Design

Conventions

- Two channels of audio are $x_L[n]$ and $x_R[n]$. Commonly without suffix $x[n]$.
- Sampling rate is f_s, thus sampling interval

$$\tau_s = \frac{1}{f_s}$$

Common definitions

- Interesting constant, $\zeta = W\left(\frac{\log 2}{5}\right) \approx 0.122630300853342312$

Frequencies

- Maximum audio frequency, $f_{\max} = 20\ \text{kHz}$
- Sweet frequency,

$$\begin{aligned} \ell &= f_{\max} \frac{\zeta}{\log 2} \\ &\approx 3538.36253807677143\ \text{Hz} \end{aligned}$$

- Sweet timespan,

$$t = 50\mu\text{day} = 4.32\ s$$

- Downsample rate, $N = 1200$

Viewport and positioning

- Viewport width, w_{vp} ~~= 4192 dip~~ = 3840 dip
- Viewport diagonal, $d = \frac{5}{4} w_{\text{vp}} = 4800\ \text{dip}$
- Viewport aspect ratio, ρ
- Viewport height, $h_{\text{vp}} = \frac{w_{\text{vp}}}{\rho} = 2160\ \text{dip}$
- Viewport area, $A_{\text{vp}} = \frac{w_{\text{vp}}^2}{\rho}$
- Channel area,

$$
\begin{aligned}
A_{\mathrm{ch}} &= h_{\mathrm{vp}}(w_{\mathrm{vp}} - h_{\mathrm{vp}}) + \frac{1}{2} h_{\mathrm{vp}}(h_{\mathrm{vp}} - 1) \\
&= \frac{1}{2} h_{\mathrm{vp}}(2w_{\mathrm{vp}} - h_{\mathrm{vp}} - 1) \\
&= 5960520 \text{ px}
\end{aligned}
$$

- Initial position,

$$
\begin{aligned}
p[0] &= h_{\mathrm{vp}}(w_{\mathrm{vp}} - h_{\mathrm{vp}}) - \frac{1}{2} h_{\mathrm{vp}} \\
&= \frac{1}{2} h_{\mathrm{vp}}(2w_{\mathrm{vp}} - 2h_{\mathrm{vp}} - 1) \\
&= 3627720 \text{ px}
\end{aligned}
$$

Miscelleneous

- $\gamma_C(x) = a_0 + a_1 \log_2(a_2 x + a_3)$, where,

$$
\begin{aligned}
a_0 &= \frac{1}{4} \\
a_1 &= \frac{\log 2}{2 \log\left(\frac{\log 2}{\zeta} - 1\right)} \\
&\approx 0.225432981868225421 \\
a_2 &= \frac{(\log 2 - 2\zeta) \log 2}{\zeta^{\frac{3}{2}} \sqrt{\log 2 - \zeta}} \\
&\approx 9.57111578549689866 \\
a_3 &= \sqrt{\frac{\zeta}{\log 2 - \zeta}} \\
&\approx 0.463622652910641416
\end{aligned}
$$

Derived with $\gamma_C(0) = 0$, $\gamma_C\left(\frac{\ell}{f_{\max}}\right) = \frac{1}{2}$ and $\gamma_C(1) = 1$

Phase (φ) calculation

1. Hilbert transform, $z[n]$, of signal $x[n]$ is computed.
2. Argument, $\varphi[n]$, of that Hilbert transform $z[n]$ is computed.

3. Argument, $\varphi[n]$, is then scaled and saved in a companion file e.g. `*.phases.ext`

For image generation:

- Angular frequency,

$$\omega[n] = \varphi[n] - \varphi[n-1]$$

  normalized, where $\varphi[-n] = 0$
  - $\varphi[n] \in (-\pi, \pi]$
  - $\omega[n] \in [0, 2\pi)$
- Linear frequency,

$$f[n] = f_s \frac{\omega[n]}{2\pi}$$

- Velocity,

$$v[n] = x[n] - x[n-1]$$

- Decay time,

$$\tau_{\text{decay}}[n] = t \cdot 2^{-\frac{|f[n]|}{f_{\max}}}$$

Layer, for $(n-k)^{\text{th}}$ sample

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- Spread coefficient,

$$\varsigma = \frac{k\tau_s}{2\tau_{\text{decay}}}$$

- Spread radius,

$$r_{\text{spread}} = \frac{d}{2}\varsigma$$

- Position,

$$p[n] = p[n-1] + v\begin{cases} p[n-1] \text{ if } v \leq 0 \\ A_{\text{ch}} - p[n-1] \text{ if } v > 0 \end{cases}$$

- Color: LCh-uv with α
- $h = \varphi + \varphi_0$
- $C = x \sec \varphi$
- $L = \gamma_C\left(\frac{|f|}{f_{\max}}\right)$
- $\alpha = \gamma_C(\varsigma)$