

Context-free grammars

Roadmap

- Last time
 - Regex == DFA
 - JLex for generating Lexers
- This time
 - CFGs, the underlying abstraction for Parsers

RegExs Are Great!

- Perfect for tokenizing a language
- They do have some limitations
 - Limited class of language that cannot specify all programming constructs we need
 - No notion of structure
- Let's explore both of these issues

Limitations of RegExs

- Cannot handle “matching”
 - Eg: language of balanced parentheses
 $L = \{ (x)^x \text{ where } x > 1 \}$
cannot be matched
 - Intuition:
An FSM can only handle a finite depth of parentheses that we can handle
let's see a diagram...

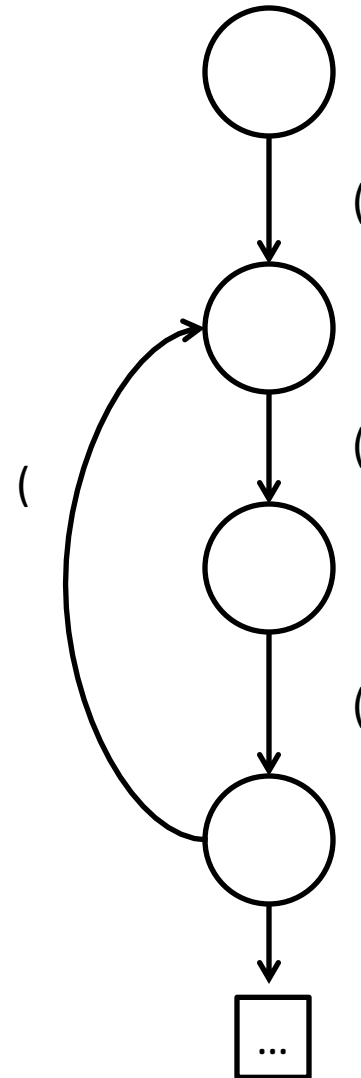
Limitations of RegExs: Balanced Parens

Assume F is an FSM that recognized L . Let N be the number of states in F .

Feed $N+1$ left parens into N

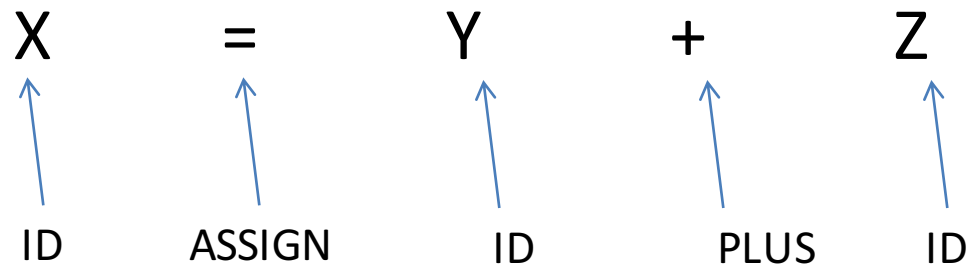
By the *pigeonhole* principle, we must have revisited some state s on two input characters i and j .

By the definition of F , there must be a path from s to a final state. But this means that it accepts some suffix of closed parens at input i and j , but both cannot be correct



Limitations of RegEx: Structure

- Our Enhanced-RegEx scanner can emit a stream of tokens:



... but this doesn't really enforce any order of operations

The Chomsky Hierarchy



Turing machine

LANGUAGE CLASS:

Recursively enumerable

Context-Sensitive

Context-Free

Regular

Happy medium?

FSM



Noam Chomsky

power

efficiency



Context Free Grammars (CFGs)

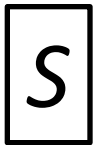
- A set of (recursive) rewriting rules to generate patterns of strings
- Can envision a “parse tree” that keeps structure

CFG: Intuition

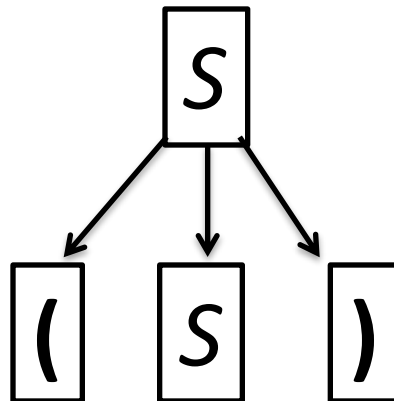
$$S \rightarrow (S)$$

A rule that says that you can rewrite S to be an S surrounded by a single set of parens

