**CS 577: Introduction to Algorithms** 11/19/2015

# Homework 10

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This assignment covers reductions between NP problems. It is due in class one week after Thanksgiving. Good luck and happy T-Day!

## Review problems

1. Consider the satisfiability problem for Boolean formulas. Show how to polynomial-time reduce search to decision.

2. Consider the variant of the Traveling Salesperson optimization problem where the tour does not need to end in the same city as where it starts. Show that this variant and the original one are equivalent under polynomial-time mapping reductions.

## Graded written problem

3. [10 points] At the National Thanksgiving Turkey Presentation the President pardons a turkey. As a token of appreciation, the turkey flies a tour of some cities, starting and ending in DC, and visiting each city at least once and no more than 3 times. As the turkey is afraid of flying new routes, the number of distinct routes has to be kept as small as possible, where the route from $A$ to $B$ is considered the same as the route from $B$ to $A$. Given the effort it takes the turkey to fly from $A$ to $B$ for all pairs of cities $A$ and $B$ that the turkey wants to visit, you need to find a tour that meets all requirements and takes the least amount of effort of the turkey.

   (a) Formulate the Turkey Tour Problem as an NP optimization problem using graph terminology only.

   (b) Show how to polynomial-time reduce the Traveling Salesperson Problem to the Turkey Tour Problem.
   *Hint:* Make use of the equivalence given in problem 2. For starters, you may want to think about the variant of the Turkey Tour Problem where the bound of 3 is replaced by 2.

   (c) Formulate a decision version of the Turkey Tour Problem, and show how to polynomial-time reduce it to and from the optimization version.

## Additional written problem

4. Some time ago, people from the programming languages group asked about the following problem.

   You are given a list of formulas of the following form:

   - $x_i = 0$,
   - $x_i = 1$,

- $x_i \geq x_j$ or $x_k < X$, and
- $x_i > x_j$ or $x_k \leq X$.

Here $X$ denotes a set of variables, and $x_k < X$ means that $x_k < x$ for all $x \in X$; $x_k \leq X$ is defined similarly. The question is whether there exists a way to assign the values 0 and 1 to the variables such that all formulas are satisfied.

Show a polynomial-time mapping reduction from 3-SAT to this problem.

## Optional programming problem

5. [2.5 points] Solve SPOJ problem The Courier (problem code COURIER).
   *Hint:* Whereas the trivial algorithm for the Traveling Salesperson Problem takes time $\Theta(n!)$, there is a dynamic programming algorithm that only takes time $O(n^2 \cdot 2^n)$.

## Challenge problem

6. Recall scheduling to minimize the maximum lateness. Suppose we want to minimize the *sum* of the latenesses instead of the *maximum* lateness. Give a polynomial-time mapping reduction from the partition problem to this problem.