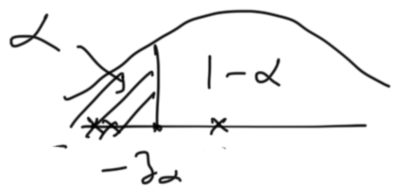
Testing of statistical hypotheses :

(1) r.v. $X \sim N(\mu, \sigma^2)$ r.s. $X_1, \to X_n$ sample mean $\overline{X}$

μ unknown, σ^2 known $(\sigma^2 = \sigma_0^2)$

H_0	H_1	C (critical region) Or rejection region for H_0
$\mu \geq \mu_0$	$\mu < \mu_0$	$Z_0 = \dfrac{\overline{X} - \mu_0}{\sigma_0/\sqrt{n}} < -z_\alpha$
$\mu \leq \mu_0$	$\mu > \mu_0$	$Z_0 = \dfrac{\overline{X} - \mu_0}{\sigma_0/\sqrt{n}} > z_\alpha$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert Z_0 \rvert > z_{\frac{\alpha}{2}}$



(2) $X \sim N(\mu, \sigma^2)$ r.s. $X_1, \to X_n$ $\quad \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$

σ^2 unknown

H_0	H_1	C
$\mu \geq \mu_0$	$\mu < \mu_0$	$t_0 = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} < -t_{\alpha, n-1}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$t_0 > t_{\alpha, n-1}$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\frac{\alpha}{2}, n-1}$

(3) $X \sim N(\mu, \sigma^2)$

σ^2 unknown

H_0	H_1	C

$\sigma^2 \le \sigma_0^2$ $\quad$ $\sigma^2 > \sigma_0^2$ $\quad$ $\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2} > \chi^2_{\alpha, n-1}$

$\sigma^2 \ge \sigma_0^2$ $\quad$ $\sigma^2 < \sigma_0^2$ $\quad$ $\chi_0^2 < \chi^2_{1-\alpha, n-1}$

$\sigma^2 = \sigma_0^2$ $\quad$ $\sigma^2 \ne \sigma_0^2$ $\quad$ $\chi_0^2 > \chi^2_{\frac{\alpha}{2}, n-1}$ or $\chi_0^2 < \chi^2_{1-\frac{\alpha}{2}, n-1}$

— x —

Two samples

(i) indep

$X_1 \sim N(\mu_1, \sigma_1^2)$ r.s $X_{11}, \ldots, X_{1n_1}$ $\quad \bar{X}_1 \quad S_1^2$

unknown (μ) known (σ)

$X_2 \sim N(\mu_2, \sigma_2^2)$ r.s $X_{21}, \ldots, X_{2n_2}$ $\quad \bar{X}_2 \quad S_2^2$

$H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \ne \mu_2$

C $\quad |Z_0| = \left| \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \right| > z_{\frac{\alpha}{2}}$

(ii) indep

$X_1 \sim N(\mu_1, \sigma_1^2)$ r.s $X_{11}, \ldots, X_{1n_1}$ $\quad \bar{X}_1 \quad S_1^2$

unknown

$X_2 \sim N(\mu_2, \sigma_2^2)$ r.s $X_{21}, \ldots, X_{2n_2}$ $\quad \bar{X}_2 \quad S_2^2$

$(ii)_I$ $\quad \sigma_1^2 = \sigma_2^2 = \sigma^2$ $\quad H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \ne \mu_2$

C $\quad |t_0| > t_{\frac{\alpha}{2}, n_1+n_2-2}$

$t_0 = \dfrac{\bar{X}_1 - \bar{X}_2}{\ldots} \sim t_{n_1+n_2-2}$

$$s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$

(ii) II $\sigma_1^2 \neq \sigma_2^2$ $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \neq \mu_2$

C $|t_0| > t_{\frac{\alpha}{2}, \nu}$

$$t_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_\nu$$

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 + 1} + \frac{(S_2^2/n_2)^2}{n_2 + 1}} - 2$$

Testing equality of variance

$H_0 : \underline{\sigma_1^2 = \sigma_2^2}$ vs $H_1 : \sigma_1^2 \neq \sigma_2^2$

Under H_0

$$F_0 = \frac{\frac{(n_1 - 1) S_1^2}{(n_1 - 1) \sigma_1^2}}{\frac{(n_2 - 1) S_2^2}{(n_2 - 1) \sigma_2^2}} = \frac{S_1^2}{S_2^2} \sim F_{n_1 - 1, n_2 - 1}$$

C $F_0 > F_{\frac{\alpha}{2}, n_1 - 1, n_2 - 1}$ or $F_0 < F_{1 - \frac{\alpha}{2}, n_1 - 1, n_2 - 1}$

Paired t-test

$$\overbrace{(X_{11}, X_{21}), \ldots, (X_{1n}, X_{2n})}^{\text{indep}}$$

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

$D_i = X_{1i} - X_{2i}$, $i = 1, \ldots, n$

$\equiv H_0: \mu_D = 0$ vs $H_1: \mu_D \neq 0$

$\bar{D}, \; S_D^2$

$$\mu_D = \mu_1 - \mu_2$$

C ✓ $|t_0| > t_{\frac{\alpha}{2}, n-1}$

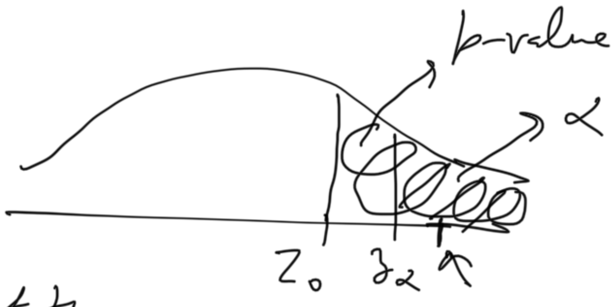
$$t_0 = \frac{\bar{D}}{S_D/\sqrt{n}}$$

—x—

p-value = $P(Z > Z_0)$

$H_0: \mu < \mu_0$ vs $H_1: \mu > \mu_0$

C $Z_0 > z_\alpha$



p-value < α → reject H_0

p-value > α → accept H_0

—x—

Contingency table test:

	Columns			
Row	1	2	— —	C
1	O_{11}	O_{12}	- -	O_{1C}
2	O_{21}	O_{22}	- -	O_{2C}

H_0: rows & columns method of classification are indep.

$\vdots$ — — —

r O_{r1} O_{r2} O_{rc}

$$p_{ij} = u_i v_j$$

mle $\hat{u}_i = \frac{1}{n} \sum_{j=1}^{c} O_{ij}$, $\hat{v}_j = \frac{1}{n} \sum_{i=1}^{r} O_{ij}$

assuming indep , the expected # in each cell is

$$E_{ij} = n \hat{u}_i \hat{v}_j = \frac{1}{n} \sum_{m=1}^{c} O_{im} \sum_{k=1}^{r} O_{kj}$$

for large n

$$\chi_0^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1)(c-1)}$$

C $\chi_0^2 > \chi^2_{\alpha,(r-1)(c-1)}$.