Discounted Payback Period

The *discounted payback period* is the time needed to pay back the original investment in terms of *discounted* future cash flows. Each cash flow is discounted back to the beginning of the investment at a rate that reflects both the time value of money and the uncertainty of the future cash flows. This rate is the cost of capital—the return required by the suppliers of capital (creditors and owners) to compensate them for the time value of money and the risk associated with the investment. The more uncertain the future cash flows, the greater the cost of capital.

From the perspective of the investor, the cost of capital is the *required rate of return* (RRR), the return that suppliers of capital demand on their investment (adjusted for tax deductibility of interest). Because the cost of capital and the RRR are basically the same concept but from different perspectives, we sometimes use the terms interchangeably in our study of capital budgeting.

Returning to Investments A and B, suppose that each has a cost of capital of 10%. The first step in determining the discounted payback period is to discount each year's cash flow to the beginning of the investment (the end of the year 2000) at the cost of capital:

	Investment A		Investment B	
Year	End of Year Cash Flow	Value at the End of 2000	End of Year Cash Flow	Value at the End of 2000
2001	$400,000	$363,636	$100,000	$90,909
2002	400,000	330,579	100,000	82,644
2003	400,000	300,526	100,000	75,131
2004	400,000	273,205	1,000,000	683,013
2005	400,000	248,369	1,000,000	620,921

How long does it take for each investment's discounted cash flows to pay back its $1,000,000 investment? The discounted payback period for A is four years:

	Investment A		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$363,640	$363,640	
2002	330,580	694,220	
2003	300,530	994,750	
2004	273,205	1,267,955	⇦ $1,000,000 investment paid back
2005	248,369	1,516,324	

The discounted payback period for B is five years:

	Investment B		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$90,910	$90,910	
2002	86,240	177,150	
2003	75,130	252,280	
2004	683,010	935,290	
2005	620,921	1,556,211	⇦ $1,000,000 investment paid back

This example shows that it takes one more year to pay back each investment with discounted cash flows than with nondiscounted cash flows.

Discounted Payback Decision Rule

It appears that the shorter the payback period, the better, whether using discounted or nondiscounted cash flows. But how short is better? We don't know. All we know is that an investment "breaks-even" in terms of discounted cash flows at the discounted payback period—the point in time when the accumulated discounted cash flows equal the amount of the investment. Using the length of the payback as a basis for selecting investments, A is preferred over B. But we've ignored some valuable cash flows for both investments.

Discounted Payback as an Evaluation Technique

Here is how discounted payback measures up against the three criteria.

Criterion 1: Does Discounted Payback Consider All Cash Flows? Look again at Investments C and D. The main difference between them is that D has a very large cash flow in 2005, relative to C. Discounting each cash flow at the 10% cost of capital,

	Investment C		Investment D	
Year	End of Year Cash Flow	Value at the End of 2000	End of Year Cash Flow	Value at the End of 2000
2001	$300,000	$272,727	$300,000	$272,727
2002	300,000	247,934	300,000	247,934
2003	300,000	225,394	300,000	225,394
2004	300,000	204,904	300,000	204,904
2005	300,000	186,276	10,000,000	6,209,213

The discounted payback period for C is four years:

	Investment C		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$272,727	$272,727	
2002	247,934	520,661	
2003	225,394	746,055	
2004	204,904	950,959	
2005	186,276	1,137,235	⇦ $1,000,000 investment paid back

The discounted payback period for D is also four years, with each year-end cash flow from 2001 through 2004 contributing the same as those of Investment C. However, D's cash flow in 2005 contributes over $6 million more in terms of the present value of the project's cash flows:

	Investment D		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$272,727	$272,727	
2002	247,934	520,661	
2003	225,394	746,055	
2004	204,904	950,959	
2005	6,209,213	7,160,172	⇦ $1,000,000 investment paid back

The discounted payback period method ignores the remaining discounted cash flows: $950,959 + $186,276 – $1,000,000 = $137,235 from Investment C in year 2005 and $950,959 + $6,209,213 – $1,000,000 = $6,160,172 from Investment D in year 2005.