Before applying rule



After applying rule



CFGs recognize the language of trees where all the leaves are terminals

Context Free Grammars (CFGs)

- Formally, a 4-tuple:
 - N is the set of nonterminal symbols
 - Σ is the set of terminal symbols
 - P is the set of productions
 - S is the start nonterminal in N

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Rules for deriving strings

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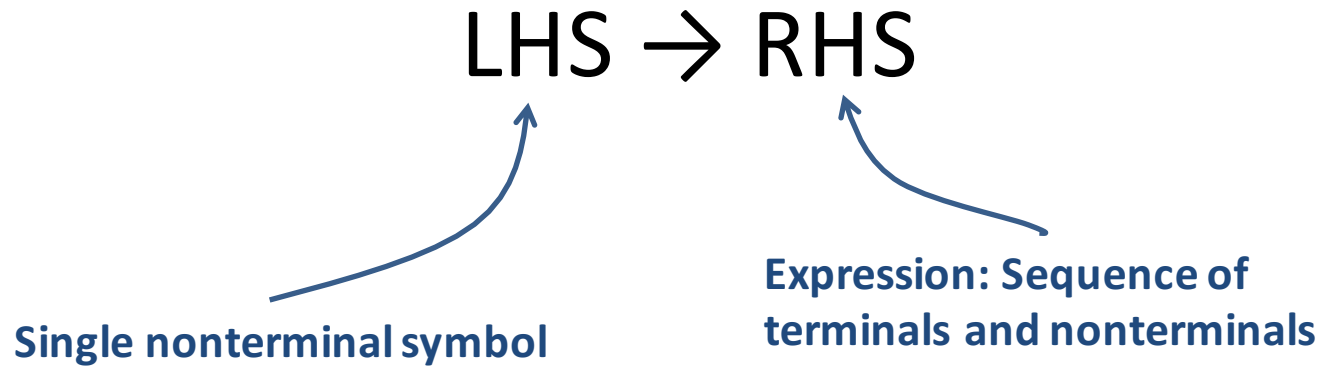
Placeholder / interior nodes
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Tokens from
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Rules for deriving strings

If not otherwise specified, use the
non-terminal that appears on the LHS
of the first production as the start

Production Syntax



Production Shorthand

Nonterm \rightarrow expression

Nonterm $\rightarrow \varepsilon$

Sequence of terms and nonterms



equivalently:

Nonterm \rightarrow expression

| ε

equivalently:

Nonterm \rightarrow expression | ε

Derivations

- To derive a string:
 - Start by setting “*Current Sequence*” to the start symbol
 - Repeat:
 - Find a Nonterminal X in the Current Sequence
 - Find a production of the form $X \rightarrow \alpha$
 - “Apply” the production: create a new “current sequence” in which α replaces X
 - Stop when there are no more nonterminals

Derivation Syntax

- We'll use the symbol \Rightarrow for *derives*
- We'll use the symbol $\overset{+}{\Rightarrow}$ for *derives in one or more steps*
- We'll use the symbol $\overset{*}{\Rightarrow}$ for *derives in zero or more steps*

An Example Grammar

An Example Grammar

Terminals

begin

end

semicolon

assign

id

plus

An Example Grammar

For readability, bold and lowercase

Terminals

begin

end

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An Example Grammar

For readability, bold and lowercase

Terminals

begin } **Program**
end } **boundary**
semicolon
assign
id
plus

An Example Grammar

For readability, bold and lowercase

Terminals

begin } **Program**
end } **boundary**

semicolon  **Represents “;”**
assign **Separates statements**




id

plus

An Example Grammar

For readability, bold and lowercase




Terminals

begin } **Program**
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semicolon  **Represents “;”**
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id
plus  **Represents “=” statement**

An Example Grammar

For readability, bold and lowercase





Terminals

begin } **Program**
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semicolon  **Represents ";"**
assign  **Represents "=" statement**
id  **Identifier / variable name**
plus

An Example Grammar

For readability, bold and lowercase

Terminals

begin } **Program**
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semicolon  **Represents ";"**
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id  **Identifier / variable name**
plus  **Represents "+" expression**

