RTSM/QUIZ/1

Fill in the blanks (Numerical)

Date of Exam : 10thSep, 2021

Time : 08:05 am to 9:00 am

Duration : 50min

No of questions: 10 out of 15 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

September 21, 2021

1. Let $X \sim N(0, 4)$ and $Y|X = x \sim N(x^2, 9)$. Then the the correlation between X and Y is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0
   ERROR RANGE: ∓ 0.005

2. Consider a bivariate normal model $N(2, 3, 4, 9, 0.5)$. Then the variance of the regression of Y for given $X = 1/3$ is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 6.75
   ERROR RANGE: ∓ 0.005

3. For the given data $\{(y_i, x_i), i = 1, 2, \cdots, n\}$, consider the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where ϵ_i are *i.i.d.* $N(0, \sigma^2)$. Find the value of $Cov(\hat{\beta}_0, \hat{\beta}_1)$, where $\hat{\beta}_0$ and $\hat{\beta}_1$ is the corresponding least squared estimator of β_0, β_1 and $\sigma = 1.2, \bar{x} = 3.5, \sum(x_i - \bar{x})^2 = 4.2$.

   [Answer only within the error range ∓ 0.005 will get the credit]

   ANS: -1.2
   ERROR RANGE: ∓ 0.005

4. Let $X_i \sim N(i, i)$ for $i = 1, 2, \ldots 10$ and they are independently distributed. What is the non-centrality parameter of the distribution of $\sum_{i=1}^{10} \frac{X_i^2}{i}$

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 55
   ERROR RANGE: ∓ 0.005

5. Let $\mathbf{Z} = (z_1, z_2)^T \sim N((3, 1)^T, \mathbf{I}_2)$, and $\mathbf{A} = \begin{pmatrix} 3 & -6 \\ 1 & -2 \end{pmatrix}$, then $\mathbf{E}(\mathbf{Z^T A Z})$ is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 11
   ERROR RANGE: ∓ 0.005
   HINT: $3(E(Z_1^2) - 5E(Z_1 Z_2) - 2E(Z_2^2)) = 3 * (1 + 9) - 5 * (3 * 1) - 2 * (1 + 1) = 11$

6. Consider the dataset $\mathbb{D} = \{(-1, -2.53), (-3, -7.51), (0, .01), (2, 5.03), (7, 17.59)\}$ and a Linear Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad where \;\; \epsilon_i \sim N(0, \sigma^2) \forall \; i = 1, 2, 3, 4, 5$$

   Then the maximum likelihood estimate of $100\sigma^2$ is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 0.23272 CORRECTED TO 0.0193931
   ERROR RANGE: ∓ 0.005

7. Let $S_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 \leq 4\}$ and S_2 be a subspace of $\mathbb{R}^3$ such that $\dim(S_2) = 2$. Then the area of $S_1 \cap S_2$ is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANS: $4\pi = 12.566$
   ERROR RANGE: ∓ 0.005 changed to ∓ 0.05
   HINT: $S_1 \cap S_2$ is a circle of radius 2.

8. A variety of summary statistics were collected for a small sample (10) of bivariate data, where the dependent variable was y and an independent variable was x. And we have, $\sum(y_i - \bar{y})(x_i - \bar{x}) = 466$,$\sum(y_i - \bar{y})^2 = 1434$, $\sum(x_i - \bar{x})^2 = 234$. Then MSResidual is

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 63.25
   ERROR RANGE: ∓ 0.005

9. Let the true model be $y_i \sim N(1 + 2x_i, 4)$ and $S_{xx} = 5.7$ Find the non-centrality parameter of $\frac{SSModel}{4}$

   [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

   ANSWER : 1.425
   ERROR RANGE: ∓ 0.005

10. Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ where ϵ_i s are independently and identically distributed $N(0, \sigma^2)$ variables. Here x variable stands for the length of a pendulum in $\log_{10}$ scale and y variable stands for the measured time period of it in the same scale.

x	1.04	1.09	1.16	1.12	1.17	1.25	1.26	1.29
y	0.818	0.845	0.899	0.865	0.890	0.946	0.938	0.935

   Find the value if y in usual scale when $x = 1.1725$ in $\log_{10}$ scale.

   [Answer only within the error range ∓ 0.005 will get the credit]

ANSWER : 7.798

ERROR RANGE: ∓ 0.005

11. The following matrix A is a positive definite covariance matrix if $\rho > c$

$$A = \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}$$

Find the value of c.

ANSWER : -0.5

ERROR RANGE: ∓ 0.005

12. Consider the observations from two populations $\bar{x}_1 = 25.1700$, $S_1^2 = 133.2010$ with sample size $n_1 = 10$ and $\bar{x}_2 = 29.3933$, $S_2^2 = 177.9093$ sample size $n_2 = 15$. Find the value of the t-statistic to test that the two populations have same mean against the means are unequal under the independent normality assumption with equal variance the of populations.

[Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

ANSWER : 2.8128 or - 2.8128 or 0.8167807or -0.8167807

ERROR RANGE: ∓ 0.005

HINT Estimated variance (133.2010+177.9093)/23=311.1103/23=13.5265. or (9*133.2010+14*177.9093)/23=160.4147

13. Consider the observations from two populations $\bar{x}_1 = 16.76$, and $\bar{x}_2 = 17.04$ with equal sample size $n = 10$ and pulled slandered deviation is 0.284. Find the lower bound of 95% confidence interval $\mu_1 - \mu_2$ under the independent normality assumption of populations.

[Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

ANSWER : -0.5468

ERROR RANGE: ∓ 0.005

14. A random sample of size 11 are drawn from the pdf $f(x) = 0.5\ e^{-|x-\mu|}$. Find the MLE of μ when samples are $\{-0.84, 1.08, -0.16, -0.66, 0.73, 0.06, -0.49, 1.64, -1.12, -1.59, 1.06\}$.

[Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

ANSWER : -0.16
ERROR RANGE: ∓ 0.005

15. Consider the observations from two populations $\bar{x}_1 = 16.76$, and $\bar{x}_2 = 17.04$ with equal sample size $n = 10$ and pulled slandered deviation is 0.284. Find the length of the of 95% confidence interval $\mu_1 - \mu_2$ under the independent normality assumption of populations.

    [Answer must be in decimal. Answer only within the error range ∓ 0.005 will get the credit]

    ANSWER : 0.5337
    ERROR RANGE: ∓ 0.005