CS 536

CFGs for Syntax Definition

Roadmap

- Last time
  - Defined context-free grammar basics
- This time
  - CFGs for syntax design
    - Language membership
    - List grammars
    - Resolving ambiguity

CFG Review

- $G = (N,\Sigma,P,S)$
- $\Rightarrow$ means *derives*
  $\overset{+}{\Rightarrow}$ means *derives in 1 or more steps*
- CFG generates a string by applying productions until no non-terminals remain

Example: Nested parens

$N = \{ Q \}$

$\Sigma = \{$ **(**, **)** $\}$

$P = Q \rightarrow \mathbf{(}\ Q\ \mathbf{)}$

$\quad\quad\ \ \mid\ \varepsilon$

$S = Q$

Formal CFG Language Definition

Let G= (N,Σ,P,S) be a CFG. Then

L(G) = $\left\{ w \middle| S \overset{+}{\Rightarrow} w \right\}$ where

S is the start nonterminal of G

w is a sequence of terminals or ε

CFGs as Language Definition

- CFG productions define the *syntax* of a language

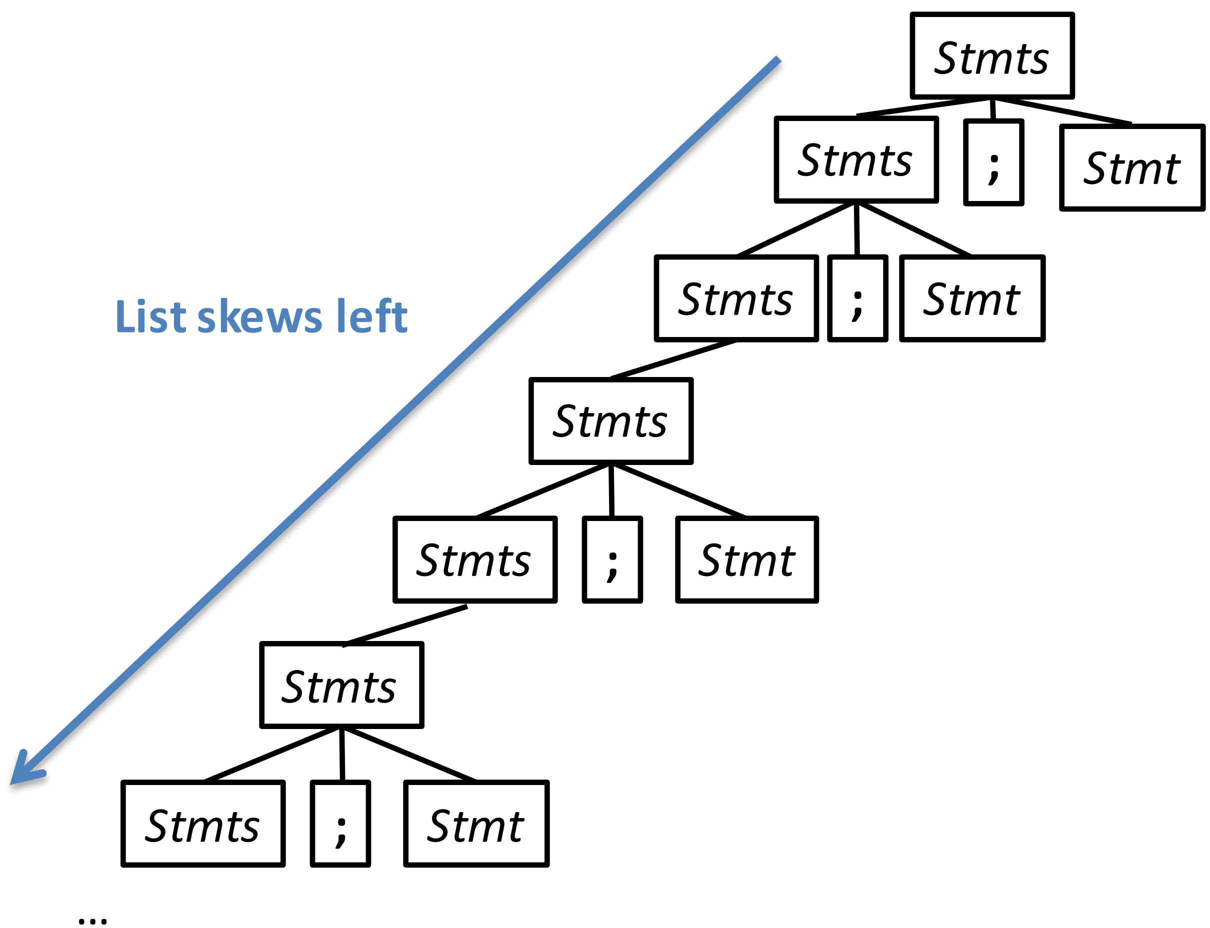
  1. *Prog* → **begin** *Stmts* **end**
  2. Stmts → *Stmts* **semicolon** *Stmt*
  3. | *Stmt*
  4. *Stmt* → **id assign** *Expr*
  5. *Expr* → **id**
  6. | *Expr* **plus id**