An Example Grammar

For readability, bold and lowercase

Terminals

begin
end
semicolon
assign
id
plus

Nonterminals

Prog
Stmts
Stmt
Expr

An Example Grammar

For readability, bold and lowercase

Terminals

begin
end
semicolon
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For readability, Italics and UpperCamelCase

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An Example Grammar

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Terminals

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Prog ————— Root of the parse tree
Stmts
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An Example Grammar

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begin
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For readability, Italics and UpperCamelCase

Nonterminals

Prog ————— **Root of the parse tree**
Stmts ————— **List of statements**
Stmt
Expr

An Example Grammar

For readability, bold and lowercase

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begin
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For readability, Italics and UpperCamelCase

Nonterminals

<i>Prog</i>	—————	Root of the parse tree
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<i>Stmt</i>	—————	A single statement
<i>Expr</i>		

An Example Grammar

For readability, bold and lowercase

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An Example Grammar

For readability, bold and lowercase

Terminals

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For readability, Italics and UpperCamelCase

Nonterminals

Prog
Stmts
Stmt
Expr

Defines the syntax of legal programs

Productions

Prog → **begin** *Stmts* **end**

Stmts → *Stmts* **semicolon** *Stmt*
 | *Stmt*

Stmt → **id** **assign** *Expr*

Expr → **id**
 | *Expr* **plus** **id**

An Example Grammar

For readability, bold and lowercase

Terminals

begin } **Program**
end } **boundary**
semicolon — **Represents “;”**
assign — **Separates statements**
id — **Represents “=” statement**
plus — **Identifier / variable name**
— **Represents “+” expression**

For readability, Italics and UpperCamelCase

Nonterminals

Prog — **Root of the parse tree**
Stmts — **List of statements**
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Productions

1. $Prog \rightarrow \mathbf{begin\ Stmts\ end}$
2. $Stmts \rightarrow Stmts\ \mathbf{semicolon}\ Stmt$
3. $\quad\quad\quad | Stmt$
4. $Stmt \rightarrow \mathbf{id\ assign}\ Expr$
5. $Expr \rightarrow \mathbf{id}$
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Derivation Sequence

Productions

Parse Tree

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Derivation Sequence

Parse Tree

Key

terminal

Nonterminal

Rule
used

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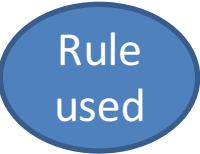
Derivation Sequence

Prog

Parse Tree



Key



Productions

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Derivation Sequence

Prog ⇒ **begin** *Stmts* **end** 1

Parse Tree



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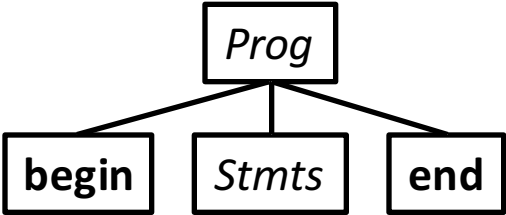
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Derivation Sequence

