MA 69204 Statistical Software Lab

Assignment No. 3

Rejection Method for Generating Random Variables

Continuous Case: Suppose we have a method for simulating a random variable having density function $\boldsymbol{g(y)}$. We can use this to simulate from a continuous distribution having density $\boldsymbol{f(x)}$.

Step 1: Simulate $\boldsymbol{Y}$ having density $\boldsymbol{g}$ and simulate a random number $\boldsymbol{U}$.

Step 2: If $\boldsymbol{U \leq \frac{f(Y)}{cg(Y)}}$, set $\boldsymbol{X = Y}$. Otherwise return to Step 1.

Then $\boldsymbol{X}$ has density $\boldsymbol{f}$.

Exercise 1: To generate a random variable having beta density

$$\boldsymbol{f(x) = 20x(1-x)^3 \quad , 0 < x < 1.}$$

Step1: Generate random numbers $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$.

Step 2: If $\boldsymbol{U_2 \leq \frac{256}{27} U_1(1-U_1)^3}$, stop and set $\boldsymbol{X = U_1}$. Otherwise return to Step 1.

Exercise 2: Generating a standard normal variable.

Step 1: Generate $\boldsymbol{Y_1}$, and exponential random variable with rate 1.

Step 2: Generate $\boldsymbol{Y_2}$, and exponential random variable with rate 1.

Step 3: If $\boldsymbol{Y_2 > \frac{1}{2}(Y_1-1)^2}$, set $\boldsymbol{Y = Y_2 - \frac{1}{2}(Y_1-1)^2}$ and go to Step 4. Else go to Step 1.

Step 4: Generate a uniform random number $\boldsymbol{U \sim U[0,1]}$ and set

$$\boldsymbol{Z = \begin{cases} Y_1, & \text{if } U \leq 0.5 \\ -Y_1 & \text{if } U > 0.5 \end{cases}}.$$

The random variables $\boldsymbol{Z}$ and $\boldsymbol{Y}$ generated by the above are independent with $\boldsymbol{Z}$ being standard normal and $\boldsymbol{Y}$ exponential with rate 1.

Generate 1000 random variates using each algorithm and apply Chi-square test for goodness of fit in each case. For calculating probabilities related to beta and normal distributions in applying goodness of fit test, use numerical integration (Simpson's one-third rule etc.). Compare this generation of normal variable with the algorithms in Experiment No. 1 in terms of computational times.

Discrete Case: Suppose we have a method for simulating a random variable having probability mass function $\{\boldsymbol{q_j}, \boldsymbol{j} \geq \boldsymbol{0}\}$**.** We can use this to simulate from a discrete distribution having probability mass function density $\{\boldsymbol{p_j}, \boldsymbol{j} \geq \boldsymbol{0}\}$.

Step 1: Simulate $\boldsymbol{Y}$ having density $\boldsymbol{q_j}$ and simulate a random number $\boldsymbol{U}$.

Step 2: If $\boldsymbol{U} \leq \frac{\boldsymbol{p_Y}}{\boldsymbol{cq_Y}}$, set $\boldsymbol{X} = \boldsymbol{Y}$. Otherwise return to Step 1.

Then $\boldsymbol{X}$ has density $\boldsymbol{p_j}$**.**

Exercise 3: To generate a random variable $\boldsymbol{X}$ that takes one of the values $\boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{10}$ with respective probabilities $\boldsymbol{0.11}, \boldsymbol{012}, \boldsymbol{0.09}, \boldsymbol{0.08}, \boldsymbol{0.12}, \boldsymbol{0.10}, \boldsymbol{0.09}, \boldsymbol{0.09}, \boldsymbol{0.10}, \boldsymbol{0.10}$**.**

Step 1: Generate a random number $\boldsymbol{U_1}$ and set $\boldsymbol{Y} = [\boldsymbol{10U_1}] + \boldsymbol{1}$.

Step 2: Generate a second random number $\boldsymbol{U_2}$.

Step3: If $\boldsymbol{U_2} \leq \frac{\boldsymbol{p_Y}}{.\boldsymbol{12}}$, set $\boldsymbol{X} = \boldsymbol{Y}$ and stop. Otherwise return to Step 1.