Multivariate Normal distn (MVN):

Let $Z_i \sim NID(0,1)$, $i = 1, \dots, n$

$$\underline{X_i = a_{i1} Z_1 + a_{i2} Z_2 + \dots + a_{in} Z_n + \mu_i}$$

$, i = 1, \dots, m$; a_{ij} and $\mu_i \in$ consts.

$i = 1, \dots, m$, $j = 1, \dots, n$

$\underset{\sim}{X} = (X_1, \dots, X_m)$ MVN

$$E(X_i) = \mu_i \quad , \quad V(X_i) = \sum_{j=1}^{m} a_{ij}^2$$

Joint mgf of $\underset{\sim}{X} = (X_1, \dots, X_m)$

$$\phi(t_1, \dots, t_m) = E\left(e^{t_1 X_1 + \dots + t_m X_m}\right)$$

$$= \exp\left[\sum_{i=1}^{m} t_i \mu_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} t_i t_j \, Cov(X_i, X_j)\right]$$

$$\sum_{i=1}^{m} t_i X_i \sim N$$

$$E\left(\sum_{i=1}^{m} t_i X_i\right) = \sum_{i=1}^{m} t_i \mu_i$$

$$V\left(\sum_{i=1}^{m} t_i X_i\right) = Cov\left(\sum_{i=1}^{m} t_i X_i, \sum_{j=1}^{m} t_j X_j\right)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{m} t_i t_j \, Cov(X_i, X_j)$$

— x —

MVN

Let $\underset{\sim}{t} \in \mathbb{R}^m$

let $\underset{\sim}{\mu} \in \mathbb{R}$, Σ p.d. matrix

$$\underset{\sim}{X} = (X_1, \ldots, X_m) \sim MVN(\underset{\sim}{\mu}, \Sigma)$$

joint density of $\underset{\sim}{X}$ is

$$f_{\underset{\sim}{X}}(\underset{\sim}{x}) = \frac{1}{(2\pi)^{m/2}\sqrt{|\Sigma|}} \exp\left\{-\frac{1}{2}(\underset{\sim}{x}-\underset{\sim}{\mu})\Sigma^{-1}(\underset{\sim}{x}-\underset{\sim}{\mu})^T\right\}$$

for $\underset{\sim}{x} \in \mathbb{R}^m$