Nondeterministic FSMs

CS 536

Explore NFAs

Claim: NFAs add no power to DFAs

Epsilon transitions

Claim: Epsilon transitions add no power

Regular expressions

NFMs, formally

$$(Q, \Sigma, \delta, q, F)$$

finite set of states

the alphabet
(characters)

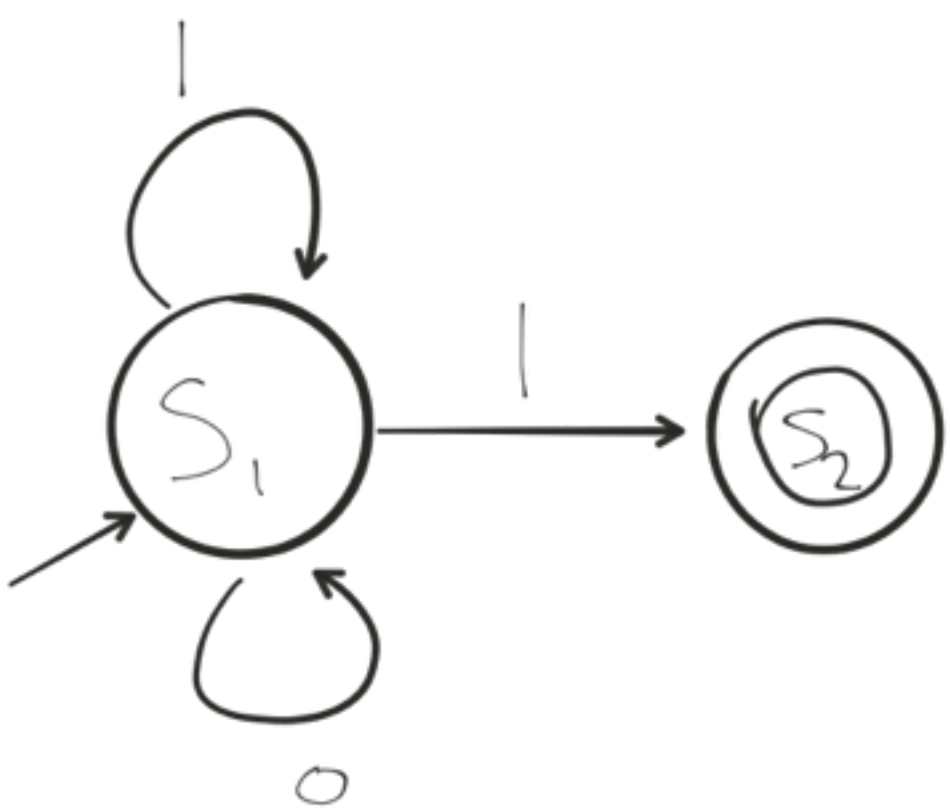
transition relation
$\delta : Q \times \Sigma \to 2^Q$

start state
$q \in Q$

final states
$F \subseteq Q$

NFA

To check if string is in *L*(*M*) *of* NFA *M*, simulate **set** of **choices** it could make



