Statistics Software Lab (MA60292)
Assignment-12

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1. A process $\{X_t\}$ is defined by $X_t = W_{t-1} + 2W_t + W_{t+1}$, where W_t are iid $N(0, \sigma_w^2)$. Now use R to generate sufficient number of observations from the series $\{W_t\}$ and use them to generate 1000 observations from the series $\{X_t\}$. Use R to plot the sample ACF for this generated series. Compare this sample ACF with the theoretical ACF.

2. Consider AR(2) process that is a stationary solution of time series $\{X_t\}$ of the equation

$$X_t - 0.5X_{t-1} = Z_t,\ t = \dots, -1, 0, 1, \dots,$$

   where $Z_t \sim\ WN(0, 9)$. Find sample ACVF $\gamma_X(h)$ for $h \geq 0$. Also find the best linear predictor of X_{n+1} in terms of $\{X_1, \dots, X_n\}$ and find its mean square error.

3. Find the sample ACVF of the time series $X_t = Z_t + 0.3Z_{t-1} - 0.4Z_{t-2}$, where $Z_t \sim\ WN(0, \sigma^2)$. Also compute the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$.

4. Consider the AR(2) process $X_t - 0.5X_{t-1} + 0.5X_{t-2} = Z_t$, where $Z_t \sim\ WN(0, \sigma^2)$. Find the sample partial autocorrelation functions.

5. (reference Peter Bartlett 2010) Consider the time series data (sunspot data below). It consists of n = 285 observations of the number of sunspots, from 1700 to 1984. This is a quantity that is believed to affect our weather patterns. This time series has been studied by many authors, including Yule (Philosophical Transactions of the Royal Society of London, Series A, 226:267–298, 1927) and Brillinger and Rosenblatt (Spectral Analysis of Time Series, B. Harris (Ed.), pp 153–188, Wiley, 1967). We will study the square root of the data (this transformation ensures that the variance is roughly constant). That is, for the series $Z_1, \dots, Z_n$ from the sunspot data, first compute the series $X_t = \sqrt{Z_t}$, and work with the series $\{X_t\}$ in what follows.

   (a) Compute the sample ACF and the sample PACF for this series.

   (b) By considering the sample ACF and sample PACF, decide which of the following would be appropriate for this data: AR(1), AR(2), MA(1) or MA(2). Use the data to estimate the parameters of the model that you choose.

   (c) Using your fitted model, calculate forecasts X_n^{n+h}, for $h = 1, 2, 3, 4$. Calculate the 95% prediction intervals (assuming Gaussian noise).

   (d) The file sunspot2 data on the website includes the number of sunspots for the years 1985 to 1988. Plot all of the data, and your forecasts and prediction intervals for the last four years. (Don't forget to undo the square root transformation by taking the square of your predictions.)

   Sunspot data: 5.00 11.00 16.00 23.00 36.00 58.00 29.00 20.00 10.00 8.00 3.00 0.00 0.00 2.00 11.00 27.00 47.00 63.00 60.00 39.00 28.00 26.00 22.00 11.00 21.00 40.00 78.00 122.00 103.00 73.00 47.00 35.00 11.00 5.00 16.00 34.00 70.00 81.00 111.00 101.00 73.00 40.00 20.00 16.00 5.00 11.00 22.00 40.00 60.00 80.90 83.40 47.70 47.80 30.70 12.20 9.60 10.20 32.40 47.60 54.00 62.90 85.90 61.20 45.10 36.40 20.90 11.40 37.80 69.80 106.10 100.80 81.60 66.50 34.80 30.60 7.00 19.80 92.50 154.40 125.90 84.80 68.10 38.50 22.80 10.20 24.10 82.90 132.00 130.90 118.10 89.90 66.60 60.00 46.90 41.00 21.30 16.00 6.40 4.10 6.80 14.50 34.00 45.00 43.10 47.50 42.20 28.10 10.10 8.10 2.50 0.00 1.40 5.00 12.20 13.90 35.40 45.80 41.10 30.10 23.90 15.60 6.60 4.00 1.80 8.50 16.60 36.30 49.60 64.20 67.00 70.90 47.80 27.50 8.50 13.20 56.90 121.50 138.30 103.20 85.70 64.60 36.70 24.20 10.70 15.00 40.10 61.50 98.50 124.70 96.30 66.60 64.50 54.10 39.00 20.60 6.70 4.30 22.70 54.80 93.80 95.80 77.20 59.10 44.00 47.00 30.50 16.30 7.30 37.60 74.00 139.00 111.20 101.60 66.20 44.70 17.00 11.30 12.40 3.40 6.00 32.30 54.30 59.70 63.70 63.50 52.20 25.40 13.10 6.80 6.30 7.10 35.60 73.00 85.10 78.00 64.00 41.80 26.20 26.70 12.10 9.50 2.70 5.00 24.40 42.00 63.50 53.80 62.00 48.50 43.90 18.60 5.70 3.60 1.40 9.60 47.40 57.10 103.90 80.60 63.60 37.60 26.10 14.20 5.80 16.70 44.30 63.90

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