Criterion 2: Does Discounted Payback Consider the Timing of Cash Flows? Look at Investments E and F. Using a cost of capital of 5% for both E and F, the discounted cash flows for each period are:

	Investment E		Investment F	
Year	End of Year Cash Flow	Value at the End of 2000	End of Year Cash Flow	Value at the End of 2000
2001	$300,000	$285,714	$0	$0
2002	300,000	272,109	0	0
2003	300,000	259,151	0	0
2004	300,000	246,811	1,200,000	987,243
2005	300,000	235,058	300,000	235,058

The discounted payback period for E is four years:

	Investment E		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$285,714	$285,714	
2002	272,109	557,823	
2003	259,151	816,974	
2004	246,811	1,063,785	⇦ $1,000,000 investment paid back
2005	235,058	1,298,843	

The discounted payback period for F is five years:

	Investment F		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$0	$0	
2002	0	0	
2003	0	0	
2004	$987,243	$987,243	
2005	235,058	1,222,301	⇦ $1,000,000 investment paid back

The discounted payback period is able to distinguish investments with different timing of cash flows. E's cash flows are expected sooner than those of F. E's discounted payback period is shorter than F's—four years versus five years.

Criterion 3: Does Discounted Payback Consider the Riskiness of Cash Flows? Look at Investments G and H. Suppose the cost of capital for G is 5% and the cost of capital for H is 10%. We are assuming that H's cash flows are more uncertain than G's. The discounted cash flows for the two investments, using the appropriate discount rate, are:

	Investment G		Investment H	
Year	End of Year Cash Flow	Value at the End of 2000	End of Year Cash Flow	Value at the End of 2000
2001	$250,000	$238,095	$250,000	$227,273
2002	250,000	226,757	250,000	206,612
2003	250,000	215,959	250,000	187,829
2004	250,000	205,676	250,000	170,753
2005	250,000	195,882	250,000	155,230

The discounted payback period for G is five years:

	Investment G		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$238,095	$238,095	
2002	226,757	464,852	
2003	215,959	680,811	
2004	205,676	886,487	
2005	195,882	1,082,369	⇦ $1,000,000 investment paid back

According to the discounted payback period method, H does not pay back its original $1,000,000 investment—not in terms of discounted cash flows:

	Investment H		
End of Year	Value at the End of 2000	Accumulated Discounted Cash Flows	
2001	$227,273	$227,273	
2002	206,612	433,885	
2003	187,829	621,714	
2004	170,753	792,467	
2005	155,230	947,697	⇦ Less than $1,000,000 paid back

Because risk is reflected through the discount rate, risk is explicitly incorporated into the discounted payback period analysis. The discounted payback period method is able to distinguish between Investment G and the riskier Investment H.

Is Discounted Payback Consistent with Owners' Wealth Maximization? Discounted payback cannot provide us any information about how profitable an investment is—because it ignores everything after the "break-even" point! The discounted payback period can be used as an initial screening device—eliminating any projects that don't pay back over the expected term of the investment. But since it ignores some of the cash flows that contribute to the present value of investment (those above and beyond what is necessary for the investment's payback), the discounted payback period technique is not consistent with owners' wealth maximization.

Is IRR Consistent with Owners' Wealth Maximization? Evaluating projects with IRR indicates the ones that maximize wealth so long as: (1) the projects are independent, and (2) they are not limited by capital rationing. For mutually exclusive projects or capital rationing, the IRR may—but not always—lead to projects that do not maximize wealth.

Multiple Internal Rates of Return

The typical project usually involves only one large negative cash flow initially, followed by a series of future positive flows. But that's not always the case. Suppose you are involved in a project that uses environmentally sensitive chemicals. It may cost you a great deal to dispose of them, which will cause a negative cash flow at the end of the project.

Suppose we are considering a project that has cash flows as follows:

Period	End of Period Cash Flow
0	−$100
1	+$474
2	−$400

What is the internal rate of return on this project? Solving for the internal rate of return:

$$\$0 = -\$100 + \frac{\$474}{(1+\text{IRR})^1} + \frac{-\$400}{(1+\text{IRR})^2}$$

One possible solution is IRR = 10%. Yet *another* possible solution is IRR = 2.65 or 265%. Therefore, there are two possible solutions, IRR = 10% per year and IRR = 265% per year.

