CS540: HW1 (P2)

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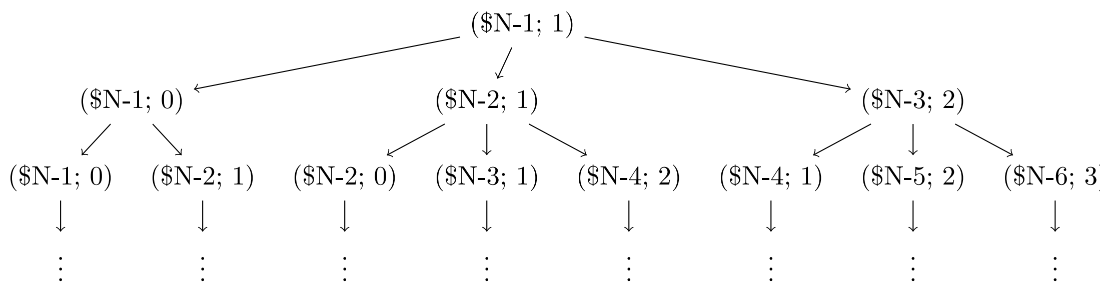
(a) (1) States: the amount of money remaining to be spent, and the amount of money spent the year prior.

(2) State space: the state space, when visualized, looks like the below graph:



(3) Cost function: the cost of each action is 1, and the total cost is the number of years taken in total.

(4) Successor states: for a given state with $x remaining to be spent and $y spent in the last year, then generate:

i. ($x - ($y+1), $y+1), if $x-($y+1) $\in$ [0,N)

ii. ($x - $y, $y), if $x-($y+1) $\in$ [0,N)

iii. ($x - ($y-1), $y-1), if $x-($y+1) $\in$ [0,N)

(b) No, this is not an admissible heuristic. Recall that an admissible heuristic h is one for which, for all nodes n in the search space, $h(n) \leq h^*(n)$ (where $h^*(n)$ is the actual cost of the minimum-cost path from n to a goal state.

Consider the initial state when $N = 4$. In this state, the minimum-cost path to a goal state has a cost of 3 (spend \$1, then \$2, then \$1). However, in this case $h(n) > h^*(n)$, and so this heuristic function is not admissible.

(c) Consider the heuristic "the amount of money spent in the previous year, minus 1 (or 1 if in the first year)". That this heuristic does not over-estimate the minimum-cost path to a goal state is clear, since the maximum amount we can decrease our spending from the previous year is \$1, i.e. spending \$X-1 instead of \$X.

For example, if we spent \$5 in the previous year, then by definition the lowest possible minimum-cost path to a goal state is 4 (spend \$4 this year, then \$3, then \$2, then \$1). This generalizes for all amounts of money spent in the previous year, and then we must only cover the base case of the first year.

(d) Given that N is a positive integer, it *is* guaranteed there will be a plan for the government to use up the money. The government can always utilize a plan in which it spends exactly \$1 each year for N years. At a high level, the proof argument follows below:

This is a satisfactory path since, if in year $t-1$ it spent $\$X$, in year t it may spend $\$X$, $\$X+1$, or $\$X-1$. So, by simply spending exactly \$1 each year (i.e. $f(t) = f(t-1) = \$1$ for all t), the government is utilizing a valid spending plan. Furthermore, if it spends \$1 each year, then trivially it will spend exactly \$1 in the last year to use up the money. Finally, this plan satisfies the condition that it must spend \$1 in the first year.

(If N is invalid, i.e. not a positive integer, then of course there is no guarantee that there will be a valid spending plan.)