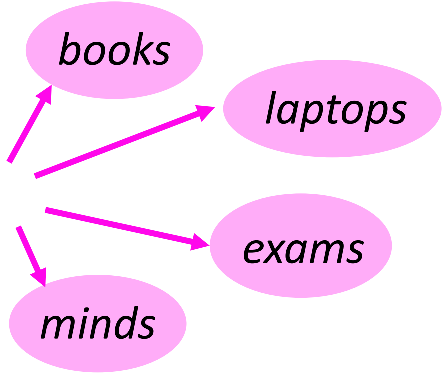
Language Models and Recurrent Neural Networks

Slides mostly from the Stanford NLP course by Prof. Chris Manning and others

Language Modeling

- **Language Modeling** is the task of predicting what word comes next

the students opened their ______



- More formally: given a sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$, compute the probability distribution of the next word $\boldsymbol{x}^{(t+1)}$:

$$P(\boldsymbol{x}^{(t+1)} \mid \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

where $\boldsymbol{x}^{(t+1)}$ can be any word in the vocabulary $V = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_{|V|}\}$

- A system that does this is called a **Language Model**

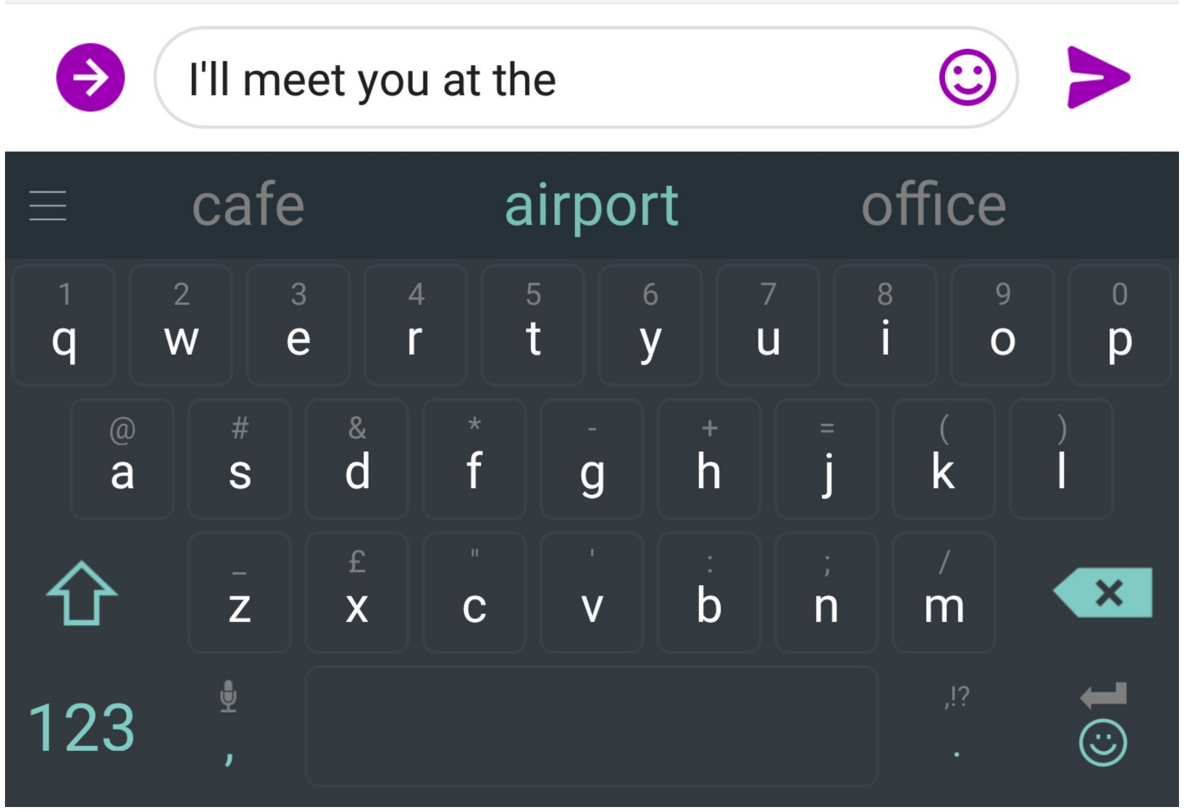
Language Modeling

- You can also think of a Language Model as a system that assigns a probability to a piece of text

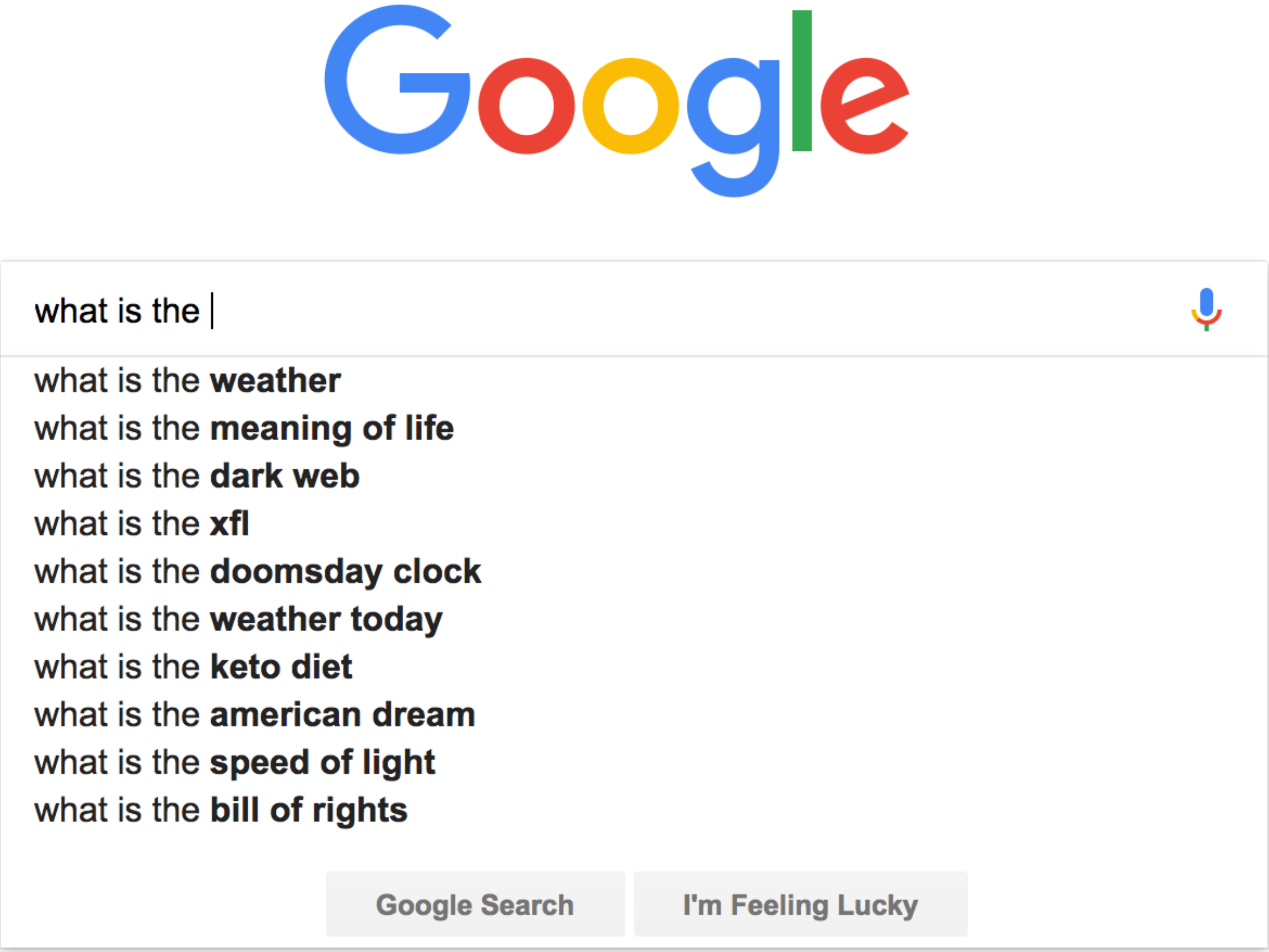
- For example, if we have some text $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$, then the probability of this text (according to the Language Model) is:

$$
\begin{aligned}
P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) &= P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \, \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \, \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)}) \\
&= \prod_{t=1}^{T} \underbrace{P(\boldsymbol{x}^{(t)} | \, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})}_{\text{This is what our LM provides}}
\end{aligned}
$$

You use Language Models every day!



You use Language Models every day!



n-gram Language Models

the students opened their ______

- **Question**: How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn an *n*-gram Language Model!

- **Definition:** An *n*-gram is a chunk of *n* consecutive words.
  - unigrams: “the”, “students”, “opened”, ”their”
  - bigrams: “the students”, “students opened”, “opened their”
  - trigrams: “the students opened”, “students opened their”
  - four-grams: “the students opened their”

- **Idea:** Collect statistics about how frequent different n-grams are and use these to predict next word.

n-gram Language Models

- First we make a Markov assumption: $x^{(t+1)}$ depends only on the preceding *n*-1 words

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\overbrace{\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(t-n+2)}}^{n\text{-1 words}})$$

(assumption)

prob of a n-gram

$$= \frac{\boxed{P(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(t-n+2)})}}{\boxed{P(\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(t-n+2)})}}$$

prob of a (n-1)-gram

(definition of conditional prob)

- **Question:** How do we get these *n*-gram and (*n*-1)-gram probabilities?
- **Answer:** By counting them in some large corpus of text!

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)},\dots,\boldsymbol{x}^{(t-n+2)})}$$

(statistical approximation)

n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~*as the proctor started the clock, the*~~ $\underbrace{\textit{students opened their}}_{\text{condition on this}}$ ______

discard

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

For example, suppose that in the corpus:

- "students opened their" occurred 1000 times
- "students opened their books" occurred 400 times
  - → P(books | students opened their) = 0.4
- "students opened their exams" occurred 100 times
  - → P(exams | students opened their) = 0.1

Should we have discarded the "proctor" context?

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their w"* never occurred in data? Then w has probability 0!

(Partial) Solution: Add small δ to the count for every $w \in V$. This is called *smoothing*.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count}(\text{students opened their } \boldsymbol{w})}}{\boxed{\text{count}(\text{students opened their})}}$$

Sparsity Problem 2

Problem: What if *"students opened their"* never occurred in data? Then we can't calculate probability for *any* w!

(Partial) Solution: Just condition on *"opened their"* instead. This is called *backoff*.

Note: Increasing *n* makes sparsity problems *worse.* Typically, we can't have *n* bigger than 5.

Storage Problems with n-gram Language Models

<u>Storage</u>: Need to store count for all *n*-grams you saw in the corpus.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\boxed{\text{count}(\text{students opened their } \boldsymbol{w})}}{\text{count}(\text{students opened their})}$$

Increasing *n* or increasing corpus increases model size!

This is a reason why the older n-gram language models could not be run over small devices - needed storage of huge tables of counts / probabilities.

Modern neural network based LMs (to be discussed next) are much more space efficient

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the ________

condition on this

get probability distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
…	

sample

Note: we are sampling the next word based on a probability distribution. Hence we may choose some word which has a lower probability. This brings diversity in the generated text.