- We call this notation "*BNF*" or "*enhanced BNF*"
- HTTP grammar using BNF:
  - http://www.w3.org/Protocols/rfc2616/rfc2616-sec2.html

List Grammars

- Useful to repeat a structure arbitrarily often

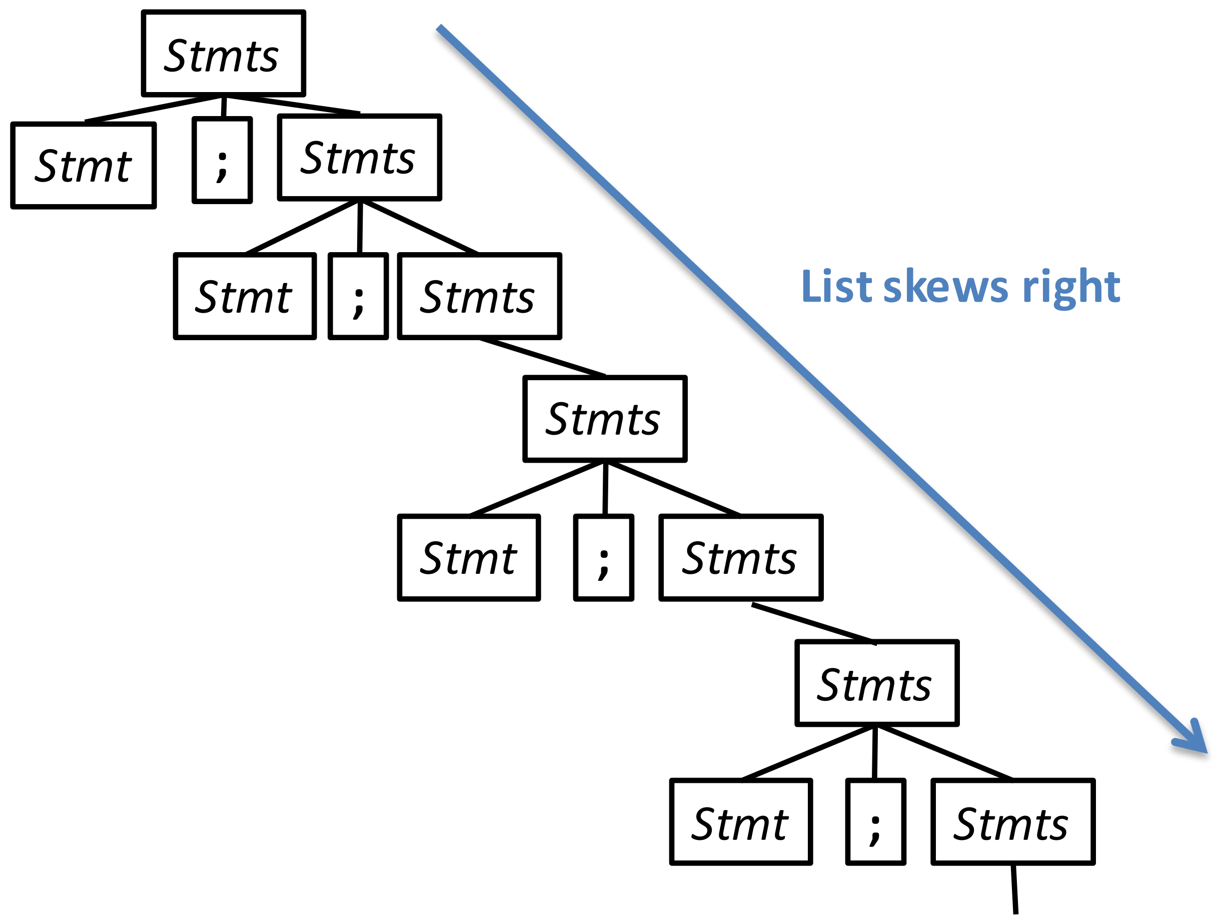
Stmts → *Stmts* **semicolon** *Stmt* | *Stmt*



List Grammars

- Useful to repeat a structure arbitrarily often

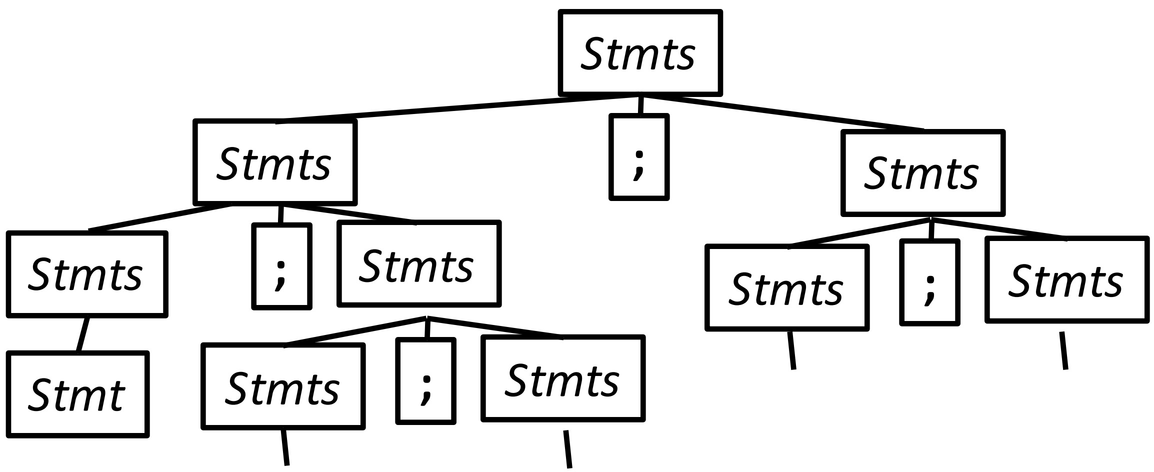
Stmts → *Stmt* **semicolon** *Stmts* | *Stmt*



