MA 69204 Statistical Software Lab

Assignment No. 4

(a) Generating a Random Permutation

Step 1: Let $p_1, p_2, \dots, p_n$ be any permutation of $1, 2, \dots, n$ (for example, $p_j = j, j = 1, \dots, n$).

Step 2: Set $k = n$.

Step 3: Generate a random number $U \sim U(0, 1)$ and let $I = [kU] + 1$.

Step 4: Interchange the values of p_I and p_k.

Step 5: Let $k = k - 1$ and if $k > 1$ go to Step 3.

Step 6: $p_1, p_2, \dots, p_n$ is the desired permutation.

(b) Generation of Stationary Poisson Process

Let the rate be $\lambda > 0$, t refer to time, I is the number of events that have occurred by time t, and $S(I)$ is the most recent time.

Step 1: $t = 0, I = 0$.

Step 2: Generate a random number U.

Step 3: $t = t - \frac{1}{\lambda} ln\, U$. If $t > T$, stop.

Step 4: $I = I + 1,\ S(I) = t$.

Step 5: Go to step 2.

(c) Generation of Non-stationary Poisson Process

Algorithm 1:

Let the rate be $\lambda(t)$ is the intensity function and λ is such that $\lambda(t) \leq \lambda$, the final value of I represents the number of events that occurred by time T and $S(1), \dots, S(I)$ are the event times.

Step 1: $t = 0, I = 0$.

Step 2: Generate a random number U.

Step 3: $t = t - \frac{1}{\lambda} ln\, U$. If $t > T$, stop.

Step 4: Generate a random number U.

Step 5: If $U \leq \frac{\lambda(t)}{\lambda}$, set $I = I + 1,\ S(I) = t$.

Step 6: Go to step 2.

Algorithm 2:

In the algorithm t represents the present time, J the present interval (i.e., $J = j$ when $t_{j-1} \leq t \leq t_j$), I is the number of events so far, and $S(1), \dots, S(I)$ are the event times.

Step 1: $t = 0,\ J = 1,\ I = 0$.

Step 2: Generate a random number U and set $X = -\frac{1}{\lambda_J} \ln U$.

Step 3: $t + X > t_J$, go to step 8.

Step 4: $t = t + X$

Step 5: Generate a random number U.

Step 6: If $U \leq \frac{\lambda(t)}{\lambda},$ set $I = I + 1,\ S(I) = t$.

Step 7: Go to step 2.

Step 8: If $J = k + 1$, stop.

Step 9: $X = \frac{(X - t_J + t)\lambda_J}{\lambda_{J+1}},\ J = J + 1$.

Step 10: Go to Step 3.