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price ________

condition on this

get probability distribution

of	0.308
for	0.050
it	0.046
to	0.046
is	0.031
…	

sample

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price of ________

condition on this

get probability distribution

the	0.072
18	0.043
oil	0.043
its	0.036
gold	0.018
…	

sample

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

…but **incoherent.** We need to consider more than three words at a time if we want to model language well.

But increasing *n* worsens sparsity problem, and increases model size…

How to build a *neural* language model?

- Recall the Language Modeling task:
  - Input: sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$
  - Output: prob. dist. of the next word $P(\boldsymbol{x}^{(t+1)} | \, \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$

- How about a window-based neural model?

Note: our problem definition (learning a LM) has not changed. We will only see a new method for the same problem.

A fixed-window neural Language Model

Discarding the words appearing much before … this part is similar to a n-gram LM

Difference in approach: we will now feed these n words into a neural network for predicting the probabilities for the next word.

~~*as the proctor started the clock*~~ *the students opened their* ______

discard

fixed window

A fixed-window neural Language Model

output distribution

$$\hat{\boldsymbol{y}} = \mathrm{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

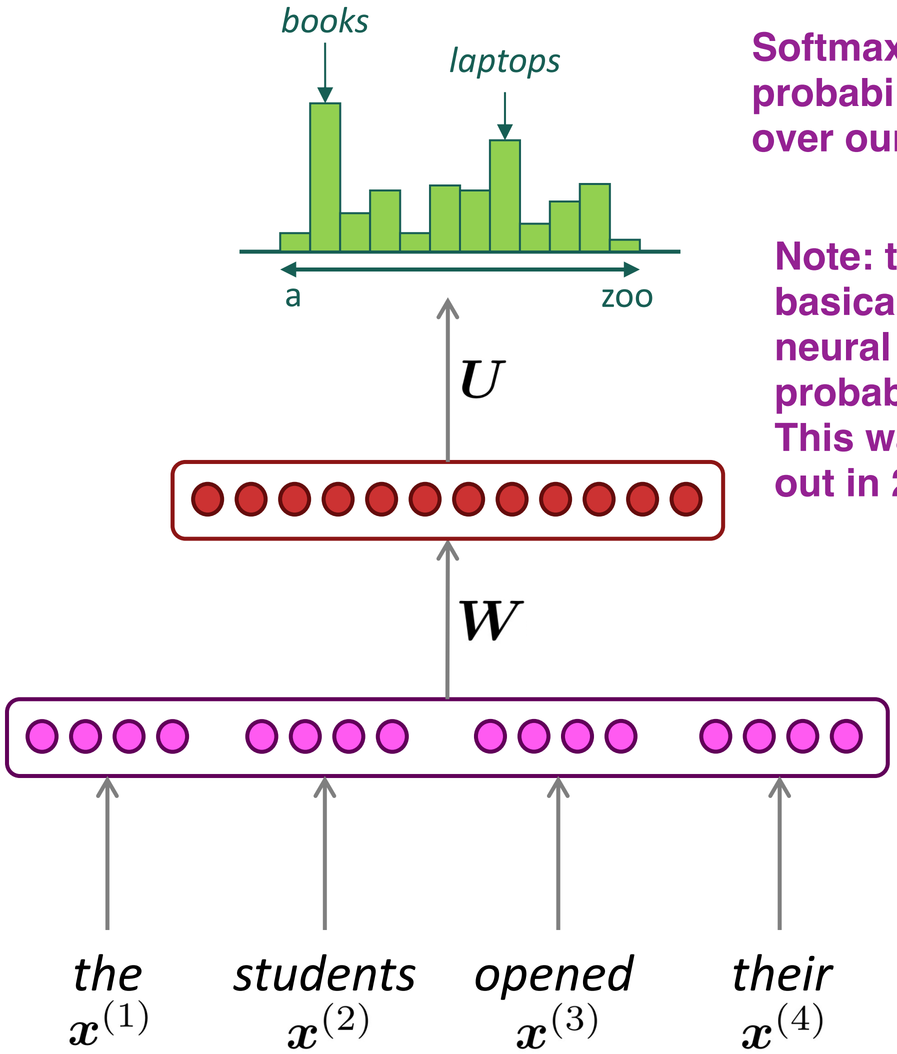
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$



Softmax generates a probability distribution over our entire vocabulary

Note: this model is basically a fixed-window neural classifier (outputs probability of classes). This was actually tried out in 2000.

A fixed-window neural Language Model

Approximately: Y. Bengio, et al. (2000/2003): A Neural Probabilistic Language Model

Improvements over *n*-gram LM:

- No sparsity problem

Distributional embeddings —> should be able to take advantage of semantic similarity of words

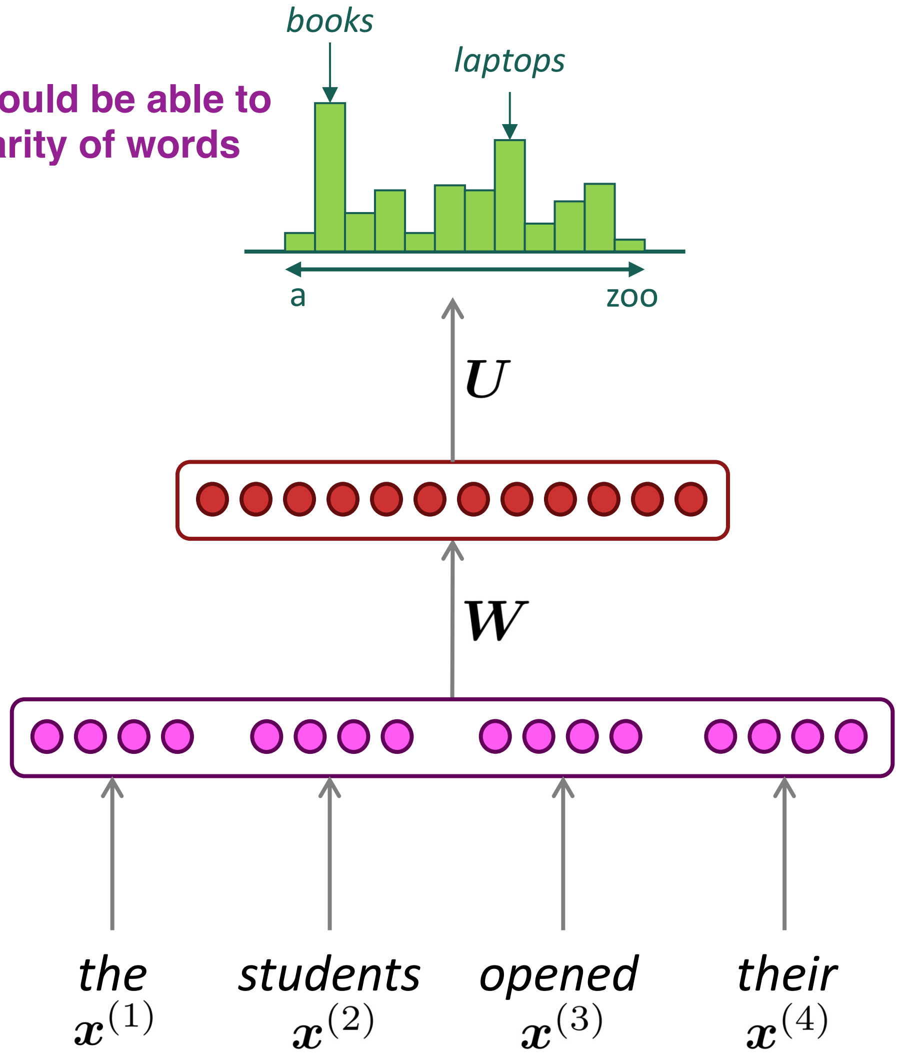
- Don't need to store all observed *n*-grams

We just need to store the learned weights

Remaining **problems**:

- Fixed window is too small
- Enlarging window enlarges W
- Window can never be large enough!
- $x^{(1)}$ and $x^{(2)}$ are multiplied by completely different weights in W. No symmetry in how the inputs are processed.

We need a neural architecture that can process *any length input*



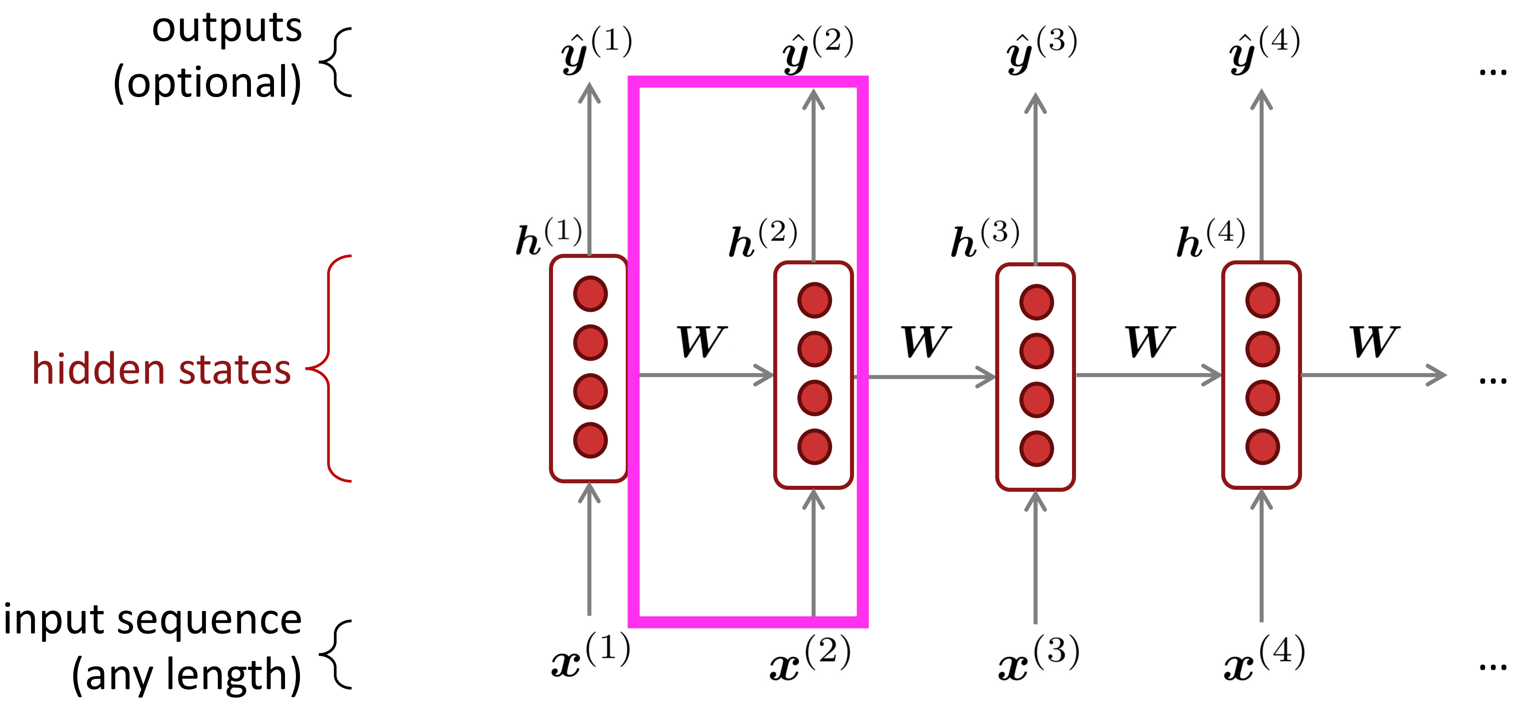
Word order, relation among the words are important for LM, but the model we discussed treats each word independently.

