AI61201: Visual Computing With AI/ML

Module 5: Multi-resolution Representations

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Multi-resolution Representation

- Multi-resolution representation involves representing an image at multiple scales or resolutions to capture different levels of detail.
- Multi-resolution representation enables feature extraction at different granularities.
- Image pyramids are a commonly used type of multi-resolution representation, where the original image is progressively downsampled to construct each successive level of the pyramids.
- Image pyramids are useful in accelerating image analysis algorithms, finding objects at different scales, etc.

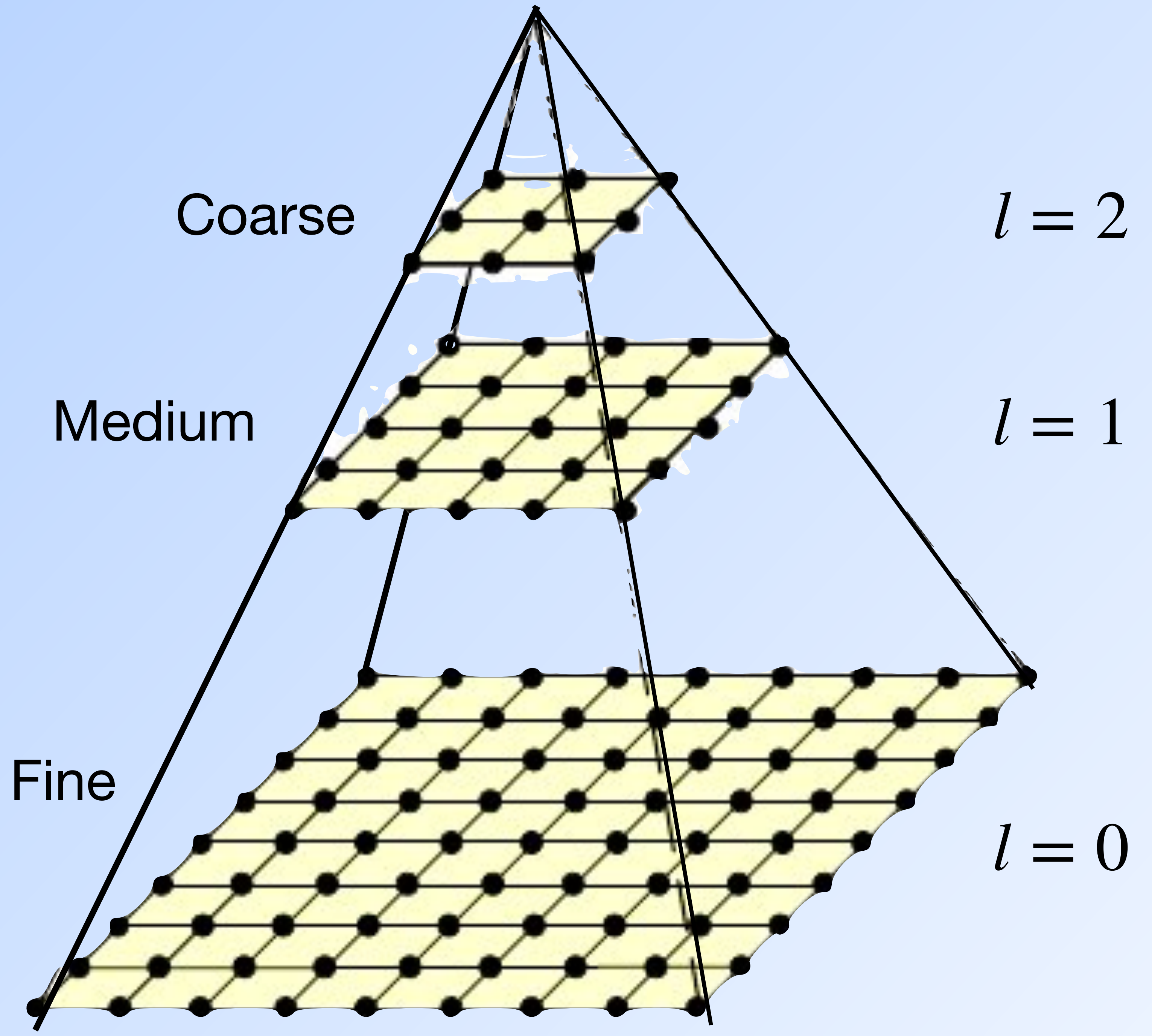


Image Pyramid Representation

Gaussian Pyramid

- In Gaussian pyramid, the image at each level is smoothed by applying a symmetric Gaussian kernel and then resampled to half the resolution to get the next level.
- Gaussian pyramid representation in useful in applications such as image fusion, motion-estimation, pattern matching etc.
- Applications:
    - Coarse to fine image analysis - useful for making the analysis algorithms faster
    - Multi-scale object detection - useful for detecting objects occurring at different scales.

A three level Gaussian Pyramid of an image

Laplacian Pyramid

- The Laplacian pyramid focuses on capturing details or high frequency information at different scales.
- The Laplacian pyramid is built on top of the Gaussian pyramid.
- The top level of the Laplacian pyramid is the same as the top level of the Gaussian pyramid, and each successive level is obtained by subtracting the upsampled version of the next higher level of the Gaussian pyramid from the current level.
- The Laplacian pyramid represents a band-pass filtering operation
- Applications: texture synthesis, image compression, noise removal, image blending.