List Grammars

- What if we allowed both "skews"?

Stmts → *Stmts* **semicolon** *Stmts* | *Stmt*



Derivation Order

- Leftmost Derivation: always expand the leftmost nonterminal
- Rightmost Derivation: always expand the rightmost nonterminal

1. *Prog* → **begin** *Stmts* **end**
2. Stmts → *Stmts* **semicolon** *Stmt*
3. | *Stmt*
4. *Stmt* → **id assign** *Expr*
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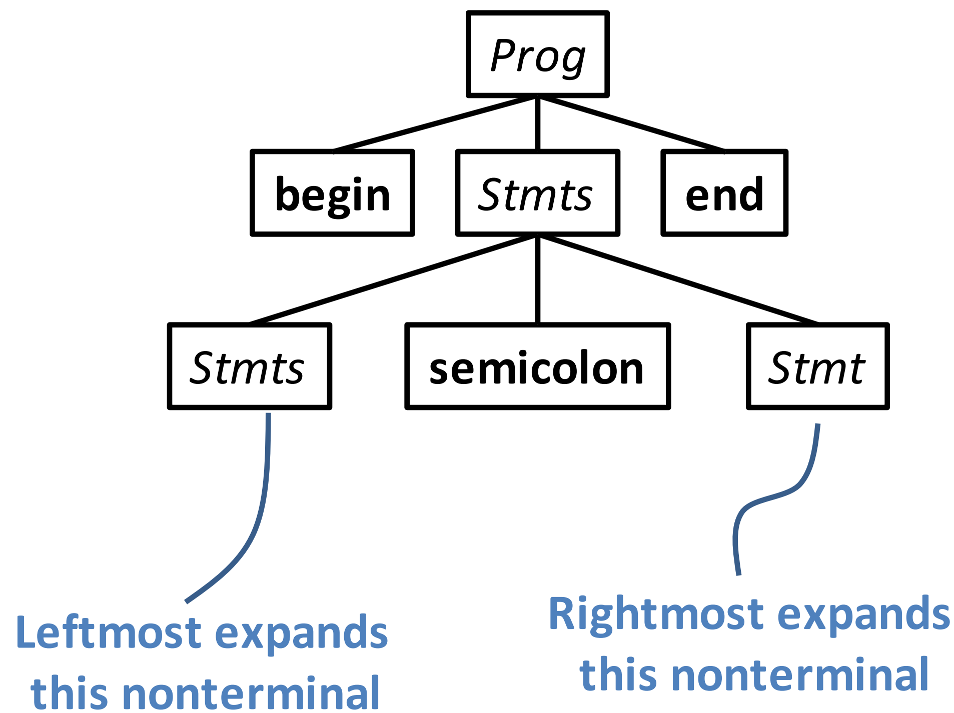


Ambiguity

- Even with a fixed derivation order, it is possible to derive the same string in multiple ways
- For Grammar G and string w
  - *G* is ambiguous if
    - >1 leftmost derivation of w
    - >1 rightmost derivation of w
    - > 1 parse tree for w