What we desire - a neural model that
- can process an arbitrarily long context
- have more sharing of the parameters that are applied to the words in that context, and
- is still sensitive to proximity (e.g., the word coming right before the target word should be given more weightage than the word coming 3 words earlier).

3. Recurrent Neural Networks (RNN)

A family of neural architectures

Core idea: Apply the same weights W *repeatedly*



To generate the k-th word, we take (i) the k-th input, (ii) the previous hidden state (multiplied by a matrix W).

This previous hidden state is a representation for the previous context.

We repeat this process at every time-step - predict the next output, and build up the hidden state (representation till now).

A Simple RNN Language Model

$$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$$

output distribution

$$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$$

Computation of hidden state uses a non-linearity function, e.g., sigmoid or tanh

hidden states

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h\boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e\boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$$

$\boldsymbol{h}^{(0)}$ is the initial hidden state

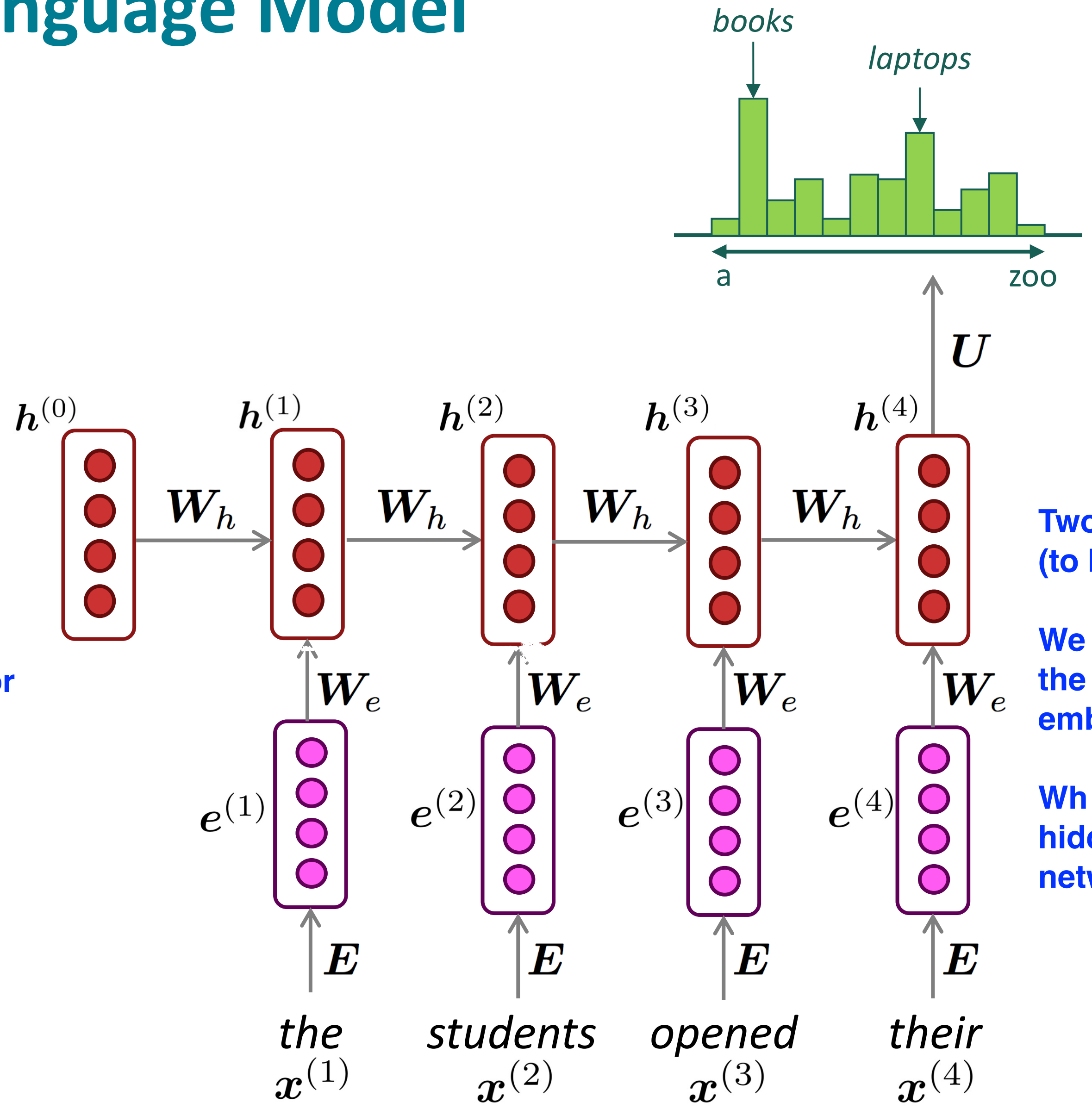
Initial hidden state can be a zero vector

word embeddings

$$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$$

words / one-hot vectors

$$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$$



Note: we can take this output based on the hidden state at any (every) time-step

Two parameter matrices (to be learned):

We - for multiplying with the input word embeddings

Wh - for updating the hidden state of the network

__Note__: this input sequence could be much longer now!

RNN Language Models

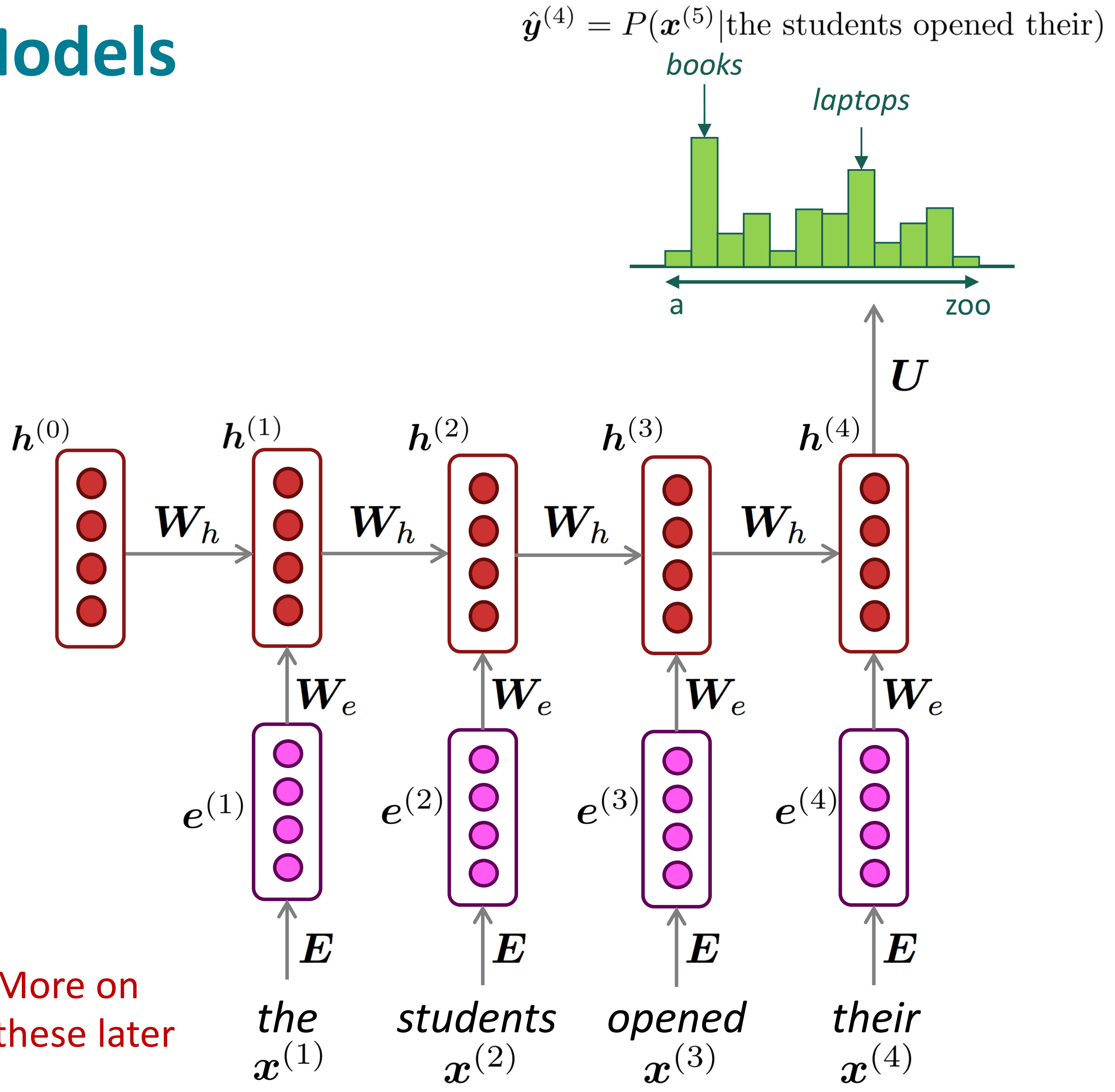
RNN **Advantages**:

- Can process any length input
- Computation for step *t* can (in theory) use information from many steps back
- Model size doesn't increase for longer input context
- Same weights applied on every timestep, so there is symmetry in how inputs are processed.

RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back

More on these later



Training an RNN Language Model

- Get a big corpus of text which is a sequence of words $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\boldsymbol{y}}^{(t)}$ for *every step t.*
  - i.e., predict probability dist of *every word*, given words so far

