

## Decisions under Risk and Uncertainty

All the analysis of managerial decision making up to this point in the text has been developed under the assumption that the manager knows with certainty the marginal benefits and marginal costs associated with a decision. While managers do have considerable information about the outcome for many decisions, they must frequently make decisions in situations in which the outcome of a decision cannot be known in advance. A manager may decide, for example, to invest in a new production facility with the expectation that the new technology and equipment will reduce production costs. Even after studying hundreds of technical reports, a manager may still not know with certainty the cost savings of the new plant until the plant is built and operating. In other words, the outcome of the decision to build the new plant is random because the reduction in costs (the outcome) is not known with certainty at the time of the decision. Another risky decision involves choosing the profit-maximizing production level or the price to charge when the marginal benefit and marginal cost can take on a range of values with differing probabilities.

In this chapter we will present some basic rules that managers, and for that matter all decision makers, can and do use to help make decisions under conditions of risk and uncertainty. In the first section, we explain the difference between decision making under risk and decision making under uncertainty. The larger portion of this chapter is devoted to analyzing decisions under risk, rather than situations of uncertainty, because, as you will see, managers facing random benefits and costs are more often confronted with situations involving risk than uncertainty. As you will also see, the rules we present in this chapter for decision making under risk and uncertainty provide only guidelines for making decisions when outcomes are not certain, because no single rule for making such decisions is, or can be, universally employed by all managers at all times. Nevertheless, the rules presented give an overview of some of the helpful methods of analyzing risk and uncertainty.

Before plunging into our presentation of decision making under uncertainty and risk, we want to address a question that may be concerning you: Why do we devote such a large portion of this text to managerial decision making under certainty or complete information, knowing full well that a large proportion of managerial decisions are made with incomplete information—that is, under risk or uncertainty? There are two good reasons. First, the theory of optimization, weighing marginal benefits and marginal costs, as explained in Chapter 3 and applied throughout the text, provides the basic foundation for all decision making regardless of the amount of information available to a decision maker about the potential outcomes of various actions. In order to learn how to do something under less-than-ideal conditions, one must first learn how to do it under ideal conditions. Second, even though a decision maker does not have complete information about the

marginal benefits and marginal costs of all levels of an activity or choice variable, the  $MB = MC$  rule from Chapter 3 is the most productive approach to profit-maximization decisions under many, if not most, relevant circumstances.

## 15.1 DISTINCTIONS BETWEEN RISK AND UNCERTAINTY

When the outcome of a decision is not known with certainty, a manager faces a decision-making problem under either conditions of risk or conditions of uncertainty. A decision is made under **risk** when a manager can make a list of all possible outcomes associated with a decision and assign a probability of occurrence to each one of the outcomes. The process of assigning probabilities to outcomes sometimes involves rather sophisticated analysis based on the manager's extensive experience in similar situations or on other data. Probabilities assigned in this way are *objective probabilities*. In other circumstances, in which the manager has little experience with a particular decision situation and little or no relevant historical data, the probabilities assigned to the outcomes are derived in a subjective way and are called *subjective probabilities*. Subjective probabilities are based upon hunches, "gut feelings," or personal experiences rather than on scientific data.

### risk

A decision-making condition under which a manager can list all outcomes and assign probabilities to each outcome.

An example of a decision made under risk might be the following: A manager decides to spend \$1,000 on a magazine ad believing there are three possible outcomes for the ad: a 20 percent chance the ad will have only a small effect on sales, a 60 percent chance of a moderate effect, and a 20 percent chance of a very large effect. This decision is made under risk because the manager can list each potential outcome and determine the probability of each outcome occurring.

In contrast to risk, **uncertainty** exists when a decision maker cannot list all possible outcomes and/or cannot assign probabilities to the various outcomes. When faced with uncertainty, a manager would know only the different decision options available and the different possible *states of nature*. The states of nature are the future events or conditions that can influence the final outcome or payoff of a decision but cannot be controlled or affected by the manager. Even though both risk and uncertainty involve less-than-complete information, there is more information under risk than under uncertainty.

### uncertainty

A decision-making condition under which a manager cannot list all possible outcomes and/or cannot assign probabilities to the various outcomes.

An example of a decision made under uncertainty would be, for a manager of a pharmaceutical company, the decision of whether to spend \$3 million on the research and development of a new medication for high blood pressure. The payoff from the research and development spending will depend on whether the president's new health plan imposes price regulations on new drugs. The two states of nature facing the manager in this problem are (1) government does impose price regulations or (2) government does *not* impose price regulations. While the manager knows the payoff that will occur under either state of nature, the manager has no idea of the probability that price regulations will be imposed on drug companies. Under such conditions, a decision is made under uncertainty.

This important distinction between conditions of uncertainty and conditions of risk will be followed throughout this chapter. The decision rules employed by managers when outcomes are not certain differ under conditions of uncertainty and conditions of risk.

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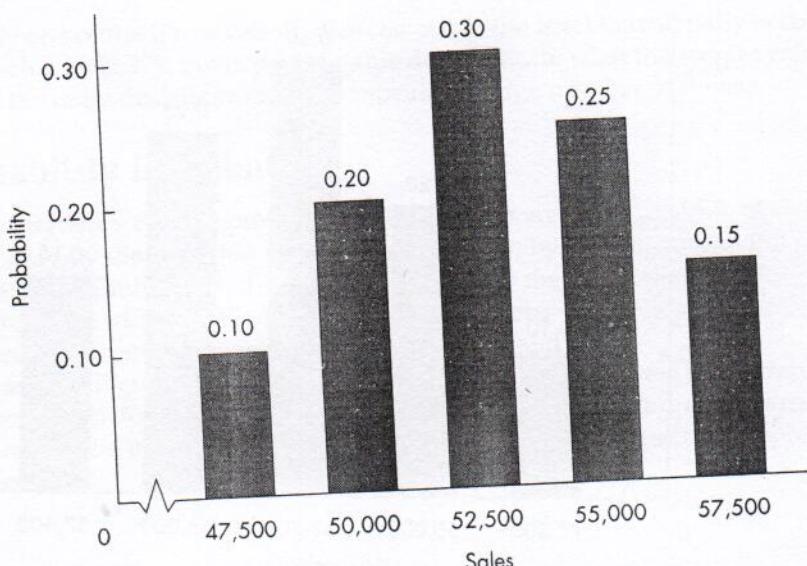


FIGURE 15.1 The Probability Distribution for Sales Following an Advertising Campaign

### Expected Value of a Probability Distribution

The **expected value** of a probability distribution of decision outcomes is the weighted average of the outcomes, with the probabilities of each outcome serving as the respective weights. The expected value of the various outcomes of a probability distribution is

$$E(X) = \text{Expected value of } X = \sum_{i=1}^n p_i X_i$$

where  $X_i$  is the  $i$ th outcome of a decision,  $p_i$  is the probability of the  $i$ th outcome, and  $n$  is the total number of possible outcomes in the probability distribution. Note that the computation of expected value requires the use of fractions or decimal values for the probabilities  $p_i$ , rather than percentages. The expected value of a probability distribution is often referred to as the **mean of the distribution**.

The expected value of sales for the advertising campaign associated with the probability distribution shown in Figure 15.1 is

$$\begin{aligned} E(\text{sales}) &= (0.10)(47,500) + (0.20)(50,000) + (0.30)(52,500) \\ &\quad + (0.25)(55,000) + (0.15)(57,500) \\ &= 4,750 + 10,000 + 15,750 + 13,750 + 8,625 \\ &= 52,875 \end{aligned}$$

While the amount of actual sales that occur as a result of the advertising campaign is a random variable possibly taking values of 47,500, 50,000, 52,500, 55,000, or 57,500 units, the expected level

**expected value**  
The weighted average of the outcomes, with the probabilities of each outcome serving as the respective weights.

**mean of the distribution**  
The expected value of the distribution.

is 52,875 units. If only one of the five levels of sales can occur, the level that actually occurs is not equal to the expected value of 52,875, but expected value does indicate what the *average* value of the outcomes would be if the risky decision were to be repeated a large number of times.

## Dispersion of a Probability Distribution

As you may recall from your statistics classes, probability distributions are generally characterized not only by the expected value (mean) but also by the variance. The **variance** of a probability distribution measures the dispersion of the distribution about its mean. Figure 15.2 shows the probability distributions for the profit outcomes of two different decisions, A and B. Both decisions, as illustrated in Figure 15.2, have identical expected profit levels but different variances. The larger variance associated with making decision B is reflected by a larger dispersion (a wider spread of values around the mean). Because distribution A is more compact (less spread out), A has a smaller variance.

### **variance**

The dispersion of a distribution about its mean.

