

**Instruction:** There are 14 questions. Every question carries 3 marks. In case of any doubt or mistake any query should not be made. All mistakes of the question paper should be written on a page by drawing a box. For identifying every mistake in a question (or in example) 1 mark will be given. Either the mistake should be corrected or justification should be given. Use of calculators is not permitted. Rough work involved in the question must be shown. However final answer should be written in a box.

1. Mohan goes to school according to following.

- (A) Mohan does not go to school if his mood is less than 38.  
 (B) Mohan does not go to school if his mood is less than 60 and there is rain.  
 (C) If his mood is more than (or equal to 60) and there is no rain then he will go to school.  
 (D) If his mood is more than 78 then he will go to school.

Assume that the mood is an integer. The smallest possible value of mood is 0 and the biggest possible value of mood is 99.

Example 1: When no information is available then the average value of mood is  $(0+99)/2=49.5$ .

Example 2: When it is known that Mohan has not gone to school then the average value of mood is  $(0+78)/2=39$ .

Example 3: When it is known that Mohan has not gone to school and there is no rain then the average value of mood is  $(0+59)/2=29.5$ .

Question: Today there is no rain and Mohan has gone to school. What is the average value of mood?

2. When a person suffers from Flu then viruses a, b, c, d, e and f are developed in blood. When a person suffers from Plague then viruses a, b, g, h and j are developed in blood. When a person suffers from Malaria then viruses a, c, k, and m are developed in blood. Ram, Hari, Gopal and Sani are experts in observing viruses. Ram and Hari observe the blood of Kapil. Ram is able to observe virus 'b'. Hari is able to observe virus 'c'. What is the evidence that Kapil is suffering from Flu? Let observing power of Ram and Hari be 0.56 and 0.3 respectively?

[Example: Let Gopal and Sani observe Jalaj. Let observing powers of Gopal and Sani are 0.4 and 0.75 respectively. Suppose Gopal observes 'h' and Sani observes 'k' then Evidence that Jalaj is suffering from Plague is  $(0.4*0.25)/(1-0.4*0.75)=0.143$ .]

3. Numbers: 4216, 3917, 3829, 2138, 1942, 1841, 1187, 327, 291, 283, 112, 94, 36 Ram is supposed to take a number from these. After that Hari is supposed to take a number which is lesser than the number taken by Ram. When Ram takes a number he gets points which are double of the number taken. When Hari takes the number, he gets points which are three times of the number taken. The difference between the points is calculated. If Ram has got more points then Hari is supposed to pay Ram some money. If Hari gets more points then Ram is supposed to pay to Hari. The amount of the money is equal to the difference. Assume both Ram and Hari want to get as much money as possible. Also assume that Ram is not supposed to take the last number.

[Example: Let Ram takes 327 and Hari takes 283 then Ram will get 654 ( $2*327$ ) points. Hari gets 849 ( $3*283$ ). Hence Ram will give 195 ( $849-654$ ) money to Hari.]

[Example: Let Ram takes 3917 and Hari 1942. In this case Hari will give 2008 ( $2*3917-3*1942$ ) money to Ram.]

4. ABCD...T is an object. The path followed by three robots is shown. What is missing?

From X:  $Xa(212,0^0)$ ,  $aA(15,90^0)$ ,  $AB(300,180^0)$ ,  $BC(300,270^0)$ ,  $CD(22,360^0)$ ,  $DE(130,450^0)$ ,  $EF(50,360^0)$ ,  $FG(90,270^0)$ ,  $GH(56,360^0)$ ,  $HI(127,450^0)$ ,  $IJ(72,360^0)$ ,  $JK(300,270^0)$ ,  $KL(300,180^0)$ ,  $LM(500,90^0)$ ,  $MN(200,0^0)$ ,  $Nb(80,0^0)$ ,  $bQ(60,90^0)$ ,  $QR(220,0^0)$ , out

From Y:  $Yc(20,0^0)$ ,  $cI(12,90^0)$ ,  $IJ(72,0^0)$  out

From Z:  $Zd(12,0^0)$ ,  $dE(21,90^0)$ , missing

Assume a, b, c, d, and e are points on edge TA, PQ, HI, DE and HI respectively.