Prog ⇒ **begin** *Stmts* **end** ①

Parse Tree



Key

terminal

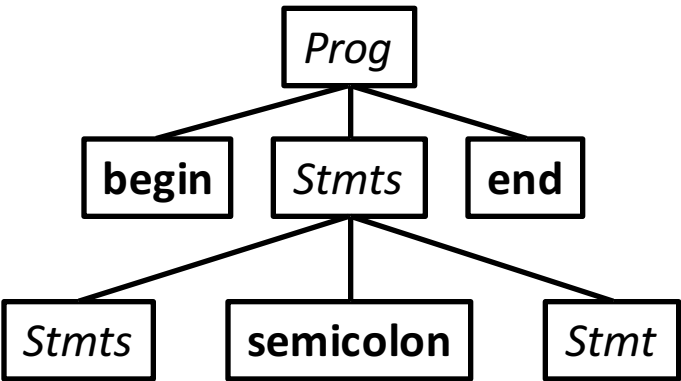
Nonterminal

Rule used

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Parse Tree



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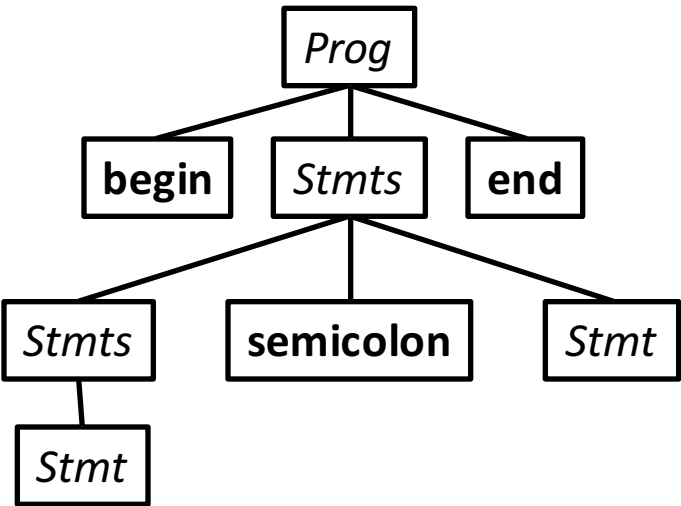
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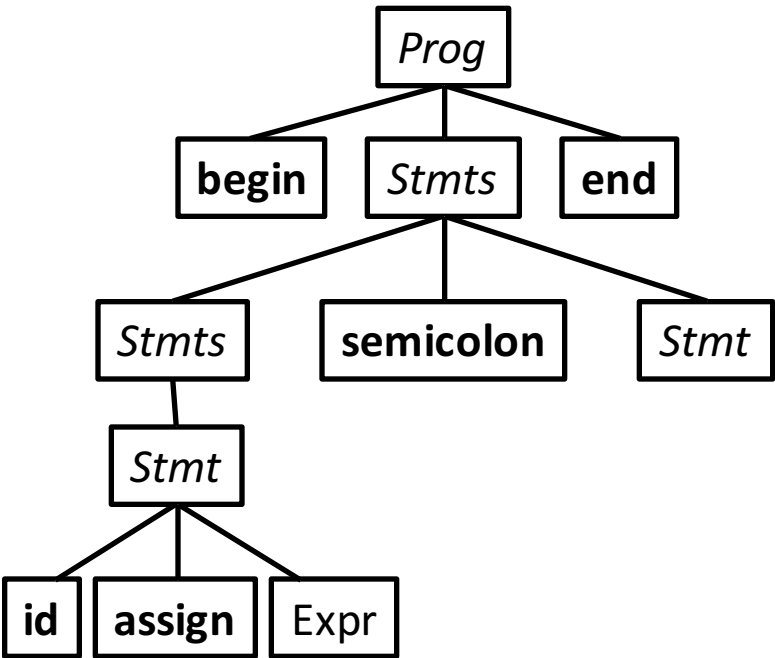
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Parse Tree