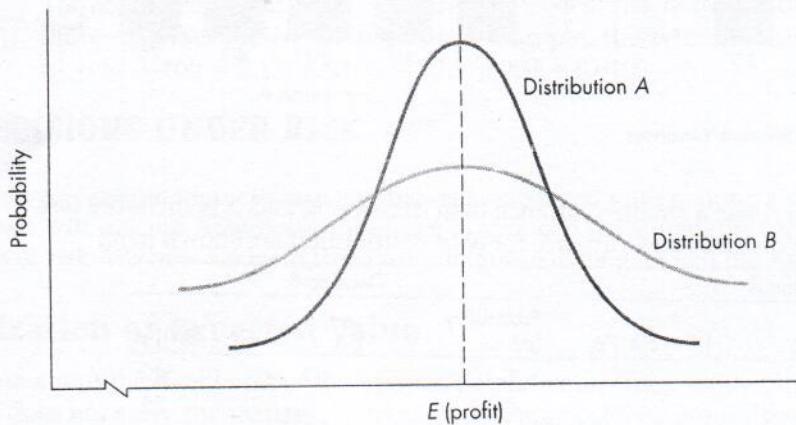


FIGURE 15.2 Two Probability Distributions with Identical Means but Different Variances

The variance of a probability distribution of the outcomes of a given decision is frequently used to indicate the level or degree of risk associated with that decision. If the expected values of two distributions are the same, the distribution with the higher variance is associated with the riskier decision. Thus in Figure 15.2, decision B has more risk than decision A. Furthermore, variance is often used to compare the riskiness of two decisions even though the expected values of the distributions differ.

Mathematically, the variance of a probability distribution of outcomes  $X_i$ , denoted by  $\sigma_x^2$ , is the probability-weighted sum of the squared deviations about the expected value of X:

$$\text{Variance}(X) = \sigma_x^2 = \sum_{i=1}^n p_i[X_i - E(X)]^2$$

As an example, consider the two distributions illustrated in Figure 15.3. As is evident from the graphs and demonstrated in the following table, the two distributions have the same mean, 50. Their

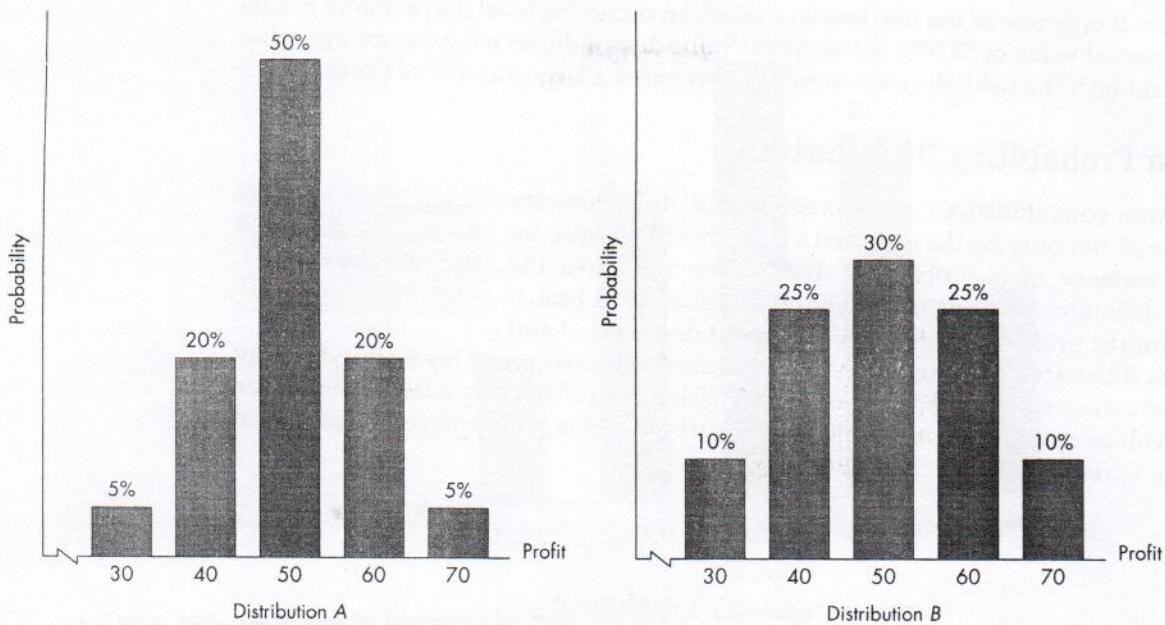


FIGURE 15.3 Probability Distributions with Different Variances

variances differ, however. Decision *A* has a smaller variance than decision *B*, and it is therefore less risky. The calculation of the expected values and variance for each distribution are shown here:

Profit ( $X_i$ )	Decision A			Decision B		
	Probability ( $p_i$ )	$p_i X_i$	$[X_i - E(X)]^2 p_i$	Probability ( $p_i$ )	$p_i X_i$	$[X_i - E(X)]^2 p_i$
30	0.05	1.5	20	0.10	3	40
40	0.20	8	20	0.25	10	25
50	0.50	25	0	0.30	15	0
60	0.20	12	20	0.25	15	25
70	0.05	3.5	20	0.10	7	40
		$E(X) = 50$	$\sigma_A^2 = 80$			$E(X) = 50$
						$\sigma_B^2 = 130$

Because variance is a squared term, it is usually much larger than the mean. To avoid this scaling problem, the standard deviation of the probability distribution is more commonly used to measure dispersion. The **standard deviation** of a probability distribution, denoted by  $\sigma_x$ , is the square root of the variance:

$$\sigma_x = \sqrt{\text{Variance}(X)}$$

The standard deviations of the distributions illustrated in Figure 15.3 and in the preceding table are  $\sigma_A = 8.94$  and  $\sigma_B = 11.40$ . As in the case of the variance of a probability distribution, the higher the standard deviation, the more risky the decision.

**standard deviation**  
The square root of the variance.

Managers can compare the riskiness of various decisions by comparing their standard deviations, as long as the expected values are of similar magnitudes. For example, if decisions *C* and *D* both have standard deviations of 52.5, the two decisions can be viewed as equally risky if their

expected values are close to one another. If, however, the expected values of the distributions differ substantially in magnitude, it can be misleading to examine only the standard deviations. Suppose decision *C* has a mean outcome of \$400 and decision *D* has a mean outcome of \$5,000 but the standard deviations remain 52.5. The dispersion of outcomes for decision *D* is much smaller *relative to its mean value of \$5,000* than is the dispersion of outcomes for decision *C* *relative to its mean value of \$400*.

When the expected values of outcomes differ substantially, managers should measure the riskiness of a decision *relative* to its expected value. One such measure of relative risk is the coefficient of variation for the decision's distribution. The **coefficient of variation**, denoted by  $v$ , is the standard deviation divided by the expected value of the probability distribution of decision outcomes:

$$v = \frac{\text{Standard deviation}}{\text{Expected value}} = \frac{\sigma}{E(X)}$$

### coefficient of variation

The standard deviation divided by the expected value of the probability distribution.



1 The coefficient of variation measures the level of risk *relative* to the mean of the probability distribution. In the preceding example, the two coefficients of variation are  $v_C = 52.5/400 = 0.131$  and  $v_D = 52.5/5,000 = 0.0105$ .

## 15.3 DECISIONS UNDER RISK

Now that we have shown how to measure the risk associated with making a particular managerial decision, we will discuss how these measures of risk can help managers make decisions under conditions of risk. We now set forth three rules to guide managers making risky decisions.

### Maximization of Expected Value

Information about the likelihood of the various possible outcomes, while quite helpful in making decisions, does not solve the manager's decision-making problem. How should a manager choose among various decisions when each decision has a variety of possible outcomes? One rule or solution to this problem, called the **expected value rule**, is to choose the decision with the highest expected value. The expected value rule is easy to apply. Unfortunately, this rule uses information about only one characteristic of the distribution of outcomes, the mean. It fails to incorporate into the decision the riskiness (dispersion) associated with the probability distribution of outcomes. Therefore, the expected value rule is not particularly useful in situations where the level of risk differs very much across decisions—unless the decision maker does not care about the level of risk associated with a decision and is concerned only with expected value. (Such a decision maker is called *risk neutral*, a concept we will discuss later in this chapter.) Also, the expected value rule is only useful to a manager when the decisions have *different* expected values. Of course, if decisions happen to have identical expected values, the expected value rule offers no guidance for choosing between them, and, considering only the mean, the manager would be indifferent to a choice among them. The expected value rule *cannot* be applied when decisions have identical expected values and *should not* be applied when decisions have different levels of risk, except in the circumstance noted earlier: that is, when the decision maker is risk neutral.

### expected value rule

Choosing the decision with the highest expected value.

To illustrate the expected value rule (and other rules to be discussed later), consider the owner and manager of Chicago Rotisserie Chicken, who wants to decide where to open one new restaurant. Figure 15.4 shows the probability distributions of possible weekly profits if the manager decides to locate the new restaurant in either Atlanta (Panel A), Boston (Panel B), or Cleveland (Panel C). The expected values, standard deviations, and coefficients of variation for each distribution are displayed in each panel.