5. Let 90% elements of the set A are in set C. Let 90% elements of the set B are in set C. Hence it can be concluded that at most x% elements of the set  $A \cup B$  are in C]. What is the value of x?  
[Example: Let 90% elements of the set A are in set C. Let 90% elements of the set B are in set C. Hence it can be concluded that at least 81.81% elements of the set  $A \cup B$  are in C]
6. In a game Ram and Dipu write numbers one after another. If 'x' is written by Ram then  $x+36$  or  $x+67$  or  $x+98$  can be written by Dipu. If 'x' is written by Dipu then  $x+59$  or  $x+a$  or  $x+b$  can be written by Ram. The game continues till a number, which is a multiple of either 'p' or 'q' or 'r' or 's' is written. If Ram writes a multiple of 'p' then Ram wins. If Dipu writes a multiple of 'q' then Dipu wins. The player who writes a multiple of 'r' wins. The player who writes a multiple of 's' loses. Let 853, 865 and 915 are found to be bad numbers for Dipu to write. Let 943 is found to be a good number for Ram to write. On the basis of this information find one more good number for Ram to write. Here a, b, p, q, r, s are some positive integers.
7. Ram has to choose one among A, B or C. Hari has to choose one among D, E or F. The profit matrix is given. What will be the profit of each of them? Assume that they are taking the decision without knowing the decision of another. [Notation: When Ram chooses A and Mohan chooses E then profit of Ram is 90 and profit of Hari is 34]. The answer of this question must be written in a box. The box is very important.

Fact				Ram Thinks				Hari Thinks			
	D	E	F		D	E	F		D	E	F
A	12,45	90,34	56,78	A	51,94	22,86	11,41	A	12,34	45,36	19,39
B	36,79	32,45	26,31	B	93,4	46,3	29,56	B	56,78	29,56	17,12
C	82,96	83,46	38,94	C	24,5	22,5	23,78	C	12,78	30,56	12,32
Ram Thinks that Hari thinks				Hari thinks that Ram thinks							
	D	E	F		D	E	F				
A	23,45	56,78	24,67	A	29,61	43,56	22,19				
B	23,67	89,34	90,23	B	23,66	23,56	12,41				
C	34,56	67,89	23,45	C	12,89	22,56	20,21				

8. Anil gives answer of a question by adding 12. Dipu gives by adding 16. Hari gives by multiplying by 2. Let A, B, ..., Z be questions. Answers of few of these questions by Anil, Dipu and Hari are given. Among these which are certainly context sensitive question? [Definition: The context sensitive sentences involve words like "you", "now", etc.]. Write answer in the form BCF.

	Anil	Dipu	Hari
A	118	121	210
B	23	26	20
C	100	104	176
D	62	66	100
E	46	50	68
F	31	33	34

9. There are two pots of capacity 73 and 89 respectively. Using them we want to measure 31 liters of milk. Following production rules are used. However if total amount of milk is 127 then three of these rules are modified. Write the modified rules.

[Hint: from (12,75) we can get (0,87) (73,14) (12,0) (0,75) or (12,89) but not (73,75)].

[Write answer in the form: 3<sup>rd</sup> rule is if  $x > y + 65$  then [0,y] else [65-x,y]]

[Notation: [x,y] means that the amount of milk in the first pot is 'x' and in the second pot is 'y']

Operation	Rule: What can be achieved from [x,y] and when?
Fill first pot	[73,y]
Fill second pot	[x,89]
Make first pot empty	[0,y]
Make second pot empty	[x,0]
Transfer from first pot to the second pot	if $(x+y > 89)$ then $[x+y-89,89]$ else $[0,x+y]$
Transfer from second pot to the first pot	if $(x+y < 73)$ then $[73,x+y-73]$ else $[x+y,0]$
Stop	When $x=31$ or $y=31$

10. A cake is supposed to be cut under some mechanism. Initially Hari is asked whether he is willing to pay amount 'x' for cutting. If Hari says "no" then Dipu is asked. If Dipu also says "no" then Anil has to cut.

(A) When Hari cuts then Hari, Dipu and Anil get profits of 100, 70 and 150 respectively.

(B) When Dipu cuts then Hari, Dipu and Anil get profits of 90, 127 and 251 respectively.

(C) When Anil cuts then Hari, Dipu and Anil get profits of 56, 107 and 110 respectively.