Example: Ambiguous Grammars

Expr → **intlit**

| *Expr* **minus** *Expr*

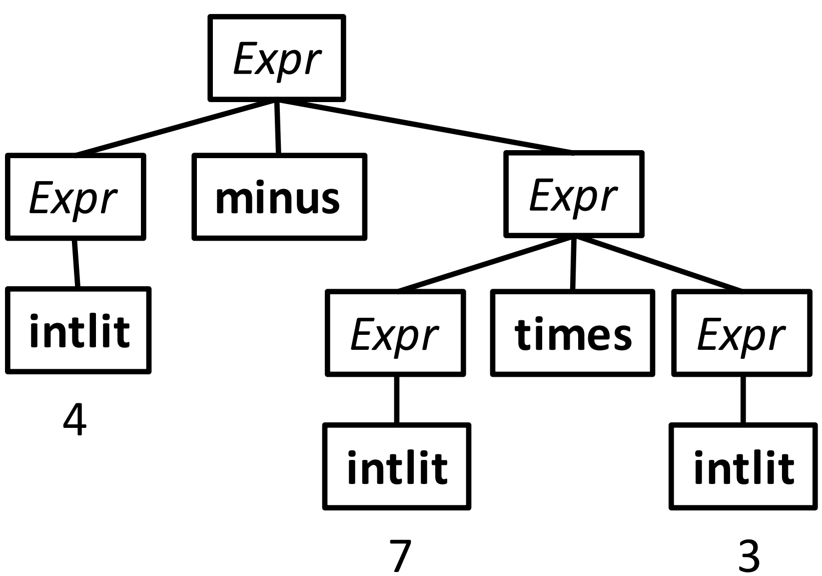
| *Expr* **times** *Expr*

| **lparen** *Expr* **rparen**

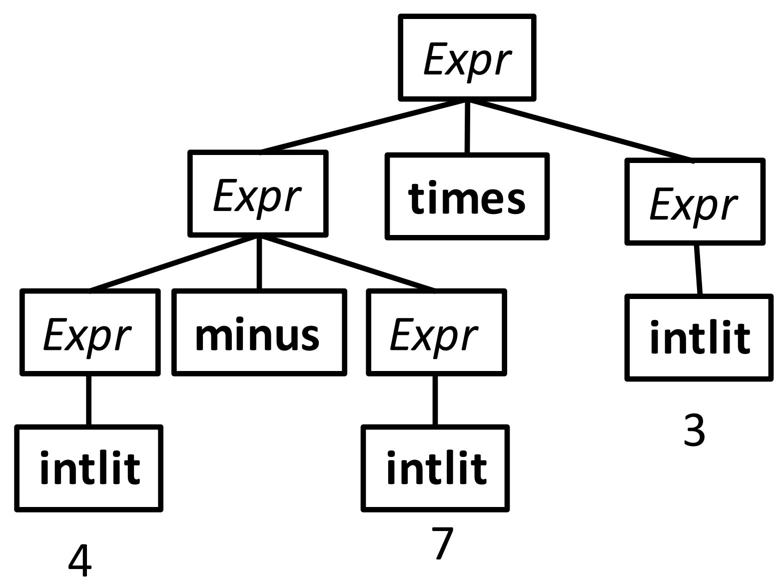
Derive the string 4 - 7 * 3

(assume tokenization)