	1	1	1
s1	s1	s1	s1
s1	s1	s1	**s2**

NFA == DFA

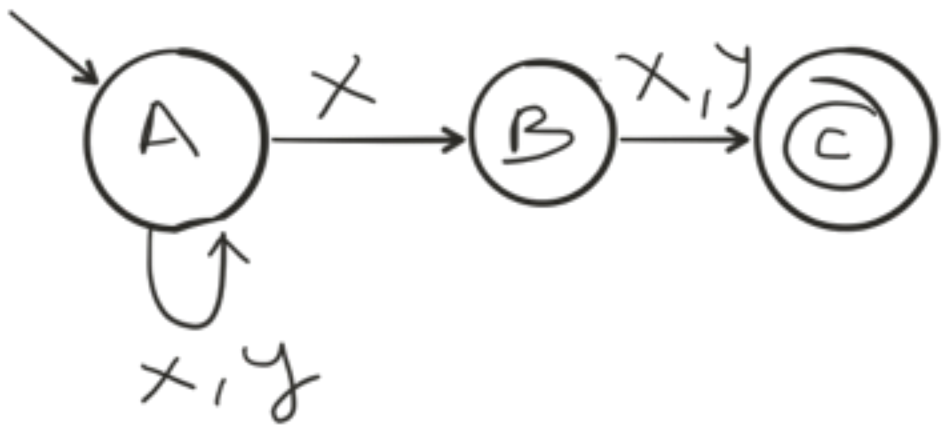
Claim: *L(NFA) = L(DFA)*

Idea: we can only be in finitely many subsets of states at any one time

$2^{|Q|}$ possible combinations of states

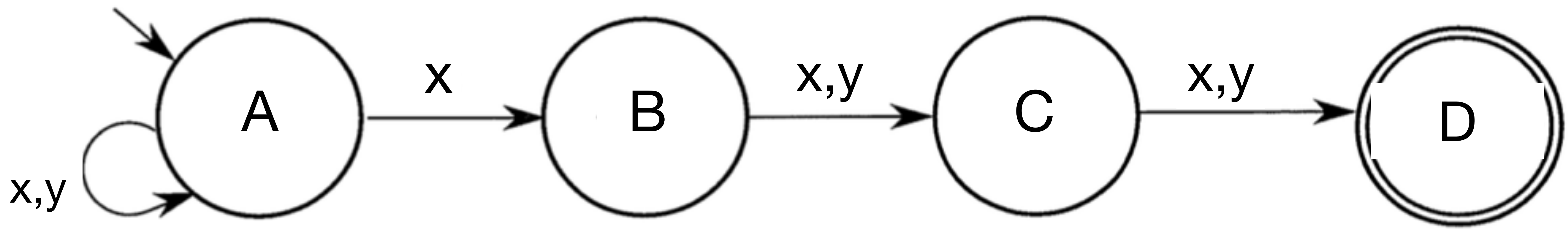
Why?

Why 2^|Q| states?



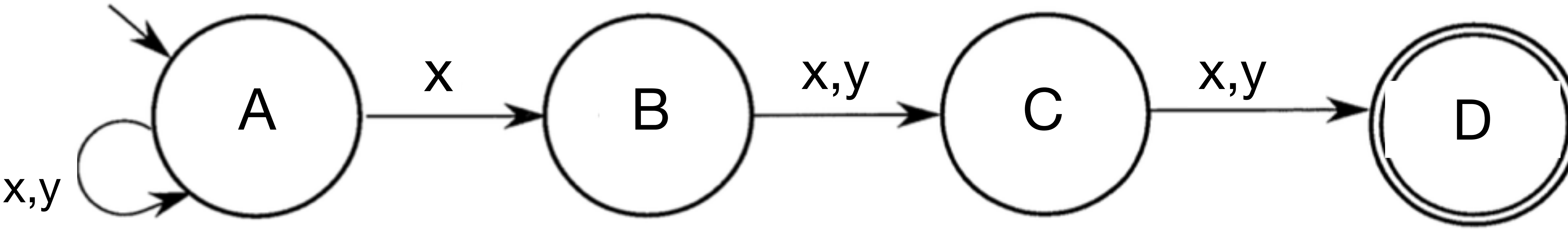
Build DFA that tracks set of states the NFA is in!