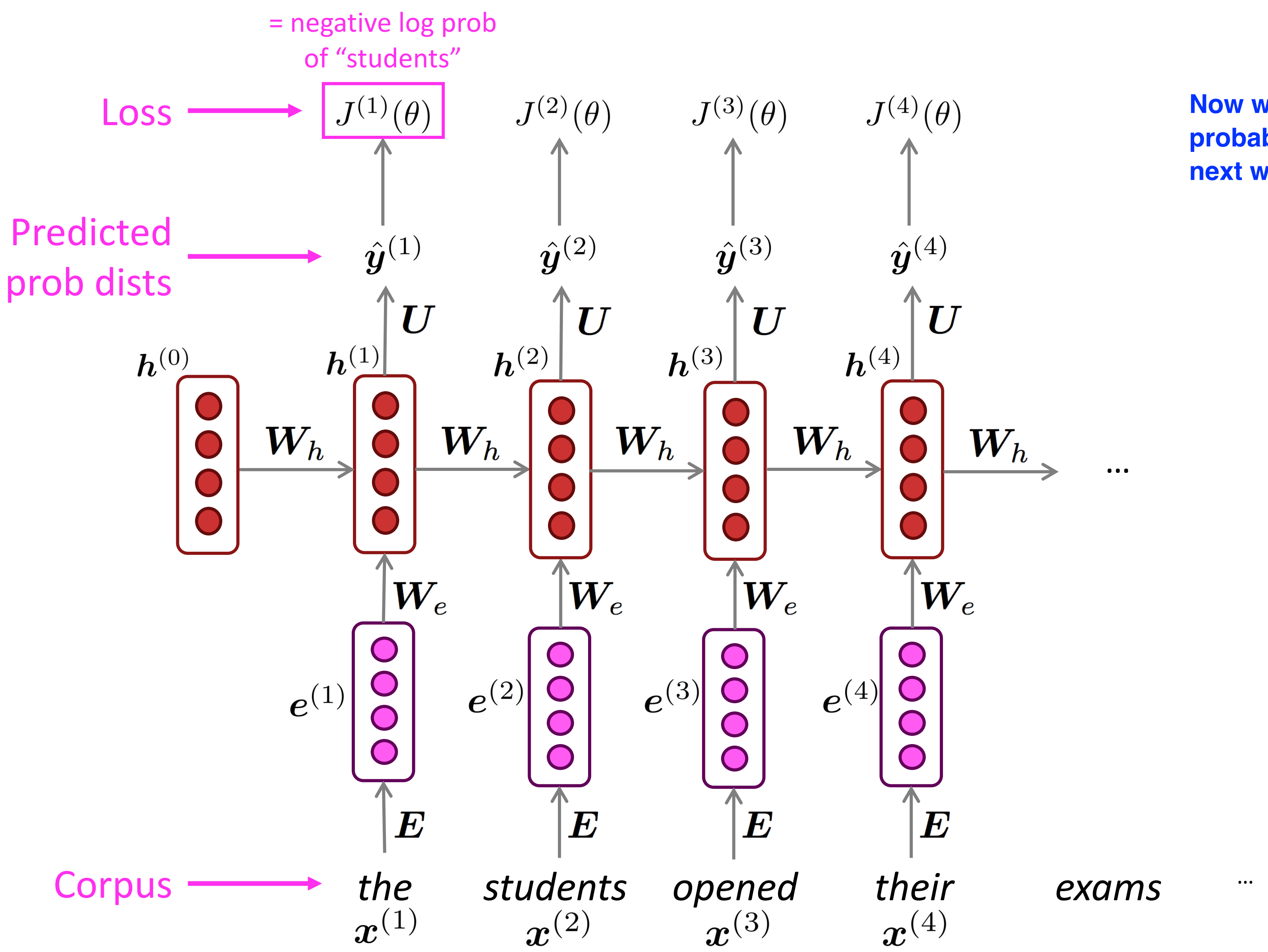
- Loss function on step *t* is cross-entropy between predicted probability distribution $\hat{\boldsymbol{y}}^{(t)}$, and the true next word $\boldsymbol{y}^{(t)}$ (one-hot for $\boldsymbol{x}^{(t+1)}$):

$$J^{(t)}(\theta) = CE(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

- Average this to get overall loss for entire training set:

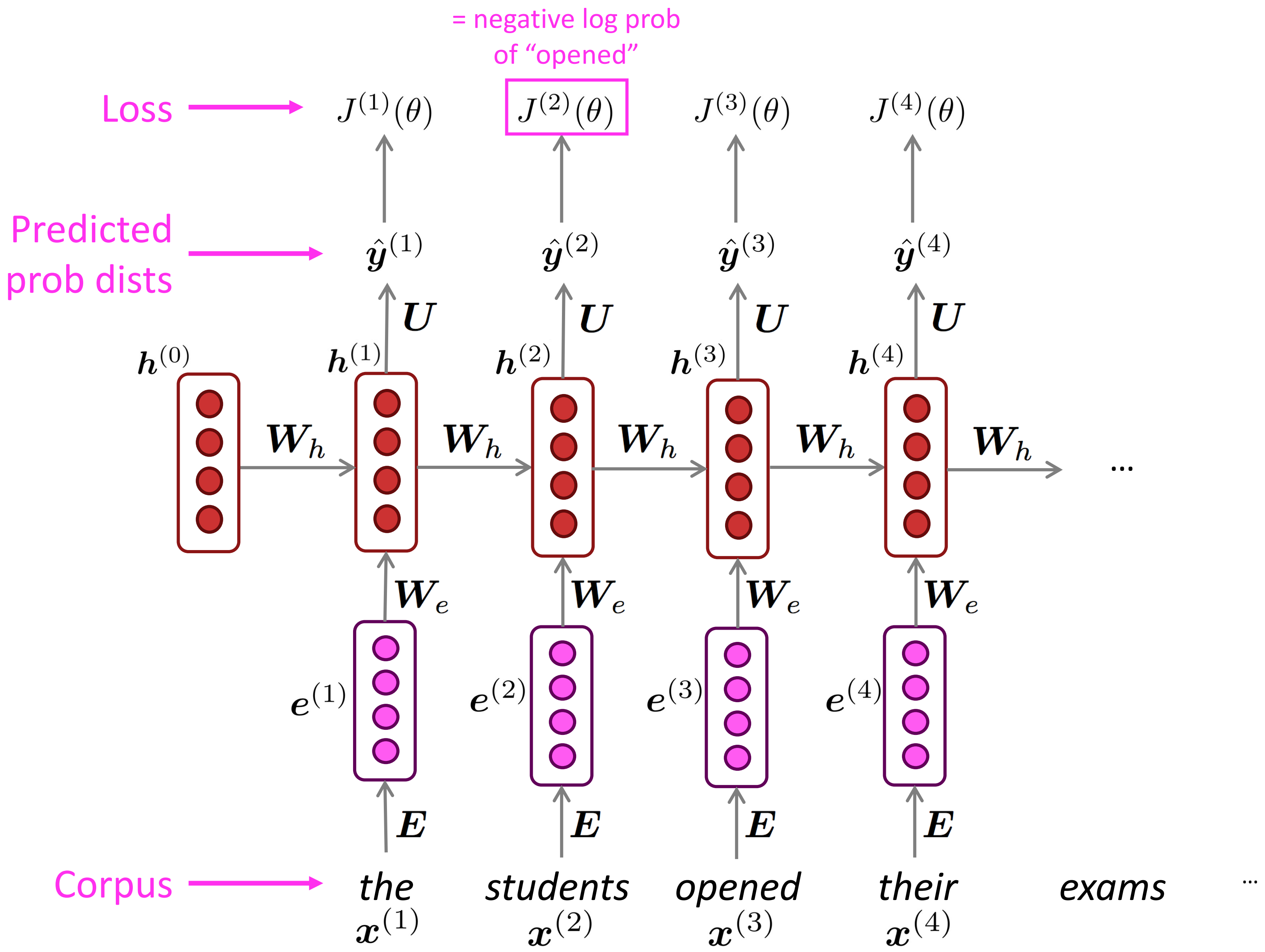
$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

Training an RNN Language Model

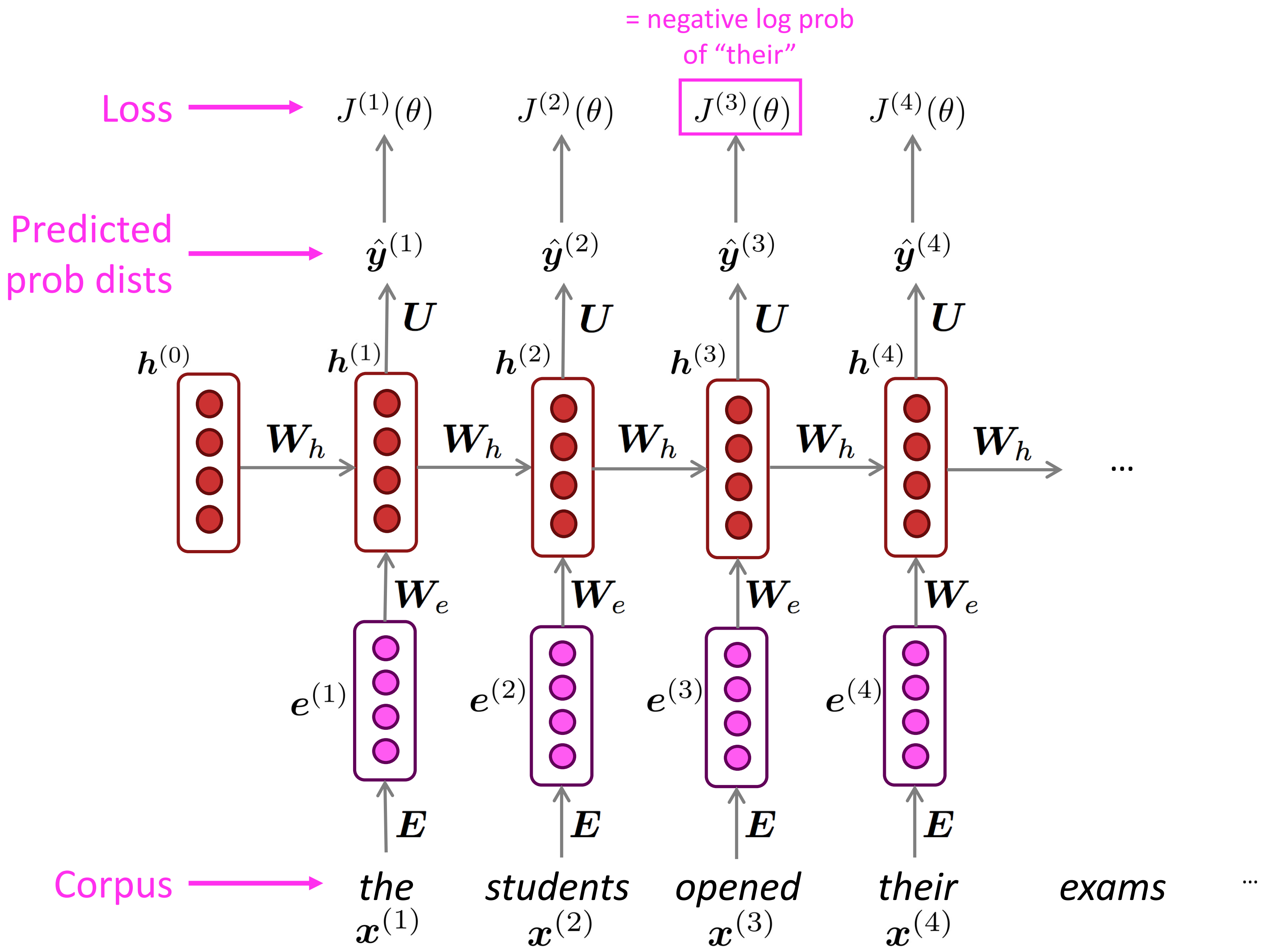


Now we are predicting a probability distribution (for the next word) at every step.