We can see this graphically in Exhibit 13.4, where the NPV of these cash flows are shown for discount rates from 0% to 300%. Remember that the IRR is the discount rate that causes the NPV to be zero. In terms of this graph, this means that the IRR is the discount rate where the NPV is $0, the point at which the present value changes sign—from positive to negative or from negative to positive. In the case of this project, the present value changes from negative to positive at 10% and from positive to negative at 265%.

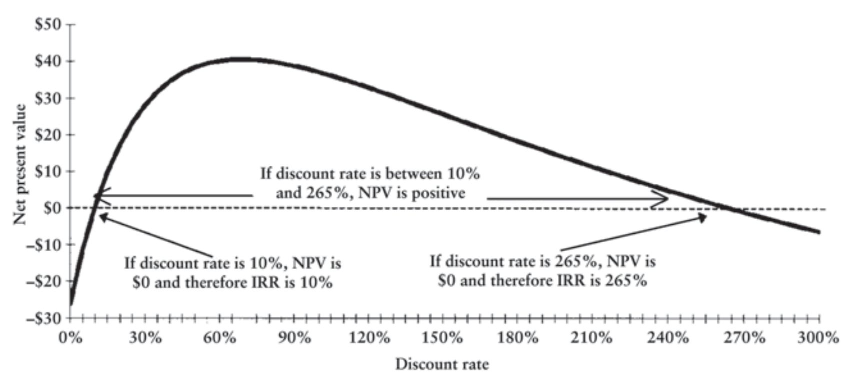
Multiple solutions to the yield on a series of cash flows occurs whenever there is more than one change from positive to negative or from negative to positive in the sequence of cash flows. For example, the cash flows in the previous example above followed a pattern of negative positive negative. There are two sign changes: from minus to plus and from plus to minus. There are also two possible solutions for IRR, one for each sign change.

If you end up with multiple solutions, what do you do? Can you use any of these? None of these? If there are multiple solutions, there is no unique internal rate of return. And if there is no unique solution, the solutions we get are worthless as far as making a decision based on IRR. This is a strike against the IRR as an evaluation technique.

Modified Internal Rate of Return

The ***modified internal rate of return*** technique is similar to the IRR, but using a more realistic reinvestment assumption. As we saw in the previous section, there are situations in which it's not appropriate to use the IRR.

EXHIBIT 13.4 **Investment Profile of a Project with an Initial Cash Outlay of $100, a First Period Cash Inflow of $474, and a Second Period Cash Outflow of $400, Resulting in Multiple Internal Rates of Return**



Let's look again at A's IRR of 28.65% per year. This means that when the first $400,000 comes into the firm, it is reinvested at 28.65% per year for four more periods, when the second $400,000 comes into the firm, it is reinvested at 28.65% per year for three more periods, and so on. If you reinvested all of A's cash inflows at the IRR of 28.65%—that is, you had other investments with the same 28.65% yield—you would have by the end of the project:

End of Year	Cash Inflow		Value at the End of the Project
2001	$400,000	$\$400,000\ (1 + 0.2865)^4 =$	$1,095,719
2002	400,000	$\$400,000\ (1 + 0.2865)^3 =$	$851,705
2003	400,000	$\$400,000\ (1 + 0.2865)^2 =$	$662,033
2004	400,000	$\$400,000\ (1 + 0.2865)^1 =$	$514,600
2005	400,000	$\$400,000\ (1 + 0.2865)^0 =$	$400,000
			$3,524,057

Investing $1,000,000 in A contributes $3,524,057 to the future value of the firm in the fifth year, providing a return on the investment of 28.65% per year. Let FV = $3,524,057, PV = $1,000,000, and $n = 5$. Using the basic valuation equation,

$$FV = PV(1 + i)^n$$

and substituting the known values for FV, PV, and n, and solving for r, the IRR,

$$\begin{aligned} \$3{,}524{,}057 &= \$1{,}000{,}000(1 + i)^5 \\ i &= 28.65\% \text{ per year} \end{aligned}$$

Therefore, by using financial math to solve for the annual return, i, we have assumed that the cash inflows are reinvested at the IRR.

