ANOVA

CRD: y_{ij} ; j^{th} response to the i^{th} treatment

$i = 1, \ldots, k$

$j = 1, \ldots, n_i$

	Treatment/Factor level (i)				
	1	2	...	k	
	y_{11}	y_{21}	...	y_{k1}	
	y_{12}	y_{22}	...	y_{k2}	
	$\vdots$	$\vdots$		$\vdots$	
	y_{1n_1}	y_{2n_2}		y_{kn_k}	
Total	$T_{1.}$	$T_{2.}$	...	$T_{k.}$	$T_{..}$
sample mean	$\bar{y}_{1.}$	$\bar{y}_{2.}$	...	$\bar{y}_{k.}$	$\bar{y}_{..}$

$$N = \sum_{i=1}^{k} n_i$$

Statistical model

$$y_{ij} = \mu_i + \epsilon_{ij}$$

$$= \mu + (\mu_i - \mu) + \epsilon_{ij}$$

$$= \mu + \alpha_i + \epsilon_{ij} \quad , i = 1, \ldots, k$$

$$j = 1, \ldots, n_i$$

$$\epsilon_{ij} \sim NID(0, \sigma^2)$$

$$SS_{Total} = \sum \sum (y_{ij} - \bar{y}_{..})^2 = \sum \sum y_{ij}^2 - \frac{T_{..}^2}{}$$

$i \quad j \qquad\qquad i \quad S \qquad N$

$$SS_{Treatment} = \sum_{i=1}^{k} n_i(\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{k} \frac{T_{i.}^2}{n_i} - \frac{T_{..}^2}{N}$$

$$SS_{Error} = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2 = SS_{Total} - SS_{Treatment}$$

ANOVA $\quad H_0: \mu_1 = \mu_2 = \cdots = \mu_k$

H_1: atleast one of them is different

SOV	d.f.	SS	MS	F
Treatment	$k-1$	SS_{Treat}	$MS_{Treat} = \frac{SS_{Treat}}{k-1}$	$\frac{MS_{Treat}}{MS_{Error}} = F_0$
Error	$N-k$	SS_{Error}	$MS_{Error} = \frac{SS_{Error}}{N-k}$	
Total	$N-1$	SS_{Total}		

reject H_0 at los α if $F_0 > F_{\alpha, k-1, N-k}$

<u>RBD</u>:

$i \longrightarrow$

	Treatment 1	2	--	k	Total	av
block j ↓ → 1	y_{11}	y_{21}	---	y_{k1}	$T_{.1}$	$\bar{y}_{.1}$
→ 2	y_{12}	y_{22}	--	y_{k2}	$T_{.2}$	$\bar{y}_{.2}$
⋮	⋮	⋮	--		⋮	⋮
→ b	y_{1b}	y_{2b}	--	y_{kb}	$T_{.b}$	$\bar{y}_{.b}$
Total	$T_{1.}$	$T_{2.}$	--	$T_{k.}$	$T_{..}$	$\bar{y}_{..}$
av	$\bar{y}_{1.}$	$\bar{y}_{2.}$	--	$\bar{y}_{k.}$		

$i = 1, \ldots, k$

treatment effect

$j = 1, \ldots, b$

$$y_{ij} = \mu + \alpha_i + \beta_j + E_{ij}$$

block effect

$$N = kb$$

df

$$kb-1 = (k-1) + (b-1) + \underset{(k-1)(b-1)}{\overset{df}{\text{error}}}$$

$$SS_{Tot} = \sum_i \sum_j y_{ij}^2 - \frac{T_{..}^2}{kb}$$

$$SS_{Treat} = \sum_i \frac{T_{i.}^2}{b} - \frac{T_{..}^2}{kb}$$

$$SS_{Blks} = \sum_j \frac{T_{.j}^2}{k} - \frac{T_{..}^2}{kb}$$

$$SS_{Error} = SS_{Tot} - SS_{Treat} - SS_{Blks}$$

$$H_0: \mu_{1.} = \mu_{2.} = \dots = \mu_{k.}$$

Latin square

k^{th} column

tire position

i^{th} treatment

	I	II	III	IV
I	A	B	C	D
II	B	C	D	A
III	C	D	A	B
IV	D	A	B	C

j^{th} row

bus

A

B

C

D

$$i, j, k = 1, \dots, r$$

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ijk}$$

$$(i,j,k) \in \mathfrak{R}^2$$

$$SS_{Treat} = \sum_i \frac{T_{i..}^2}{} - \frac{T_{...}^2}{}$$

$$1 \quad r \quad r^2$$

$$SS_{Rows} = \sum_j \frac{T_{.j.}^2}{r} - \frac{T_{...}^2}{r^2}$$

$$SS_{colum} = \sum_k \frac{T_{..k}^2}{r} - \frac{T_{...}^2}{r^2}$$

$$SS_{Total} = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{T_{...}^2}{r^2}$$

$$SS_{Err} = SS_{Tot} - SS_{Trt} - SS_{Rows} - SS_{colum}$$

Graeco-Latin square :

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \Phi_l + e_{ijkl}$$

$i \to i^{th}$ row effect

$j \to j^{th}$ Latin letter effect $\quad i,j,k,l \in (1, ---, p)$

$k \to k^{th}$ greek letter effect

$l \to l^{th}$ column effect

	1	2	3	4
1	$A\alpha$	$B\beta$	$C\gamma$	$D\delta$
2	$B\delta$	$A\gamma$	$D\beta$	$C\alpha$
3	$C\beta$	$D\alpha$	$A\delta$	$B\gamma$
4	$D\gamma$	$C\delta$	$B\alpha$	$A\beta$

$$H_0 : \mu_{.1..} = \mu_{.2..} = -- = \mu_{.p..}$$

dd

$$SS_{.} = \frac{1}{} \sum^{b} T_{.i}^2 - T_{....}^2$$

$$SS_L = \frac{1}{p}\sum_{j=1} T_{.j..} - \frac{T_{....}}{N} \qquad p-1$$

$$SS_G = \frac{1}{p}\sum_{j=1}^{p} T_{..k.}^2 - \frac{T_{....}^2}{N} \qquad p-1$$

$$SS_{Rows} = \frac{1}{p}\sum_{i=1}^{p} T_{i...}^2 - \frac{T_{....}^2}{N} \qquad p-1$$

$$SS_{Column} = \frac{1}{p}\sum_{l=1}^{p} T_{...l}^2 - \frac{T_{....}^2}{N} \qquad p-1$$

$$SS_E = SS_{Total} - SS_L - SS_G - SS_{Rows} - SS_{Column}$$ difference

$$SS_{Total} = \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 - \frac{T_{....}^2}{N} \qquad p^2-1$$