Random permutation

$k = n = 4$ $\quad$ $P_3 \; P_4$

$\boxed{1, 2, 3, 4}$

$I = 3$ $\quad$ $k = n-1 = 3$ $\quad$ $1, 2, 4, 3$

$P_2 \; P$

$I = 2$ $\quad$ $1, 4, 2, 3$

$I = 2$ $\quad$ $1, 4, 2, 3$

Algo Step 1 Let $P_1, P_2, \ldots, P_n$ be any permutation of $1, 2, \ldots, n$

Step 2 $\quad$ Set $k = n$

Step 3 $\quad$ U, $\quad I = [kU] + 1$

Step 4 $\quad$ Interchange P_I and P_k

Step 5: Let $k = k-1$, and if $k > 1$ go to step 3

Step 6 $\quad$ $P_1, \ldots, P_n$ is the desired random permutation

→ Follow the algo until the positions $n, n-1, \ldots, n-r+1$ are filled

random subset, say of size r.

— x —

P.P. (λ)

$N(t) = \#$ of event $(0, t] \sim P.P.(\lambda)$

(i) $N(t)$ has stationary & indep. increment

$N(t_1, t_2]$ in $N(0, t_1]$

$\| \qquad\qquad \|$

$N(t_2) - N(t_1) \qquad N(t_1)$

indep

$\overset{d}{=}$

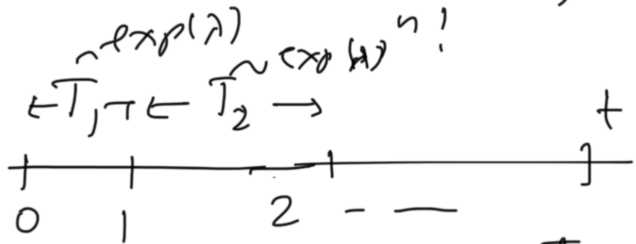
$N(t_2 - t_1) = N(0, t_2 - t_1]$

$h \to$ small

(ii) $P(N(h) = 1) = \lambda h + o(h)$

$P(N(h) \geq 2) = o(h)$

$$P(N(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \quad n = 0, 1, 2, \ldots$$



$T_1 > t \equiv N(0, t] = 0$

$$P(T_1 > t) = P(N(t) = 0)$$
$$= e^{-\lambda t}$$

$T_1 \sim \exp(\lambda)$