Rejection method:

known method $Y \overset{pdf}{\sim} g(x)$

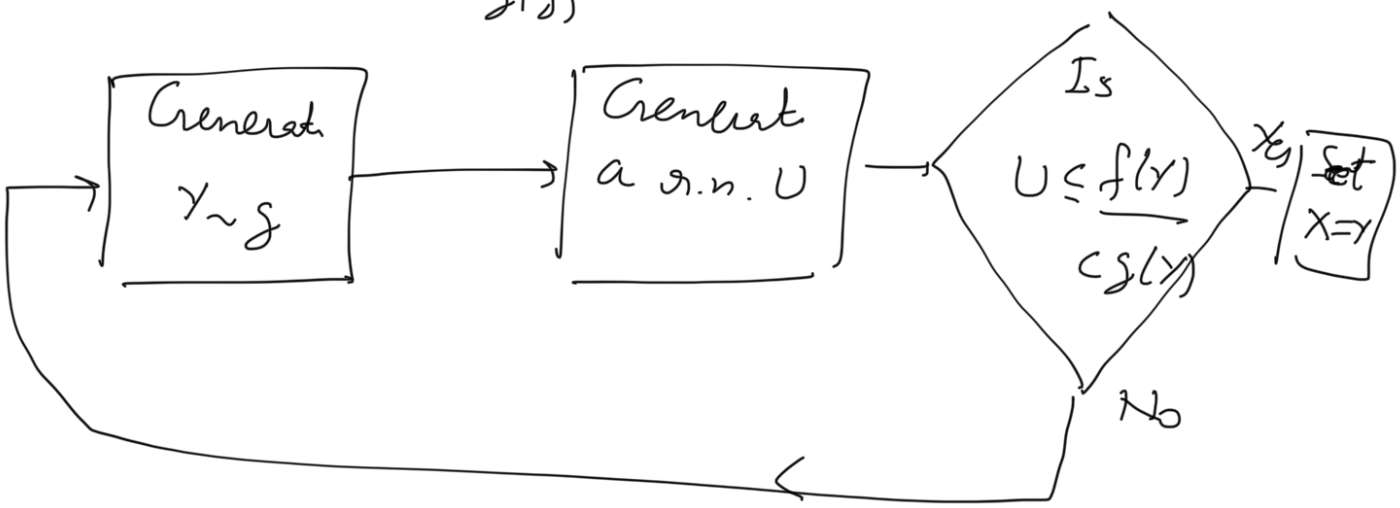
simulate $X \overset{pdf}{\sim} f(x)$

generate Y

accept this generated value with prob. $\propto \frac{f(Y)}{g(Y)}$

Specifically, let $C \in$ const

$$\frac{f(y)}{g(y)} \leq C \quad \forall y$$



pf:

$$P(X \leq x) = P\left(Y \leq x \;\middle|\; U \leq \frac{f(Y)}{Cg(Y)}\right)$$

$$= \frac{P\left(Y \leq x, U \leq \frac{f(Y)}{Cg(Y)}\right)}{k},$$

where $k = P\left(U \leq \frac{f(Y)}{Cg(Y)}\right)$

$$= \frac{1}{k} \int P\left(Y \leq x, U \leq \frac{f(Y)}{Cg(Y)} \;\middle|\; Y = y\right) g(y)\,dy$$

$$= \frac{1}{k} \int_{-\infty}^{x} P\left(U \leq \frac{f(y)}{Cg(y)}\right) g(y)\,dy$$

$$= \frac{1}{k} \int_{-\infty}^{x} P\left(U \le \frac{f(y)}{c\,g(y)}\right) g(y)\,dy$$

$$= \frac{1}{k} \int_{-\infty}^{x} \frac{f(y)}{c\,g(y)}\, g(y)\,dy$$

$$= \frac{1}{kc} \int_{-\infty}^{x} f(y)\,dy$$

let $x \to \infty$

$$\therefore \quad 1 = \frac{1}{kc} \Rightarrow k = \frac{1}{c}$$

$$\therefore \quad k = P\left(U \le \frac{f(y)}{c\,g(y)}\right) = \frac{1}{c}$$

$$\Rightarrow \quad \frac{f(y)}{c\,g(y)} = \frac{1}{c} \Rightarrow \quad f(y) = g(y)$$