Assuming that cash inflows are reinvested at the IRR is strike two against IRR as an evaluation technique if it is an unrealistic rate. One way to get around this problem is to modify the reinvestment rate built into the mathematics.

Suppose you have an investment with the following expected cash flows:

Year	End of Year Cash Flow
0	−\$10,000
1	+\$3,000
2	+\$3,000
3	+\$6,000

The IRR of this project is 8.55% per year. This IRR assumes you can reinvest each of the inflows at 8.55% per year. To see this, consider what you would have at the end of the third year if you reinvested each cash flow at 8.55%:

Year	End of Year Cash Flow	Future Value at End of Third Year, Using 8.55%
1	+\$3,000	$\$3,000\ (1 + 0.0855)^2 = \$3,534.93$
2	+\$3,000	$\$3,000\ (1 + 0.0855)^1 = \$3,256.50$
3	+\$6,000	$\$6,000\ (1 + 0.0855)^0 = \$6,000.00$
FV_3		\$12,791.43

Investing \$10,000 today produces a value of \$12,791.43 at the end of the third year. The return on this investment is calculated using the present value of the investment (the \$10,000), the future value of the investment (the \$12,791.43) and the number of periods (3 in this case):

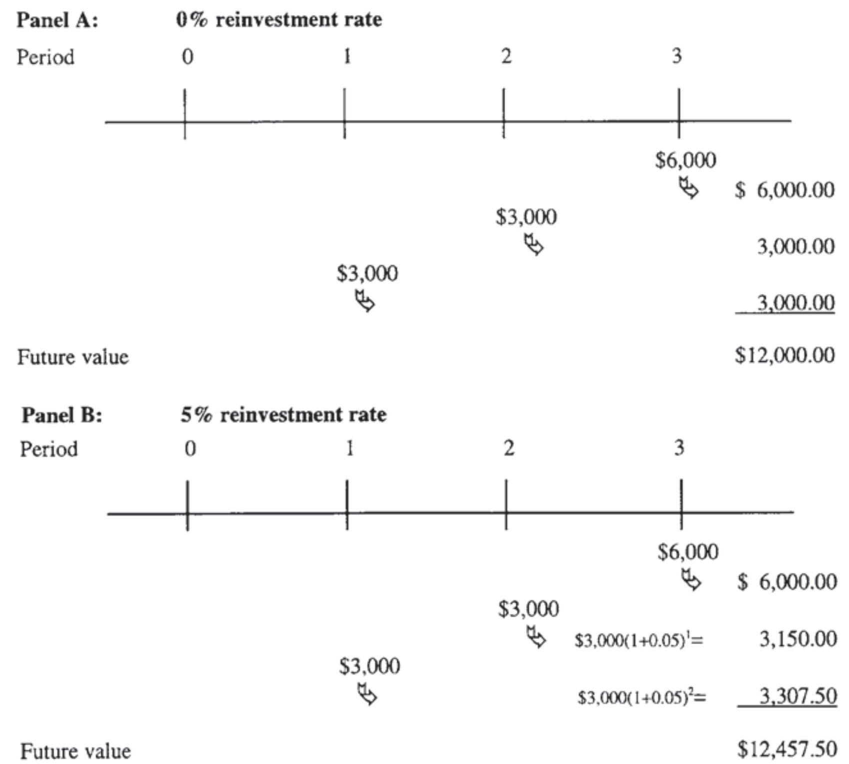
$$\text{Return on investment} = \sqrt[3]{\frac{\$12,791.43}{\$10,000.00}} - 1 = 8.55\%$$

Let's see what happens when we change the reinvestment assumption. If you invest in this project and each time you receive a cash inflow you stuff it under your mattress, you accumulate \$12,000 by the end of the third year: \$3,000 + 3,000 + 6,000 = \$12,000. What return do you earn on your investment of \$10,000? You invest \$10,000 and end up with \$12,000 after three years. The \$12,000 is the future value of the investment, which is also referred to as the investment's *terminal value*.