On the basis of past experience, the manager calculates that weekly profit in Atlanta will take one of four values: \$3,000 or \$4,000 per week each with a 30 percent chance of occurring, and \$2,000 or \$5,000 a week each with a 20 percent chance of occurring. The expected weekly profit in Atlanta is \$3,500. If the manager decides to open a restaurant in Boston, the weekly profits may be any of six indicated values ranging from \$1,000 to \$6,000 weekly with the indicated probabilities and an expected value of \$3,750. For Cleveland, the manager assigns a probability of 30 percent to weekly profits of \$1,000 and \$6,000 and a probability of 10 percent to each of the profits \$2,000, \$3,000, \$4,000, and \$5,000, with an expected value of \$3,500 for the distribution. If the manager is not concerned with risk (is risk neutral) and follows the expected value rule, the new restaurant will be opened in Boston, with the highest expected profit of \$3,750. Note that if the manager had been choosing between only the Atlanta and Cleveland locations, the expected value rule could not have been applied because each has an expected value of \$3,500. In such cases some other rule may be used.

## Mean–Variance Analysis

Managers who choose among risky alternatives using the expected value rule are, in effect, ignoring risk (dispersion) and focusing exclusively on the mean outcome. An alternative method of making decisions under risk uses both the mean *and* the variance of the probability distribution, which incorporates information about the level of risk into the decisions. This method of decision making, commonly known as **mean–variance analysis**, employs both the mean and the variance (or standard deviation) to make decisions according to the rules listed below.

Given two risky decisions (designated *A* and *B*), the *mean–variance rules* for decisions under risk are

1. If decision *A* has a higher expected outcome *and* a lower variance than decision *B*, decision *A* should be made.
2. If both decisions *A* and *B* have identical variances (or standard deviations), the decision with the higher expected value should be made.
3. If both decisions *A* and *B* have identical expected values, the decision with the lower variance (standard deviation) should be made.

**mean–variance analysis**  
Method of decision making that employs both the mean and the variance to make decisions.

The mean–variance rules are based on the assumption that a decision maker prefers a higher expected return to a lower, other things equal, and a lower risk to a higher, other things equal. It therefore follows that the *higher* the expected outcome and the *lower* the variance (risk), the more desirable a decision will be. Under rule 1, a manager would always choose a particular decision if it has *both* a greater expected value *and* a lower variance than other decisions being considered. With the same level of risk, the second rule indicates managers should choose the decision with the higher expected value. Under rule 3, if the decisions have identical expected values, the manager chooses the less risky (lower standard deviation) decision.

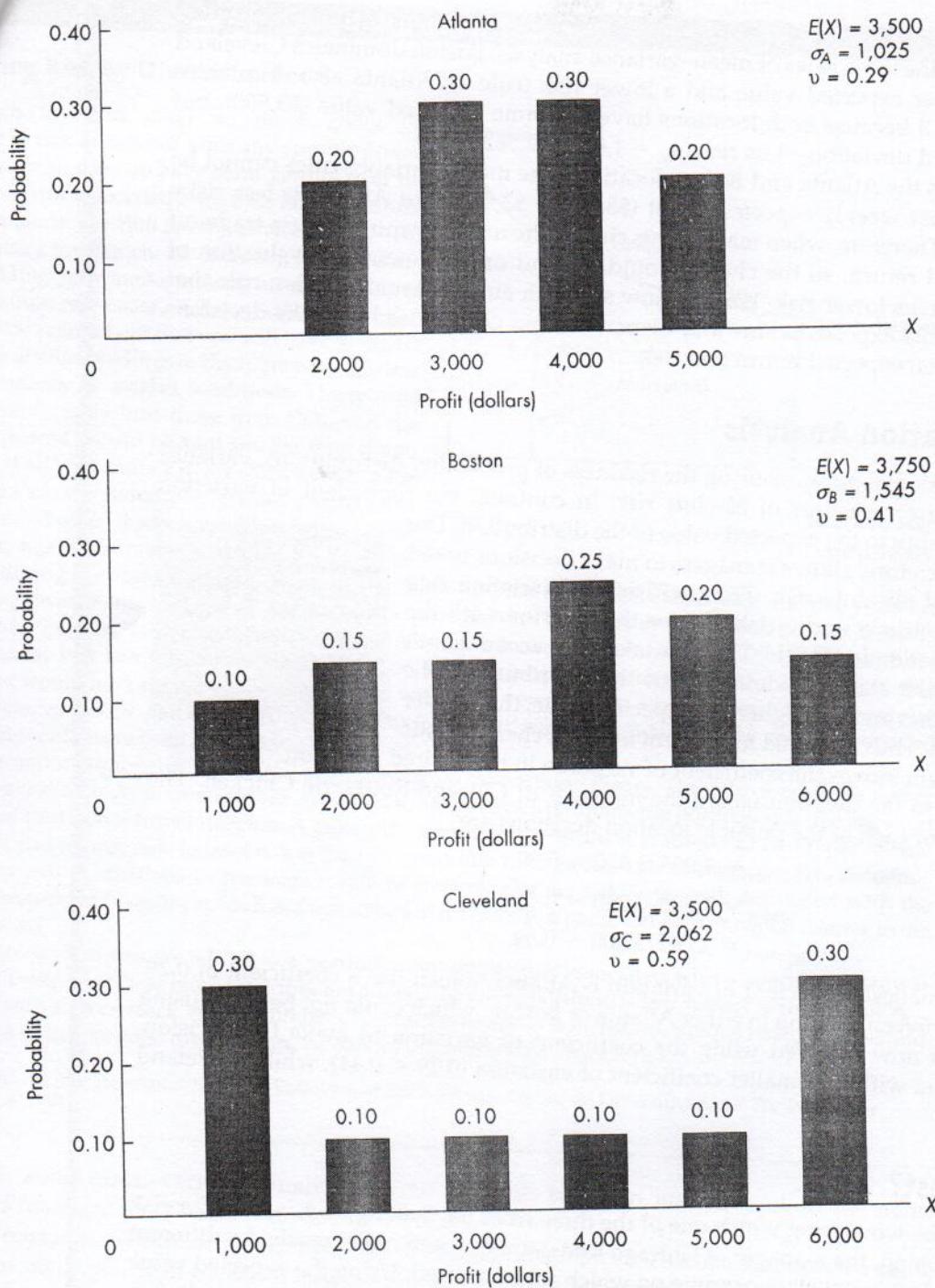


FIGURE 15.4 Probability Distributions for Weekly Profit at Three Restaurant Locations

Returning to the problem of Chicago Rotisserie Chicken, no location dominates both of the other locations in terms of any of the three rules of mean-variance analysis. Boston dominates Cleveland because it has both a higher expected value and a lower risk (rule 1). Atlanta also dominates Cleveland in terms of rule 3 because both locations have the same expected value (\$3,500), but Atlanta has a lower standard deviation—less risk ( $\sigma_A = 1,025 < 2,062 = \sigma_C$ ).

If the manager compares the Atlanta and Boston locations, the mean-variance rules cannot be applied. Boston has a higher weekly expected profit (\$3,750 > \$3,500), but Atlanta is less risky ( $\sigma_A = 1,025 < 1,545 = \sigma_B$ ). Therefore, when making this choice, the manager must make a trade-off between risk and expected return, so the choice would depend on the manager's valuation of higher expected return versus lower risk. We will now set forth an additional decision rule that uses information on both the expected value and dispersion and can be used to make decisions involving trade-offs between expected return and risk.

## Coefficient of Variation Analysis

As we noted in the discussion about measuring the riskiness of probability distributions, variance and standard deviation are measures of *absolute risk*. In contrast, the coefficient of variation [ $\sigma/E(X)$ ] measures risk *relative* to the expected value of the distribution. The coefficient of variation, therefore, allows managers to make decisions based on relative risk instead of absolute risk. The **coefficient of variation rule** states: "When making decisions under risk, choose the decision with the smallest coefficient of variation [ $\sigma/E(X)$ ]." This rule takes into account both the expected value and the standard deviation of the distribution. The lower the standard deviation and the higher the expected value, the smaller the coefficient of variation. Thus a desired movement in either characteristic of a probability distribution moves the coefficient of variation in the desired direction.

**coefficient of variation rule**  
Decision-making rule that the decision to be chosen is the one with the smallest coefficient of variation.

We return once more to the decision facing the manager of Chicago Rotisserie Chicken. The coefficients of variation for each of the possible location decisions are

$$v_{\text{Atlanta}} = 1,025/3,500 = 0.29$$

$$v_{\text{Boston}} = 1,545/3,750 = 0.41$$

$$v_{\text{Cleveland}} = 2,062/3,500 = 0.59$$

The location with the smallest coefficient of variation is Atlanta, which has a coefficient of 0.29. Notice that the choice between locating in either Atlanta or Boston, which could not be made using mean-variance rules, is now resolved using the coefficient of variation to make the decision. Atlanta wins over Boston with the smaller coefficient of variation ( $0.29 < 0.41$ ), while Cleveland comes in last.