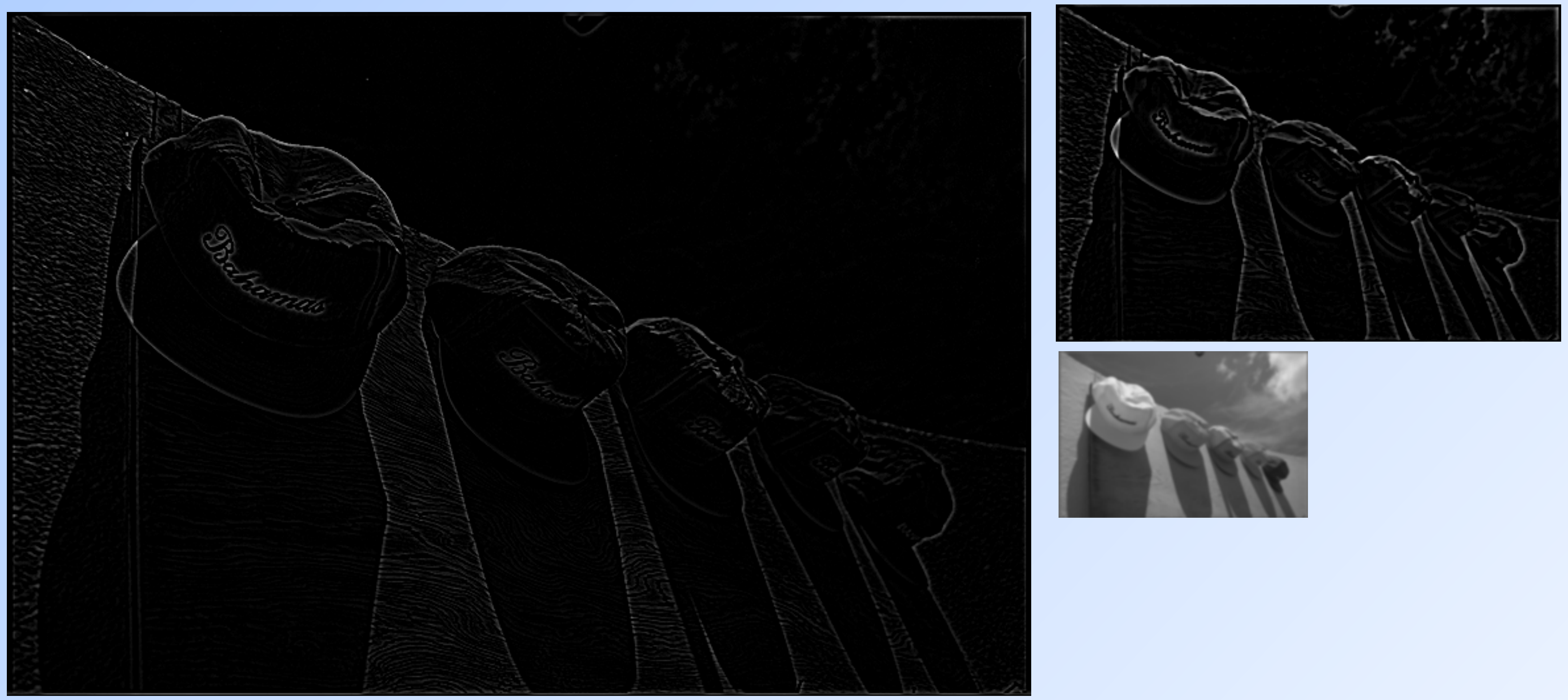


Image 1

Laplacian Pyramid

- The Laplacian pyramid can be used to blend two images as follows:
  - The Laplacian pyramids are created for both the images to be blended.
  - A Gaussian pyramid is created for the blending mask, which defines how the blending should occur.
  - At each level of the pyramid, the blended Laplacian is computed as follows:

$$L_n(x, y) = G_n^{mask}(x, y)L_n^1(x, y) + (1 - G_n^{mask}(x, y))L_n^2(x, y)$$

  - The image is reconstructed level by level using the blended Laplacians, until the finest level of the pyramid is reached.

