MA 69204 Statistical Software Lab

Assignment No. 1

(a) Generation of a random sample from an exponential distribution

$$f(x) = \frac{1}{\sigma}\, Exp\left\{-\left(\frac{x-\theta}{\sigma}\right)\right\}, x > \theta, \sigma > 0.$$

Use Probability Integral Transform for generation. Apply a Chi-square test to justify the sample.

(b) Generation of a random sample from a Cauchy Distribution

The density function is

$$f(x) = \frac{\sigma}{(\pi\{\sigma^2+(x-\mu)^2\}}\ , \qquad x, \mu \in \mathbb{R}, \sigma > 0.$$

Use Probability Integral Transform for generation. Further apply Chi-square test to justify the sample.

(c) Generation of a random sample from a Double Exponential (Laplace) Distribution

The density function is

$$f(\mathrm{x}) = \frac{1}{2\sigma}\,\mathrm{Exp}\left\{-\frac{|x-\mu|}{\sigma}\right\}, \qquad x, \mu \in \mathbb{R}, \sigma > 0\,.$$

Algorithm: Generate U_1 and $U_2 \sim U(0,1)$. If $U_1 \leq 0.5$, $X = \ln U_2$, else, $X = -\ln U_2$.

Apply a Chi-square test to justify the sample.

(d) Generation of a Right Trapezoidal Distribution

The density function is:

$$f(x) = \begin{cases} a + 2(1-a)x, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Algorithm: Generate $U_1 \sim U(0,1)$. If $U_1 \leq a$, generate $U_2 \sim U(0,1)$ and $X = U_2$.

If $U_1 > a$, generate $U_2, U_3 \sim U(0,1)$ and $X = max\,(U_2, U_3)$.

Apply a Chi-square test to justify the sample.

(e) Two Algorithms for Generation of Standard Normal Variables

Algorithm 1: Generate $U_1, U_2 \sim U(0,1)$. Define

$X_1 = (-2\ln U_1)^{\frac{1}{2}}\cos(2\pi U_2)$ and $X_2 = (-2\ln U_2)^{\frac{1}{2}}\sin(2\pi U_2)$.

Then $X_1, X_2 \sim N(0,1)$.

Algorithm 2:

Step I: Generate $U_1, U_2 \sim U(0,1)$ and let $V_i = 2U_i - 1, i = 1,2, W = V_1^2 + V_2^2$.

If $W > 1$, go to Step I, else go to Step II.

Step II: Let $Y = \left(-\frac{2\ln W}{W}\right)^{\frac{1}{2}}$, and $X_1 = V_1 Y, X_2 = V_2 Y$. Then $X_1, X_2 \sim N(0,1)$.

Remark: To generate $N(\mu, \sigma^2)$, transform $N(0,1)$ variable X by $\sigma X + \mu$.

(f) Two Algorithms for Generation of Gamma Random Variables

Gamma (α, β) density:

$$f(\mathrm{x}) = \frac{1}{\beta^\alpha \Gamma(\alpha)} Exp\left\{-\frac{x}{\beta}\right\} x^{\alpha-1}, x > 0, \alpha > 0, \beta > 0.$$

To generate Gamma (α, 1)

Algorithm 1: For $0 < \alpha < 1$; let $b = 1 + \frac{\alpha}{e}$.

Step I: Generate $U_1 \sim U(0,1)$ and let $P = bU_1$.
If $P > 1$, go to Step III, else go to Step II.

Step II: Let $Y = P^{\frac{1}{\alpha}}$ and generate $U_2 \sim U(0,1)$. If $U_2 \le e^{-Y}$, return $X = Y$, otherwise go back to Step I.

Step III: Let $Y = -\ln\left(\frac{b-P}{\alpha}\right)$ and generate $U_2 \sim U(0,1)$. If $U_2 \le Y^{\alpha-1}$, return $X = Y$, otherwise go back to Step I.

Algorithm 2: For $\alpha > 1$; let $a = (2\alpha - 1)^{-\frac{1}{2}}$, $b = \alpha - ln\, 4$, $q = \alpha + \frac{1}{a}$, $\theta = 4.5$ and $d = 1 + \ln\theta$

Step I: Generate $U_1, U_2 \sim U(0,1)$.

Step II: Let $V = a\ln\frac{U_1}{1-U_1}$, $Y = \alpha\, e^V$, $Z = U_1^2 U_2$ and $W = b + qV - Y$.

Step III: If $W + d - \theta Z \ge 0$, return $X = Y$, otherwise go to Step IV.

Step IV: If $W \ge \ln Z$, return $X = Y$, otherwise go back to Step I.

Remark: If $X \sim Gamma\ (\alpha, 1)$, then $\beta X \sim Gamma\ (\alpha, \beta)$.

Generate 1000 random variates using each algorithm and apply Chi-square test for goodness of fit in each case. For calculating probabilities related to normal and gamma distributions in applying goodness of fit test, use numerical integration (Simpson's one-third rule etc.).