1. The call price $C(K)$ is given in Example 13.3. It is obvious that $C(K)$ is a piece-wise linear function of K with the following features: $C(0) = S_0$, the slope is equal to $-1/(1+r)$ for $K < dS_0$ (in that interval, both $(\cdots)^+ > 0$), to $-p^*/(1+r) > -1/(1+r)$ for $dS_0 < K < uS_0$ (in that interval, only the first of $(\cdots)^+$ is positive), to 0 for $K > uS_0$, where $C(K) = 0$ (plot it yourself!).

The features common to all calls: (i) $C(0) = S_0$ (zero strike price means one just receives the share at maturity, so the current price of such call should be equal to the current share price); (ii) $C(K)$ is a decreasing function of K (the larger K, the smaller the payoff, so it should cost less); (iii) $C(K) \to 0$ as $K \to \infty$ (as the payoff then vanishes).

2. The payoff function is equal to the sum of those of the call and put, so, from (13.5) and (13.6), it is given by $(s-K)^+ + (s-K)^- = |s-K|$. You must be able to plot that function yourself!

3. (i) The payoff function is $g(s) = (s-K_1)^+ - (s-K_2)^+$. This is a piece-wise linear function with zero slope outside the interval $[K_1, K_2]$, such that $g(0) = 0$ and $g(\infty) = K_2 - K_1$. Plot it.

(ii) The spread must be cheaper as its payoff is equal to that of the call for $s \le K_2$ and is strictly smaller than the call's for $s > K_2$.

(iii) Since $S_0 = 5$, $dS_0 = 4$ and $uS_0 = 6$, one has $d = \frac{4}{5}$, $u = \frac{6}{5}$ (NA as $d < 1+r = 1.1 < u$) and $p^* = \frac{1+r-d}{u-d} = \frac{3}{4}$. So, from the pricing formula (13.11), the spread price equals $\frac{1}{1+r}\left[p^* g(6) + (1-p^*)g(4)\right] = \frac{1}{1.1}\left[\frac{3}{4}\times 2 + \frac{1}{4}\times 1\right] = \frac{35}{22} \approx 1.59$.

(iv) One has $\Delta = \frac{g(6)-g(4)}{6-4} = \frac{1}{2}$, $b = \frac{1.2g(4)-0.8g(6)}{1.1\times 0.4} = -\frac{10}{11}$. Verifying replication: $V_1(u) = \frac{1}{2}\times 6 - \frac{10}{11}\times 1.1 = 3 - 1 = 2$, $V_1(d) = \frac{1}{2}\times 4 - \frac{10}{11}\times 1.1 = 2 - 1 = 1$, which coincide with the values of $g(6)$ and $g(4)$, respectively. OK!

4. (i) One has to verify (13.2). Here $d = S_1(\omega_2)/S_0 = \frac{8}{9}$, $u = S_1(\omega_1)/S_0 = \frac{4}{3} = \frac{12}{9}$, so indeed $d < 1+r = \frac{10}{9} < u$.

(ii) One has $p^* = \frac{10/9-8/9}{12/9-8/9} = \frac{1}{2}$, so that the claim value is $X^* = \frac{1}{1+r}\mathbf{E}^* X = \frac{9}{10}\left[\frac{1}{2}\times 7 + \frac{1}{2}\times 2\right] = \frac{81}{20} = 4.05$.

(iii) One has $\Delta = \frac{7-2}{20/3-40/9} = \frac{9}{4} = 2.25$, $b = \frac{(12/9)\times 2-(8/9)\times 7}{(10/9)\times(4/9)} = \frac{36}{5} = -7.2$, so that the replicating portfolio is $(\Delta, b) = \left(\frac{9}{4}, -\frac{36}{5}\right)$. Its time $t=0$ value is $V_0 = \Delta S_0 + b = \frac{9}{4}\times 5 - \frac{36}{5} = \frac{81}{20}$, which agrees with the result of part (ii). Its time $t=1$ values are:

$$V_1 = \Delta S_1 + b(1+r) = \begin{cases} \frac{9}{4}\times\frac{20}{3} - \frac{36}{5}\times\frac{10}{9} = 7 & \text{if } \omega = \omega_1, \\ \frac{9}{4}\times\frac{40}{9} - \frac{36}{5}\times\frac{10}{9} = 2 & \text{if } \omega = \omega_2, \end{cases} \qquad \text{OK!}$$

5. (i) We computed the price in Example 13.4. Like the call price discussed in Problem 1, it is a piece-wise linear function, changing its slope value at the points dS_0 and uS_0. Plot it. Alternatively, one can express the put price in terms of the call price using the put-call parity (13.15) and use the plot from Problem 1 (flip it upside down!).

(ii) Here $d = \frac{3.6}{4} = 0.9$, $u = \frac{4.6}{4} = 1.15$, so that $d < 1+r = 1.05 < u$, this is an NA market, one can use the pricing formula with $p^* = \frac{1.05-0.9}{1.15-0.9} = 0.6$: $P = \frac{1}{1.05}\left[0.6\times(4.6-3.8)^- + 0.4\times(3.6-3.8)^-\right] = \frac{8}{105} \approx 0.076$.

(iii) From the put-call parity (13.15), $C = S_0 + P - \frac{K}{1+r} = \frac{16}{35} \approx 0.457$.