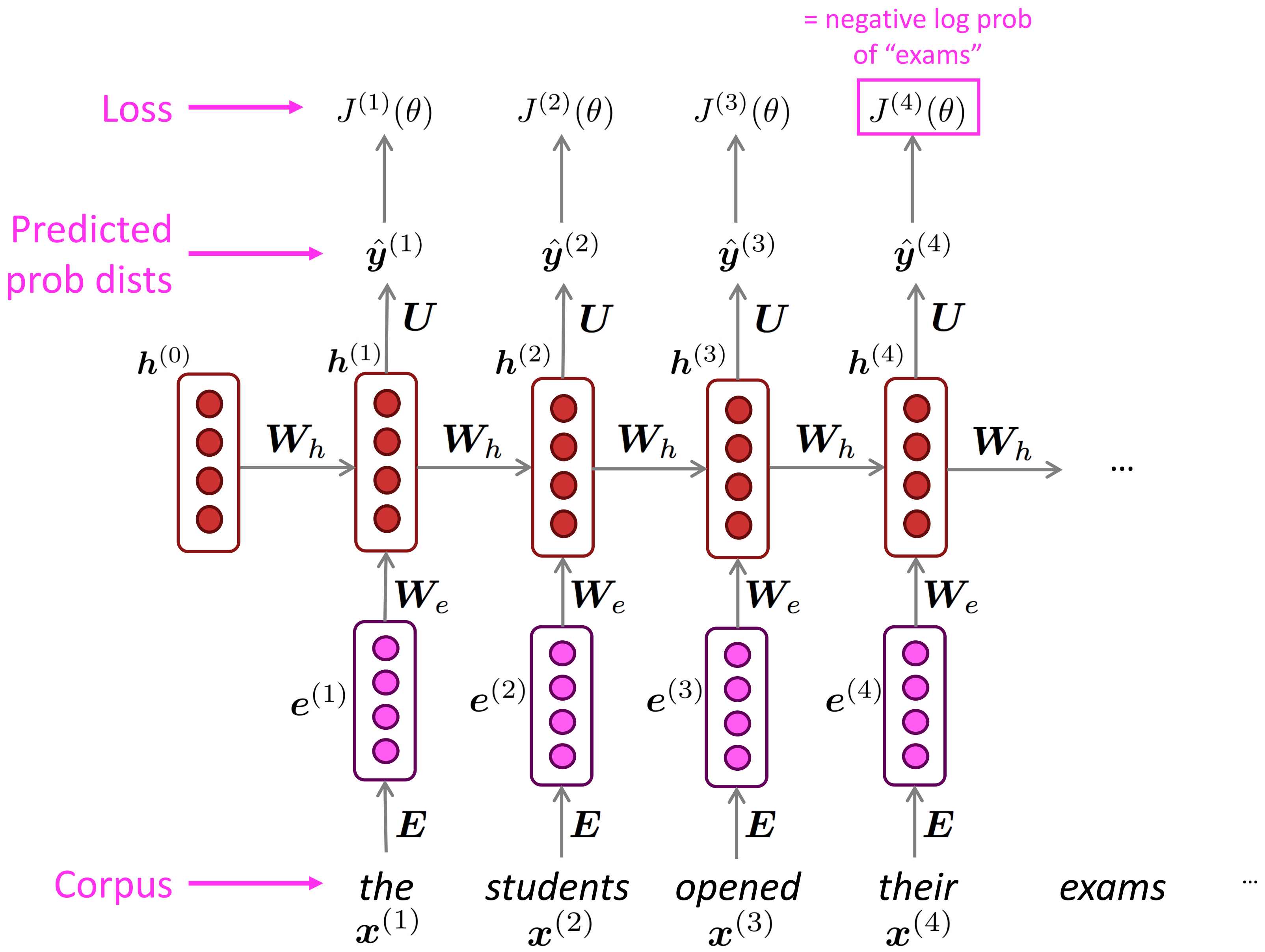
Training an RNN Language Model



Training an RNN Language Model

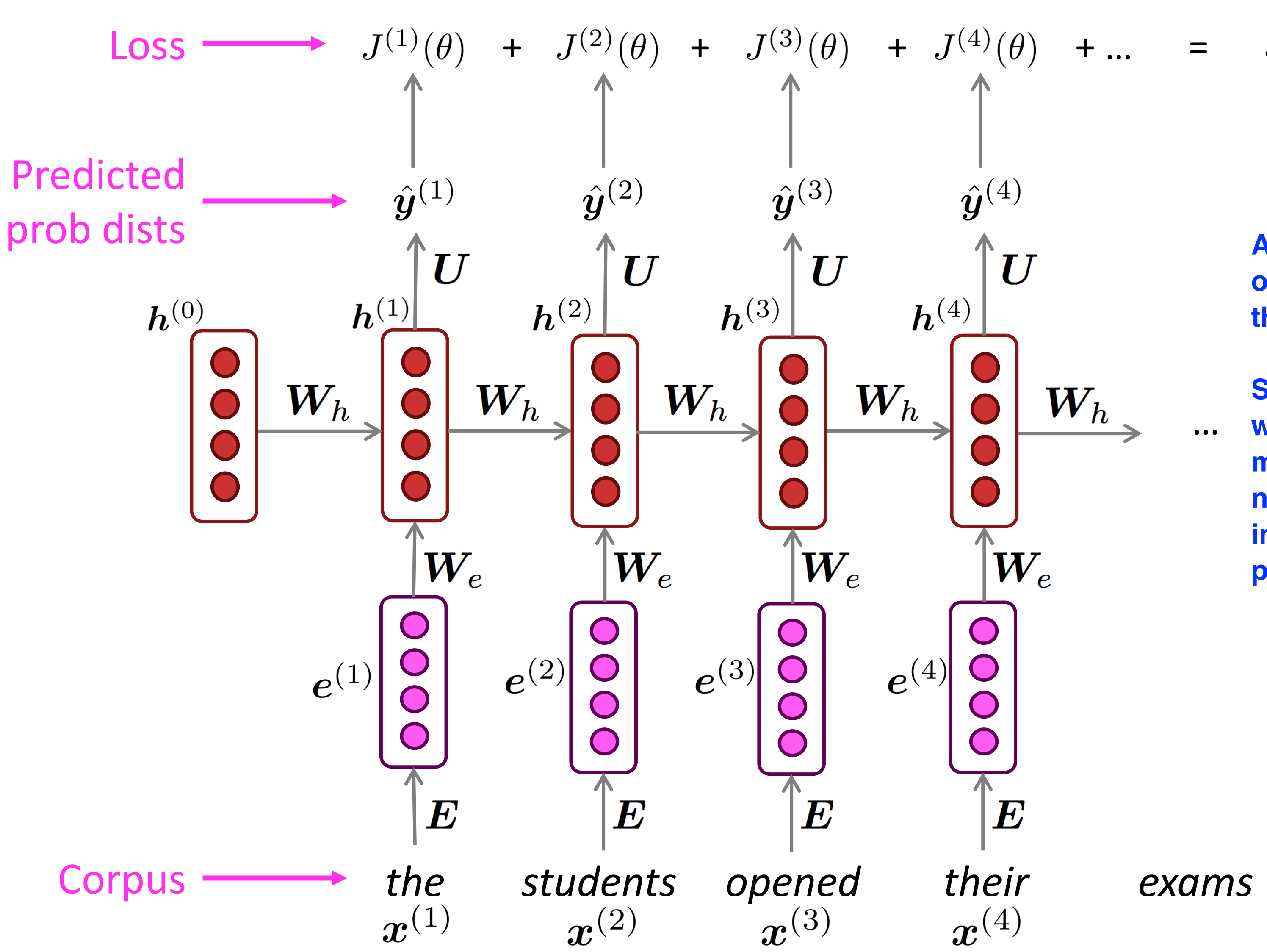


Training an RNN Language Model



Training an RNN Language Model

"Teacher forcing"



$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J^{(t)}(\theta)$$

At each step, we are looking to generate only the next word. We are not asking the LM to generate continuous text.

Suppose the model predicts the next word wrongly. We will penalize the model (though the loss function). Our next input will be the correct next word in the corpus, NOT what the model predicted.

Training a RNN Language Model

- However: Computing loss and gradients across entire corpus $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ at once is too expensive (memory-wise)!

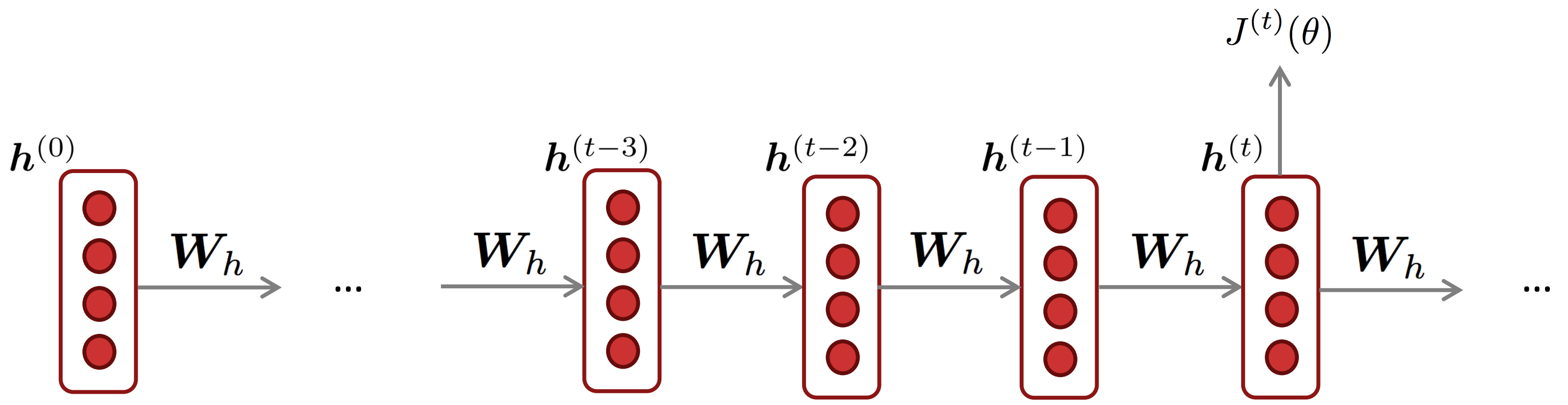
$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta)$$

- In practice, consider $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ as a sentence (or a document)

- Recall: Stochastic Gradient Descent allows us to compute loss and gradients for small chunk of data, and update.

- Compute loss $J(\theta)$ for a sentence (actually, a batch of sentences), compute gradients and update weights. Repeat on a new batch of sentences.

Backpropagation for RNNs



<u>Question:</u> What's the derivative of $J^{(t)}(\theta)$ w.r.t. the repeated weight matrix $\boldsymbol{W}_h$?

<u>Answer:</u> $$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}$$

This is NOT a t-times product of the same partial derivative. The partial derivative at each step is different.

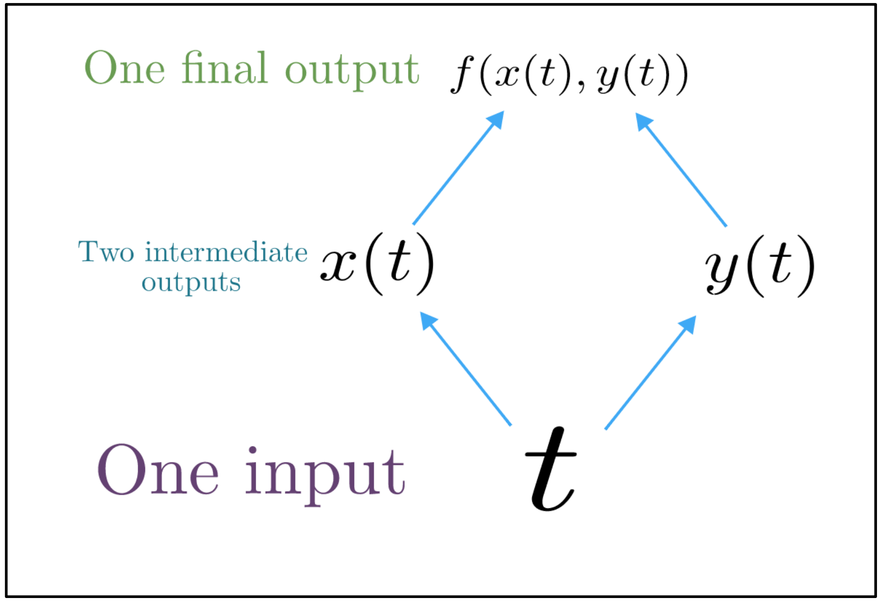
"The gradient w.r.t. a repeated weight is the sum of the gradient w.r.t. each time it appears"

Why?