Parse Tree 1

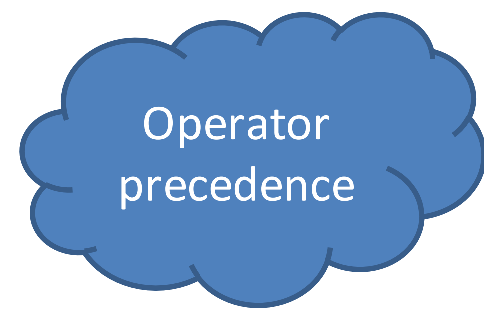


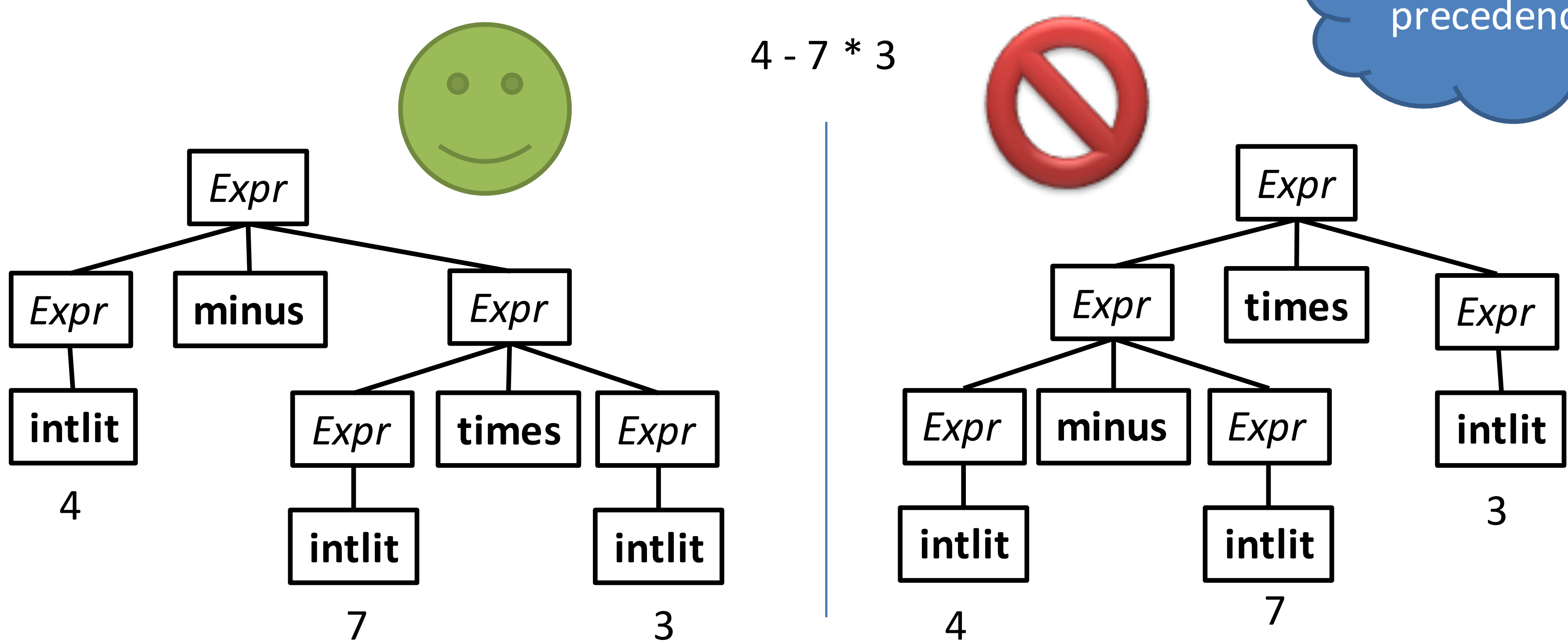
Parse Tree 2



Why is Ambiguity Bad?

- Eventually, we'll be using CFGs as the basis for our parser
  - Parsing is much easier when there is no ambiguity in the grammar
  - The parse tree may mismatch user understanding!





Resolving Grammar Ambiguity: Precedence

$Expr \rightarrow$ **intlit**

| *Expr* **minus** *Expr*

| *Expr* **times** *Expr*

| **lparen** *Expr* **rparen**

- Intuitive problem
  - "Context-freeness"
  - Nonterminals are the same for both operators
- To fix precedence
  - 1 nonterminal per precedence level
  - Parse lowest level first

