RTSM/QUIZ/2

Fill in the blanks (Numerical)

Date of Exam : 8th Oct, 2021

Time : 08:00 am to 09:00 am

Duration : 50min

No of questions: 10 out of 14 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

October 20, 2021

1. Let $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for all $i = 1, 2, \ldots 10$. Observed values of $\hat{\beta}_0 = 1.2$, $MSError = 3.5$, $\bar{x} = 2.3$, $S_{xx} = 5.7$. Then observed absolute value of the t-statistic for $H_0 : \beta_0 = 1.5$ vs $H_1 : \beta_0 \neq 1.5$ is

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0.1581524

   ERROR RANGE: ∓ 0.005

   Soln: $abs(1.2 - 1.5)/sqrt(3.5 * (1/10 + 2.3^2/5.7)) = 0.1581524$

2. Let for a simple linear regression model $MSError = 3.5$, $n = 12$. The upper bound of 95% confidence interval of σ^2

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANS: 1.708719 CORRECTED TO (12-2)*3.5/qchisq(0.025,df=10)= 10.77927

   ERROR RANGE: ∓ 0.005

   Soln: $(12 - 2) * 3.5/qchisq(0.975, df = 10) = 1.708719$

3. Let for a simple linear regression model $MSError = 5.3$, $n = 10$, $\sum(x_i - \bar{x})^2 = 5.7$, $\bar{x} = 2.3$. Find the estimated variance of the estimator of the expected value of y for $x = 2.5$

   [Answer only within the error range ∓ 0.005 will get the credit]

ANS: 0.567193

ERROR RANGE: ∓ 0.005

Soln: $5.3/10 + 5.3 * (2.5 - 2.3)^2/5.7 = 0.567193$

4. Let for a simple linear regression model $MSError = 0.35$, $n = 10$, $S_{xx} = 5.7$, $\bar{x} = 2.3$. Find the length of the 95% prediction interval of y for $x = 2.5$

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANS: 2.87079

   ERROR RANGE: ∓ 0.005

   Soln: $2 * qt(0.975, df = 8) * sqrt(0.35 * (1 + 1/10 + (2.5 - 2.3)^2/5.7)) = 2.87079$

5. Suppose for a multiple linear regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2\mathbf{I}_6), \mathbf{Y} \in \mathbb{R}^6$, $\beta \in \mathbb{R}^4$ observed $\mathbf{X}$ matrix is

$$\mathbf{X} = \begin{bmatrix} 1 & 3.0 & -1.5 & -0.4 \\ 1 & 1.5 & 1.0 & 2.0 \\ 1 & 0.5 & 2.0 & 0.8 \\ 1 & 1.0 & 1.0 & -2.0 \\ 1 & 7.0 & 4.0 & 0.9 \\ 1 & 0.0 & 2.0 & 6.0 \end{bmatrix}$$

   Find $E(Y_1 - 2Y_2 + 3Y_3 - 1.5Y_4 - 0.5Y_6)$.

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0

   ERROR RANGE: ∓ 0.005

6. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2\mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$use Jackknife method to test at 5% level for the null hypothesis that the observation y_5 is not an outlier based on the following estimates. Residual $e_5 = 2.01$, $MSResudual = 1.4$ and 5^{th} diagonal element of projection matrix $h_{55} = 0.036$, where $n = 25$, $k = 6$. Find the absolute value of t-statistics

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 1.858582 CORRECTED TO 1.841531

   ERROR RANGE: ∓ 0.005

   Soln: $2.01/sqrt(((18 * (1.4) - 2.01^2/(1 - 0.036))/17) * (1 - 0.036)) = 1.841531$

7. Let $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for all $i = 1, 2, \ldots 10$. Observed values of $h_{33} = 0.67$, $MSError = 3.5$, $e_3 = 0.57$. Find the value of r_3 (studentized residual)

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0.530376

   ERROR RANGE: ∓ 0.005

   Soln: (0.57)/sqrt(3.5*(1-0.67))= 0.530376

8. Let $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for all $i = 1, 2, \ldots 10$. Observed values of $h_{33} = 0.67$, $MSError = 3.5$, $e_3 = 0.57$. Find the value of d_3 (Standardized residual)

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0.3046778

   ERROR RANGE: ∓ 0.005

   Soln: (0.57)/sqrt(3.5)= 0.3046778

9. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ find the value of $\frac{R^2}{R^2_{adjusted}}$ when $R^2 = 0.82$, $k = 6$, $n = 25$.

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0.76 CORRECTED TO 1.078947

   ERROR RANGE: ∓ 0.005

   Soln: $R^2/(1-(1-R^2)*(n-1)/(n-k-1)) = 0.82/0.76 = 1.078947$

10. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ under LS method find the estimated value of $E(\|\beta - \hat{\beta}\|_2^2)$ when $SSError = 5.67$, and eigenvalues of $\mathbf{X}^T\mathbf{X}$ are $\{3.8, 3.3, 2.1, 1.3, 0.5\}$ and $n = 25$

    [Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 1.080591

    ERROR RANGE: ∓ 0.005

    Soln: $5.67/(25-5)*(1/3.8+1/3.3+1/2.1+1/1.3+1/0.5) = 1.080591$

11. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ under Ridge regression find the estimated value of $Trace(D(\hat{\beta}_R))$ when $MSError = 5.67$, and eigenvalues of $\mathbf{X}^T\mathbf{X}$ are $\{3.8, 3.3, 2.1, 1.3, 0.5\}$, sample size $n = 25$ and Lagrange multiplayer $\lambda = 0.5$

    [Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 9.332445
    ERROR RANGE: ∓ 0.005
    Soln: $5.67 * (3.8/(4.3)^2 + 3.3/(3.8)^2 + 2.1/(2.6)^2 + 1.3/(1.8)^2 + 0.5/(1)^2) = 9.332445$

12. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ if $R^2 = 0.82$, $k = 6$, $n = 25$ find the value of F-statistic for the ANOVA of regression model.

    [Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 13.66667
    ERROR RANGE: ∓ 0.005
    Soln: $R^2/(1 - R^2) * (n - k - 1)/k = 13.66667$

13. Consider a bivariate normal model with estimated sample correlation coefficient $r = 0.089$ with sample size 18. Find the absolute value of t-statistic to test $H_0 : \rho = 0$ vs $H_0 : \rho \neq 0$

    [Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 0.3574184
    ERROR RANGE: ∓ 0.005
    Soln: $r * sqrt(n - 2)/sqrt(1 - r^2) = 0.3574184$

14. Consider a bivariate normal model with estimated sample correlation coefficient $r = 0.89$ with sample size 103. Find the absolute value of large sample z-statistic to test $H_0 : \rho = 0.8$ vs $H_0 : \rho \neq 0.8$

    [Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 1.252188 CORRECTED TO 3.23314
    ERROR RANGE: ∓ 0.005
    Soln: $(atanh(0.89) - atanh(0.8))/sqrt(1/(n - 3)) = 3.23314$