Multivariate Normal dists (MVN):

$$Z_i \sim NID(0,1)\ ,\ i=1,\dots,n$$

Const $a_{ij}\ ,\ i=1,\dots,m,\ j=1,\dots,n$ and $\mu_i\ ,\ i=1,\dots,m$

If $X_i = a_{i1}Z_1 + \dots + a_{in}Z_n + \mu_i\ ,\ i=1,\dots,m$

then the r.v's $X_1,\dots,X_m$ are said to have a MVN.

$$E(X_i) = \mu_i\ ;\ V(X_i) = \sum_{j=1}^{n} a_{ij}^2$$

mgf of $X_1,\dots,X_m$

$$\phi(t_1,\dots,t_m) = E\left(e^{t_1X_1 + \dots + t_mX_m}\right)$$

$$E\left(\sum_{i=1}^{m} t_iX_i\right) = \sum_{i=1}^{m} t_i\mu_i$$

$$V\left(\sum_{i=1}^{m} t_iX_i\right) = Cov\left(\sum_{i=1}^{m} t_iX_i\ ,\ \sum_{j=1}^{m} t_jX_j\right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{m} t_it_j\,Cov(X_i,X_j)$$

$$\phi(t_1,\dots,t_m) = e^{\sum_{i=1}^{n} t_i\mu_i + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} t_it_j\,Cov(X_i,X_j)}$$

joint dists of $X_1,\dots,X_m$ is completely determined from the knowledge of $\mu_i = E(X_i)$ and $Cov(X_i,X_j)$

$i,j = 1,\dots,m$

Gaussian Process or normal process:

A SP $(X(t), t \geq 0)$ is Gaussian process if $X(t_1), \ldots, X(t_n)$ has MVN $\forall t_1, \ldots, t_n$.

$\longrightarrow$ $(B(t), t \geq 0)$ BM process

$$\left.\begin{array}{l} B(t_1) \\ B(t_2) = B(t_1) + \underbrace{(B(t_2) - B(t_1))}_{\text{indep}} \\ \vdots \end{array}\right] \Rightarrow \text{BM is a Gaussian process}$$

SBM/Weiner process $W(t)$ is Gaussian process

$W(t) \sim N(0,t)$

$$E(W(t)) = 0$$

$$V(W(t)) = t$$

Fact $\longrightarrow$ $Cov(W(s), W(t)) = Cov(W(s), W(t) - W(s) + W(s))$

$$= Cov(W(s), W(t) - W(s)) + Cov(W(s), W(s))$$

$$= 0 + V(W(s)) = 0 + s$$

—x—

$$Cov(W(s), W(t)) = s \wedge t = \min(s,t)$$

$\wedge \to \min$

$\vee \to \max$