Resolving Grammar Ambiguity: Precedence

Expr → **intlit**

| *Expr* **minus** *Expr*

| *Expr* **times** *Expr*

| **lparen** *Expr* **rparen**

Expr → *Expr* **minus** *Expr*

| *Term*

Term → *Term* **times** *Term*

| *Factor*

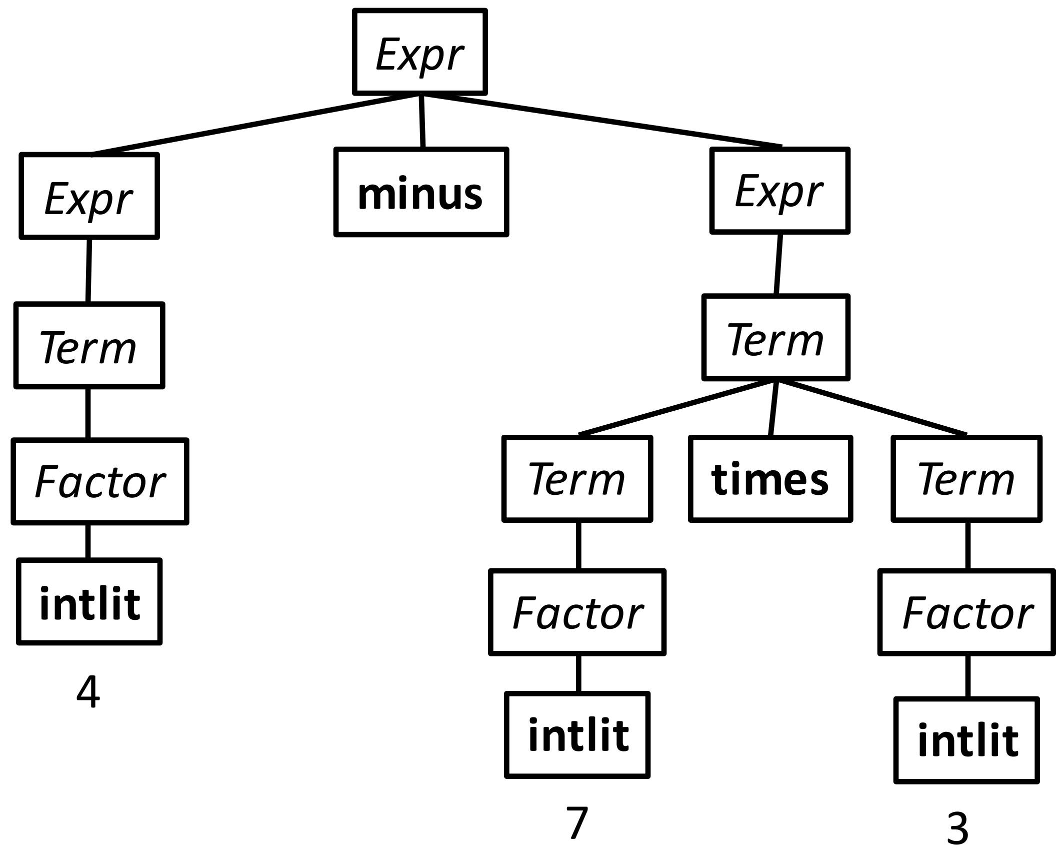
Factor → **intlit**

| **lparen** *Expr* **rparen**

lowest precedence level first

1 nonterm per precedence level

Derive the string 4 - 7 * 3



Resolving Grammar Ambiguity: Precedence

Fixed Grammar

Expr → **expr minus expr**

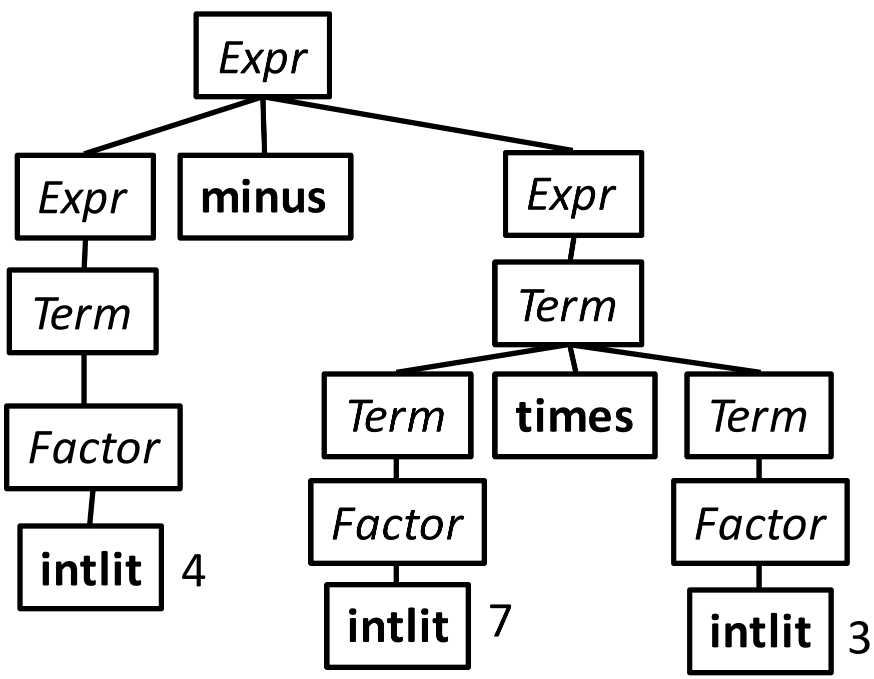
| *Term*

Term → *Term* **times** *Term*

| *Factor*

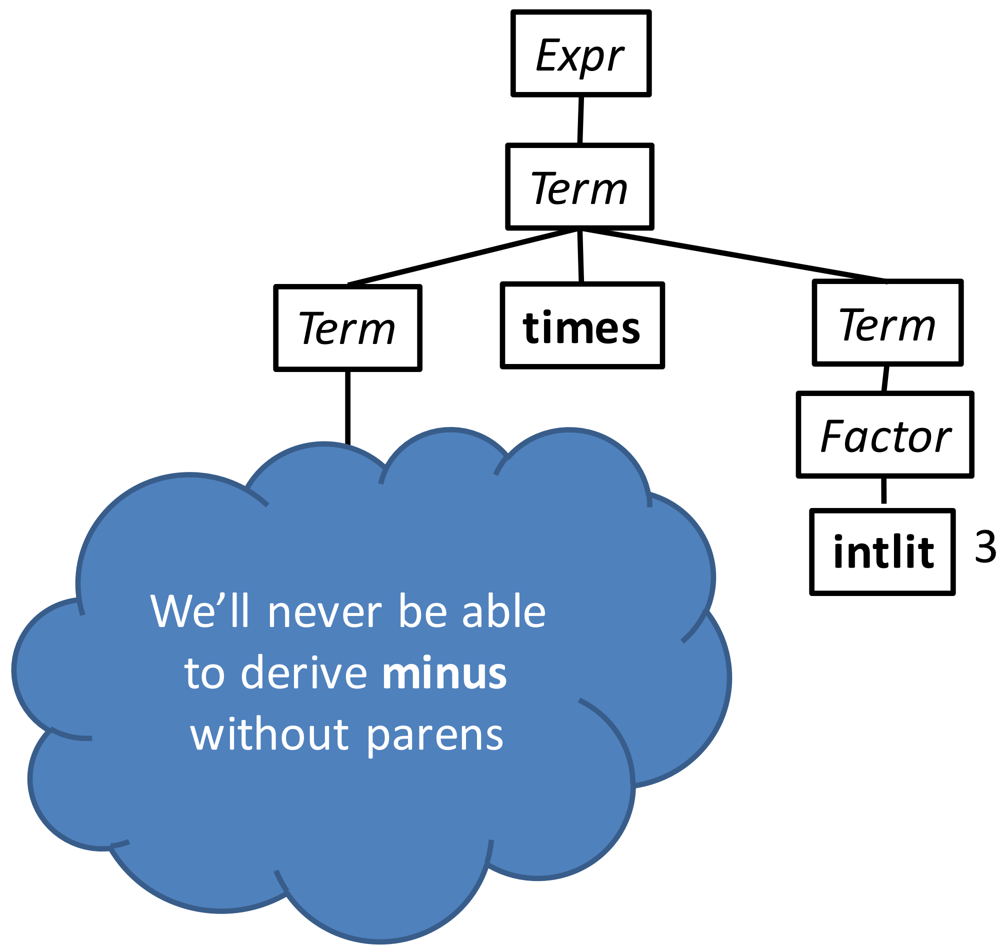
Factor → **intlit**

| **lparen** *Expr* **rparen**



Derive the string 4 - 7 * 3

Let's try to re-build the wrong parse tree



Did we fix all ambiguity?

