CS 577: Introduction to Algorithms 12/13/15

Homework 10 (Rubric)

Instructor: Dieter van Melkebeek TA: Bryce Sandlund

Problem 3 [10 points]

- [2 points] (a)
  - [.5 points] spanning tree, max degree 3, minimal effort among all such spanning trees statement
  - [1 point] explanation for why this is the turkey tour problem
  - [.5 points] how the problem fits in the NP-Optimization framework (solutions are polynomial length in input size, validity can be checked in polynomial time, objective value can be evaluated in polynomial time)
- [5 points] (b)
  - [3 points] correctness of reduction
  - [1 point] minimum spanning tree is a valid TSP path (circuit)
  - [1 point] TSP path constructed is minimal
- [2 points] (c)
  - [.5 points] decision problem statement
  - [.5 points] TTP-decision $\leq$ TTP-optimization
  - [2 points] TTP-optimization $\leq$ TTP-decision
    - [1 point] binary search for k
    - [1 point] construct spanning tree from decision

Many points were lost in part (a). Some people did not explain why the graph formulation was the turkey tour problem. All claims should be justified in this class. Furthermore, the deduction that a maximum degree of 3 constrains the turkey to only visit 3 cities is not true for a general turkey tour. This is because the turkey can go in a cycle. Each vertex has degree 2, but is only visited 1 time. Therefore the turkey could visit a city only 3 times and the degree could be more than 3. Instead, you must first deduce the solution is a spanning tree, then since every path is unique in a spanning tree, each edge is traversed exactly once in each direction, and the degree is equivalent to restricting the number of visits of each city. Furthermore, some people skipped showing the problem fit the NP-optimization framework. My guess is people didn't realize this required justification beyond the graph formulation.

Another common issue was arguing minimality of the TSP path (or circuit) constructed from the reduction from TSP to TTP. Note that even if you show the path constructed has the same cost as the spanning tree, this does not guarantee it is the minimal TSP path unless you show that all TSP paths are present as valid spanning trees in the graph G'. For some solutions, this wasn't even true (DC should have only one dummy vertex), but for many others, the argument was not present.

A third common issue was in the polynomial-time argument of binary search for the optimal value of k (tour cost) in the reduction from TTP-optimization to TTP-decision. Although binary

search takes logarithmic time, the range is exponential in the number of bits of the input, because it only takes $\log_2(w)$ bits to specify a weight w. Therefore the binary search takes time linear in the number of bits of the input.

Finally, although in max-flow we use ∞ without much thought, in the reductions, you really should specify a finite value of ∞ that has bit-length polynomial in the length of the input. This shows an appropriate value exists while keeping the reduction polynomial-time.