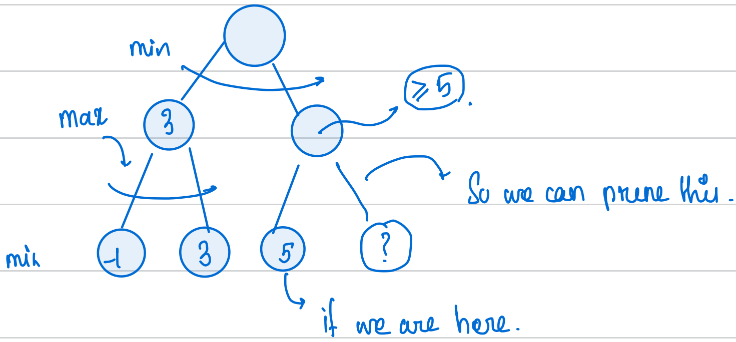
Minimax Algorithm:

Convention: Suppose that we're playing chess then white tries to maximize the score while black tries to minimize it. Leaf nodes ⇒ static evaluation of nodes.

```
minimax ( pos, depth , playerOne)
        if depth == 0 or gameOver in pos :   <------ This is for the leaf nodes.
              return static evaluation of pos

         if playerOne :
              maxEval = -∞
              for each child of pos :
                     eval = minimax( child , depth-1, False)
                     maxEval = max( eval , maxEval)
              return maxEval.

          else :
               minEval = +∞
               for each child of pos :
                      eval = minimax( child , depth-1, True)
                      minEval = min( eval, minEval)
               return minEval
```



maximising Player :

$\alpha = \max(\alpha, eval)$

if $\alpha \geq \beta$

break

minimizing Player :

$\beta = \min(\beta, eval)$

if $\alpha \geq \beta$

break

Constraint Satisfaction Problems : (Part - i)

Map Coloring : We want to color each of the countries but we don't want neighbouring countries to have the same color.

State : It's a black box

Goal test : It is a function that checks if a particular state is a goal state or not.

CSPs :

State → variables X_i with values from domain D.

Goal test → set of constraints specifying allowable combinations of values for subset of variables.

Map Coloring :

Each country → variables.

Domains : D = { color set }

Constraints : Adjacent regions should have different colors.

- Explicit : Constraints in terms of domain
- Implicit : Direct inequalities.

Sol^n : Assignment of variables that passes all constraints.

N-Queens :

Variables : Position of queen in each row Q_k

Domains: $\{1, 2, \dots n\} \rightarrow$ row numbers

Constraints:

$\forall i,j \quad (Q_i, Q_j) \Rightarrow$ non-threatening. (Implicit)

$(Q_1, Q_2) \in \{ \dots \}$ (Explicit)

Constraint Graphs:

Binary CSP: Each constraint relates at most 2 variables

Nodes are variables & edges are constraints.

Cryptarithmetic Puzzles:

$$\begin{array}{r} {\scriptstyle x_3\ x_2\, x_1} \\ TWO \\ +\ TWO \\ \hline FOUR \end{array}$$

Variables: $X_1, X_2, X_3, T, W, O, F, U, R$

Domain: $\{0, 1, \dots 9\}$

Constraints

↳ Graph won't be binary

Waltz Algorithm: (Example of CSP)

Used for the interpretation of line drawings of solid polyhedra as 3D objects.

Approach:

Each intersection is a variable.

Adjacent intersections impose constraints on each other.

Sol's are physically realizable 3D interpretations.

Types of CSPs:

a) Discrete Variables

finite domain → $d \rightarrow O(d^n)$

eg: Boolean CSPs

infinite domains → integers, strings

eg: Job Scheduling

b) Continuous Variables

eg: linear constraints solvable in polynomial time by LP methods.

Types of Constraints:

Unary Constraints: eg: $var \neq x$ ← value.

Binary Constraints: eg: $var_1 \neq var_2$.

Higher ordered: (involving 3 or more variables)

Preferences ⇒ x is better than y.

Solving CSPs:

Backtracking Search: [Bad name] — cuz we backtrack only we don't have more options !!

→ One variable at a time ! (Only need to consider assignments to one variable at a time s.t that doesn't break the constraints)

→ Check constraints on the go ! (Incremental goal test)

Pseudo code:

function Backtrack (csp)
    return recursive Search ({ }, csp)   initially assignment is empty.

function recursive-Search (asgn , csp)
    if asgn is complete => passes all constraints.
        return asgn

    var ← select un-assigned-variable (variables [csp] , asgn , csp)
    for each value in Order-domain-values (var, asgn, csp) :
        if value == consistent given Constraints [csp] :
            add { var = value } to asgn
            result ← recursive-Search (asgn , csp)
            if result ≠ failure
                return result
            } Normal backtracking with this check.
            remove { var = value } from asgn
    return failure.
        cuz of this

Improving :
Ordering : Order of assignment
Filtering : Can we detect a failure early on.   Ruling out suspects.
Structure : Can we exploit the problem structure.