## Which Rule Is Best?

At this point, you may be wondering which one of the three rules for making decisions under risk is the "correct one." After all, the manager of Chicago Rotisserie Chicken either reached a different decision or reached no decision at all depending on which rule was used. Using the expected value rule, Boston was the choice. Using the coefficient of variation rule, Atlanta was chosen. According to mean-variance analysis, Cleveland was out, but the decision between Atlanta and Boston could not

**Illustration 15.1****Reducing Risk by Diversification**

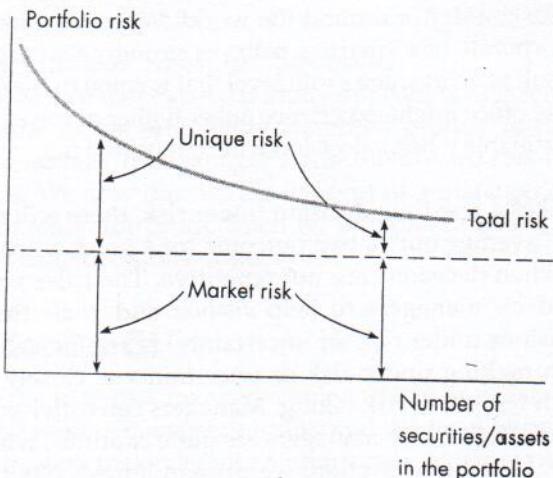
though investors can't do much about the amount of risk associated with any specific project or investment, they do have some control over the amount of risk associated with their entire portfolio of investments. *The Wall Street Journal* advised: "The best strategy, investment advisors say, is to diversify by spreading your money among a wide variety of stocks, bonds, real estate, cash, and other holdings."

The WSJ pointed out that you will have to expect the value of your holdings to fluctuate with changes in the economy or market conditions. The returns should comfortably beat those from CDs, and the ups and downs should be a lot smaller than if you simply put all your money in the stock market. One investment adviser stated, "Diversified portfolios of stocks and bonds had much less risk while providing nearly as much return as an all-stock portfolio during the past 15, 20, and 25 years." During the period since 1968, stocks soared in five years but were losing investments in six years. Investors who put a third of their money in stocks, a third in Treasury bonds, and a third in "cash equivalent" investments would have lost money in only four years, with the largest annual loss being less than 5 percent. The annual compound return over the 25 years in that investment would have been 9 percent, compared with 10.56 percent in an all-stock portfolio, 8.26 percent in all bonds, and 9.89 percent in 60 percent stock and 40 percent bonds. But the more diversified investment would have been less risky.

The theoretical arguments in the WSJ article are based on portfolio theory. The core of portfolio theory is deceptively simple: As more securities are added to an investor's portfolio, the portfolio risk (the standard deviation of portfolio returns) declines. A particular security or investment is subject to two types of risk: market risk and unique risk. Market risk is the risk faced due to economywide changes, such as economic fluctuations and fluctuations in the market rate of interest. Unique risk is the risk associated with the particular security or investment, such as fluctuations in the sales of a particular firm or region relative to the entire economy.

As different securities are added to a portfolio, the unique risk associated with a specific security is diversified away. That is, as more securities are added, the entire portfolio is less subject to the unique risk associated with a given stock. As the number of securities or assets is increased, unique risk decreases and the total risk of the portfolio (the standard deviation) approaches the market risk.

**Source:** Based on Tom Herman, "The First Rollovers of Spring Bring Advice on Diversification," *The Wall Street Journal*, Apr. 8, 1993.



be resolved using mean-variance analysis. If the decision rules do not all lead to the same conclusion, a manager must decide which rule to follow.

When a decision is to be made repeatedly, with identical probabilities each time, the expected value rule provides managers with the most reliable rule for maximizing (expected) profit. The average return of a given risky course of action repeated many times will approach the expected value of that action. Therefore, the average return of the course of action with the highest expected value will tend

to be higher than the average return of any course of action with a lower expected value, when carried out a large number of times. Situations involving repeated decisions can arise, for example, when a manager must make the same risky decision once a month or even once every week. Or a manager at corporate headquarters may make a decision that directs activities of dozens, maybe even hundreds, of corporate offices in the country or around the world. When the risky decision is repeated many times, the manager at corporate headquarters believes strongly that each of the alternative decision choices will probably result in an average profit level that is equal to the expected value of profit, even though any one corporate office might experience either higher or lower returns. In practice, then, the expected value rule is justifiable when a decision will be repeated many times under identical circumstances.

When a manager makes a one-time decision under risk, there will not be any follow-up repetitions of the decision to "average out" a bad outcome (or a good outcome). Unfortunately, there is no best rule to follow when decisions are not repetitive. The rules we present for risky decision making should be used by managers to help *analyze* and *guide* the decision-making process. Ultimately, making decisions under risk (or uncertainty) is as much an art as it is a science.

The "art" of decision making under risk or uncertainty is closely associated with a decision maker's preferences with respect to risk taking. Managers can differ greatly in their willingness to take on risk in decision making. Some managers are quite cautious, while others may actually seek out high-risk situations. In the next section, we present a theory, not a rule, of decision making under risk that formally accounts for a manager's attitude toward risk. This theory, usually referred to as *expected utility theory*, postulates that managers make risky decisions with the objective of

 [2] [3] maximizing the expected *utility* of profit. The theory can, in some situations, provide a more powerful tool for making risky decisions than the rules presented in this section.

## 15.4 EXPECTED UTILITY: A THEORY OF DECISION MAKING UNDER RISK

As we just mentioned, managers differ in their willingness to undertake risky decisions. Some managers avoid risk as much as possible, while other managers actually prefer more risk to less risk in decision making. To allow for different attitudes toward risk taking in decision making, modern decision theory treats managers as deriving utility or satisfaction from the profits earned by their firms. Just as consumers derived utility from the consumption of goods in Chapter 5, in **expected utility theory**, managers are assumed to derive utility from earning profits. Expected utility theory postulates that managers make risky decisions in a way that maximizes the expected utility of the profit outcomes. While expected utility theory does provide a tool for decisions under risk, the primary purpose of the theory, and the reason for presenting this theory here, is to explain why managers make the decisions they do make when risk is involved. We want to stress that expected utility theory is an economic model of how managers *actually* make decisions under risk, rather than a rule dictating how managers *should* make decisions under risk.

Suppose a manager is faced with a decision to undertake a risky project or, more generally, must make a decision to take an action that may generate a range of possible profit outcomes,  $\pi_1, \pi_2, \dots$ ,

### expected utility theory

A theory of decision making under risk that accounts for a manager's attitude toward risk.

$\pi_n$ , that the manager believes will occur with probabilities  $p_1, p_2, \dots, p_n$ , respectively. The **expected utility** of this risky decision is the sum of the probability-weighted utilities of each possible profit outcome:

$$E[U(\pi)] = p_1U(\pi_1) + p_2U(\pi_2) + \dots + p_nU(\pi_n)$$

**expected utility**  
The sum of the probability-weighted utilities of each possible profit outcome.

where  $U(\pi)$  is a utility function for profit that measures the utility associated with a particular level of profit. Notice that expected *utility* of profit is different from the concept of expected *profit*, which is the sum of the probability-weighted profits. To understand expected utility theory, you must understand how the manager's attitude toward risk is reflected in the manager's utility function for profit. We now discuss the concept of a manager's utility of profit and show how to derive a utility function for profit. Then we demonstrate how managers could employ expected utility of profit to make decisions under risk.

## A Manager's Utility Function for Profit

Since expected utility theory is based on the idea that managers enjoy utility or satisfaction from earning profit, the nature of the relation between a manager's utility and the level of profit earned plays a crucial role in explaining how managers make decisions under risk. As we now show, the manager's attitude toward risk is determined by the manager's *marginal utility of profit*.

It would be extremely unusual for a manager *not* to experience a higher level of total utility as profit increases. Thus the relation between an index of utility and the level of profit earned by a firm is assumed to be an upward-sloping curve. The amount by which total utility increases when the firm earns an additional dollar of profit is the **marginal utility of profit**:

$$MU_{\text{profit}} = \Delta U(\pi)/\Delta \pi$$

**marginal utility of profit**  
The amount by which total utility increases with an additional dollar of profit earned by a firm.

where  $U(\pi)$  is the manager's utility function for profit. The utility function for profit gives an index value to measure the level of utility experienced when a given amount of profit is earned. Suppose, for example, the marginal utility of profit is 8. This means a \$1 increase in profit earned by the firm causes the utility index of the manager to increase by eight units. Studies of attitudes toward risk have found most business decision makers experience *diminishing marginal utility of profit*. Even though additional dollars of profit increase the level of total satisfaction, the additional utility from extra dollars of profit typically falls for most managers.