For  $0 < x < 10$  Hari will say "yes" because  $100 - x > 90$ .

For  $10 < x < 20$  Hari will say "no" because  $100 - x < 90$  and  $127 - x > 107$ .

For  $20 < x < 44$  Hari will say "yes" because  $100 - x > 56$  and  $127 - x < 107$ .

For  $x > 44$  Hari will say "no" because  $100 - x > 56$  and  $127 - x < 107$ .

Hence the range of 'x' values for which Hari will say "yes" is  $0 < x < 10$  and  $20 < x < 44$ .

When the cost of cutting is 6 (i.e.  $x=6$ ) then Hari thinks if he (Hari) cuts then he will get  $100 - 6 = 94$ .

If either Dipu or Anil cuts then he will get 90 and 56 respectively. Hence cutting is profitable.

When  $x=16$  then Hari thinks if he (Hari) cuts then he will get  $100 - 16 = 86$ . If he does not cut then Dipu will become agree to cut. Hence Hari will get 90.

When  $x=26$  then Hari thinks if he (Hari) cuts then he will get  $100 - 26 = 74$ . If he does not cut then Dipu will also not cut. When Anil will cut then Hari will get only 56.

For  $x=16$  Dipu will become agree to cut. It is because by cutting Dipu will get  $127 - 16 = 111$ . By not cutting Dipu will be getting 107.

For  $x=26$  Dipu will not become agree to cut. It is because by cutting he will get  $127 - 26 = 101 < 107$ .

Question: Let the profit distribution be according to following. Give the range of 'x' values for which Hari will say "yes".

(A) When Hari cuts then Hari, Dipu and Anil get profits of 100, 70 and 150 respectively.

(B) When Dipu cuts then Hari, Dipu and Anil get profits of 110, 500 and 251 respectively.

(C) When Anil cuts then Hari, Dipu and Anil get profits of 67, 484 and 110 respectively.

11. The fuel supply to an engine is controlled by following rules.

Rule 1: If temperature is  $(22 \pm 16)^{\circ}\text{C}$  then fuel should be supplied with a rate  $(300 \pm 40)$ .

Rule 2: If temperature is  $(48 \pm 10)^{\circ}\text{C}$  then fuel should be supplied with a rate  $(600 \pm 200)$ .

Rule 3: If temperature is  $(60 \pm 20)^{\circ}\text{C}$  then fuel should be supplied with a rate  $(540 \pm 50)$ .

Rule 4: If temperature is  $(75 \pm 5)^{\circ}\text{C}$  then fuel should be supplied with a rate  $(483 \pm 100)$ .

Let temperature be  $49^{\circ}\text{C}$ . What should be the fuel supply rate?

Solution: The rules 1 and 4 are not applicable since  $49^{\circ}\text{C}$  is not  $(22 \pm 16)^{\circ}\text{C}$  and  $(75 \pm 5)^{\circ}\text{C}$ .

The rule 2 is 90% applicable because  $49^{\circ}\text{C}$  is  $[1 - (49 - 48)/10] = 90\%$  true in  $(48 \pm 10)^{\circ}\text{C}$ .

The rule 3 is 45% applicable because  $49^{\circ}\text{C}$  is  $[1 - (60 - 49)/20] = 45\%$  true in  $(60 \pm 20)^{\circ}\text{C}$ .

Suppose a person decides to supply fuel at the rate of 570.

According to rule 2 the fuel rate should be  $(600 \pm 200)$ . It is dishonored by  $(600 - 570)/200 = 15\%$ .

According to rule 3 the fuel rate should be  $(540 \pm 50)$ . It is dishonored by  $(570 - 540)/50 = 60\%$ .

Since the applicability of rules 2 and 3 is 90% and 45% hence the overall dishonor is maximum of  $15 \times 90$  and  $60 \times 45$ . It is  $\max(13.5\%, 27\%) = 27\%$ . We would like to minimize it.

Suppose a person decides to supply fuel at the rate of 560.

According to rule 2 the fuel rate should be  $(600 \pm 200)$ . It is dishonored by  $(600 - 560)/200 = 20\%$ .

According to rule 3 the fuel rate should be  $(540 \pm 50)$ . It is dishonored by  $(560 - 540)/50 = 40\%$ .

The overall dishonor is  $\max(20 \times 90, 40 \times 45) = 18\%$ . It is the best possible answer.