We solve for the return on the investment by inserting the known values (PV = \$10,000, FV = \$12,000, n = 3) into the basic valuation equation and solving for the discount rate, r:

$$\begin{aligned} \$12,000 &= \$10,000(1+r)^3 \\ (1+r)^3 &= \$12,000/\$10,000 \\ (1+r) &= \sqrt[3]{1.2} = 1.0627 \\ r &= 0.0627 \text{ or } 6.27\% \text{ per year} \end{aligned}$$

EXHIBIT 13.5 **Modified Internal Rate of Return**



The return from this investment, with no reinvestment of cash flows, is 6.27%. We refer to this return as a *modified internal rate of return* (MIRR) because we have *modified* the reinvestment assumption. In this case, we modified the reinvestment rate from the IRR of 8.55% to 0%.

But what if, instead, you could invest the cash inflows in an investment that provides an annual return of 5%? Each cash flow earns 5% annually compounded interest until the end of the third period. We can represent this problem in a time line, shown in Exhibit 13.5. The future value of the cash inflows, with reinvestment at 5% annually, is:

$$\begin{aligned} FV &= \$3{,}000(1+0.05)^2 + \$3{,}000(1+0.05)^1 + \$6{,}000 \\ &= \$3{,}307.50 + \$3{,}150.00 + \$6{,}000 = \$12{,}457.50 \end{aligned}$$

The MIRR is the return on the investment of $10,000 that produces $12,457.50 in three years:

$$\$12{,}457.50 = \$10{,}000(1 + \text{MIRR})^5$$
$$\text{MIRR} = 0.0760 \text{ or } 7.60\% \text{ per year}$$

A way to think about the modified return is to consider breaking down the return into its two components:

1. the return you get if there is no reinvestment (our mattress stuffing), and
2. the return from reinvestment of the cash inflows.

We can also represent MIRR in terms of a formula that combines terms we are already familiar with. Consider the two steps in the calculation of MIRR:

Step 1: Calculate the present value of all cash outflows, using the reinvestment rate as the discount rate.
Step 2: Calculate the future value of all cash inflows reinvested at some rate.
Step 3: Solve for rate—the MIRR—that causes future value of cash inflows to equal present value of outflows:

In this last example,

Reinvestment Rate	Modified Internal Rate of Return (MIRR)
0.00%	6.27%
5.00%	7.60%
8.55%	8.55%

If instead of reinvesting each cash flow at 0%, we reinvest at 5% per year, then the reinvestment adds 7.60% – 6.27% = 1.33% to the investment's return. But wait—we reinvested at 5%. Why doesn't reinvestment add 5%? Because you only earn on reinvestment of intermediate cash flows—the first $3,000 for two periods at 5% and the second $3,000 for one period at 5%—not all cash flows.

Let's calculate the MIRR for Investments A and B, assuming reinvestment at the 10% cost of capital.

Step 1: Calculate the present value of the cash outflows. In both A's and B's case, this is $1,000,000.

Step 2: Calculate the future value by figuring the future value of each cash flow as of the end of 2005:[3]

	Investment A		Investment B	
Year	End of Year Cash Flows	End of Year 2005 Value of Cash Flow	End of Year Cash Flow	End of Year 2005 Value of Cash Flow
2001	$400,000	$585,640	$100,000	$146,410
2002	400,000	532,400	100,000	133,100
2003	400,000	484,000	100,000	121,000
2004	400,000	440,000	1,000,000	1,100,000
2005	400,000	400,000	1,000,000	1,000,000
Future value		$2,442,040		$2,500,510

Step 3: For A, solve for the rate that equates $2,442,040 in five years with $1,000,000 today:

$$\begin{aligned} \$2{,}442{,}040 &= \$1{,}000{,}000(1 + \text{MIRR})^5 \\ \text{MIRR} &= 0.1955 \text{ or } 19.55\% \text{ per year} \end{aligned}$$

Following the same steps, the MIRR for Investment B is 20.12% per year.

Modified Internal Rate of Return Decision Rule

The modified internal rate of return is a return on the investment, assuming a particular return on the reinvestment of cash flows. As long as the MIRR is greater than the cost of capital—that is, MIRR > cost of capital—the project should be accepted. If the MIRR is less than the cost of capital, the project does not provide a return commensurate with the amount of risk of the project.