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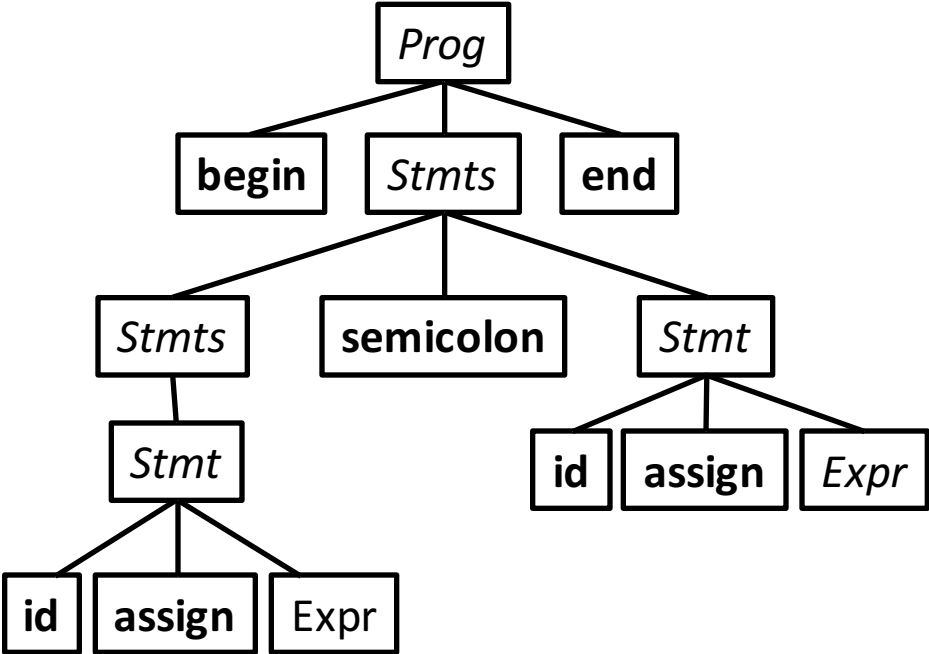
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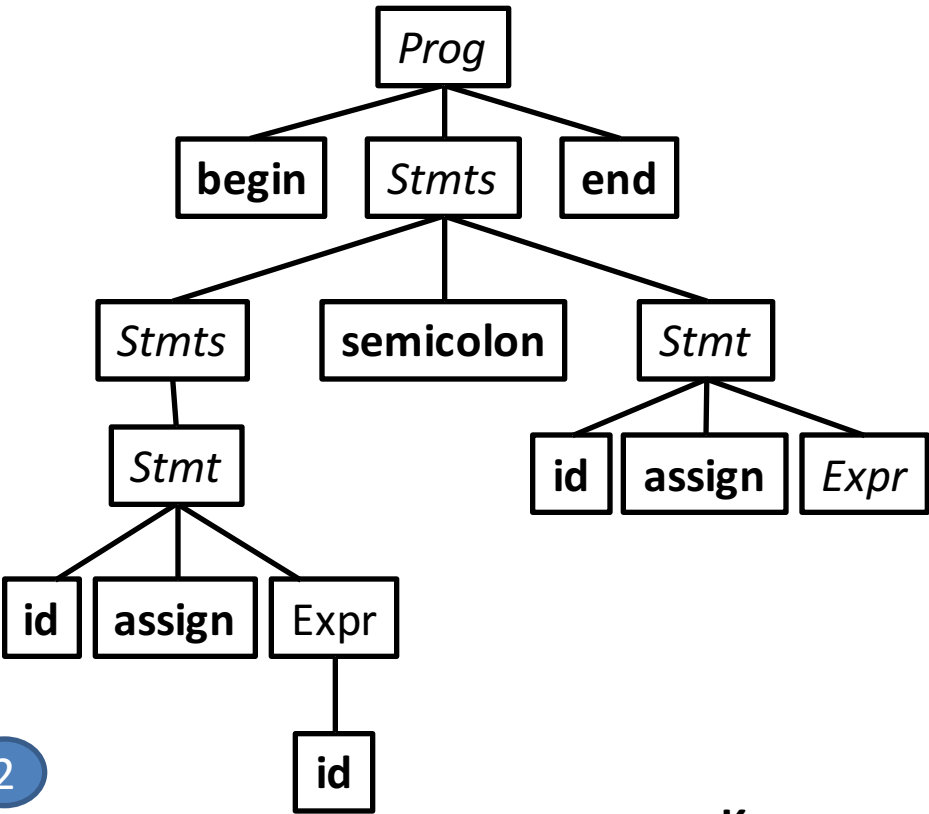
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Parse Tree



