MA 69204 Statistical Software Lab
Assignment No. 5

Generating a Random Sample from a Multivariate Normal Distribution

We want to generate $\underline{\mathbf{X}} \sim N_p(\underline{\boldsymbol{\mu}}, \boldsymbol{\Sigma})$. For each of the following $(\underline{\boldsymbol{\mu}}, \boldsymbol{\Sigma})$ generate 5000 random vectors. Find the sample mean and dispersion matrices in each case. Validate your sample by calculating the norm error for mean as well as dispersion matrix in each case.

Step I: Decompose $\boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}^{\mathbf{T}}$, where p×p matrix $\mathbf{C}$ is lower triangular.

Step II: Generate $Z_1, Z_2, \ldots, Z_p$ as IID N(0, 1) random variates.

Step III: For i = 1, 2, …, p, let $\mathbf{X_i = \mu_i + \sum_{j=1}^{i} c_{ij} Z_j}$, where c_{ij} is the (i, j)th element of $\mathbf{C}$.

Step IV: Return $\underline{\mathbf{X}} = (X_1, \ldots, X_p)$.

1. $\underline{\mu}^{\mathbf{T}} = \mathbf{(1, -1, 2)}$,
$$\Sigma = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

2. $\underline{\mu}^{\mathbf{T}} = \mathbf{(1, 1, 1)}$
$$\Sigma = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

3. $\underline{\boldsymbol{\mu}} = \underline{\mathbf{0}}$,
$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

4. $\underline{\mu}^{\mathbf{T}} = \mathbf{(4, 3, 2, 1)}$
$$\Sigma = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

5. $\underline{\mu}'$ **= (2, 4, -1, 3, 0)**

$$\Sigma = \begin{bmatrix} 4 & -1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ -1 & 3 & 1 & -1 & 0 \\ \frac{1}{2} & 1 & 6 & 1 & -1 \\ -\frac{1}{2} & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$