**risk averse**  
Term describing a decision maker who makes the less risky of two decisions that have the same expected value

The shape of the utility curve for profit plays a pivotal role in expected utility theory because the shape of  $U(\pi)$  determines the manager's attitude toward risk, which determines which choices a manager makes. Attitudes toward risk may be categorized as *risk averse*, *risk neutral*, or *risk loving*. People are said to be **risk averse** if, facing two risky decisions with equal expected profits, they choose the less risky decision. In contrast, someone choosing the more risky decision, when the expected profits are identical, is said to be **risk loving**. The third type of attitude toward risk arises for someone who is indifferent between risky situations when the expected profits are identical. In this last case, a manager ignores risk in decision making and is said to be **risk neutral**.

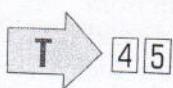
**risk loving**  
Term describing a decision maker who makes the riskier of two decisions that have the same expected value.

**risk neutral**  
Term describing a decision maker who ignores risk in decision making and considers only expected values of decisions.

Figure 15.5 shows the shapes of the utility functions associated with the three types of risk preferences. Panel A illustrates a utility function for a risk-averse manager. The utility function for profit is upward-sloping, but its slope diminishes as profit rises, which corresponds to the case of diminishing marginal utility. When profit increases by \$50,000 from point A to point B, the manager experiences an increase in utility of 10 units. When profit falls by \$50,000 from point A to point C, utility falls by 15 units. A \$50,000 loss of profit creates a larger reduction in utility than a \$50,000 gain would add to utility. Consequently, risk-averse managers are more sensitive to a dollar of lost profit than to a dollar of gained profit and will place an emphasis in decision making on avoiding the risk of loss.

In Panel B, the marginal utility of profit is constant ( $\Delta U / \Delta \pi = 15/50 = 0.3$ ), and the loss of \$50,000 reduces utility by the same amount that a gain of \$50,000 increases it. In this case, a manager places the same emphasis on avoiding losses as on seeking gains. Managers are risk neutral when their utility functions for profit are linear or, equivalently, when the marginal utility of profit is constant.

Panel C shows a utility function for a manager who makes risky decisions in a risk-loving way. The extra utility from a \$50,000 increase in profit (20 units) is greater than the loss in utility suffered when profit falls by \$50,000 (10 units). Consequently, a risk-loving decision maker places a greater weight on the potential for gain than on the potential for loss. We have now developed the following relation.



**Relation** A manager's attitude toward risky decisions can be related to his or her marginal utility of profit. Someone who experiences diminishing (increasing) marginal utility for profit will be a risk-averse (risk-loving) decision maker. Someone whose marginal utility of profit is constant is risk neutral.

## Deriving a Utility Function for Profit

As discussed earlier, when managers make decisions to maximize expected utility under risk, it is the utility function for profit that determines which decision a manager chooses. We now show the steps a manager can follow to derive his or her own utility function for profit,  $U(\pi)$ . Recall that the utility function does not directly measure utility but does provide a number, or index value, and that it is the magnitude of this index that reflects the desirability of a particular profit outcome.

The process of deriving a utility function for profit is conceptually straightforward. It does, however, involve a substantial amount of subjective evaluation. To illustrate the procedure, we return to the decision problem facing the manager of Chicago Rotisserie Chicken (CRC). Recall that CRC must decide where to locate the next restaurant. The profit outcomes for the three locations range from \$1,000 to \$6,000 per week. Before the expected utilities of each location can be calculated, the manager must derive her utility function for profits covering the range \$1,000 to \$6,000.

The manager of CRC begins the process of deriving  $U(\pi)$  by assigning minimum and maximum values that the index will be allowed to take. For the lower bound on the index, suppose the manager assigns a utility index value of 0—although any number, positive or negative, will do—to the lowest profit outcome of \$1,000. For the upper bound, suppose a utility index value of 1 is assigned—any value greater than the value of the lower bound will do—to the highest profit outcome of \$6,000. Again, we emphasize, choosing 0 and 1 for the upper and lower bounds is completely arbitrary, just as long as the upper bound is greater algebraically than the lower bound. For example, lower and upper bounds of -12 and 50 would also work just fine. Two points on the manager's utility function for profit are

$$U(\$1,000) = 0 \quad \text{and} \quad U(\$6,000) = 1$$

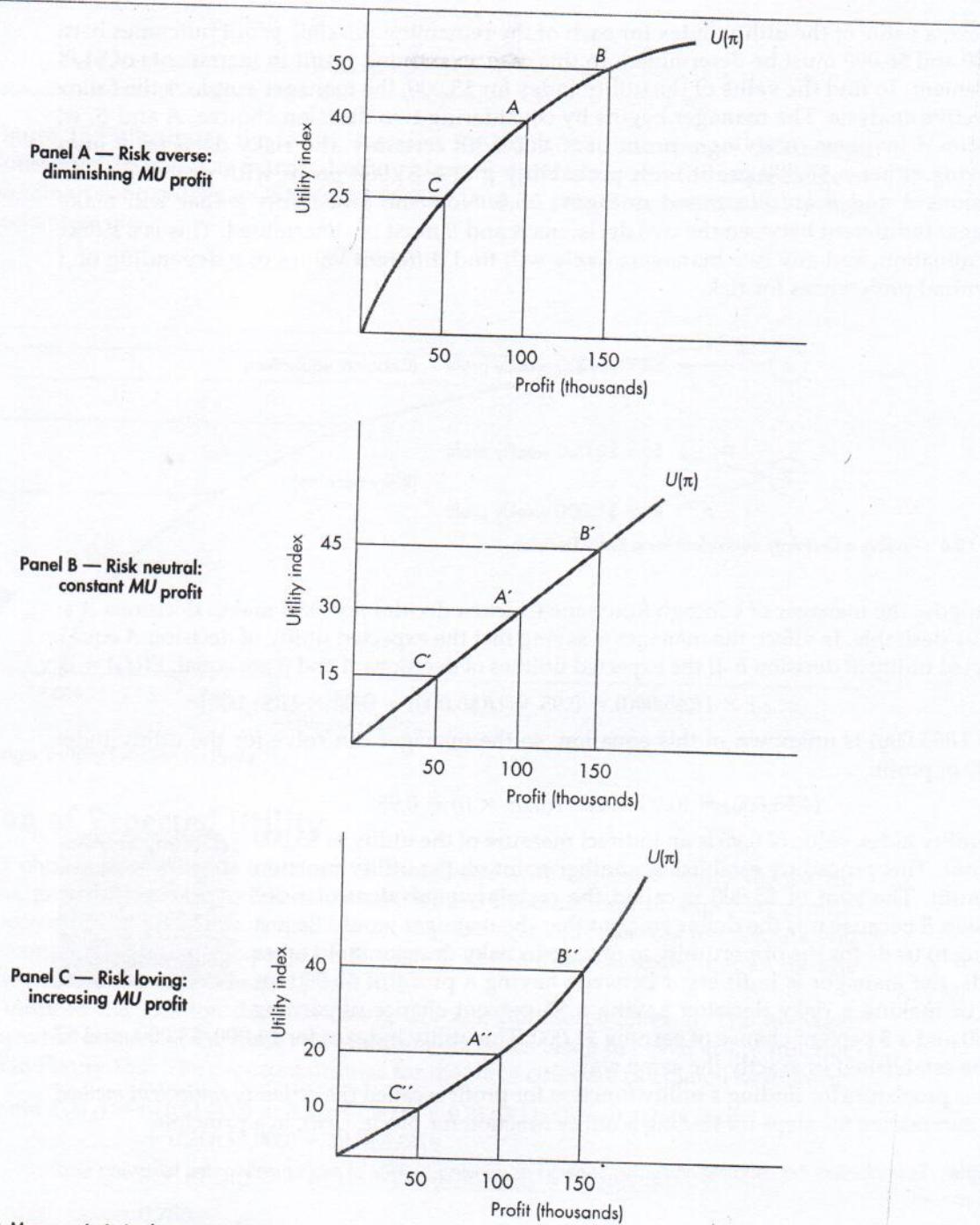


FIGURE 15.5 A Manager's Attitude toward Risk

Next, a value of the utility index for each of the remaining possible profit outcomes between \$1,000 and \$6,000 must be determined. In this case, examining profit in increments of \$1,000 is convenient. To find the value of the utility index for \$5,000, the manager employs the following subjective analysis: The manager begins by considering two decision choices, A and B, where decision A involves receiving a profit of \$5,000 with certainty and risky decision B involves receiving either a \$6,000 profit with probability  $p$  or a \$1,000 profit with probability  $1 - p$ . Decisions A and B are illustrated in Figure 15.6. Now the probability  $p$  that will make the manager indifferent between the two decisions A and B must be determined. This is a subjective determination, and any two managers likely will find different values of  $p$  depending on their individual preferences for risk.