<u>Fixed Grammar</u>

Expr → *Expr* **minus** *Expr*

| *Term*

Term → *Term* **times** *Term*

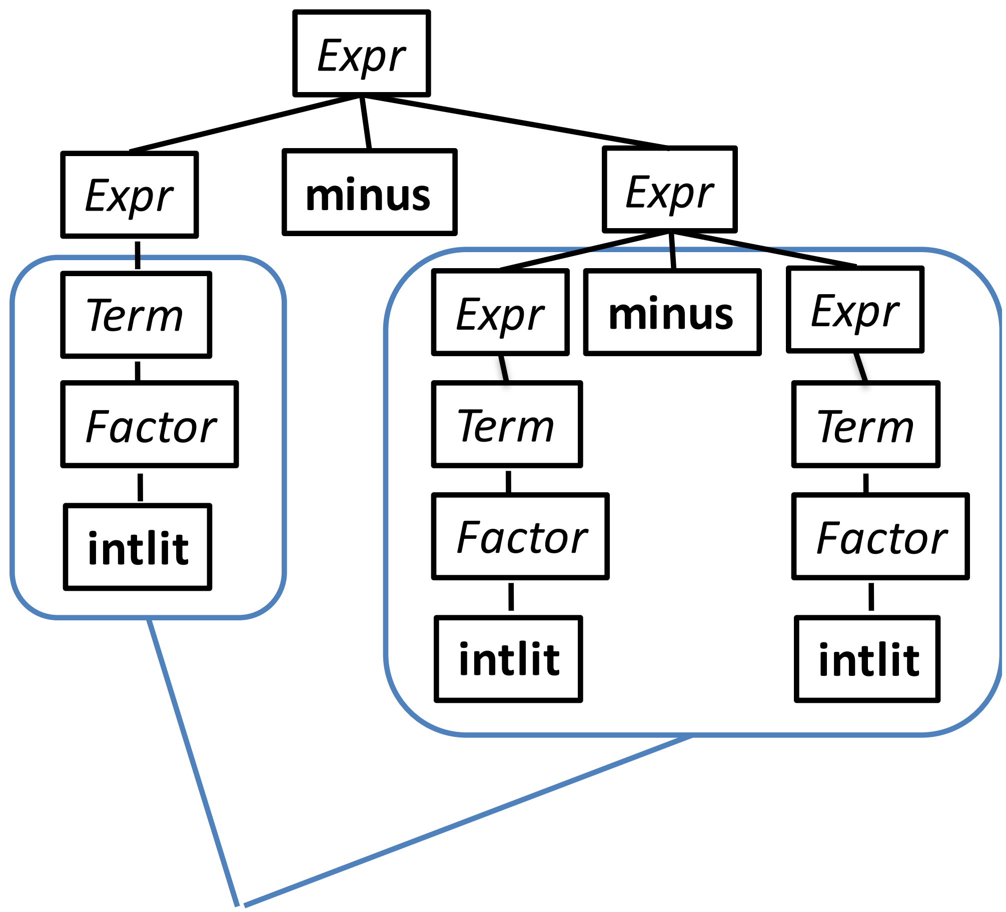
| *Factor*

Factor → **intlit**

| **lparen** *Expr* **rparen**

NO!

Derive the string 4 - 7 - 3



These subtrees could have been swapped!

Where we are so far

- Precedence
  - We want correct behavior on 4 – 7 * 9
  - A new nonterminal for each precedence level
- Associativity
  - We want correct behavior on 4 – 7 – 9
  - Minus should be *left associative*: a – b – c = (a – b) – c
  - Problem: the *recursion* in a rule like

Expr → *Expr* **minus** *Expr*

Definition: Recursion in Grammars

- **A grammar is *recursive* in (nonterminal) *X* if**

  $X \overset{+}{\Rightarrow} \alpha X \gamma$ for non-empty strings of symbols α and γ

- **A grammar is *left-recursive* in *X* if**

  $X \overset{+}{\Rightarrow} X\gamma$ for non-empty string of symbols γ

- **A grammar is *right-recursive* in *X* if**

  $X \overset{+}{\Rightarrow} \alpha X$ for non-empty string of symbols α

Resolving Grammar Ambiguity: Associativity

- We'll recognize left-associative operators with left-associative productions
- We'll recognize right-associative operators with right-associative productions