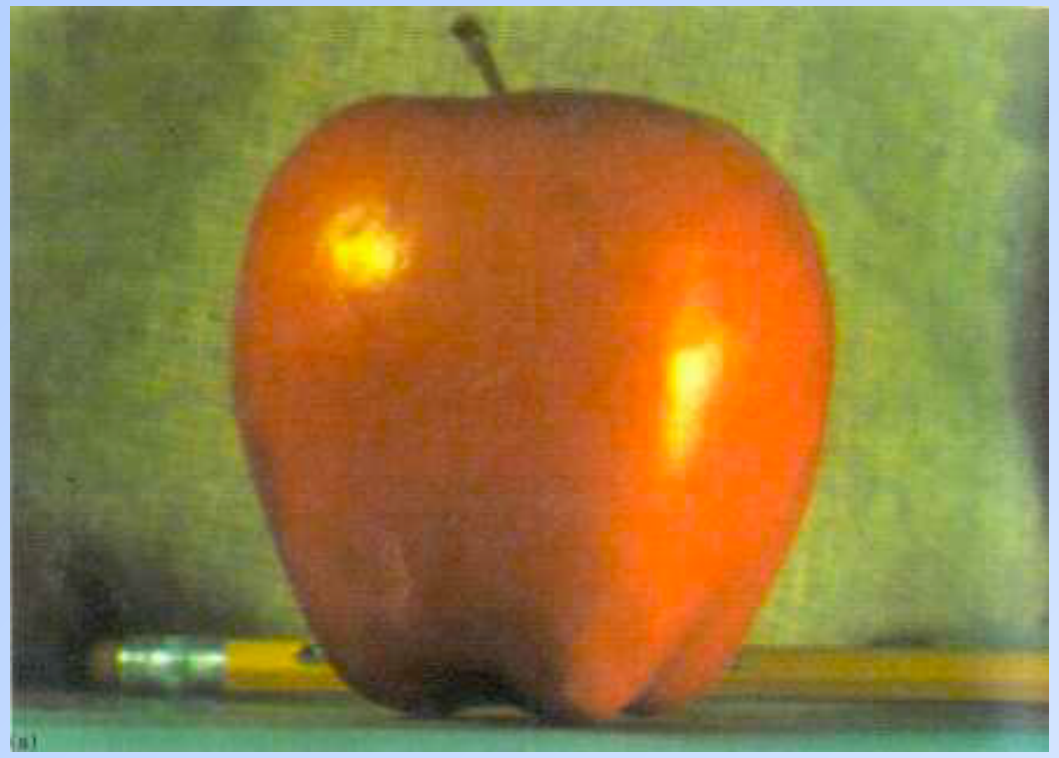


Image 1

