Finite single period market:

NA $\longrightarrow$ $S_0 \in \left(\frac{S_1(d)}{1+r}, \frac{S_1(u)}{1+r}\right)$

$\exists$ equivalent martingale measure (EMM) p^* on $\Omega = \{u, d\}$ with property

$$S_0 = p^* \frac{S_1(d)}{1+r} + (1-p^*) \frac{S_1(d)}{1+r} = E^*\left(\frac{S_1}{1+r}\right)$$

→ $t=0$, $t=1$

→ $t=1$, $\Omega = \{\omega_1, \dots, \omega_N\}$ outcome space has $N<\infty$ elts

→ there are $(n+1)$ traded assets in the market: n stocks with time t prices $S_t := (S_t^1, \dots, S_t^n)$ and a bond (or bank account) $B_t = (1+r)^t$, $t=0,1$.

→ A set $D \subset \mathbb{R}^m$ is called convex if for any $\underset{\sim}{x}_1, \underset{\sim}{x}_2 \in D$, all the pts on the st. line segment connecting the $\underset{\sim}{x}_i$'s also lie in D;

$$\beta \underset{\sim}{x}_1 + (1-\beta)\underset{\sim}{x}_2 \in D \quad , \quad \beta \in (0,1)$$

→ Convex hull of a set $B \subseteq \mathbb{R}^m$, denoted by $\operatorname{conv}(B)$, is the smallest convex set D st $B \subset D$.

For a finite $B := \{\underset{\sim}{y}_1, \dots, \underset{\sim}{y}_N\} \subset \mathbb{R}^m$, one has

$$D := \operatorname{conv}(B) = \{ \underset{\sim}{x} \in \mathbb{R}^m ;\ \underset{\sim}{x} = \sum_{j=1}^{N} \alpha_j \underset{\sim}{y_j} ,\ \alpha_j \geq 0 ,\ \sum_{j=1}^{N} \alpha_j = 1 \}$$

→ Denote by D_0 the relative interior of convex hull D:

$$D_0 = \{ \underset{\sim}{x} \in \mathbb{R}^m :\ \underset{\sim}{x} = \sum_{j=1}^{N} \alpha_j \underset{\sim}{y_j} ,\ \alpha_j > 0 ,\ \sum_{j=1}^{N} \alpha_j = 1 \}$$

→ By a hyperplane in $\mathbb{R}^m$ one means a linear manifold of dim $m-1$. Analytically, a hyperplane can be specified by one linear equation

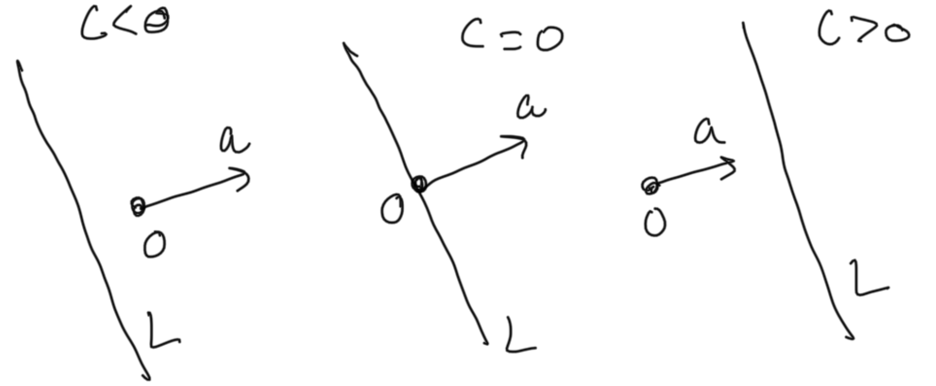
$$L := \{ \underset{\sim}{x} : (x_1, \ldots, x_m) \in \mathbb{R}^m : \underset{\sim}{a} \cdot \underset{\sim}{x} = c \}$$

where $\underset{\sim}{a} \cdot \underset{\sim}{x} = \sum_{j=1}^{m} x_j a_j$, $c \in \mathbb{R}$, $\underset{\sim}{a} = (a_1, \ldots, a_m) \neq 0$ are fixed

$\underset{\sim}{a}$ is orthogonal to L

→ $\underset{\sim}{x_1}, \underset{\sim}{x_2} \in L$ one has $\underset{\sim}{a}(\underset{\sim}{x_1} - \underset{\sim}{x_2}) = 0$

$\underset{\sim}{x_0} \in L$ $\quad \underset{\sim}{a}(\underset{\sim}{x} - \underset{\sim}{x_0}) = 0$



<u>Seperation thm</u>: Assume that $D \in \mathbb{R}^m$ is convex set, D_0 its relative interior. For any pt $x_0 \notin D_0$, $\exists$ a vector $\underset{\sim}{a} \in \mathbb{R}^m$ s.t. $\underset{\sim}{a}(\underset{\sim}{x} - \underset{\sim}{x_0}) > 0$, $\forall\, x \in D_0$