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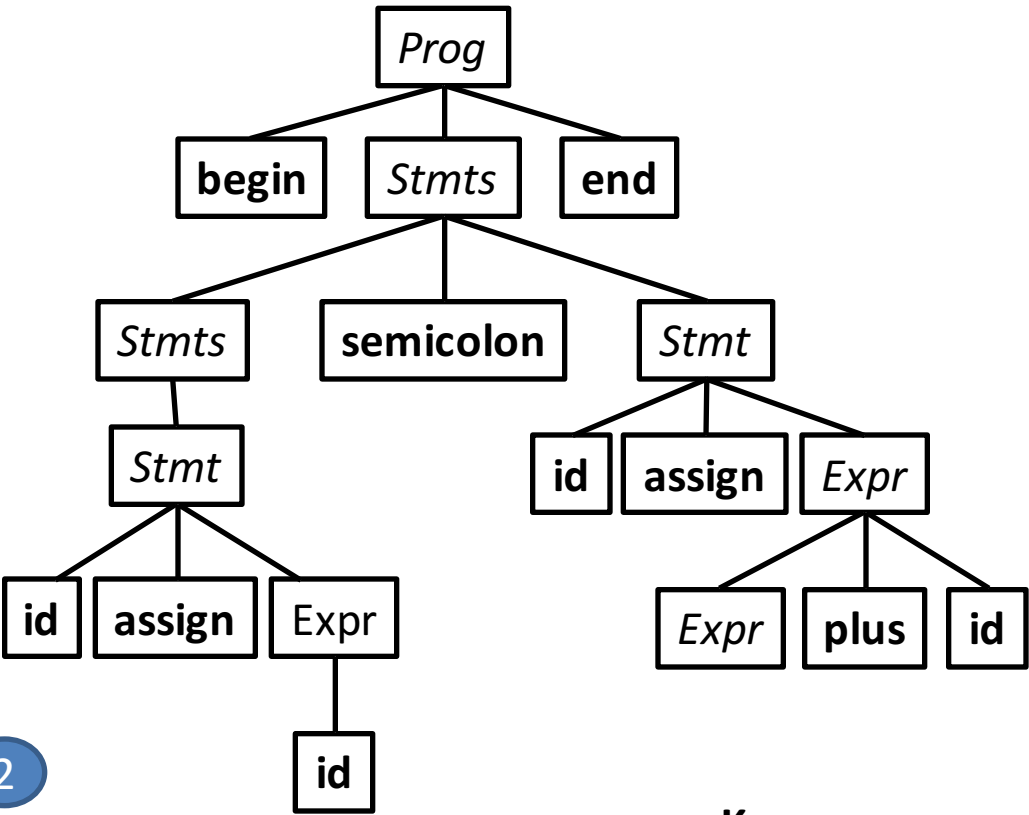
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Parse Tree



Key

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Nonterminal

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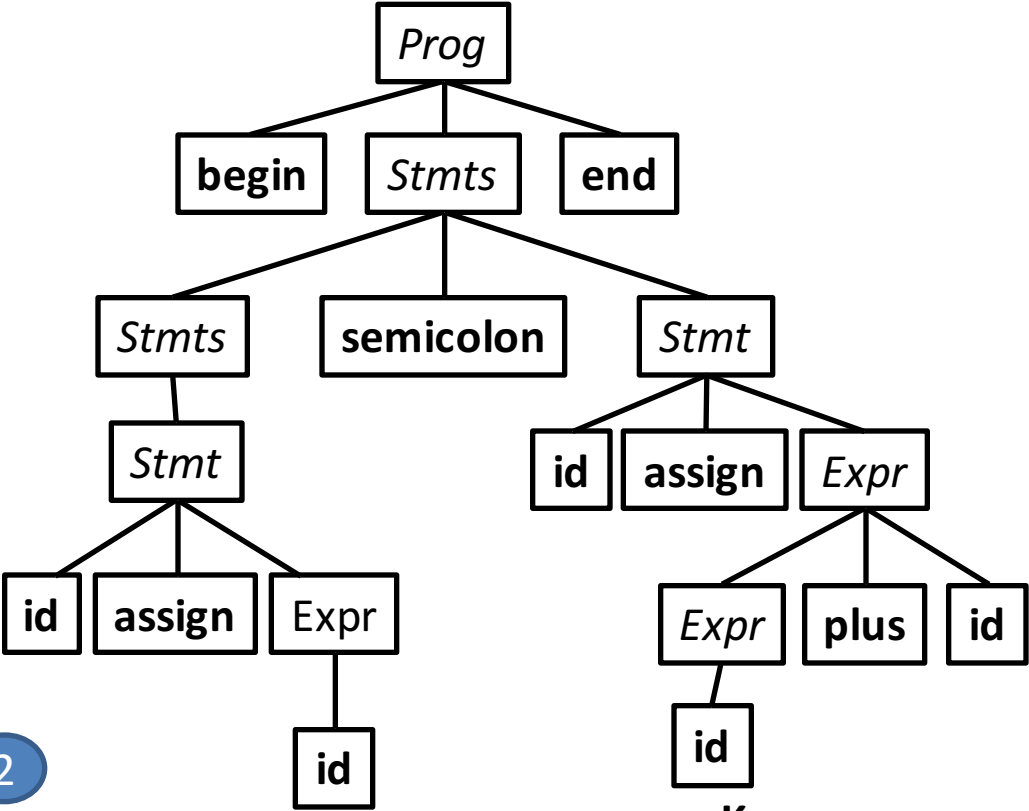
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Parse Tree



