

CS540: HW3 (P1)

Keith Funkhouser
wfunkhouser@cs.wisc.edu

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(a) (i) Distances from [3, 3] to...

```
class1:
--> [2, 2]: 2.0 ( $|3.0 - 2.0| + |3.0 - 2.0| = 2.0$ )
--> [4, 4]: 2.0 ( $|3.0 - 4.0| + |3.0 - 4.0| = 2.0$ )
--> [2, 4]: 2.0 ( $|3.0 - 2.0| + |3.0 - 4.0| = 2.0$ )
class2:
--> [6, 5]: 5.0 ( $|3.0 - 6.0| + |3.0 - 5.0| = 5.0$ )
--> [5.4, 5.6]: 5.0 ( $|3.0 - 5.4| + |3.0 - 5.6| = 5.0$ )
--> [3.6, 6.4]: 4.0 ( $|3.0 - 3.6| + |3.0 - 6.4| = 4.0$ )
class3:
--> [1.8, 8]: 6.2 ( $|3.0 - 1.8| + |3.0 - 8.0| = 6.2$ )
--> [5.6, 8.2]: 7.8 ( $|3.0 - 5.6| + |3.0 - 8.2| = 7.8$ )
```

The 3 closest points are [2,2], [4,4], and [2,4], all of which are in class 1. The classification for [3,3] is thus class 1.

(ii) Distances from [6, 4.4] to...

```
class1:
--> [2, 2]: 6.4 ( $|6.0 - 2.0| + |4.4 - 2.0| = 6.4$ )
--> [4, 4]: 2.4 ( $|6.0 - 4.0| + |4.4 - 4.0| = 2.4$ )
--> [2, 4]: 4.4 ( $|6.0 - 2.0| + |4.4 - 4.0| = 4.4$ )
class2:
--> [6, 5]: 0.6 ( $|6.0 - 6.0| + |4.4 - 5.0| = 0.6$ )
--> [5.4, 5.6]: 1.8 ( $|6.0 - 5.4| + |4.4 - 5.6| = 1.8$ )
--> [3.6, 6.4]: 4.4 ( $|6.0 - 3.6| + |4.4 - 6.4| = 4.4$ )
class3:
--> [1.8, 8]: 7.8 ( $|6.0 - 1.8| + |4.4 - 8.0| = 7.8$ )
--> [5.6, 8.2]: 4.2 ( $|6.0 - 5.6| + |4.4 - 8.2| = 4.2$ )
```

The 3 closest points are [6,5], [5.4,5.6], and [4,4], the majority class of which is class 2.

(iii) Distances from [2.6, 6] to...

```
class1:
--> [2, 2]: 4.6 ( $|2.6 - 2.0| + |6.0 - 2.0| = 4.6$ )
--> [4, 4]: 3.4 ( $|2.6 - 4.0| + |6.0 - 4.0| = 3.4$ )
--> [2, 4]: 2.6 ( $|2.6 - 2.0| + |6.0 - 4.0| = 2.6$ )
class2:
--> [6, 5]: 4.4 ( $|2.6 - 6.0| + |6.0 - 5.0| = 4.4$ )
--> [5.4, 5.6]: 3.2 ( $|2.6 - 5.4| + |6.0 - 5.6| = 3.2$ )
--> [3.6, 6.4]: 1.4 ( $|2.6 - 3.6| + |6.0 - 6.4| = 1.4$ )
class3:
--> [1.8, 8]: 2.8 ( $|2.6 - 1.8| + |6.0 - 8.0| = 2.8$ )
--> [5.6, 8.2]: 5.2 ( $|2.6 - 5.6| + |6.0 - 8.2| = 5.2$ )
```

The 3 closest points are [3.6,6.4], [2,4], and [1.8,8]. Since each class has one closest neighbor, the output class will be the one with the highest index, or class 3.

