CS 577: Introduction to Algorithms 10/8/2015

Homework 5

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This assignment covers greed based on exchange arguments. Good luck!

Review problems

1. Let $G = (V, E)$ be an undirected graph with a cost c_e for edge $e \in E$. When these edge costs are not distinct, there can be many different minimum spanning trees.

   Consider the following claim:

   > Suppose (V, T) with $T \subseteq E$ is a spanning tree of G such that for every edge $e \in T$, e belongs to some minimum spanning tree of G. Then T is a minimum spanning tree of G.

   Decide whether you think the claim is true or false. If it is true, give a proof. If it is false, give a counterexample.

2. You see the following special offer by the convenience store: "A bottle of cola for every 3 empty bottles returned."

   Now you decide to buy N bottles of cola from the store. You would like to know how you can get the most cola from them.

   For example, consider the case where $N = 8$. The straightforward strategy is as follows: After finishing your 8 bottles of cola, you have 8 empty bottles. Take 6 of them and you get 2 new bottles of cola. Now after drinking them you have 4 empty bottles, so you take 3 of them to get yet another new cola. Finally, you have only 2 bottles in hand, so you cannot get new cola any more. Hence, you have enjoyed $8 + 2 + 1 = 11$ bottles of cola.

   You can actually do better! You first borrow an empty bottle from your friend, then you can enjoy $8 + 3 + 1 = 12$ bottles of cola! Of course, to be fair, you have to return your remaining empty bottle back to your friend.

   Design an algorithm to finds the largest number of bottles you can get when buying N of them and can borrow as many bottles as you want. Your algorithm should run in time bounded by a polynomial in the bit length of N, i.e., a polynomial in $\log N$.

Graded written problems

3. [10 points] In a city there are n bus drivers. There are also n morning bus routes and n afternoon bus routes, each with various lengths. Each driver is assigned one morning route and one evening route. For any driver, if his total route length for a day exceeds d, he has to be paid overtime for every hour after the first d hours at a fixed rate per hour. Your task is to assign one morning route and one evening route to each bus driver so that the total overtime amount that the city authority has to pay is minimized.

   Develop an efficient algorithm for this problem.

Additional written problem

4. Your friend is working as a camp counselor, and he is in charge of organizing activities for a set of junior-high-school-age campers. One of his plans is the following mini-triathlon exercise: each contestant must swim 20 laps of a pool, then bike 10 miles, then run 3 miles. The plan is to send the contestants out in a staggered fashion, via the following rule: the contestants must use the pool one at a time. In other words, first one contestant swims the 20 laps, gets out, and starts biking. As soon as this first person is out of the pool, a second contestant begins swimming the 20 laps; as soon as he or she is out and starts biking, a third contestant begins swimming, ..., and so on.

   Each contestant has a projected swimming time (the expected time it will take him or her to complete the 20 laps), a projected biking time (the expected time it will take him or her to complete the 10 miles of bicycling), and a projected running time (the time it will take him or her to complete the 3 miles of running). Your friend wants to decide on a schedule for the triathlon: an order in which to sequence the starts of the contestants. Let's say that the completion time of a schedule is the earliest time at which all contestants will be finished with all three legs of the triathlon, assuming they each spend exactly their projected swimming, biking, and running times on the three parts. (Again, note that participants can bike and run simultaneously, but at most one person can be in the pool at any time.)

   You want to determine the best order for sending people so that the whole competition is over as early as possible. Give an efficient algorithm that produces a schedule whose completion time is as small as possible.

Optional programming problem

5. [2.5 points] Solve SPOJ problem Island Hopping (problem code ISLHOP).

Challenge problem

6. Suppose you are given a connected graph G in which the edge weights depend on a parameter t, where t ranges over the reals. More specifically, the cost of an edge e is given by a function of the form $c_e(t) = a_e t^2 + b_e t + d_e$, where a_e, b_e, d_e are reals depending on e, and $a_e > 0$. Your goal is to find the minimum cost of a minimum spanning tree of G over all values of t. Your algorithm should run in time polynomial in the number of nodes and edges of G. You may assume that you can perform standard arithmetic operations (including computing square roots) at unit cost.

   Would your approach also work for the shortest paths problem (assuming the weight functions are nonnegative over their entire range)?