

Homework 9 (Rubric)

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Problem 3 [10 points]

- [3 points] for an attempted reduction to min cut
- [4 points] for a correct reduction to min cut
 - [3 points] for a correct flow network
 - [1 point] for correctly translating a cut into a purchasing strategy
- [2 points] for a correctness argument
- [1 point] for run-time analysis

A number of submissions attempted a greedy strategy for this problem where starting from some purchasing strategy, at each step you switch vendors for the product that gives you the greatest overall decrease in cost. This algorithm does not work because you can easily get stuck in “local minima”—non-optimal configurations where no swap decreases the total cost. One such example would be where you start off purchasing everything from Alpha but purchasing everything from Omega is cheaper, and the incompatibility costs are all extremely high.

A number of solutions used bidirectional infinite-capacity edges between vertices to force the vertices to always be on the same side of any cut. Although this is valid and works as intended, it’s almost always easier in these cases to replace the connected vertices with just a single vertex—since they always have to be on the same side of any cut, that is exactly what they act like. A similar observation exists for infinite-capacity edges from the source and infinite-capacity edges to the sink.