A	B	C		
0	0	0	=	{}
0	0	1	=	{C}
0	1	0	=	{B}
0	1	1	=	{B,C}
1	0	0	=	{A}
1	0	1	=	{A,C}
1	1	0	=	{A,B}
1	1	1	=	{A,B,C}

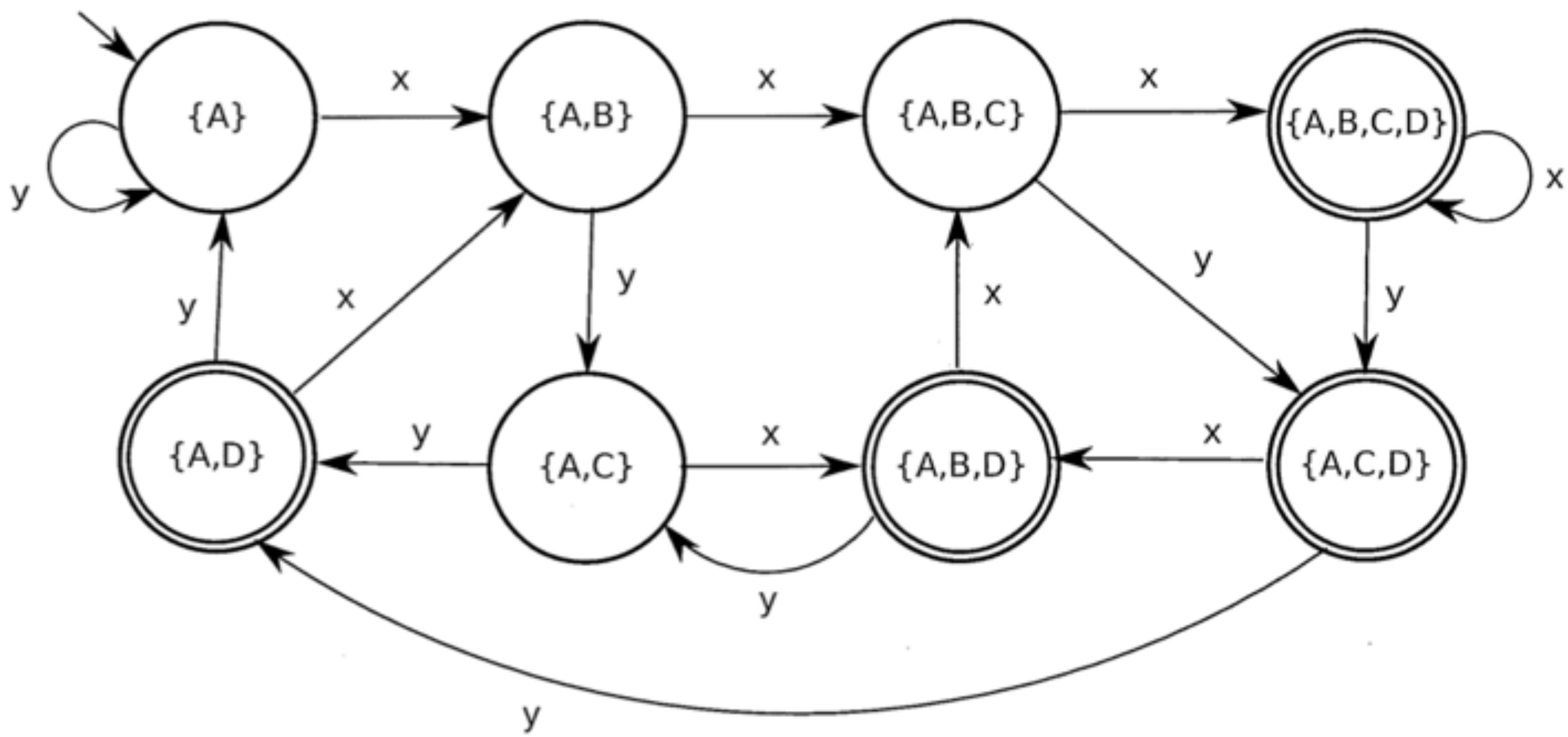


Defn: let succ(s,c) be the set of choices the NFA could make in state s with character c