If ...	this means that ...	and you ...
MIRR > cost of capital	the investment is expected to return more than required	should accept the project.
MIRR < cost of capital	the investment is expected to return less than required	should reject the project.
MIRR = cost of capital	the investment is expected to return what is required	should be indifferent between accepting or rejecting the project.

[3] We have taken each cash flow and determined its value at the end of the year 2005. We could cut down our work by recognizing that these cash inflows are even amounts—simplifying the first step to the calculation of the future value of an ordinary annuity.

Consider Investments A and B and their MIRRs with reinvestment at the cost of capital:

Investment	MIRR	IRR	NPV
A	19.55%	28.65%	$516,315
B	20.12%	22.79%	$552,619

Assume for now that these are mutually exclusive investments. We saw the danger trying to rank projects on their IRRs if the projects are mutually exclusive. But what if we ranked projects according to MIRR? In this example, there seems to be a correspondence between MIRR and NPV. In the case of Investments A and B, MIRR and NPV provide identical rankings.

Modified Internal Rate of Return as an Evaluation Technique

Now we'll go through our usual drill of assessing this technique according to the three criteria.

Criterion 1: Does MIRR Consider All Cash Flows? Assume the cash inflows from Investments C and D are reinvested at the cost of capital of 10% per year. We find that the modified internal rate of return for C is 12.87% per year and for D is 63.07% per year.[4] D's larger cash flow in year 2005 is reflected in the larger MIRR. MIRR does consider all cash flows.

Criterion 2: Does MIRR Consider the Timing of Cash Flows? To see whether the MIRR can distinguish investments whose cash flows occur at different points in time, calculate the MIRR for Investments E and F. Using the terminal values for E and F of $1,831,530 and $1,620,000, respectively, we solve for the rate that equates the terminal value in five years with each investment's $1,000,000 outlay. The MIRR of E is 12.87% per year and the MIRR of F is 10.13% per year. E's cash flows are expected sooner than F's. This is reflected in the higher MIRR. Both E and F are acceptable investments because they provide a return above the cost of capital. If we had to choose between E and F, we would choose E because it has a higher MIRR. MIRR does consider the timing of cash flows.

Criterion 3: Does MIRR Consider the Riskiness of Cash Flows? Let's look at the MIRR for Investments G and H, which have identical expected cash flows, although H's inflows are riskier. Assuming that cash flows are reinvested at the 5% per year cost of capital for G and 10% per year for H, the

[4] The terminal values for C and D are $1,831,530 and $11,531,530, respectively.

future values are $1,381,408 and $1,526,275, respectively. The MIRR for G is 6.68%, calculated using the investment of $1,000,000 as the present value and the terminal value of $1,381,408. Using the same procedure, the MIRR for H is 8.82% per year. Comparing the MIRRs with the costs of capital,

Investment	MIRR	Cost of Capital	Decision
G	6.68%	5%	Accept
H	8.82%	10%	Reject

If we reinvest cash flows at the cost of capital and if the costs of capital are different, we get different terminal values and hence different MIRRs for G and H. If we then compare each project's MIRR with the project's cost of capital, we can determine the projects that would increase owners' wealth.

MIRR distinguishes between the investments, but choosing the investment with the highest MIRR may not give the value maximizing decision. In the case of G and H, H has a higher MIRR. But, when each project's MIRR is compared to the cost of capital, we see that Investment H should not be accepted. This points out the danger of using MIRR when capital is rationed or when choosing among mutually exclusive projects: Ranking and selecting projects on the basis of their MIRR may lead to a decision that does not maximize owners' wealth. If projects are not independent or if capital is rationed, we are faced with some of the same problems we encountered with the IRR in those situations: MIRR may not produce the decision that maximizes owners' wealth.

Is MIRR Consistent with Owners' Wealth Maximization? MIRR can be used to evaluate whether to invest in independent projects and identify the ones that maximize owners' wealth. However, decisions made using MIRR are not consistent with maximizing wealth when selecting among mutually exclusive projects or when there is capital rationing.