Confidence interval estimation:

One sample

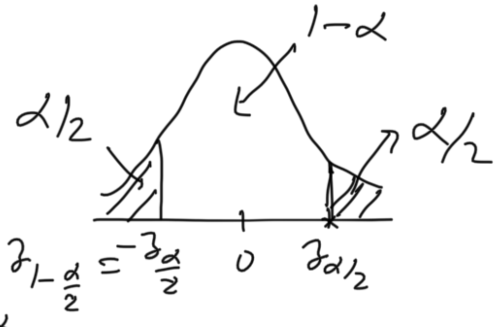
$X \sim N(\mu, \sigma^2)$ r.s. $X_1, \longrightarrow X_n$ $\bar{X}$ (sample mean)

CI for μ

(i) σ^2 known

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$P\left(-z_{\frac{\alpha}{2}} \le Z \le z_{\frac{\alpha}{2}}\right) = 1-\alpha$$



$$P\left(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

$100(1-\alpha)\%$ C.I. for μ is

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$$

(ii) σ^2 unknown

indep

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$U = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1},$$

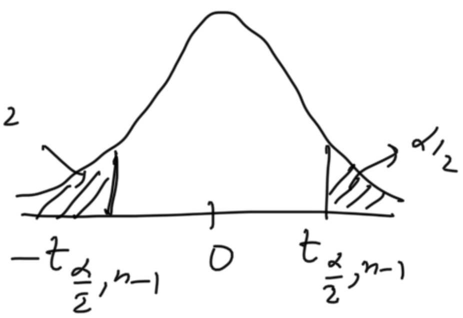
where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$$T = \frac{Z}{\sqrt{U/\cdot}} = \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$

$\sqrt{\quad} \quad (n-1)$

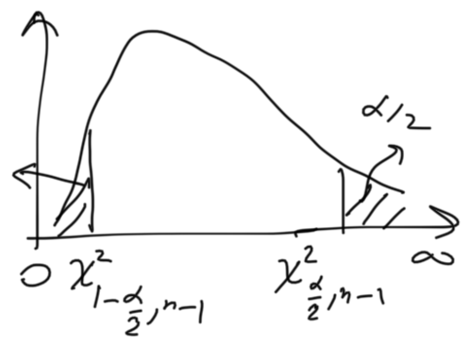
$100(1-\alpha)\%$ C.I for μ

$$\left(\bar{x} - t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}, \ \bar{x} + t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right)$$



<u>C.I. for σ^2</u>

$$U = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$



$100(1-\alpha)\%$ C.I. for σ^2

$$\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}}, \ \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}\right)$$

—x—

Two <u>sample CI</u>

		sample mean	sample var
indep	$X_1 \sim N(\mu_1, \sigma_1^2)$ rs $X_{11}, \ldots, X_{1n_1}$	$\bar{X}_1$	S_1^2
	$X_2 \sim N(\mu_2, \sigma_2^2)$ rs $X_{21}, \ldots, X_{2n_2}$	$\bar{X}_2$	S_2^2

(i) σ_1^2, σ_2^2 known

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \ \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$100(1-\alpha)\%$ C.I. for ...

$100(1-\alpha)\%$ C.I for $\mu_1-\mu_2$

$$\left(\bar{x}_1-\bar{x}_2-z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\ ,\ \bar{x}_1-\bar{x}_2+z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right)$$

(ii) σ_1^2, σ_2^2 unknown

(a) $\sigma_1^2=\sigma_2^2=\sigma^2$ (unknown)

$100(1-\alpha)\%$ C.I. for $\mu_1-\mu_2$

$$\left(\bar{x}_1-\bar{x}_2-t_{\frac{\alpha}{2},n_1+n_2-2}\,S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right.$$

$$\left.,\ \bar{x}_1-\bar{x}_2+t_{\frac{\alpha}{2},n_1+n_2-2}\,S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right),$$

where $$S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$$

(b) $\sigma_1^2\neq\sigma_2^2$ (unknowns)

$100(1-\alpha)\%$ C.I. for $\mu_1-\mu_2$ is

$$\left(\bar{x}_1-\bar{x}_2-t_{\frac{\alpha}{2},\nu}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\ ,\ \bar{x}_1-\bar{x}_2+t_{\frac{\alpha}{2},\nu}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right)$$

, where $$\nu=\frac{\left(S_1^2/n_1+S_2^2/n_2\right)^2}{\frac{(S_1^2/n_1)^2}{n_1+1}+\frac{(S_2^2/n_2)^2}{n_2+1}}-2.$$

Paired t-test

indep.

data contains n pairs $(X_{11}, X_{21}), \ldots, (X_{1n}, X_{2n})$

$X_1 \sim N(\mu_1,$ $\qquad D_i = X_{1i} - X_{2i}, \; i = 1, \dots, n$

$X_2 \sim N(\mu_2,$

CI on $\mu_1 - \mu_2 = \mu_D = E(X_1) - E(X_2)$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^{n} D_i \sim N\left(\mu_D, \frac{\sigma}{\sqrt{n}}\right)$$

$$\frac{(n-1) S_D^2}{\sigma^2} \sim \chi^2_{n-1}$$

$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$ is

$$\left(\bar{D} - t_{\frac{\alpha}{2}, n-1} \frac{S_D}{\sqrt{n}}, \; \bar{D} + t_{\frac{\alpha}{2}, n-1} \frac{S_D}{\sqrt{n}}\right).$$