Multivariable Chain Rule

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$



Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

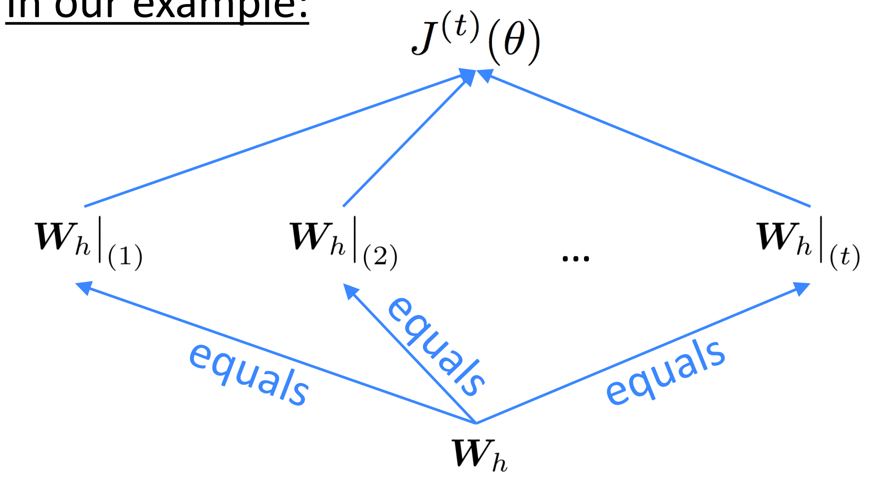
Backpropagation for RNNs: Proof sketch

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

The same Wh matrix being updated at every time step

In our example:



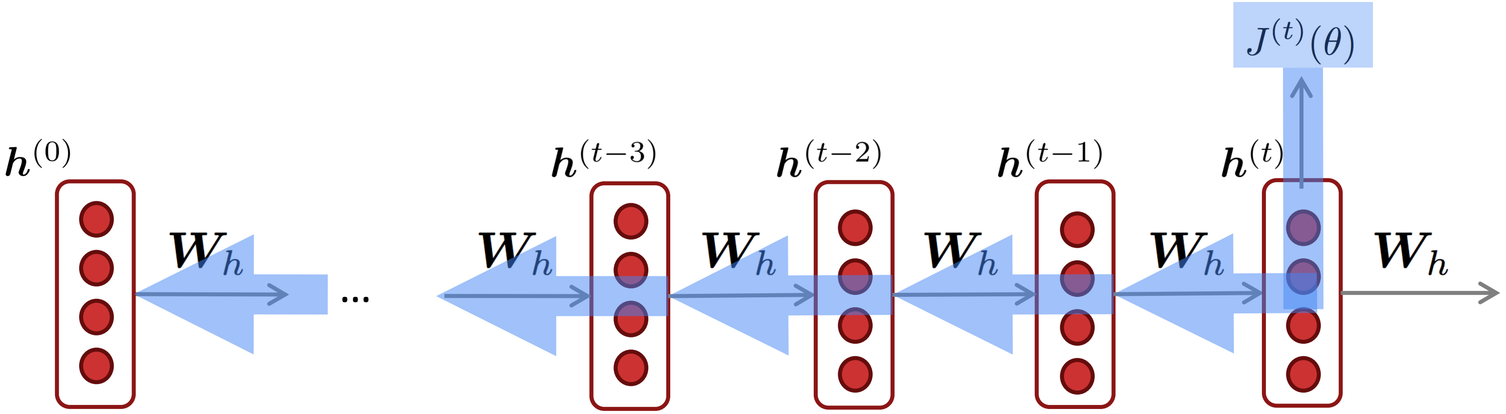
Apply the multivariable chain rule:

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h} = \sum_{i=1}^{t} \frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\bigg|_{(i)} \overset{= 1}{\boxed{\frac{\partial \boldsymbol{W}_h\big|_{(i)}}{\partial \boldsymbol{W}_h}}}$$

$$= \sum_{i=1}^{t} \frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\bigg|_{(i)}$$

Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

Backpropagation for RNNs



Compute the derivative w.r.t. Wh at the last time-step —> one update for Wh

Pass the gradient back to the (t-1) time step, compute the derivative w.r.t. Wh at (t-1) time step —> another update for Wh; sum with the previous update.

...

Likewise, we will get more updates for Wh at different time steps —> sum up all updates to get the final update for Wh

If our text is short, e.g., a sentence, we can run this BPTT till the start of the sentence.
If our text is really long, the backpropagation is usually truncated after, say, 20 time-steps, so we will sum just 20 updates — Truncated BPTT.

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \boxed{\sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}}$$

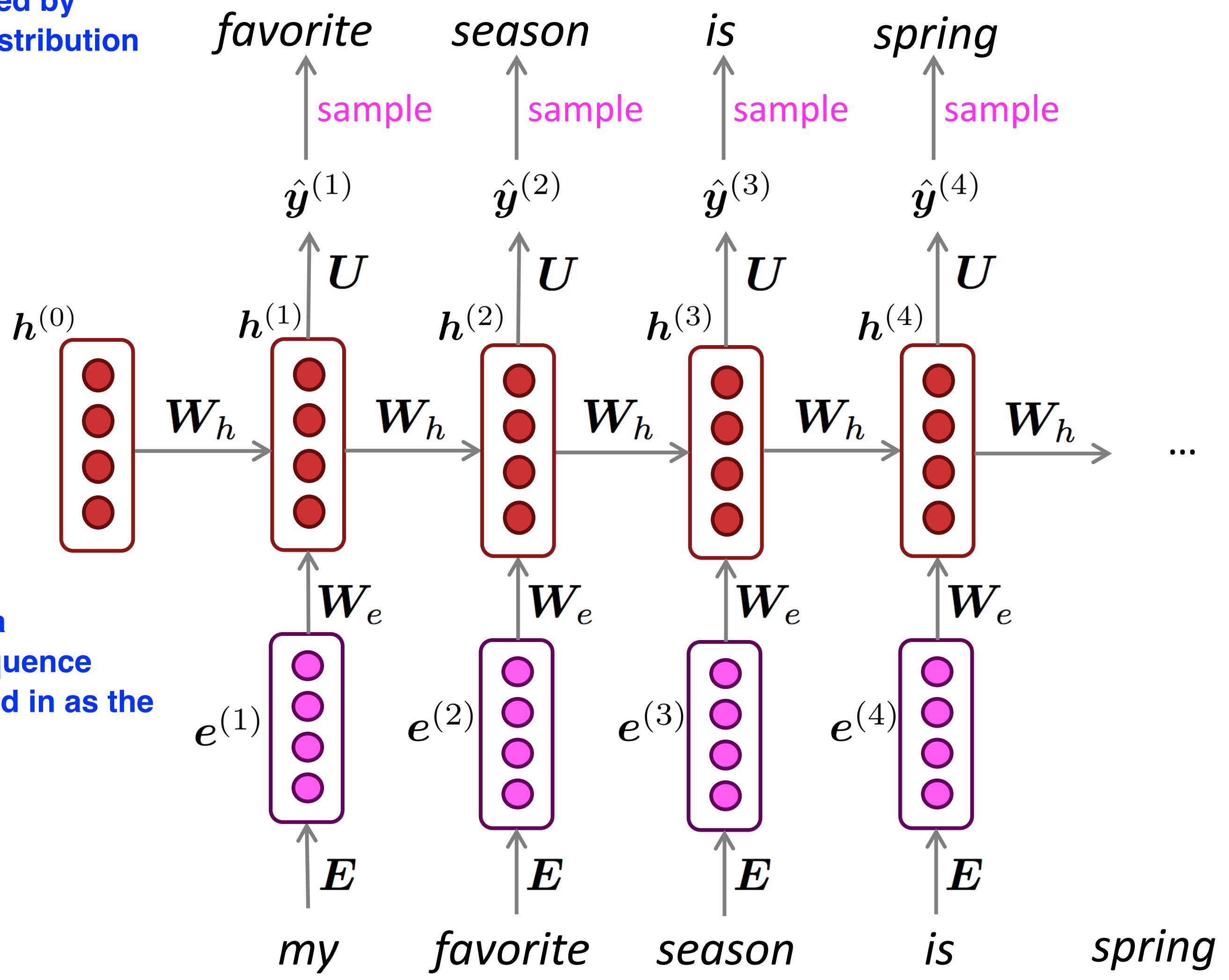
Question: How do we calculate this?

Answer: Backpropagate over timesteps *i=t*,…,0, summing gradients as you go.
This algorithm is called **"backpropagation through time"**
[Werbos, P.G., 1988, *Neural Networks* **1**, and others]

Generating text with a RNN Language Model

Just like a n-gram Language Model, you can use a RNN Language Model to generate text by repeated sampling. Sampled output becomes next step's input.

Each word is generated by sampling the prob. distribution output at that step.



Usually there is a beginning-of-sequence token which is fed in as the first token.

Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on Obama speeches:

The United States will step up to the cost of a new challenges of the American people that will share the fact that we created the problem. They were attacked and so that they have to say that all the task of the final days of war that I will not be able to get this done.

Source: https://medium.com/@samim/obama-rnn-machine-generated-political-speeches-c8abd18a2ea0

Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on recipes:

```
     Title: CHOCOLATE RANCH BARBECUE
Categories: Game, Casseroles, Cookies, Cookies
     Yield: 6 Servings

     2 tb Parmesan cheese -- chopped
     1 c  Coconut milk
     3    Eggs, beaten

 Place each pasta over layers of lumps. Shape mixture into the moderate oven and simmer
 until firm. Serve hot in bodied fresh, mustard, orange and cheese.

 Combine the cheese and salt together the dough in a large skillet; add the ingredients
 and stir in the chocolate and pepper.
```

At least the RNN has understood the structure of a recipe.

Source: https://gist.github.com/nylki/1efbaa36635956d35bcc

Evaluating Language Models

- The standard evaluation metric for Language Models is perplexity.

$$\text{perplexity} = \underbrace{\prod_{t=1}^{T} \left(\frac{1}{P_{\text{LM}}(\boldsymbol{x}^{(t+1)} | \, \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})} \right)}_{\text{Inverse probability of corpus, according to Language Model}}^{1/T}$$

Normalized by number of words

- This is equal to the exponential of the cross-entropy loss $J(\theta)$:

$$= \prod_{t=1}^{T} \left(\frac{1}{\hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}}} \right)^{1/T} = \exp\left(\frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}} \right) = \exp(J(\theta))$$

Lower perplexity is better!

RNNs have greatly improved perplexity

n-gram model

Increasingly complex RNNs

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

Perplexity improves
(lower is better)

Source: https://research.fb.com/building-an-efficient-neural-language-model-over-a-billion-words/

Why should we care about Language Modeling?

- Language Modeling is a benchmark task that helps us measure our progress on understanding language **(as well as understanding real world knowledge)**

- Language Modeling is a subcomponent of many NLP tasks, especially those involving generating text or estimating the probability of text:

    - Predictive typing
    - Speech recognition
    - Handwriting recognition
    - Spelling/grammar correction
    - Authorship identification
    - Machine translation
    - Summarization
    - Dialogue
    - etc.

