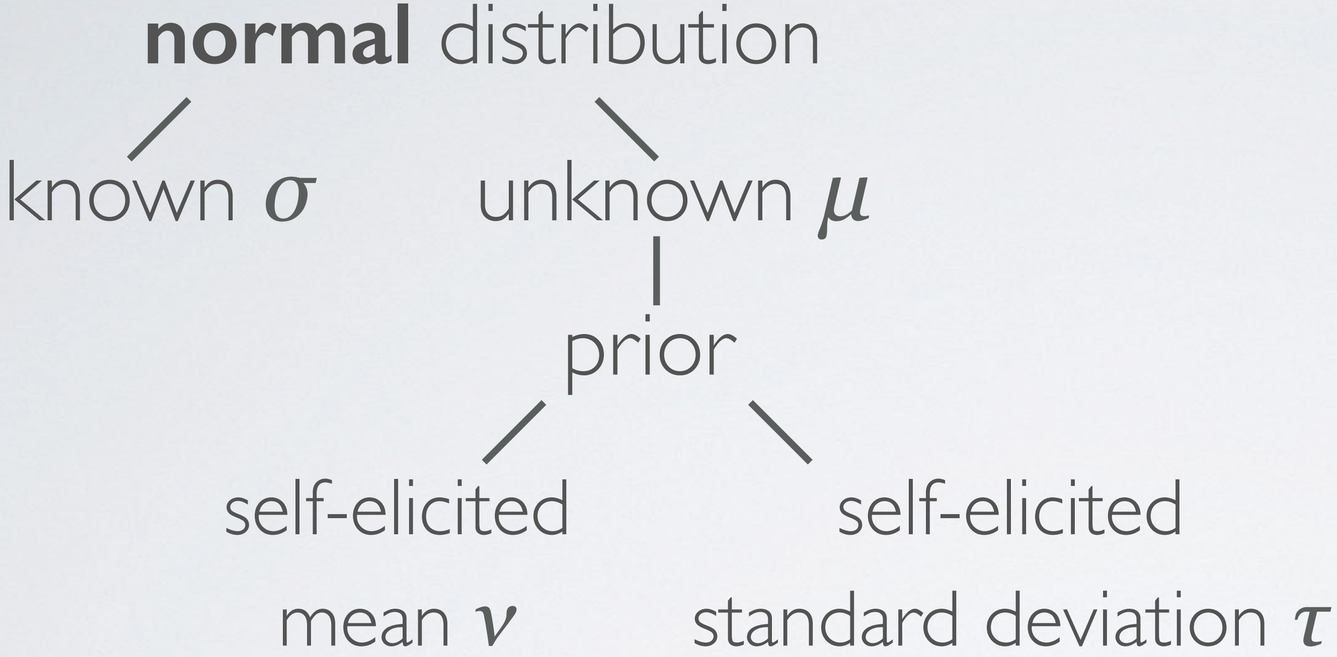
normal-normal pair



posterior density also **normal**

sometimes σ is known

https://pixabay.com/en/scale-old-weight-vintage-retro-951496/

normal-normal conjugate families

assume

- prior on unknown μ is normal
  - mean ν
  - standard deviation τ
- data $x_1, x_2, \ldots, x_n$, are independent,
  - come from normal with standard deviation σ

$$\nu^* = \frac{\nu\sigma^2 + n\bar{x}\tau^2}{\sigma^2 + n\tau^2} \qquad \tau^* = \sqrt{\frac{\sigma^2\tau^2}{\sigma^2 + n\tau^2}}$$

mass of ammonium nitrate

known standard deviation = **0.2** milligrams

mass = **10** milligrams?

|

standard deviation

of prior = **2**

prior is $\mathbf{N}(\mathbf{10}, \mathbf{2})$

mass of ammonium nitrate

10.5

$$\nu^* = \frac{\nu\sigma^2 + n\bar{x}\tau^2}{\sigma^2 + n\tau^2}$$

$$= \frac{10 \times (0.2)^2 + 5 \times 10.5 \times 2^2}{(0.2)^2 + 5 \times 2^2}$$

$$= 10.499$$

Jug by Creative Stall at the Noun Project

uncertainty of mass of ammonium nitrate

$$\tau^* = \sqrt{\frac{\sigma^2\tau^2}{\sigma^2+n\tau^2}}$$

$$= \sqrt{\frac{(0.2)^2\times 2^2}{(0.2)^2+5\times 2^2}}$$

uncertainty $= 2.0$

uncertainty of mass of ammonium nitrate

$$\tau^* = \sqrt{\frac{\sigma^2\tau^2}{\sigma^2+n\tau^2}}$$

$$= \sqrt{\frac{(0.2)^2\times 2^2}{(0.2)^2+5\times 2^2}}$$

uncertainty $= 0.089$

what has the chemist learned?

mass	uncertainty
10mg	2

what has the chemist learned?

mass	uncertainty
10.499mg	0.089

summary

1. a new conjugate family, the **normal-normal**
2. the updating formulae for the **posterior mean** and **standard deviation**
3. worked example for a case in which one might reasonably know the **standard deviation** σ for the normal distribution