Hence the fuel rate is 560.

Question: What should be the fuel rate when the temperature is  $72^{\circ}\text{C}$ ?

P.T.O.

## Certainty about uncertainty

(17,49) (15,49) (15,47) (12,45) (12,44) (11,41) (11,43) (13,41) (13,43) (18,42) (16,48) (14,48) (14,49)
One of the pairs from above is randomly taken. The first number of the pair is given to Anil. The second number is given to Bimu.
Let Anil gets 17. Anil knows that Bimu has got 49. [With 17 only 49 is attached. With 49 three numbers 17, 14 and 15 are attached.] Now Bimu should be thinking whether Anil's number is 14, 15 or 17. Conclusion: Anil knows Bimu's number. Anil also knows that Bimu does not know Anil's number.
Let Anil gets 18. Anil knows that Bimu has got 42. Now Bimu knows that Anil has got 18. Conclusion: Anil knows Bimu's number. Anil also knows that Bimu knows Anil's number.
Let Anil gets 12. Anil knows that Bimu has either got 44 or 45. In both cases Bimu should be thinking that Anil's number is 12. Conclusion: Anil does not know Bimu's number. But Anil knows that Bimu knows Anil's number.
Let Anil gets 11. Anil knows that Bimu has either got 41 or 43. In both cases Bimu should be thinking that Anil's number is 11 or 13. Conclusion: Anil does not know Bimu's number. Anil knows that Bimu does not know Anil's number. Anil knows that Bimu is confused whether Anil's number is 11 or 13.
Let Anil gets 14. Anil knows that Bimu has either got 48 or 49. If Bimu has got 48 then Bimu should be thinking that Anil's number is 14 or 16. If Bimu has got 49 then Bimu should be thinking that Anil's number is 14 or 15 or 17. Conclusion: Anil does not know Bimu's number. Anil knows that Bimu does not know Anil's number. But Anil does not know Bimu's confusion. Anil thinks that there are following two possible confusions of Bimu * (A) Bimu is confused whether Anil's number is 14 or 16. (B) Bimu is confused whether Anil's number is 14 or 15 or 17.
Let Anil gets 15. Anil knows that Bimu has either got 47 or 49. If Bimu has got 47 then Bimu should be thinking that Anil's number is 15. If Bimu has got 49 then Bimu should be thinking that Anil's number is 14 or 15 or 17. Conclusion: Anil does not know Bimu's number. Anil also does not know whether Bimu knows Anil's number or not.

Pairs	Ram's thinking when his number is 6. Ram is confused about Hari's number.
(6,2)(6,3)	Ram is certain that Hari is not confused about Ram's number.
(6,2)(6,3)(5,2)(5,3)	Ram is certain that Hari's confusion about Ram's number is (5 or 6)
(6,2)(6,3)(5,2)(7,3)	Ram is certain that Hari is confused. But he does not know Hari's confusion. Hari confusion about Ram's number is either between 5,6 or between 6,7.
(6,2)(6,3)(5,3)	Ram is confused whether Hari is confused or not.

12. Pairs: (16,29) (16,44) (16,61) (76,29) (61,29) (81,44) (83,44) (11,61) (22,61) A pair is taken. The first number is given to Hari and the second number is given to Dipu. Let the pair is (16,44). What is Dipu thinking about Hari?

[Example: Hari is thinking about Dipu that he has got either 29, 44 or 61]

13. Pairs: (11,27) (11,42) (48,42) (61,29) (61,82) (98,29) A pair is taken. The first number is given to Gyan and the second number is given to Jalaj. Here Gyan knows the number of Jalaj but Jalaj does not know the number of Gyan. What is the Pair? Write all possible answers in the same box. [Example: When Jalaj knows the number of Gyan, but Gyan does not know the number of Jalaj then possible pairs are (11,27) or (61,82). ]

14. Pairs: (26,29) (26,47) (26,61) (26,21) (19,29) (45,29) (19,47) (84,21) (58,21) A pair is taken. The first number is given to Kapil and the second number is given to Lalit. Let Kapil gets 26. Now Kapil is thinking that Lalit is either confused between 26, 19 and 45 or confused between 26 and 19 or confused between \_\_\_\_\_ or knows the number of Kapil. [Fill in the blanks]