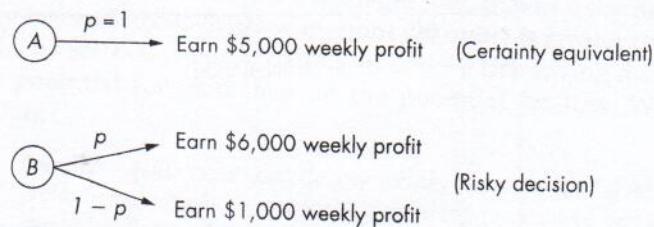


FIGURE 15.6 Finding a Certainty Equivalent for a Risky Decision

Suppose the manager of Chicago Rotisserie Chicken decides  $p = 0.95$  makes decisions A and B equally desirable. In effect, the manager is saying that the expected utility of decision A equals the expected utility of decision B. If the expected utilities of decisions A and B are equal,  $E(U_A) = E(U_B)$

$$1 \times U(\$5,000) = 0.95 \times U(\$6,000) + 0.05 \times U(\$1,000)$$

Only  $U(\$5,000)$  is unknown in this equation, so the manager can solve for the utility index of \$5,000 of profit:

$$U(\$5,000) = (0.95 \times 1) + (0.05 \times 0) = 0.95$$

The utility index value of 0.95 is an indirect measure of the utility of \$5,000 of profit. This procedure establishes another point on the utility function for profit. The sum of \$5,000 is called the **certainty equivalent** of risky decision B because it is the dollar amount that the manager would be just willing to trade for the opportunity to engage in risky decision B. In other words, the manager is indifferent between having a profit of \$5,000 for sure or making a risky decision having a 95 percent chance of earning \$6,000 and a 5 percent chance of earning \$1,000. The utility indexes for \$4,000, \$3,000, and \$2,000 can be established in exactly the same way.

**certainty equivalent**  
The dollar amount the manager would be just willing to trade for the opportunity to engage in a risky decision.

This procedure for finding a utility function for profit is called the *certainty equivalent method*. We now summarize the steps for finding a utility function for profit,  $U(\pi)$ , in a principle.

**Principle** To implement the certainty equivalent method of deriving a utility of profit function, the following steps are employed:

1. Set the utility index equal to 1 for the highest possible profit ( $\pi_H$ ) and 0 for the lowest possible profit ( $\pi_L$ ).
2. Define a risky decision to have probability  $p_0$  of profit outcome  $\pi_H$  and probability  $(1 - p_0)$  of profit outcome  $\pi_L$ . For each possible profit outcome  $\pi_0$  ( $\pi_H < \pi_0 < \pi_L$ ), the manager determines subjectively the probability  $p_0$  that gives that risky decision the same expected utility as receiving  $\pi_0$  with certainty:

$$p_0 U(\pi_H) + (1 - p_0) U(\pi_L) = U(\pi_0)$$

The certain sum  $\pi_0$  is called the certainty equivalent of the risky decision. Let the subjective probability  $p_0$  serve as the utility index for measuring the level of satisfaction the manager enjoys when earning a profit of  $\pi_0$ .

**Figure 15.7** illustrates the utility function for profit for the manager of Chicago Rotisserie Chicken. The marginal utility of profit diminishes over the entire range of possible profit outcomes (\$1,000 to \$6,000), and so this manager is a risk-averse decision maker.

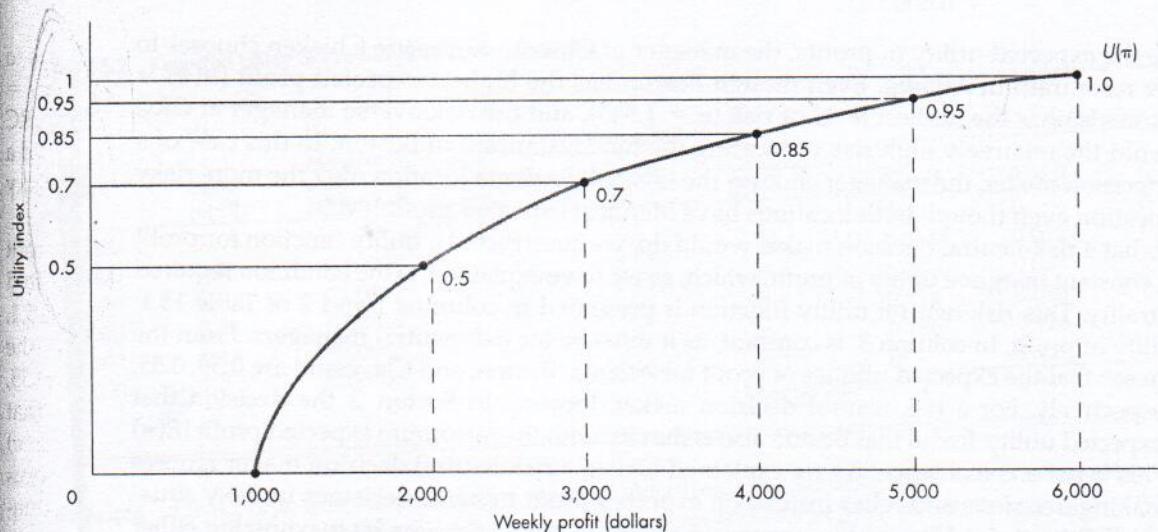


FIGURE 15.7 A Manager's Utility Function for Profit

## Maximization of Expected Utility

When managers choose among risky decisions in accordance with expected utility theory, the decision with the greatest expected utility is chosen. Unlike maximization of expected profits, maximizing expected utility takes into consideration the manager's preferences for risk. As you will see in this example, maximizing expected utility can lead to a different decision than the one reached using the maximization of expected profit rule.

Return once more to the location decision facing Chicago Rotisserie Chicken. The manager calculates the expected utilities of the three risky location decisions using her own utility function for profit shown in Figure 15.7. The expected utilities for the three cities are calculated as follows:

$$\begin{aligned} \text{Atlanta } E(U_A) &= 0U(\$1,000) + 0.2U(\$2,000) + 0.3U(\$3,000) + 0.3U(\$4,000) \\ &\quad + 0.2U(\$5,000) + 0U(\$6,000) \\ &= 0 + (0.2)(0.5) + (0.3)(0.7) + (0.3)(0.85) + (0.2)(0.95) + 0 \\ &= 0.755 \end{aligned}$$

$$\begin{aligned} \text{Boston } E(U_B) &= 0.1U(\$1,000) + 0.15U(\$2,000) + 0.15U(\$3,000) \\ &\quad + 0.25U(\$4,000) + 0.2U(\$5,000) + 0.15U(\$6,000) \end{aligned}$$

$$\begin{aligned}
 &= (0.1)(0) + (0.15)(0.50) + (0.15)(0.7) + (0.25)(0.85) \\
 &\quad + (0.2)(0.95) + (0.15)(1) \\
 &= 0.733
 \end{aligned}$$

$$\begin{aligned}
 \text{Cleveland } E(U_C) &= 0.3U(\$1,000) + 0.1U(\$2,000) + 0.1U(\$3,000) + 0.1U(\$4,000) \\
 &\quad + 0.1U(\$5,000) + 0.3U(\$6,000) \\
 &= (0.3)(0) + (0.1)(0.5) + (0.1)(0.7) + (0.1)(0.85) + (0.1)(0.95) \\
 &\quad + (0.3)(1.0) \\
 &= 0.600
 \end{aligned}$$

To maximize the expected utility of profits, the manager of Chicago Rotisserie Chicken chooses to open its new restaurant in Atlanta. Even though Boston has the highest expected profit [ $E(\pi) = \$3,750$ ], Boston also has the highest level of risk ( $\sigma = 1,545$ ), and the risk-averse manager at CRC prefers to avoid the relatively high risk of locating the new restaurant in Boston. In this case of a risk-averse decision maker, the manager chooses the less risky Atlanta location over the more risky Cleveland location even though both locations have identical expected profit levels.

