

Homework 11

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This assignment covers NP-completeness. It is the last assignment. Congrats on finishing the course!

Review problems

1. Recall that Graph Isomorphism is a problem in NP that is not known to be in P or NP-complete. Show that the following variant, Subgraph Isomorphism, is NP-complete: Given two graphs G and H , decide whether G contains an induced subgraph (i.e., a subset of the vertices of G together with all edges in G between those vertices) that is isomorphic to H .
2. Recall the Interval Scheduling problem from class. Consider the variant, Multiple Interval Scheduling, in which each job may require the machine to be reserved for multiple time intervals. Show that this variant is NP-hard.

Graded written problem

3. [10 points] At this year's National Thanksgiving Turkey Presentation, President Obama pardoned two turkeys, Abe and Honest. They want to fly their tour of appreciation together. Like their predecessors, they want to keep the number of distinct routes as small as possible, but they are fine visiting cities more than 3 times. The effort it takes to fly from A to B may differ for Abe and Honest. Among all tours satisfying the requirements, you want to find one such that the maximum of the total effort of Abe and of Honest is minimized.
 - (a) Formulate an NP-complete decision problem that is equivalent to the above problem under polynomial-time reductions.
 - (b) Prove that your decision problem is NP-complete.

Additional written problem

4. Show that the following problem is NP-complete: Given a 3-CNF formula, does there exist an assignment that satisfies exactly one literal of every clause?

Challenge problem

5. Given a Boolean circuit C , you want to find a Boolean circuit C' that behaves the same as C on every input but has as few gates as possible. Show that if $P = NP$, then this problem can be solved in polynomial time.