Recap

- **Language Model**: A system that predicts the next word

- **Recurrent Neural Network**: A family of neural networks that:
  - Take sequential input of any length
  - Apply the same weights on each step
  - Can optionally produce output on each step

- Recurrent Neural Network $\neq$ Language Model

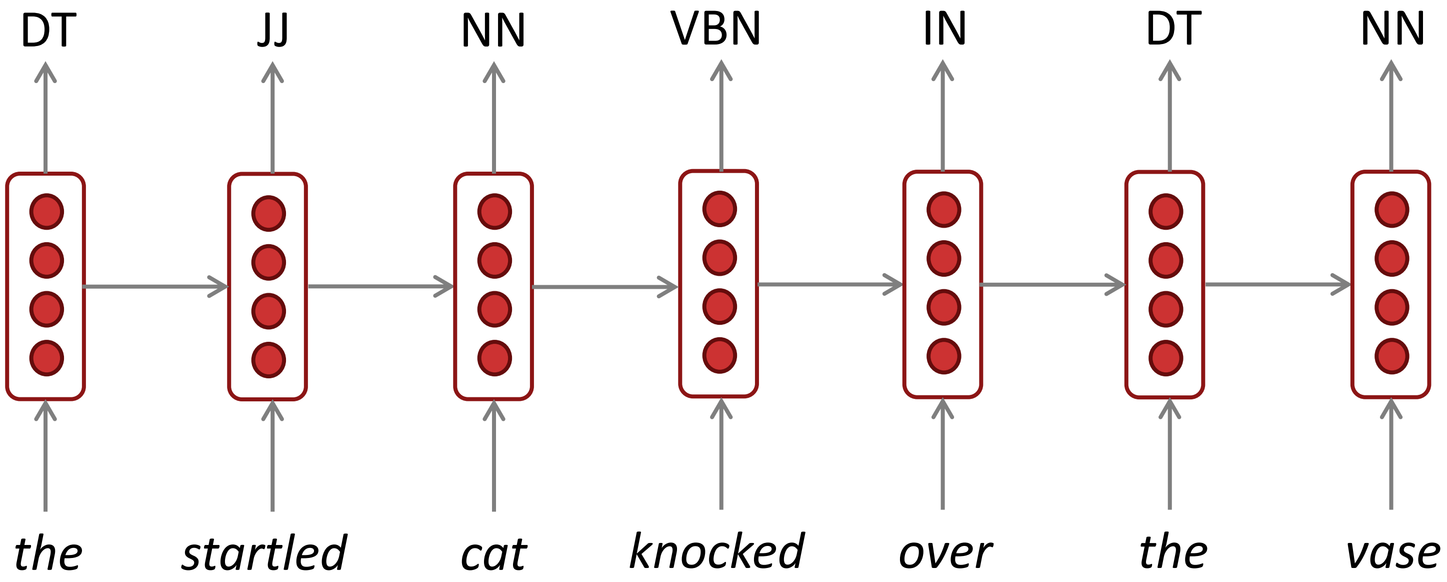
- We’ve shown that RNNs are a great way to build a LM.

- But RNNs are useful for much more!

RNNs can be used for tagging

e.g., **part-of-speech tagging**, named entity recognition

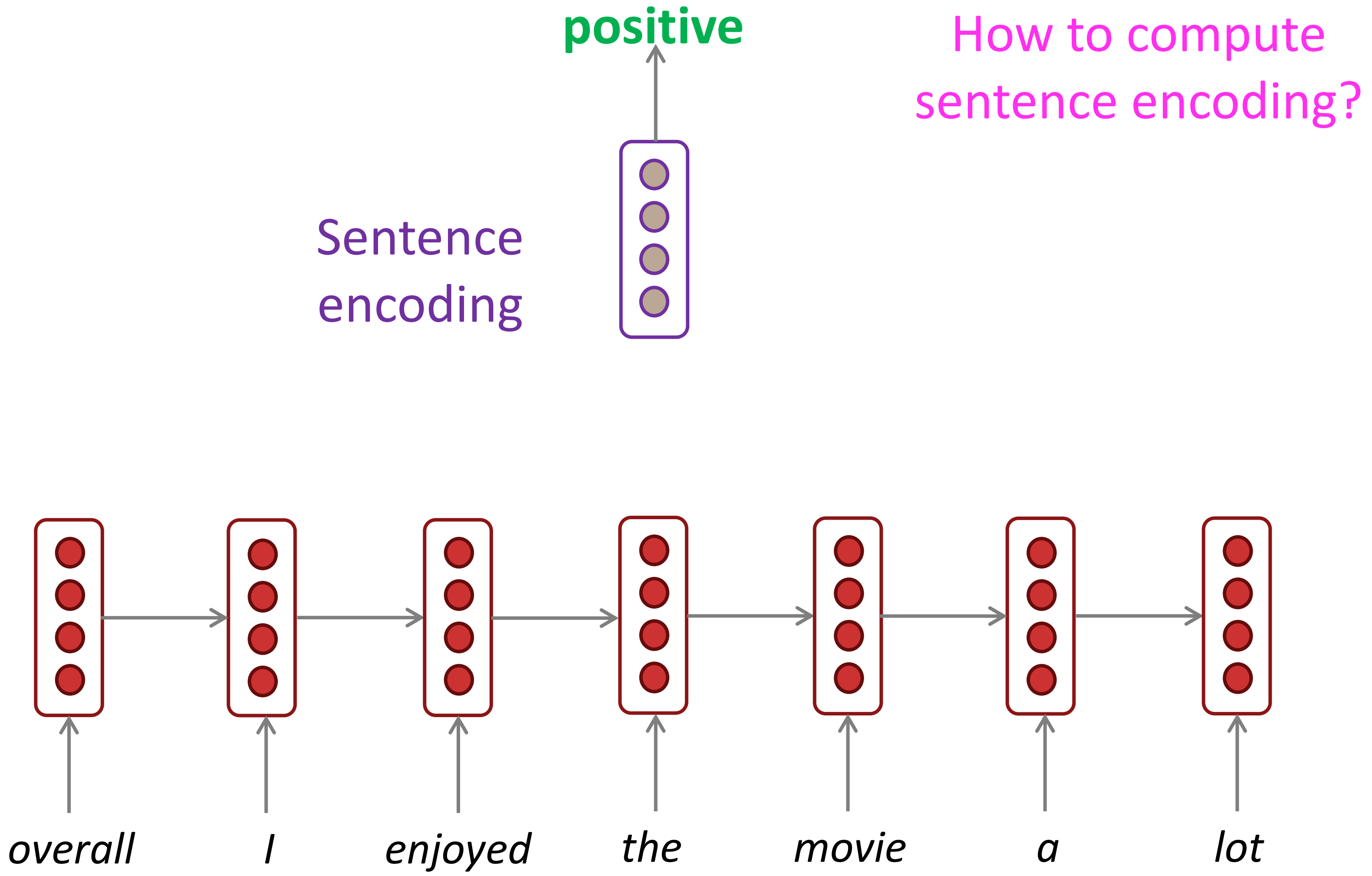
Sequence tagging tasks: where we want to label / classify every token in a sequence



This RNN is trained to output the POS tag, instead of the next word

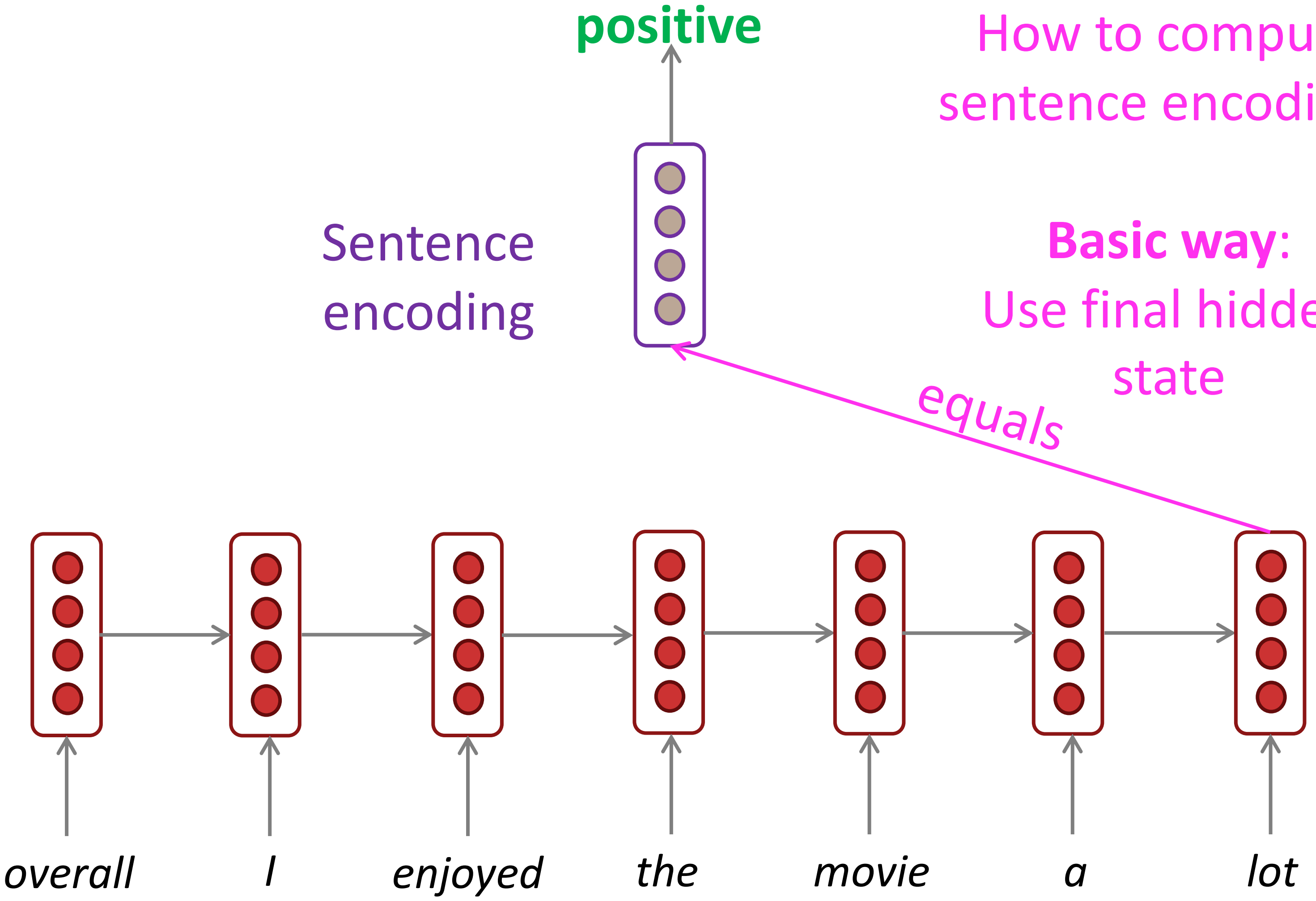
RNNs can be used for sentence classification

