1. Let $\{W(t),\ t \geq 0\}$ be a Weiner (standard BM) process . Then, find the value of
   (a) $E\left((W(t) - W(s))^2\right)$; Ans. $t - s$
   (b) $E(W(s)|W(t) = x)$ for $0 < s < t$; Ans. sx/t
   (c) $Cov(W(s), W(t))$ for $0 < s < t$; Ans. s
   (d) $E(W(t)|W(s) = x)$ for $0 < s < t$. Ans. x

2. Let $\{Y(t),\ t \geq 0\}$ be a geometric Brownian motion with $Y(0) = a$. Then, find the value of $E(Y(t))$.
   Ans. $ae^{t(\mu+\sigma^2/2)}$, where $Y(t)$ is a BM with drift coefficient μ and variance parameter σ^2.

3. Suppose that $\{Y(t),\ t \geq 0\}$, is a geometric Brownian motion with drift parameter $\mu = 0.01$ and volatility parameter $\sigma = 0.2$. If $Y(0) = 100$. Then, find the value of $E[Y(10)]$ Ans. 134.98

4. Consider the random walk that in each Δt time unit either goes up or down the amount $\sqrt{\Delta t}$ with respective probabilities p and $1 - p$, where $p = \frac{1}{2}(1 + \mu\sqrt{\Delta t})$. Argue that as $\Delta t \to 0$ the resulting limiting process is a Brownian motion process with drift rate μ. See class notes

5. Let $\{X(t),\ t \geq 0\}$ be a Brownian motion process with drift coefficient μ and variance parameter σ^2. What is the conditional distribution of $X(t)$ given that $X(s) = c$ when (i) $s < t$? (ii) $t < s$?
   Ans. (i) For $s < t$, $[X(t)|X(s) = c] \sim N(c + \mu(t - s),\ (t - s)\sigma^2)$
   (ii) For $t < s$, $[X(t)|X(s) = c] \sim N\left(\mu t + \sigma tc/s,\ \sigma^2 t(s - t)/s\right)$