Generating discrete distn:

Inverse Transform method:

r.v. $X \overset{pmf}{\sim} P(X = x_j) = p_j$; $j = 0, 1, 2, \ldots$

$$, \sum_{j=0}^{\infty} p_j = 1$$

$U \sim U(0,1)$

Set
$$X = \begin{cases} x_0 & \text{if} \quad U < p_0 \\ x_1 & \text{if} \quad p_0 \le U < p_0 + p_1 \\ \vdots & \\ x_j & \text{if} \quad \underline{\sum_{i=1}^{j-1} p_i \le U < \sum_{i=1}^{j} p_j} \\ \vdots & \end{cases}$$

$$P(X = x_j) = P\left(\sum_{i=1}^{j-1} p_i \le U < \sum_{i=1}^{j} p_j \right)$$

$$= \int_{\sum_{i=1}^{j-1} p_i}^{\sum_{i=1}^{j} p_i} 1 \, du$$

$$= p_j$$

Example ① $X = 1$ wp $P(X=1) = 0.2$

$2 \qquad P(X=2) = 0.3$

$3 \qquad P(X=3) = 0.5$

U

Set $X = \begin{cases} 1 & \text{if} \quad U < 0.2 \\ 2 & \text{if} \quad 0.2 \le U < 0.5 \\ 3 & \text{if} \quad 0.5 \le U \le 1 \end{cases}$

(2) $P(X=i) = \frac{1}{n}, \quad i = 1, \dots, n$

$$X = i \equiv \frac{i-1}{n} \le U < \frac{i}{n}$$

$$\equiv i-1 \le nU < i$$

U then set $X = [nU] + 1$