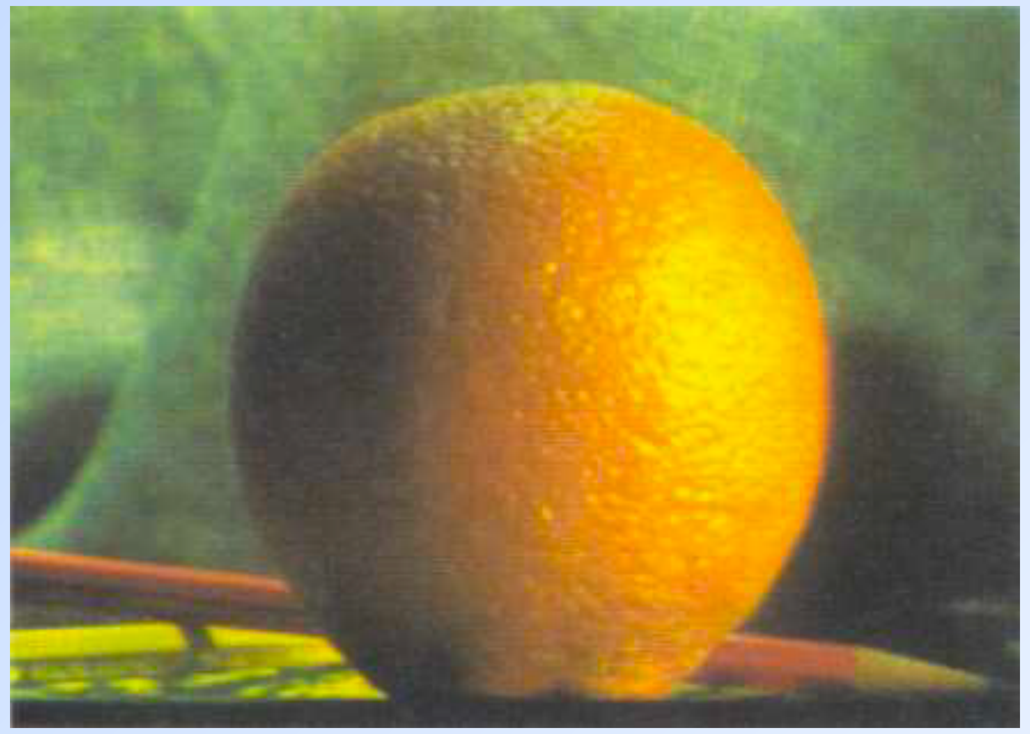
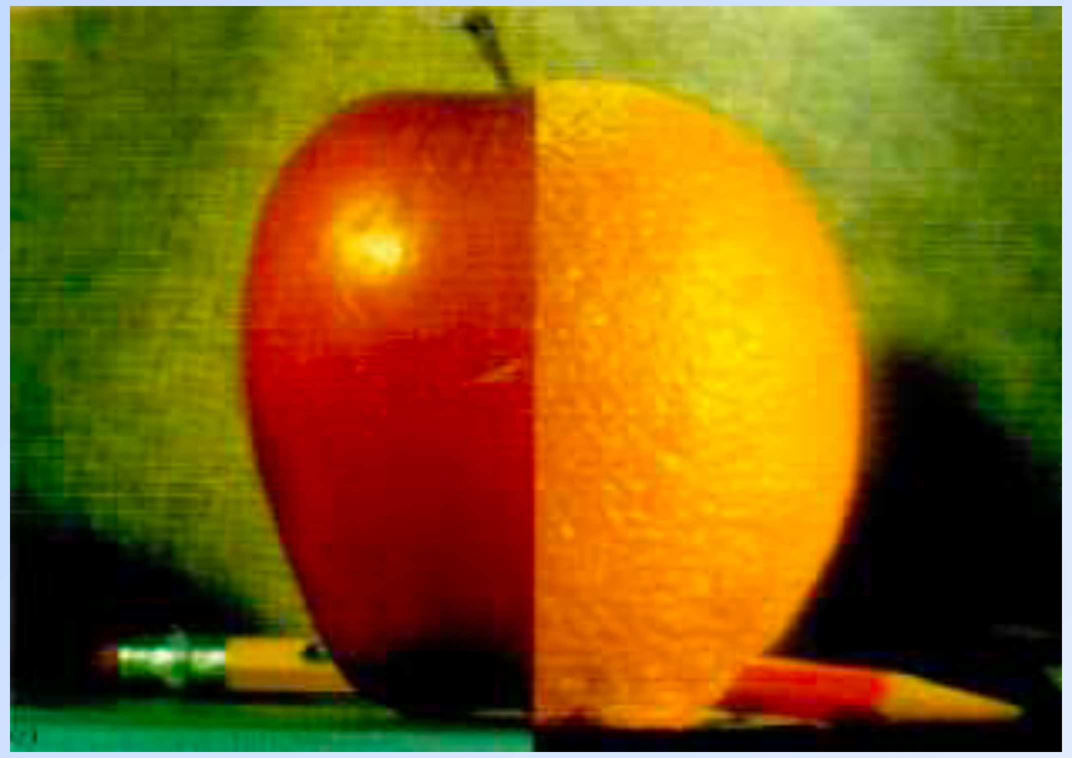