Expr → *Expr* **minus** ~~*Expr*~~ *Term*

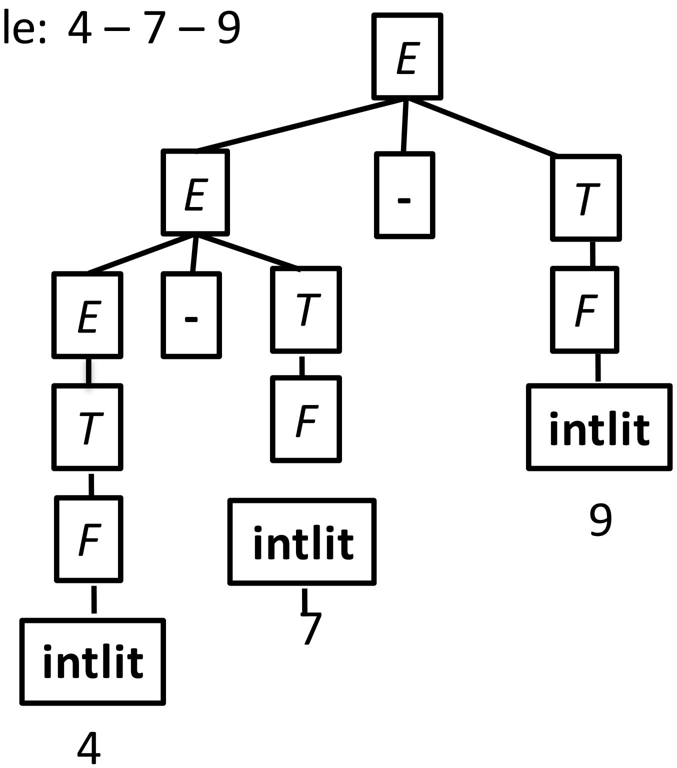
| *Term*

Term → *Term* **times** ~~*Term*~~ *Factor*

| *Factor*

Factor → **intlit** | **lparen** *Expr* **rparen**

Example: 4 – 7 – 9



Resolving Grammar Ambiguity: Associativity

Expr → *Expr* **minus** *Term*

| *Term*

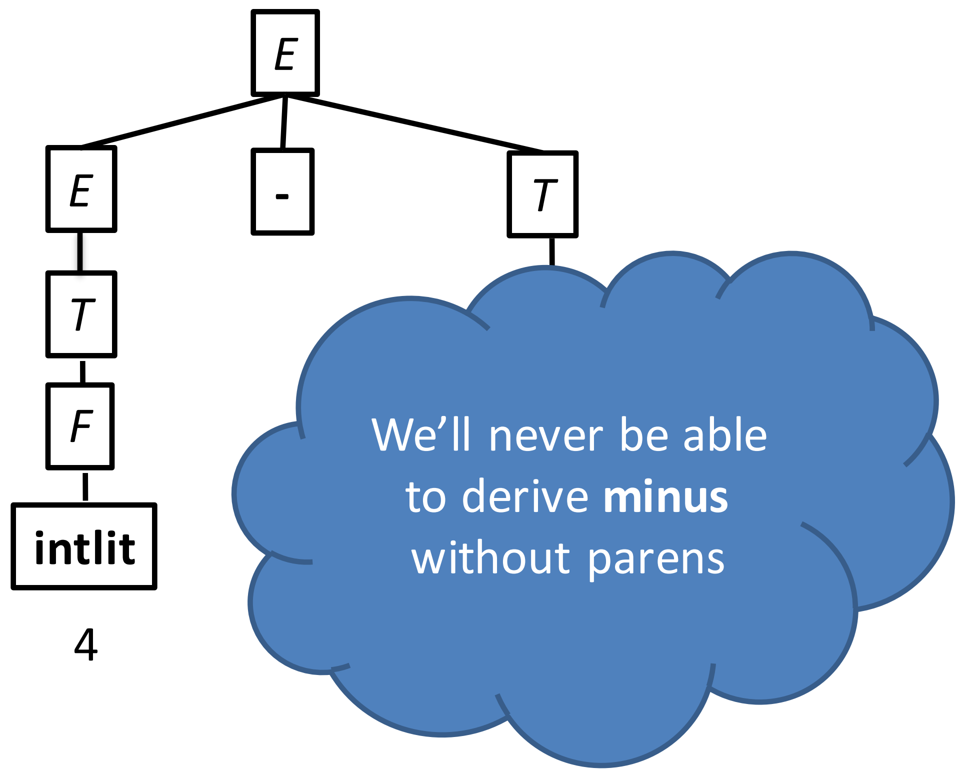
Term → *Term* **times** *Factor*

| *Factor*

Factor → **intlit** | **lparen** *Expr* **rparen**

Example: 4 – 7 – 9

Let's try to re-build the wrong parse tree again



Example

- Language of Boolean expressions
  - bexp → TRUE
    bexp → FALSE
    bexp → bexp OR bexp
    bexp → bexp AND bexp
    bexp → NOT bexp
    bexp → LPAREN bexp RPAREN
- Add nonterminals so that **OR** has lowest precedence, then **AND**, then **NOT**. Then change the grammar to reflect the fact that both **AND** and **OR** are left associative.
- Draw a parse tree for the expression:
  - true AND NOT true.

Another ambiguous example

Stmt $\rightarrow$

if Cond **then** Stmt |

if Cond **then** Stmt **else** Stmt | …

Consider this word in this grammar:

if a **then if** b **then** s **else** s2

How would you derive it?

Summary

- To understand how a parser works, we start by understanding **context-free grammars**, which are used to define the language recognized by the parser. terminal symbol
  - (non)terminal symbol
  - grammar rule (or production)
  - derivation (leftmost derivation, rightmost derivation)
  - parse (or derivation) tree
  - the language defined by a grammar
  - ambiguous grammar