To show what a risk-neutral decision maker would do, we constructed a utility function for profit that exhibits constant marginal utility of profit, which, as we have explained, is the condition required for risk neutrality. This risk-neutral utility function is presented in columns 1 and 2 of Table 15.1. Marginal utility of profit, in column 3, is constant, as it must be for risk-neutral managers. From the table you can see that the expected utilities of profit for Atlanta, Boston, and Cleveland are 0.50, 0.55, and 0.50, respectively. For a risk-neutral decision maker, locating in Boston is the decision that maximizes expected utility. Recall that Boston also is the city with the maximum expected profit [ $E(\pi) = \$3,750$ ]. This is not a coincidence. As we explained earlier, a risk-neutral decision maker ignores risk when making decisions and relies instead on expected profit to make decisions in risky situations. Under conditions of risk neutrality, a manager makes the same decision by maximizing either the expected value of profit,  $E(\pi)$ , or the expected utility of profit,  $E[U(\pi)]$ .<sup>2</sup>

TABLE 15.1 Expected Utility of Profit: A Risk-Neutral Manager

(1) Profit ( $\pi$ )	(2) Utility [ $U(\pi)$ ]	(3) Marginal utility [ $\Delta U(\pi)/\Delta\pi$ ]	(4)			(5) Probabilities	(6) Atlanta ( $P_A$ )	(7) $P_A \times U$	(8) $P_B \times U$	(9) $P_C \times U$
			(4)	(5)	(6)					
			Boston ( $P_B$ )	Cleveland ( $P_C$ )						
\$1,000	0	—	0	0.1	0.3		0	0	0	0
\$2,000	0.2	0.0002	0.2	0.15	0.1		0.04	0.03	0.02	
\$3,000	0.4	0.0002	0.3	0.15	0.1		0.12	0.06	0.04	
\$4,000	0.6	0.0002	0.3	0.25	0.1		0.18	0.15	0.06	
\$5,000	0.8	0.0002	0.2	0.2	0.1		0.16	0.16	0.08	
\$6,000	1.0	0.0002	0	0.15	0.3		0	0.15	0.3	
Expected utility = 0.50								0.55	0.50	

<sup>2</sup>The appendix to this chapter demonstrates the equivalence for risk-neutral decision makers of maximizing expected profit and maximizing expected utility of profit.

Finally, consider how a manager who is risk loving decides on a location for CRC's new restaurant. In Table 15.2, columns 1 and 2 show a utility function for profit for which marginal utility of profit is increasing. Column 3 shows the marginal utility of profit, which, as it must for a risk-loving manager, increases as profit increases. The expected utilities of profit outcomes for Atlanta, Boston, and Cleveland are 0.32, 0.41, and 0.43, respectively. In the case of a risk-loving decision maker, Cleveland is the decision that maximizes expected utility. If Atlanta and Cleveland were the only two sites being considered, then the risk-loving manager would choose Cleveland over Atlanta, a decision that is consistent with the definition of risk loving. We now summarize our discussion in the following principle.



7 8

TABLE 15.2 Expected Utility of Profit: A Risk-Loving Manager

(1) Profit ( $\pi$ )	(2) Utility [ $U(\pi)$ ]	(3) Marginal utility [ $\Delta U(\pi)/\Delta \pi$ ]	(4) Atlanta ( $P_A$ )	(5) Boston ( $P_B$ )	(6) Cleveland ( $P_C$ )	(7) (8) (9) Probability-weighted utility		
						(7) $P_A \times U$	(8) $P_B \times U$	(9) $P_C \times U$
\$1,000	0	—	0	0.1	0.3	0	0	0
\$2,000	0.08	0.00008	0.2	0.15	0.1	0.016	0.012	0.008
\$3,000	0.2	0.00012	0.3	0.15	0.1	0.06	0.03	0.02
\$4,000	0.38	0.00018	0.3	0.25	0.1	0.114	0.095	0.038
\$5,000	0.63	0.00025	0.2	0.2	0.1	0.126	0.126	0.036
\$6,000	1.0	0.00037	0	0.15	0.3	0	0.15	0.3
Expected utility = 0.32						0.41	0.43	

**Principle** If a manager behaves according to expected utility theory, decisions are made to maximize the manager's expected utility of profits. Decisions made by maximizing expected utility of profit reflect the manager's risk-taking attitude and generally differ from decisions reached by decision rules that do not consider risk. In the case of a risk-neutral manager, the decisions are identical under either maximization of expected utility or maximization of expected profit.

## 15.5 DECISIONS UNDER UNCERTAINTY

Practically all economic theories about behavior in the absence of complete information deal with risk rather than uncertainty. Furthermore, decision science has little guidance to offer managers making decisions when they have no idea about the likelihood of various states of nature occurring. This should not be too surprising, given the nebulous nature of uncertainty. We will, however, present four rather simple decision rules that can help managers make decisions under uncertainty.

### The Maximax Criterion

For managers who tend to have an optimistic outlook on life, the **maximax rule** provides a guide for making decisions when uncertainty prevails. Under the maximax rule, a manager identifies for each possible decision the best outcome that could occur and then chooses the decision that would give the maximum payoff of all the best outcomes. Under this rule a manager ignores all possible outcomes except the best outcome from each decision.

**maximax rule**  
Decision-making that calls for identifying the best outcome of all possible decisions and choosing the decision with the maximum payoff of all the best outcomes.

To illustrate the application of this rule, suppose the management at Dura Plastic is considering changing the size (capacity) of its manufacturing plant. Management has narrowed the decision to three choices. The plant's capacity will be (1) expanded by 20 percent, (2) maintained at the current capacity, or (3) reduced by 20 percent. The outcome of this decision depends crucially on how the economy performs during the upcoming year. Thus the performance of the economy is the "state of nature" in this decision problem. Management envisions three possible states of nature occurring: (1) The economy enters a period of recovery, (2) economic stagnation sets in, or (3) the economy falls into a recession.

For each possible decision and state of nature, the managers determine the profit outcome, or payoff, shown in the *payoff matrix* in Table 15.3. A **payoff matrix** is a table with rows corresponding to the various decisions and columns corresponding to the various states of nature. Each cell in the payoff matrix in Table 15.3 gives the outcome (payoff) for each decision when a particular state of nature occurs. For example, if management chooses to expand the manufacturing plant by 20 percent and the economy enters a period of recovery, Dura Plastic is projected to earn profits of \$5 million. Alternatively, if Dura Plastic expands plant capacity but the economy falls into a recession, it is projected that the company will lose \$3 million. The managers do not know which state of nature will actually occur, or the probabilities of occurrence, so the decision to alter plant capacity is made under conditions of uncertainty. To apply the maximax rule to this decision, management first identifies the best possible outcome for each of the three decisions. The best payoffs are

\$5 million for expand plant size by 20 percent.

\$3 million for maintain plant size.

\$2 million for reduce plant size by 20 percent.

### payoff matrix

A table with rows corresponding to various decisions and columns corresponding to various states of nature, with each cell giving the outcome or payoff associated with that decision and state of nature.

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TABLE 15.3 The Payoff Matrix for Dura Plastic, Inc.

Decisions	States of nature		
	Recovery	Stagnation	Recession
Expand plant capacity by 20%	\$5 million	-\$1 million	-\$3.0 million
Maintain same plant capacity	3 million	2 million	0.5 million
Reduce plant capacity by 20%	2 million	1 million	0.75 million

Each best payoff occurs if the economy recovers. Under the maximax rule, management would decide to expand its plant.

While the maximax rule is simple to apply, it fails to consider "bad" outcomes in the decision-making process. The fact that two out of three states of nature result in losses when management decides to expand plant capacity, and neither of the other decisions would result in a loss, is overlooked when using the maximax criteria. Only managers with optimistic natures are likely to find the maximax rule to be a useful decision-making tool.

**Illustration 15.2****Floating Power Plants Lower Risks and Energize Developing Nations**

Two crucial industries in developing countries are agriculture and manufacturing. A third-world nation cannot emerge from poverty without achieving a significant ability to feed itself and to manufacture both durable goods for consumption and capital goods for production. Neither of these two crucial industries can develop without energy. Domestically generated electricity can provide a versatile source of energy capable of meeting many of the most fundamental energy demands of a developing country.

A serious roadblock to construction of electric power plants in developing countries has been the risk of default on the financing required to purchase power plants. With prices beginning in the hundreds of millions of dollars, investors are understandably reluctant to lend these enormous amounts when repossession of the asset is, for all practical purposes, impossible. Donald Smith, president of Smith Cogeneration, found a solution to the problem of default risk: Build floating power plants on huge barges that can be relocated in the event of a default.

*The Wall Street Journal* reported that Smith's idea of building power plants on barges spawned a niche industry that "could become a significant portion of the world's [electricity] generating capacity." Nations such as the Dominican Republic, Ghana, India, and Haiti have signed agreements with producers of floating power plants that would not have been financed without the risk reduction created by the mobility of a floating platform. Indeed, the *WSJ* estimated that the floating nature of the power plant not only makes financing possible but also probably "lower(s) the financing costs by two or three percentage points"—no small change on a half-a-billion-dollar loan.

This illustration highlights the importance of risk in decision making. If financial institutions were managed by risk-loving managers, land-based power plants would likely be common in developing nations. Apparently, developing nations can expect to generate most of their electricity on barges anchored in their harbors—evidence that large financial lenders are indeed risk-averse.

Source: Inspired by William M. Bulkley, "Building Power Plants That Can Float," *The Wall Street Journal*, May 22, 1996.