succ(A,x) = {A,B}
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}



Build new DFA M' where Q' = 2^Q



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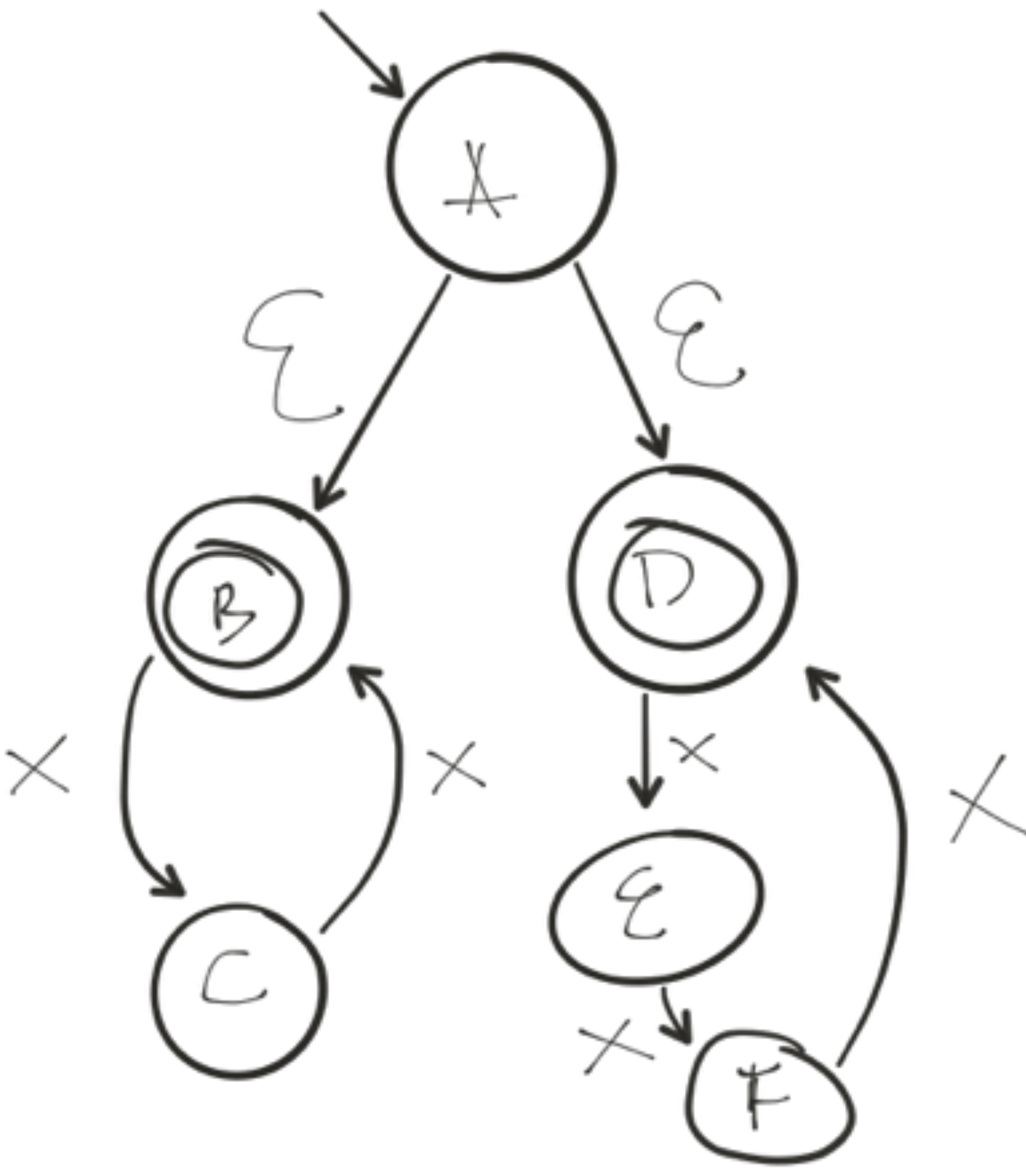
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}

To build DFA: Add an edge from state S on character c to state S' if S' represents the union of states that all states in S could possibly transition to on input c

ε-transitions

Eg: x^n, where n is even **or** divisible by 3



Useful for taking union of two FSMs

In example, left side accepts even n; right side accepts n divisible by 3

	x		x	
AB		C		B
AD		E		F
A				

Eliminating ε-transitions

We want to construct ε-free FSM M’ that is equivalent to M

Def: eclose(s) = set of all states reachable from s in zero or more epsilon transitions

First, make s an accepting state of M’ iff eclose(s) contains an accepting state

Second, put s,c—>t in transition relation of M’ iff there is a q,c—>t for some q in eclose(s)