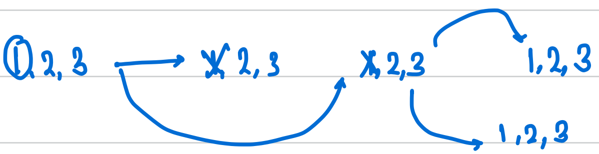
Filtering:

We keep a track of domains for unassigned variables & cross-off the bad options.

Forward Checking: (Its re-filtering of all variables after each assignment)

We cross-off values that could violate a constraint when added to existing assignment

eg:



(Map Coloring).

Essentially forward checking doesn't contribute to early detection of failures which is precisely what we want!! It doesn't care about rel^n b/w assigned & unassigned variables

Constraint Propagation: Reason from constraint to constraint.
↳ its slow!!

Consistency of a Single Arc:

Def^n: An arc $X \to Y$ is consistent if and only if for every x in the tail, there is some y in the head which could be assigned without violating a constraint.

To make $X \to Y$ consistent, we need to delete smth from the tail.

A simple form of propagation makes sure that all arcs (or edges) are consistent.

In forward filtering, we are just checking the consistency of all nodes with the new assignment.

Note:

If X loses a value, then neighbours of X need to be re-checked!

Limitation: There can 1, 0 or multiple sol^{n}s

Enforcing Arc Consistency in a CSP:

$\boxed{T = O(n^2 d^3) \Rightarrow O(n^2 d^2) \text{ can be done}}$

function AC-3 (csp) returns CSP

    q → queue of arcs (initially all arcs in CSPs)
    while q ≠ ϕ do:
        (x_i, x_j) ← remove-first(q)
        if re-move-inconsistent-values(x_i, x_j) then:
            for each x_u in Neighbours[x_i] do:
                add (x_i, x_u) to the q.

function re-move-inconsistent-values (x_i, x_j)

    removed ← false
    for each x in Domain[x_i]
        if no value y in Domain[x_j] allows (x, y) to satisfy $x_i \leftrightarrow x_j$
            remove x from Domain[x_i]
            remove ← true
    return removed

CSPs Part-II

Ordering:

What's a good place to start?

MRV - Minimum Remaining Variables: Choose the variable with the fewest legal values left in the domain post assignment.

Why min? ⇒ Most constrained variable!! (Hard ones first)

Choose a variable that doesn't cross-off too many things ⇒ Least constraining value.

Picking a value ⇒ <u>choose the easy one</u>.

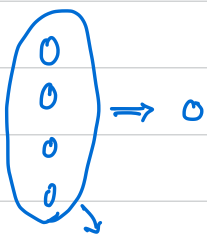
K- Consistency: (Doesn't violate constraints)

1- Consistency: (Node): Each single node's domain has a value which meets the <u>node's unary constraints</u>.

2- Consistency: (Arc): For each <u>pair of nodes</u>, any consistent assignment to one can be extended to the other. (delete from the tail after assignment)

⋮

k- consistency: For each k nodes, any consistent assignment to (k-1) can be extended to the k^{th} node.



can be extended to one more node.

legal move

Strong k-consistency : $\Rightarrow$ also $k-1 \Rightarrow k-2 \dots \Rightarrow$ 1-consistent

Claim : Strong n-consistency means we can solve without backtracking !

Why ?

Choose any assignment to a variable

Choose a new variable

By 2-consistent, the given choice is consistent with the first

Bottom to up.

⋮ ⋮

$k=3 \Rightarrow$ path consistency.

How to utilize the structure of problems to gain advantages ?

Dividing a given problem into smaller sub-problems.

eg: If originally we have n variables $\Rightarrow$ divided into sub-problems of only c variables

$T = O((n/c)\, d^c)$, linear in n.

<u>Theorem</u>: If consistent graph has no loops, then CSP can be solved in $O(nd^2)$

Compared to general case, $T = O(d^n)$

Tree-Structured CSP

<u>Algo</u>:

Topological Sorting

- <u>Order</u> : Choose a root, order the variables so that parents preceed children
- Remove backward : for $i = n$ to 2 : RemoveInconsistent ($Par(X_i)$, X_i)

- Assign Forward: For $i = 1$ to n: assign X_i consistently with Parent(X_i)

Run-time: $O(nd^2)$ $\longrightarrow$ Refer the book for the proof.

1 Adversarial Search.