(b) (i) Distances from [3, 3] to...

```
class1:
--> [2, 2]: 2.0 ( $|3.0 - 2.0| + |3.0 - 2.0| = 2.0$ )
--> [4, 4]: 2.0 ( $|3.0 - 4.0| + |3.0 - 4.0| = 2.0$ )
--> [2, 4]: 2.0 ( $|3.0 - 2.0| + |3.0 - 4.0| = 2.0$ )
class2:
--> [6, 5]: 5.0 ( $|3.0 - 6.0| + |3.0 - 5.0| = 5.0$ )
--> [5.4, 5.6]: 5.0 ( $|3.0 - 5.4| + |3.0 - 5.6| = 5.0$ )
--> [3.6, 6.4]: 4.0 ( $|3.0 - 3.6| + |3.0 - 6.4| = 4.0$ )
--> [4.4, 6]: 4.4 ( $|3.0 - 4.4| + |3.0 - 6.0| = 4.4$ )
class3:
--> [1.8, 8]: 6.2 ( $|3.0 - 1.8| + |3.0 - 8.0| = 6.2$ )
--> [5.6, 8.2]: 7.8 ( $|3.0 - 5.6| + |3.0 - 8.2| = 7.8$ )
```

The 3 closest points are [2,2], [4,4], and [2,4], all of which are in class 1. The classification for [3,3] is thus class 1.

(ii) Distances from [6, 4.4] to...

```
class1:
--> [2, 2]: 6.4 ( $|6.0 - 2.0| + |4.4 - 2.0| = 6.4$ )
--> [4, 4]: 2.4 ( $|6.0 - 4.0| + |4.4 - 4.0| = 2.4$ )
--> [2, 4]: 4.4 ( $|6.0 - 2.0| + |4.4 - 4.0| = 4.4$ )
class2:
--> [6, 5]: 0.6 ( $|6.0 - 6.0| + |4.4 - 5.0| = 0.6$ )
--> [5.4, 5.6]: 1.8 ( $|6.0 - 5.4| + |4.4 - 5.6| = 1.8$ )
--> [3.6, 6.4]: 4.4 ( $|6.0 - 3.6| + |4.4 - 6.4| = 4.4$ )
--> [4.4, 6]: 3.2 ( $|6.0 - 4.4| + |4.4 - 6.0| = 3.2$ )
class3:
--> [1.8, 8]: 7.8 ( $|6.0 - 1.8| + |4.4 - 8.0| = 7.8$ )
--> [5.6, 8.2]: 4.2 ( $|6.0 - 5.6| + |4.4 - 8.2| = 4.2$ )
```

The 3 closest points are [6,5], [5.4,5.6], and [4,4], the majority class of which is class 2.

(iii) Distances from [2.6, 6] to...

```
class1:
--> [2, 2]: 4.6 ( $|2.6 - 2.0| + |6.0 - 2.0| = 4.6$ )
--> [4, 4]: 3.4 ( $|2.6 - 4.0| + |6.0 - 4.0| = 3.4$ )
--> [2, 4]: 2.6 ( $|2.6 - 2.0| + |6.0 - 4.0| = 2.6$ )
class2:
--> [6, 5]: 4.4 ( $|2.6 - 6.0| + |6.0 - 5.0| = 4.4$ )
--> [5.4, 5.6]: 3.2 ( $|2.6 - 5.4| + |6.0 - 5.6| = 3.2$ )
--> [3.6, 6.4]: 1.4 ( $|2.6 - 3.6| + |6.0 - 6.4| = 1.4$ )
--> [4.4, 6]: 1.8 ( $|2.6 - 4.4| + |6.0 - 6.0| = 1.8$ )
class3:
--> [1.8, 8]: 2.8 ( $|2.6 - 1.8| + |6.0 - 8.0| = 2.8$ )
--> [5.6, 8.2]: 5.2 ( $|2.6 - 5.6| + |6.0 - 8.2| = 5.2$ )
```

The 3 closest points are [3.6,6.4], [4.4,6], and [2,4], the majority class of which is class 2.