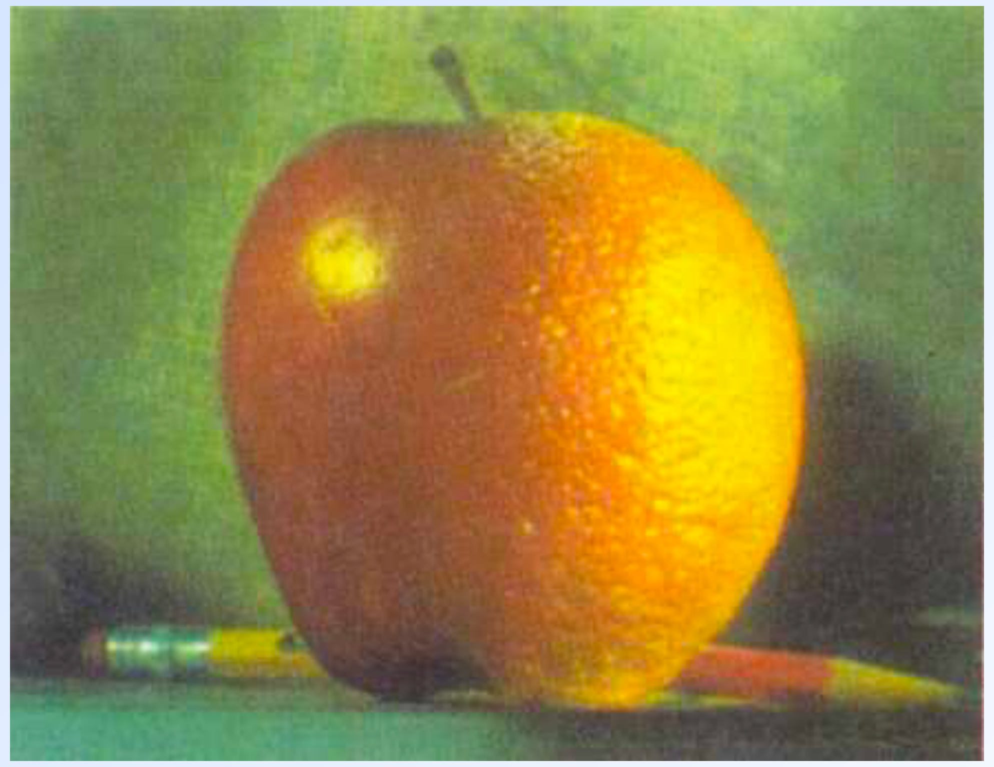