Key

terminal

Nonterminal

Rule used

Makefiles: Motivation

- Typing the series of commands to generate our code can be tedious
 - Multiple steps that depend on each other
 - Somewhat complicated commands
 - May not need to rebuild everything
- Makefiles solve these issues
 - Record a series of commands in a script-like DSL
 - Specify dependency rules and Make generates the results

Makefiles: Basic Structure

<target>: <dependency list>

(tab) <command to satisfy target>

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Example

```
Example.class: Example.java IO.class  
    javac Example.java
```

```
IO.class: IO.java  
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```

Makefiles: Basic Structure

<target>: <dependency list>

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Example

Example.class depends on example.java and IO.class

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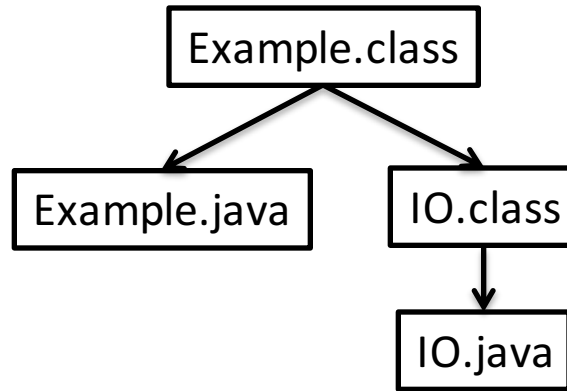
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Example.class is generated by
javac Example.java

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IO.class: IO.java
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```

Makefiles: Dependencies



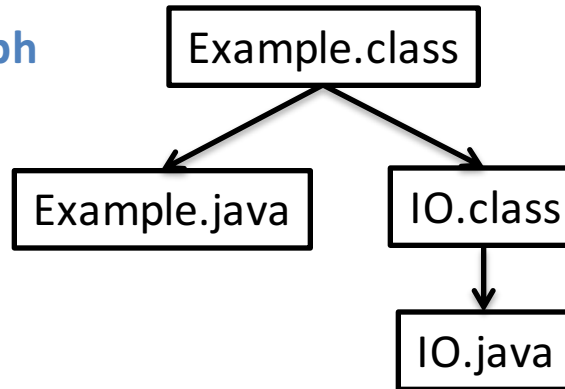
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Makefiles: Dependencies

Internal Dependency graph



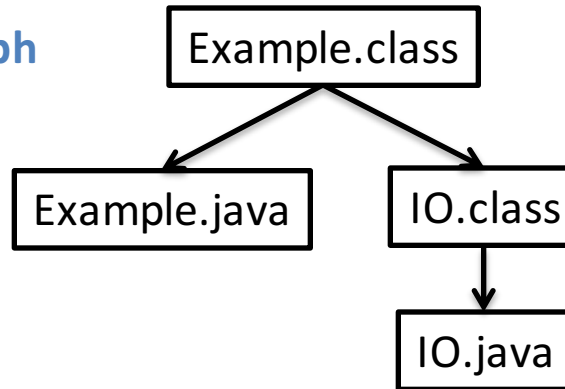
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Makefiles: Dependencies

Internal Dependency graph



A file is rebuilt if one of its dependencies changes

Example

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Example.class: Example.java IO.class  
    javac Example.java
```

```
IO.class: IO.java  
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```


Makefiles: Variables

You can thread common configuration values through your makefile

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Example

JC = /s/std/bin/javac

JFLAGS = -g

Makefiles: Variables

You can thread common configuration values through your makefile

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JFLAGS = -g **Build for debug**

Makefiles: Variables

You can thread common configuration values through your makefile

Example

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JC = /s/std/bin/javac
```

```
JFLAGS = -g Build for debug
```

```
Example.class: Example.java IO.class  
    $(JC) $(JFLAGS) Example.java
```

```
IO.class: IO.java  
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```

Makefiles: Phony Targets

- You can run commands through make.
 - Write a target with no dependencies (called phony)
 - Will cause it to execute the command every time



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clean:  
    rm -f *.class
```



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- You can run commands through make.
 - Write a target with no dependencies (called phony)
 - Will cause it to execute the command every time

Example

clean:

```
rm -f *.class
```

test:

```
java -cp . Test.class
```



Recap

- We've defined context-free grammars
 - More powerful than regular grammars
- Submit P1
- P2 will come out tonight
- Next time we'll look at grammars in more detail