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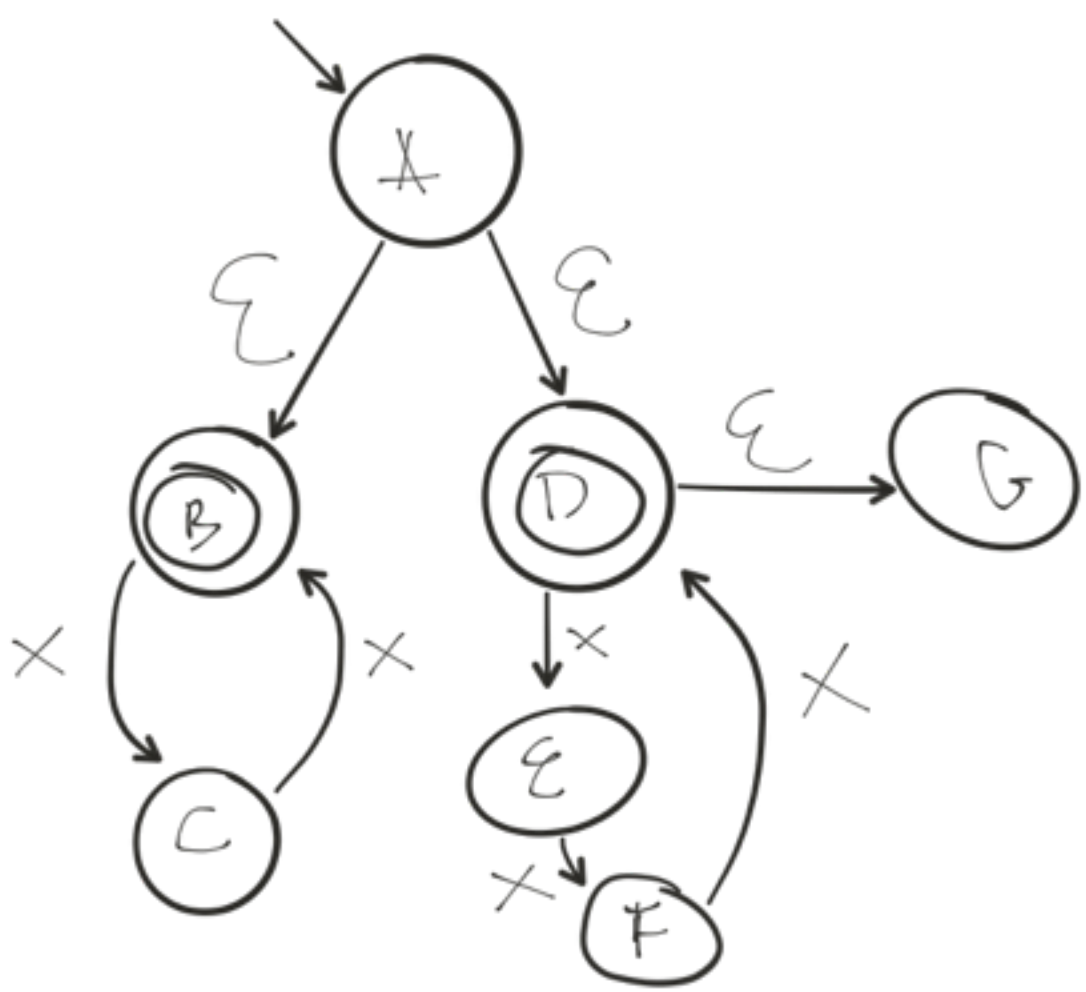