e.g., **sentiment classification**



RNNs can be used for sentence classification

e.g., **sentiment classification**



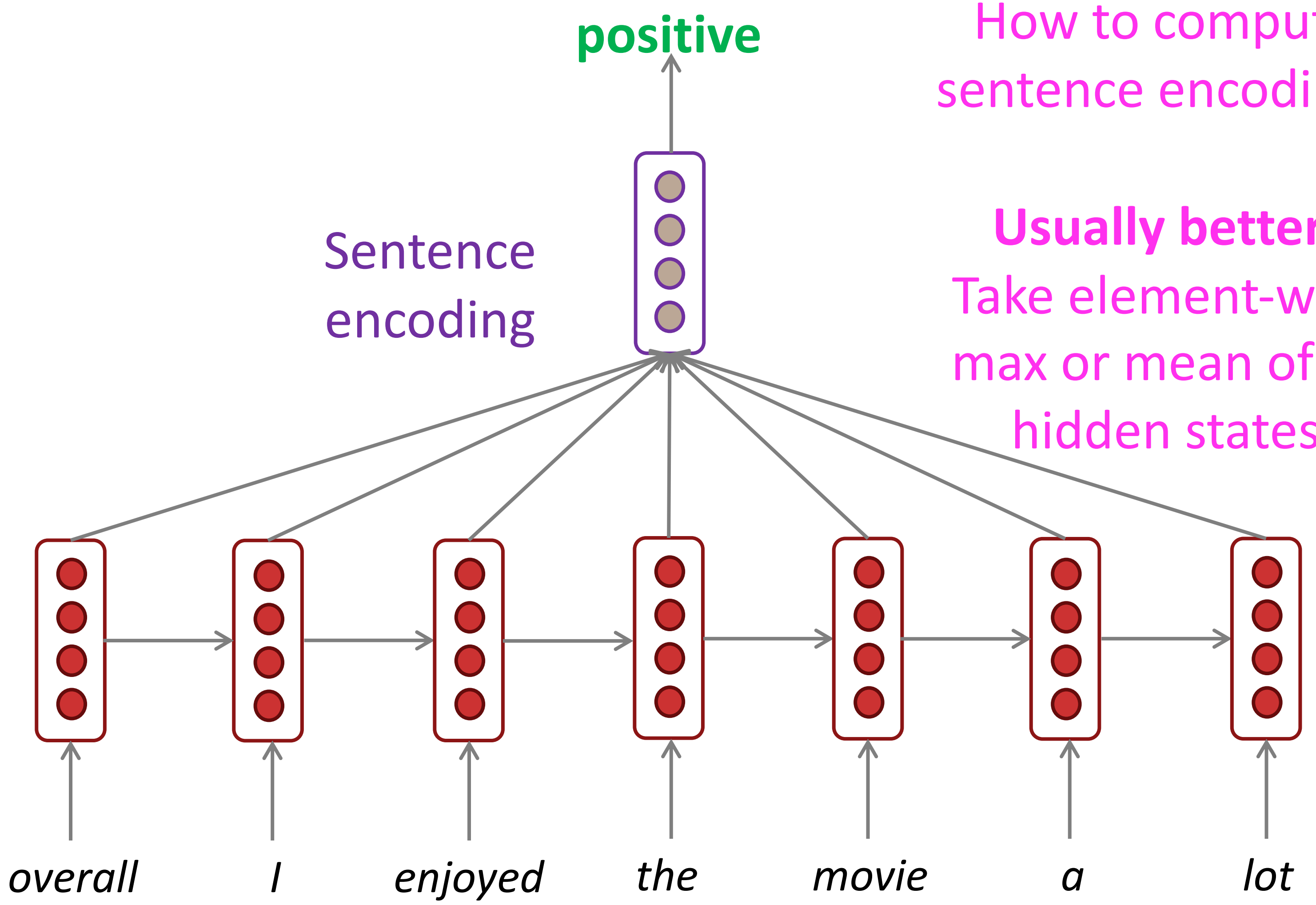
How to compute sentence encoding?

Basic way:
Use final hidden state

Such a model, if trained end-to-end, can learn to preserve sentiment-related information in the hidden states of the RNN

RNNs can be used for sentence classification

e.g., **sentiment classification**



How to compute sentence encoding?

Usually better:
Take element-wise max or mean of all hidden states

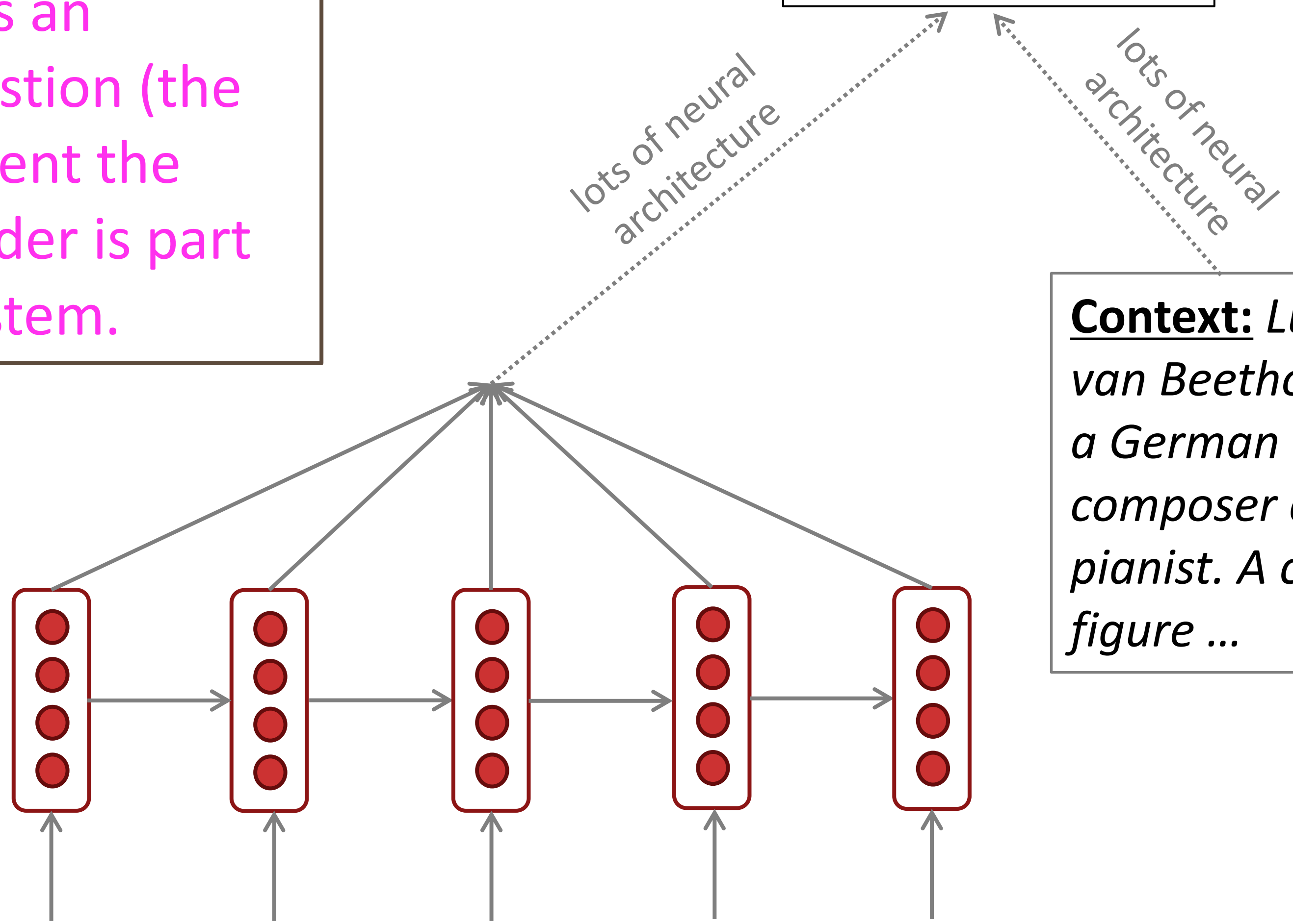
RNNs can be used as an encoder module

e.g., **question answering**, machine translation, *many other tasks!*

Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.

Answer: German

QA task: Given a context and a question, extract the answer from the context



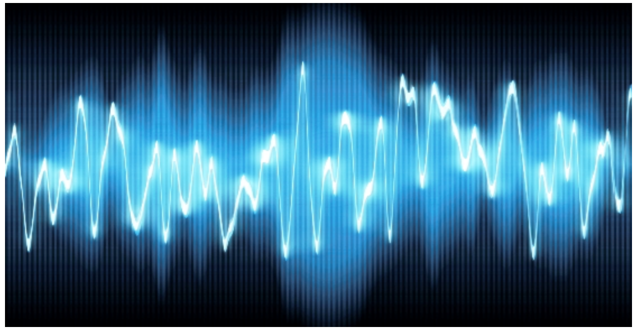
Context: *Ludwig van Beethoven was a German composer and pianist. A crucial figure …*

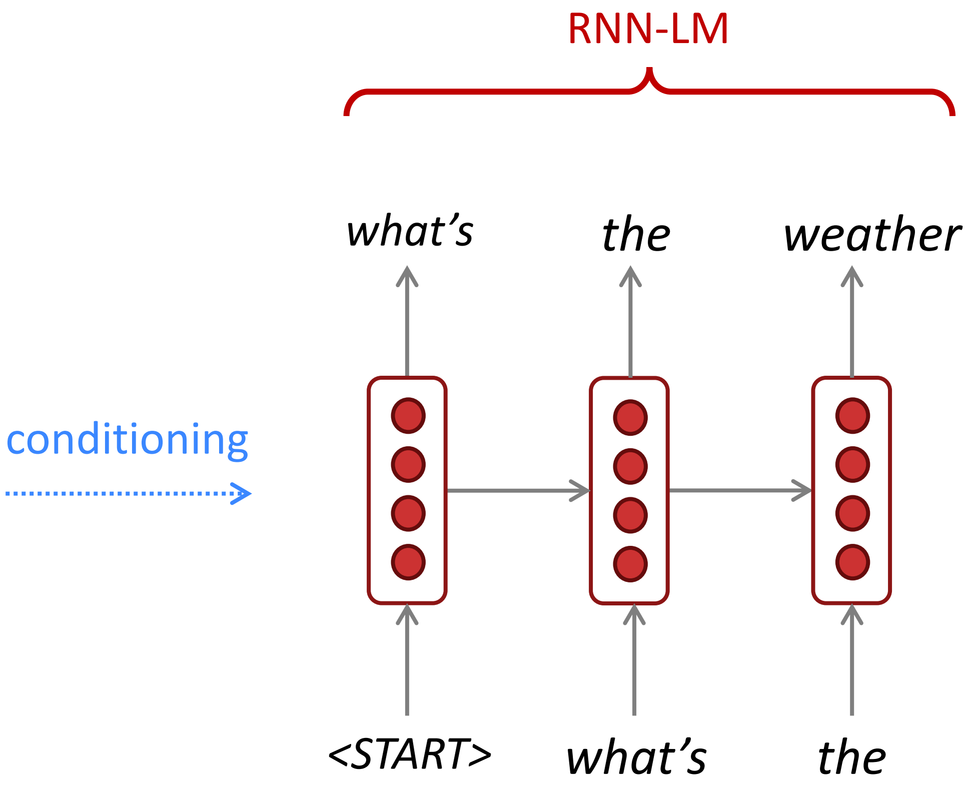
Question: *what nationality was Beethoven ?*

RNN-LMs can be used to generate text

e.g., **speech recognition**, machine translation, summarization

Input (audio)





Input is an audio signal. We use some representation of the input audio as the initial hidden state of the RNN LM, and then ask it to generate text.

This is an example of a *conditional language model*.

We'll see Machine Translation in much more detail later.

Terminology and a look forward

The RNN described in this lecture = **simple**/vanilla/**Elman** RNN

Next lecture: You will learn about other RNN flavors

like GRU  and LSTM  and multi-layer RNNs