Image 2



Blending by directly replacing pixels



Blending using Laplacian Pyramid

Wavelet Decomposition

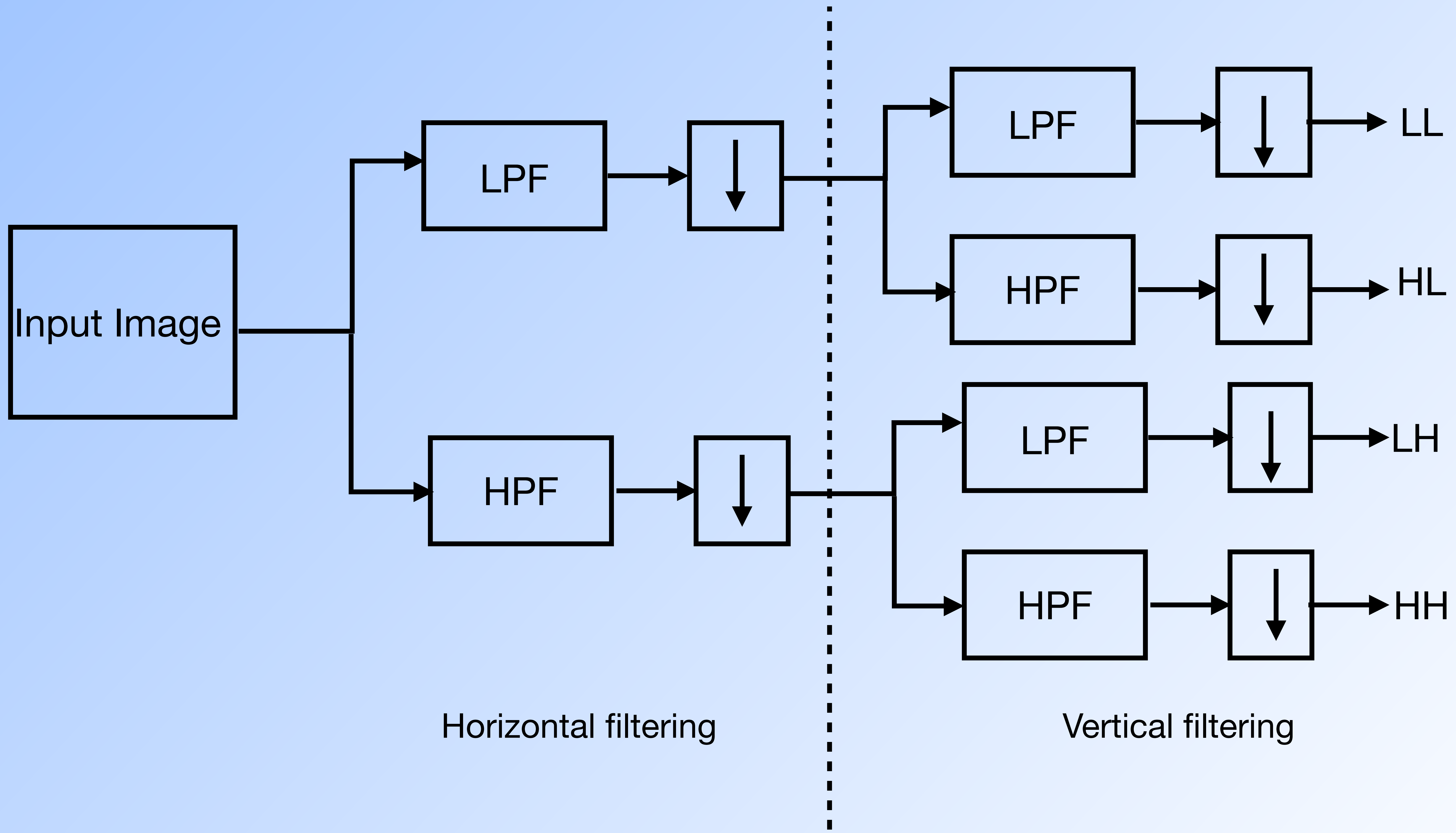
- Wavelet decomposition is a technique for breaking down an image into different frequency components, that capture both spatial and frequency information.
- Wavelet decomposition involves decomposing an image into multiple levels or scales using discrete wavelet transform (DWT).
- Unlike DFT, which decomposes signals into sinusoids, DWT decomposes signals into *wavelets,* which are designed to achieve efficient localization the spatial dimension as well as frequency dimension.
- DWT involves recursively filtering an image with a series of wavelet filters and downsampling to decompose the signal into multiple levels of resolution, capturing both coarse and fine details.
- The sub-images that represent details at different scales and frequency components are called subbands.
- While both the Gaussian and Laplacian pyramid provide over-complete representation of the image, wavelet decomposition provide a more compact and orientation selective representation, allowing for efficient storage and compression.