6. (i) NA is equivalent to existence of $p_j^* > 0$, $\sum_{j=1}^3 p_j^* = 1$, such that $S_0 = \frac{1}{1+r}(p_1^* dS_0 + p_2^* mS_0 + p_3^* uS_0)$ or, which is the same, $p_1^* d + p_2^* m + p_3^* u = 1+r$. Dividing both sides by $p_1^* + p_3^* = 1 - p_2^*$, we get

$$\underbrace{\frac{p_1^*}{p_1^*+p_3^*}}_{=:p} d + \underbrace{\frac{p_3^*}{p_1^*+p_3^*}}_{=1-p} u = \frac{1+r}{1-p_2^*} - \frac{p_2^*}{1-p_2^*} m.$$

Note that, for $p_2^* > 0$ small enough, the right-hand side will be arbitrary close to $1+r$, and so will still lie in the interval (d,u). Hence, according to our argument in the binomial case, there will exist a $p \in (0,1)$ such that $pd + (1-p)u =$ the right-hand side of the displayed formula. This proves existence of the EMM (p_j^*).

(ii) DIY. The set of all hedges is the intersection of the three half-planes in the (Δ, b)-plane that are bounded (from below) by the straight lines given by the equations $\Delta k S_0 + b(1+r) = X_k$, $k = d, m, u$. The perfect hedge would be the point at the intersection of these three lines, but they do not intersect at a common point (why?), so no perfect hedge exists.

7. (i) DIY.

(ii) NA holds iff there exists an EMM $\mathbf{P}^*$. That, in turn, is equivalent to having the point $\boldsymbol{S}_0 = (S_0^1, S_0^2)$ inside the triangle conv $\{\boldsymbol{S}_1(\omega_1), \boldsymbol{S}_1(\omega_2), \boldsymbol{S}_1(\omega_3)\}$, which is the case (see the plot you made in part (i)).

(iii) Need to find p_j^*, $j = 1,2,3$, such that $\boldsymbol{S}_0 = \sum_{j=1}^3 p_j^* \boldsymbol{S}_1(\omega_j)$, or, component-wise,

$$\begin{cases} 4 = 6p_1^* + 4p_2^* + 2p_3^* \\ 5 = 6p_1^* + 4p_2^* + 7p_3^* \\ 1 = p_1^* + p_2^* + p_3^* \end{cases}$$

the last equation just meaning that the p_j^*'s form a probability distribution. The system has unique solution $p_1^* = 0.2$, $p_2^* = 0.6$, $p_3^* = 0.2$. Thus, the EMM is unique, so that the market is complete.

(iv) For the claim $X := (S_1^1 - K)^+ = (S_1^1 - 5)^+$, its arbitrage free price is given by $X^* = \frac{1}{1+r}\mathbf{E}^*X = p_1^*(S_1^1(\omega_1) - 5)^+ + p_2^*(S_1^1(\omega_2) - 5)^+ + p_3^*(S_1^1(\omega_3) - 5)^+ = 0.2 \times (6-5)^+ + 0.6 \times (4-5)^+ + 0.2 \times (2-5)^+ = 0.2$.

(v) Need to find (Δ^1, Δ^2, b) such that $\Delta^1 S_1^1(\omega) + \Delta^2 S_1^2(\omega) + b(1+r) = X(\omega)$ for all $\omega \in \Omega$. That is, we have three equations (corresponding to the three possible states of the world):

$$\begin{cases} 6\Delta^1 + 6\Delta^2 + b = 1 \\ 4\Delta^1 + 4\Delta^2 + b = 0 \\ 2\Delta^1 + 7\Delta^2 + b = 0. \end{cases}$$

The system has unique solution $(\Delta^1, \Delta^2, b) = (0.3, 0.2, -2)$.

Yes, we do need stock 2 for replication, as the above strategy is the unique solution of the linear system equivalent to the replication condition.

8. (i) Diagrams: please DIY. The time $t = 0$ put price is $P = 66.56$.

(ii) The replicating portfolio has the following form: at time $t = 0$, use $(\Delta_1, b_1) = (-0.256, 168.96)$; at time $t = 1$, if $S = 200$ then use $(\Delta_2, b_2) = (-1, 288)$, while if $S = 700$ then use $(\Delta_2, b_2) = (-\frac{4}{35}, 89.6)$.

(iii) We simply have to verify that at time $t = 1$ one has $\Delta_1 S_1 + b_1(1+r) = \Delta_2 S_1 + b_2(1+r)$, whatever the state of the world. This is so indeed, the common value being 160 if $S_1 = 200$ and 32 if $S_1 = 700$.

(iv) Use the put-call parity.

9. Diagrams: please DIY. Here $p^* = \frac{1-2/3}{4/3-2/3} = 0.5$, the call payoff $X = (S_4 - 120)^+$. Working backwards, from the five possible payoff values (after four periods) $(16-120)^+ = 0$, $(32-120)^+ = 0$, $(64-120)^+ = 0$, $(128-120)^+ = 8$ and $(256-120)^+ = 136$, we find that the time $t = 0$ call price is 10.5.

10. (i) The Black-Scholes formula gives $C \approx 0.1184$. (ii) The implied volatility is approx. 0.2594.

11. (i) $C \approx 17.95$, $P \approx 21.54$. (ii) $C \approx 1.46$, $P \approx 11.09$. Observe that both prices are close to 20 in (i), while the call is much cheaper in (ii), the put price being close to the value $(S_0 - K)^- = 10$. One can explain that by noting that, in the latter case, the time to maturity is small and so it is unlikely that the stock price will change much. If S_T is close to 100, the call is next to worthless (its price is positive as it is still possible that $S_T > 110$), while the put value $(S_T - K)^-$ is close to $(100 - 110)^- = 10$.