**The Maximin Criterion**

For managers with a pessimistic outlook on business decisions, the *maximin rule* may be more suitable than the maximax rule. Under the **maximin rule**, the manager identifies the worst outcome for each decision and makes the decision associated with the maximum worst payoff. For Dura Plastic, the worst outcomes for each decision from Table 15.3 are

**maximin rule**  
Decision-making guide that calls for identifying the worst outcome for each decision and choosing the decision with the maximum worst payoff.

-\$3 million for expand plant size by 20 percent.

\$0.5 million for maintain plant size.

\$0.75 million for reduce plant size by 20 percent.

Using the maximin criterion, Dura Plastic would choose to reduce plant capacity by 20 percent. The maximin rule is also simple to follow, but it fails to consider any of the "good" outcomes.

**The Minimax Regret Criterion**

Managers concerned about their decisions not turning out to be the best *once the state of nature is known* (i.e., after the uncertainty is resolved) may make their decisions by minimizing the potential regret that

To illustrate the application of this rule, suppose the management at Dura Plastic is considering changing the size (capacity) of its manufacturing plant. Management has narrowed the decision to three choices. The plant's capacity will be (1) expanded by 20 percent, (2) maintained at the current capacity, or (3) reduced by 20 percent. The outcome of this decision depends crucially on how the economy performs during the upcoming year. Thus the performance of the economy is the "state of nature" in this decision problem. Management envisions three possible states of nature occurring: (1) The economy enters a period of recovery, (2) economic stagnation sets in, or (3) the economy falls into a recession.

For each possible decision and state of nature, the managers determine the profit outcome, or payoff, shown in the *payoff matrix* in Table 15.3. A **payoff matrix** is a table with rows corresponding to the various decisions and columns corresponding to the various states of nature. Each cell in the payoff matrix in Table 15.3 gives the outcome (payoff) for each decision when a particular state of nature occurs. For example, if management chooses to expand the manufacturing plant by 20 percent and the economy enters a period of recovery, Dura Plastic is projected to earn profits of \$5 million. Alternatively, if Dura Plastic expands plant capacity but the economy falls into a recession, it is projected that the company will lose \$3 million. The managers do not know which state of nature will actually occur, or the probabilities of occurrence, so the decision to alter plant capacity is made under conditions of uncertainty. To apply the maximax rule to this decision, management first identifies the best possible outcome for each of the three decisions. The best payoffs are

\$5 million for expand plant size by 20 percent.

\$3 million for maintain plant size.

\$2 million for reduce plant size by 20 percent.

#### **payoff matrix**

A table with rows corresponding to various decisions and columns corresponding to various states of nature, with each cell giving the outcome or payoff associated with that decision and state of nature.

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TABLE 15.3 The Payoff Matrix for Dura Plastic, Inc.

Decisions	States of nature		
	Recovery	Stagnation	Recession
Expand plant capacity by 20%	\$5 million	-\$1 million	-\$3.0 million
Maintain same plant capacity	3 million	2 million	0.5 million
Reduce plant capacity by 20%	2 million	1 million	0.75 million

Each best payoff occurs if the economy recovers. Under the maximax rule, management would decide to expand its plant.

While the maximax rule is simple to apply, it fails to consider "bad" outcomes in the decision-making process. The fact that two out of three states of nature result in losses when management decides to expand plant capacity, and neither of the other decisions would result in a loss, is overlooked when using the maximax criteria. Only managers with optimistic natures are likely to find the maximax rule to be a useful decision-making tool.

may occur. The **potential regret** associated with a particular decision and state of nature is the improvement in payoff the manager could have experienced had the decision been the best one when that state of nature actually occurred. To illustrate, we calculate from Table 15.3 the potential regret associated with Dura Plastic's decision to maintain the same level of plant capacity if an economic recovery occurs. The best possible payoff when recovery occurs is \$5 million, the payoff for expanding plant capacity. If a recovery does indeed happen and management chooses to maintain the same level of plant capacity, the payoff is only \$3 million, and the manager experiences a regret of \$2 million ( $= \$5 - \$3$  million).

Table 15.4 shows the potential regret for each combination of decision and state of nature. Note that every state of nature has a decision for which there is no potential regret. This occurs when the correct decision is made for that particular state of nature. To apply the **minimax regret rule**, which requires that managers make a decision with the minimum worst potential regret, management identifies the maximum possible potential regret for each decision from the matrix:

\$3.75 million for expand plant size by 20 percent.

\$2 million for maintain plant size.

\$3 million for reduce plant size by 20 percent.

**potential regret**  
For a given decision and state of nature, the improvement in payoff the manager could have experienced had the decision been the best one when that state of nature actually occurs.

**minimax regret rule**  
Decision-making guide that calls for determining the worst potential regret associated with each decision, then choosing the decision with the minimum worst potential regret.

TABLE 15.4 Potential Regret Matrix for Dura Plastic, Inc.

Decisions	States of nature		
	Recovery	Stagnation	Recession
Expand plant capacity by 20%	\$0 million	\$3 million	\$3.75 million
Maintain same plant capacity	2 million	0 million	0.25 million
Reduce plant capacity by 20%	3 million	1 million	0 million

Management chooses the decision with the lowest worst potential regret: maintain current plant capacity. For Dura Plastic, the minimax regret rule results in management's choosing to maintain the current plant capacity.

## The Equal Probability Criterion

In situations of uncertainty, managers have no information about the probable state of nature that will occur and sometimes simply assume that each state of nature is equally likely to occur. In terms of the Dura Plastic decision, management assumes each state of nature has a one-third probability of occurring. When managers assume each state of nature has an equal likelihood of occurring, the decision can be made by considering the *average payoff* for each equally possible state of nature. This approach to decision making is often referred to as the **equal probability rule**. To illustrate, the manager of Dura Plastic calculates the average payoff for each decision as follows:

**equal probability rule**  
Decision-making guide that calls for assuming each state of nature is equally likely to occur, computing the average payoff for each equally likely possible state of nature, and choosing the decision with the highest average payoff.

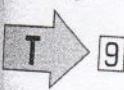
\$0.33 million [ $= (5 + (-1) + (-3))/3$ ] for expand plant size.

\$1.83 million [ $= (3 + 2 + 0.5)/3$ ] for maintain plant size.

\$1.25 million [ $= (2 + 1 + 0.75)/3$ ] for reduce plant size.

Under the equal probability rule, the manager's decision is to maintain the current plant capacity, since this decision has the maximum average return.

The four decision rules discussed here do not exhaust the possibilities for managers making decisions under uncertainty. We present these four rules primarily to give you a feel for decision making under uncertainty and to show the imprecise or "unscientific" nature of these rules. Recall that management could choose any of the courses of action depending upon which rule was chosen. These and other rules are meant only to be guidelines to decision making and are not substitutes for the experience and intuition of management.



## 15.6 SUMMARY

When managers make choices or decisions under risk or uncertainty, they must somehow incorporate this risk into their decision-making process. This chapter presented some basic rules for managers to help them make decisions under conditions of risk and uncertainty. Conditions of *risk* occur when a manager must make a decision for which the outcome is not known with certainty. Under conditions of risk, the manager can make a list of all possible outcomes and assign probabilities to the various outcomes. *Uncertainty* exists when a decision maker cannot list all possible outcomes and/or cannot assign probabilities to the various outcomes. To measure the risk associated with a decision, the manager can examine several characteristics of the probability distribution of outcomes for the decision. The various rules for making decisions under risk require information about several different characteristics of the probability distribution of outcomes: (1) the expected value (or mean) of the distribution, (2) the variance and standard deviation, and (3) the coefficient of variation.

While there is no single decision rule that managers can follow to guarantee that profits are actually maximized, we discussed a number of decision rules that managers can use to help them make decisions under risk: (1) the expected value rule, (2) the mean-variance rules, and (3) the coefficient of variation rule. These rules can only guide managers in their analysis of risky decision making. The actual decisions made by a manager will depend in large measure on the manager's willingness to take on risk. Managers' propensity to take on risk can be classified in one of three categories: risk averse, risk loving, or risk neutral.

Expected utility theory explains how managers can make decisions in risky situations. The theory postulates that managers make risky decisions with the objective of maximizing the expected utility of profit. The manager's attitude for risk is captured by the shape of the utility function for profit. If a manager experiences diminishing (increasing) marginal utility for profit, the manager is risk averse (risk loving). If marginal utility for profit is constant, the manager is risk neutral.

If a manager maximizes expected utility for profit, the decisions can differ from decisions reached using the three decision rules discussed for making risky decisions. However, in the case of a risk-neutral manager, the decisions are the same under maximization of expected profit and maximization of expected utility of profit. Consequently, a risk-neutral decision maker can follow the simple rule of maximizing the expected value of profit and simultaneously also be maximizing utility of profit.

In the case of uncertainty, decision science can provide very little guidance to managers beyond offering them some simple decision rules to aid them in their analysis of uncertain situations. We discussed four basic rules for decision making under uncertainty in this chapter: (1) the maximax rule, (2) the maximin rule, (3) the minimax regret rule, and (4) the equal probability rule.