**CS 577: Introduction to Algorithms** 12/1/15

# Homework 8 (Rubric)

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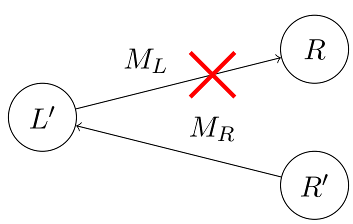
## Problem 3 [10 points]

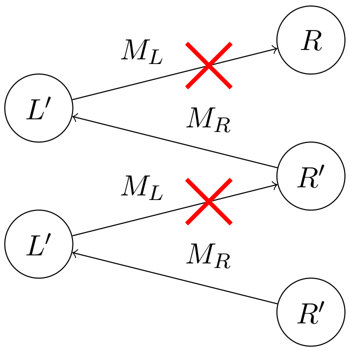
- [5 points] correctness
  - [1 point] correctly handles path with odd number of edges
  - [1 point] correctly handles path with even number of edges
  - [1 point] correctly handles cycle
  - [1 point] solution is a valid matching
  - [1 point] solution covers $L' \cup R'$
- 2 points] correctness argument (at most 1/2 if algorithm misses any above cases)
- [2 points] $O(|M_L| \cup |M_R|)$ algorithm (minus 1 pt for $O(n \log n)$ or $O(n+m)$, minus 2 for $\Omega(n^2)$)
- [1 point] runtime argument

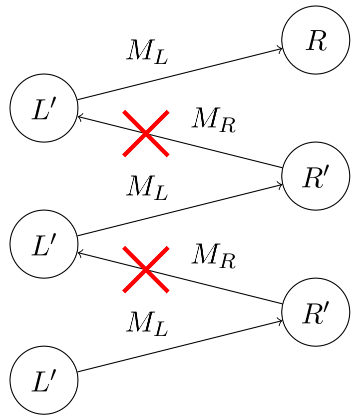
I think this homework was challenging. Any approach you take requires a careful argument and dealing with multiple cases. Points were lost evenly throughout all items of the rubric. I wrote many counterexamples on submissions, but let me explain them in more detail here. In general, I created a directed graph, directing edges of $M_L$ from $L$ to $R$ and edges of $M_R$ from $R$ to $L$. I also labelled vertices with $L$, $R$, $L'$, or $R'$, denoting which set they belong to. Thinking about the problem this way makes analysis much more straightforward.

The first counterexample breaks a direct reduction to Ford-Fulkerson (as well as some other solutions). Ford-Fulkerson does not necessarily break the tie between choosing the $M_R$ edge or the $M_L$ edge correctly, as both will be a maximum matching, but only the $M_R$ edge covers both the $L'$ and the $R'$ node. This tie only occurs in paths with an even number of edges.




The next common issue was deciding degree 2 vertices before a conclusive decision can be made about which edge to delete. The following two cases highlight this choice:




In the graph on the left, the $M_R$ edges need to be chosen to cover $L' \cup R'$. However, in the right, the $M_L$ edges need to be chosen to cover $L' \cup R'$. From the perspective of the middle $R'$ vertex, however, it is not clear whether the $M_L$ or $M_R$ edge should be taken.

A final common mistake was in correctness arguments. Note that the above case on the left, and the first case (or in general all paths with an even number of edges), the last vertex does not get covered. This is okay because it is not in $R' \cup L'$. Some solutions did not show why exactly one endpoint in a path is in $R' \cup L'$ and the other not, so more points were lost here.

Additionally, some solutions used the fact that there is a perfect matching covering $L' \cup R'$ in their arguments. This cannot be assumed, and instead must be shown, so these solutions lost a point.

In terms of grading, there were a few minor points I'd like to highlight in terms of time complexity. For starters, when using $O(n)$ or $O(n+m)$, it is not clear which graph is being referred to, $M_L \cup M_R$ or $G$. It could be that $M_L \cup M_R$ is a very small subset of $G$. I did not take off for this, unless you did not make the realization that in $M_L \cup M_R$, $m$ is $O(n)$, and so the complexity would really just be $O(n)$, not $O(n+m)$.

Also, the rubric has a point designated for constructing a valid matching and also covering $L' \cup R'$. If you lost a point in one of the three cases (path of odd # of edges, even # of edges, or a cycle), then you will lose one of these points. Some solutions lost both.