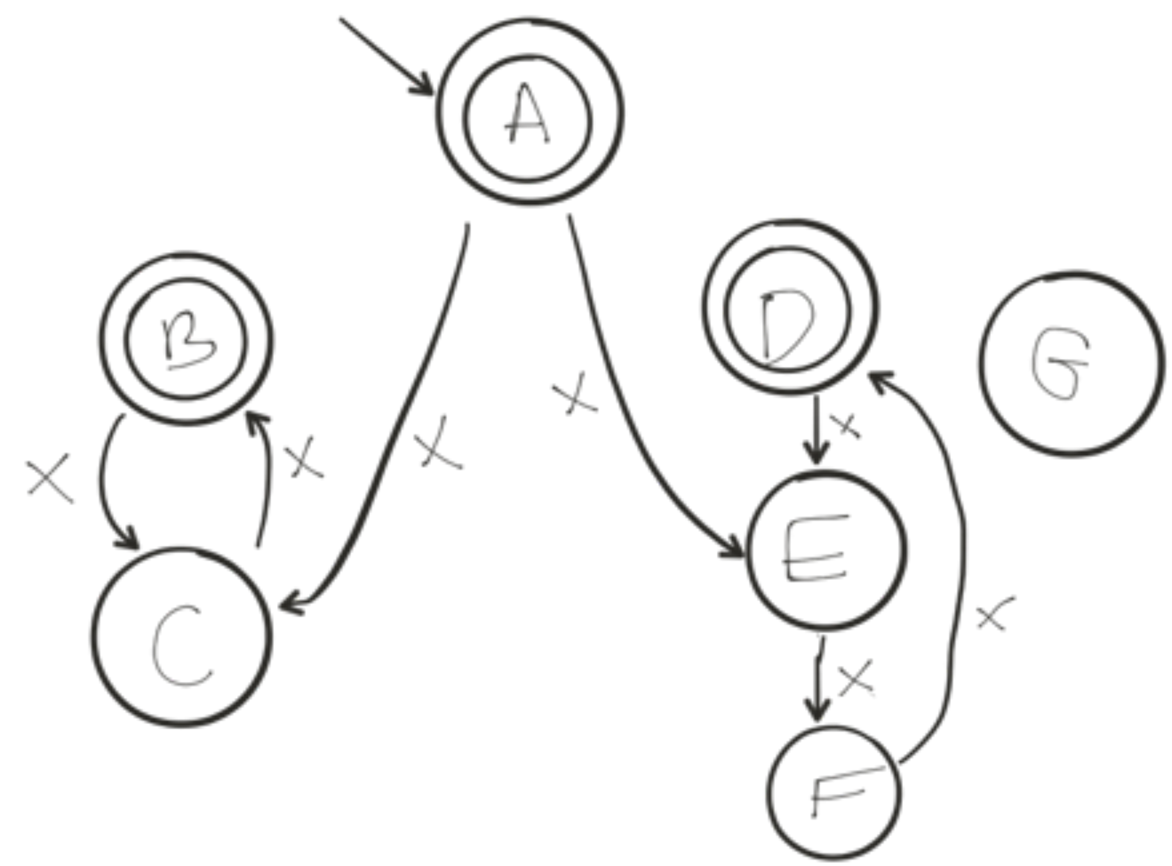

Recap

NFAs and DFAs are equally powerful

- any language definable as an NFA is definable as a DFA

ε-transitions do not add expressiveness to NFAs

- we showed a simple algorithm to remove epsilons

Regular expressions

Pattern describing a language

operands: single characters, epsilon

operators: from low to high precedence

alternation “or”: a | b

catenation: a.b, ab, a^3 (which is aaa)

iteration: a* (0 or more a’s) aka Kleene star

Regexp, cont'd

Conventions:

a+ is a.a*

letter is a|b|c|d|…|y|z|A|B|…|Z

digit is 0|1|2|…|9

not(x) all characters except x

. is any character

parentheses for grouping, e.g., (ab)*

ε, *ab, abab, ababab*

Regexp, example

Hex strings

start with 0x or 0X

followed by one or more hexadecimal digits

optionally end with l or L

0(x|X)hexdigit+(L|l|ε)

where hexdigit = digit|a|b|c|d|e|f|A|…|F

OR:

(0(x|X)hexdigit_lowercase+(L|l|ε)) | (0(x|X)hexdigit_uppercase+(L|l|ε))

Regexp, example

Single-line comments in Java/C/C++

// this is a comment

//(not(‘\n’))*’\n’