Indian Institute of Technology, Kharagpur
Mid-Autumn Semester Examination: 2018–2019

Date of Examination:........-11-2018 Session (FN/AN), *Duration: 3 Hrs,*
Subject. No. MA31020/MA41025 *No. of Registered Students (123+64)=187*
Subject Name: REGRESSION AND TIME SERIES MODEL
Department: Mathematics *TOTAL MARKS: 50*
Specific Chart, graph paper log book etc. required.... STATISTICAL TABLE...
Special Instruction: ***Begin to answer each question in a new page. Answer all parts of a question in a coherent place. Full credit will be given for the answers which are correct up to FOUR decimal places.*** **ANSWER ALL THE QUESTIONS**

1. For the model $\underset{\sim}{\mathbf{Y}} = \mathbf{X}\underset{\sim}{\beta} + \underset{\sim}{\epsilon}$, where $\underset{\sim}{\epsilon} \sim N(\underset{\sim}{0}, \sigma^2 \mathbf{I}_n), \underset{\sim}{\mathbf{Y}} \in \mathbb{R}^n$, $\underset{\sim}{\beta} \in \mathbb{R}^{(k+1)}$use Jackknife method to test at 5% level for the null hypothesis that the observation y_5 is not an outlier based on the following estimates. Residual $e_5 = 2.10$, $MSResudual = 1.04$ and 5^{th} diagonal element of projection matrix $h_{55} = 0.036$, where $n = 25$, $k = 6$. State the conclusion. [6]

2. (a) What is the definition of positive definite matrix?
   (b) What is the definition of positive semidefinite function?
   (c) Show that an auto-covariance function of a time series is a positive semidefinite function.
   . [2+2+2]

3. Consider an $AR(1)$ process $X_t = 0.35X_{t-1} + Z_t$ where $Z_t \sim WN(0, 2.5)$. Suppose the observed values of $X_3 = 3.7, X_4 = 2.4$, $X_6 = 1.3$, but X_5 is missing.
   (a)Approximate the missing value of X_5 based on the available information which minimizes the least squared error.
   (b) Find the mean squared error in approximation of X_5. [3+3]

4. Let $X_t = Z_t + \theta Z_{t-1}$, $Z_t \sim WN(0, \sigma^2)$ and $Y_t = W_t + \frac{1}{\theta} W_{t-1}$, $W_t \sim WN(0, \theta^2\sigma^2)$ be two independent MA(1) processes with $|\theta| < 1, \sigma > 0$. Find the value of of $\gamma_X(h) - \gamma_Y(h)$ for all $h = 0, 1, 2, \ldots$ [6]

5. For a given set of input data $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ consider a set of orthogonal polynomials of corresponding degrees $\{P_0, P_1, P_2, \cdots, P_k\}$ such that $y_i = \sum_{j=0}^{k} \alpha_j P_j(x_i) + \epsilon_i \; \forall i = 1, 2, \cdots n$. Show that $SSRes = SSTotal - \sum_{j=1}^{k} (\hat{\alpha}_j \sum_{i=1}^{n} P_j(x_i) y_i)$. Where ϵ_i are independent errors and $\hat{\alpha}_j$ are the least squared estimates of the corresponding coefficients. [6]

6. Let $\{X_t\}$ be a stationary time series with mean zero. Suppose that the coefficients of (X_4, X_3, X_2, X_1) are (a_1, a_2, a_3, a_4) to estimate X_5 . Now to estimate the same if $(X_4, X_3, X_2, X_1, X_0)$ are used then the coefficient of X_0 is a_5. If all coefficients are estimated by Durbin-Levinson Algorithm then show that $|a_5| \le 1$. [6]

7. Let X_{n+1} is predicted by $\hat{\mathbf{a}}^T \mathbf{X}_n$ as a best linear predictor under square error for a stationary time series $\{X_t\}$ with mean zero. Then find the value of $cov((X_{n+1} - \hat{\mathbf{a}}^T \mathbf{X}_n), X_2)$ where $\mathbf{X}_n = (X_n, X_{n-1}, \cdots X_1)$. [6]

[P.T.O]

8. Stat TRUE or FALSE to the following statements. **[Justification not needed].** [8 × 1]

(a) If A is an projection matrix then $Rank(A) = Trace(A)$

(b) If $Var(y)$ is proportional to $E(y)(1-E(y))$, then the variance stabilizing transformation is $\sin^{-1}\sqrt{y}$ where $y \in (0,1)$.

(c) If $\{X_t\}$ is a strongly stationary time series, then $\{X_t\}$ is always weakly stationary.

(d) If $\{W_t\}$ is white noise , then W_i and W_j are always independently distributed for $i \neq j$.

(e) When $X_t - 0.5X_{t-1} = Z_t + 2Z_{t-1}$ and $\{Z_t\}$ is WN then $\{X_t\}$ is an invertible time series

(f) Innovations algorithm for prediction uses the prediction error of the past data.

(g) $\nabla_2 X_t = (1-B)(1+B)X_t$

(h) For an seasonal $ARIMA(1,2,3) \times (4,5,6)_7$ representation the degrees of B operator associated to time series X_t and white noise W_t are 25 and 39 respectively.

****************** THE END ******************