$(\Omega, \mathcal{F}, \mathbb{F}, P)$ filtered prob. space

A r.v. $\tau$ on that space is called "stopping time" ST if one has _____ ①

$$\{\tau \leq t\} \in \mathcal{F}_t \text{ for each } t = 0, 1, 2, \dots$$

for an ST $\tau$

$$\underbrace{\{\tau = t\}}_{} = \underbrace{\{\tau \leq t\}}_{\in \mathcal{F}_t} \cap \underbrace{\{\tau \leq t-1\}^c}_{\in \mathcal{F}_{t-1} \subset \mathcal{F}_t} \in \mathcal{F}_t \qquad ②$$

$$\{\tau \leq t\} = \bigcup_{s=0}^{t} \underbrace{\{\tau = s\}}_{\in \mathcal{F}_s \subset \mathcal{F}_t} \qquad \text{since } ② \Rightarrow ①$$

About name.

$\tau$ (random) time when we decide to stop doing something (stop gambling or to sell a block of share at a stock exchange)

$\tau = t$, you act on the basis of you already know by that time $\therefore \{\tau = t\} \in \mathcal{F}_t$