Wavelet Decomposition

Wavelet decomposition uses filter pairs as follows

- Low-pass filter (LPF): Extracts the approximation coefficients (which represent the image at a coarse scale).
- High-pass filter (HPF): Extracts the detail coefficients (which represent the image at a fine scale).

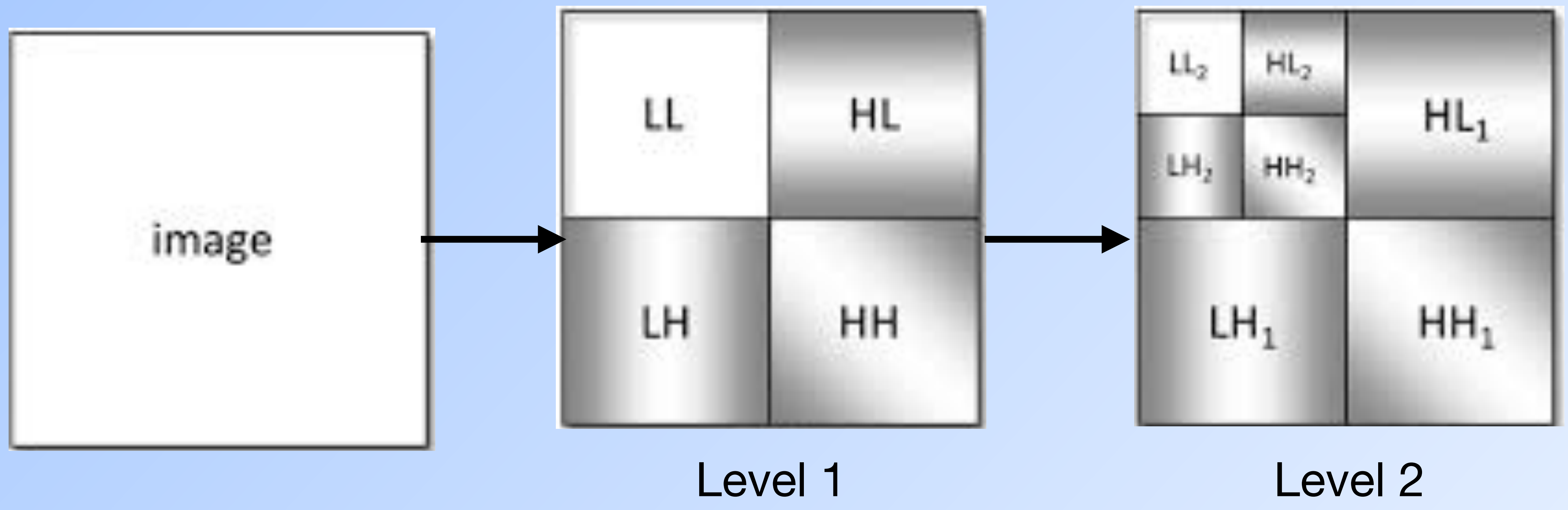
- The filters are separately applied row-wise and column-wise.
- The resulting decomposition splits the image into subbands, where each subband captures a different range of frequencies.



Wavelet Decomposition

- Multi-Scale Decomposition: each subband provides a different scale of detail:
  - Approximation Subbband (LL): Represent the low-frequency content of the image.
  - Vertical Detail Subband (HL): Capture high-frequency details in the vertical direction, i.e. horizontal edges.
  - Horizontal Detail Subband (LH): Capture high-frequency details in the horizontal direction, i.e. vertical edges.
  - Diagonal Detail Subband (HH): Capture high-frequency details in the diagonal direction, i.e. diagonal edges.

- Recursive Decomposition: decomposition can be applied recursively to the approximation subband to further break them down into more detailed levels.



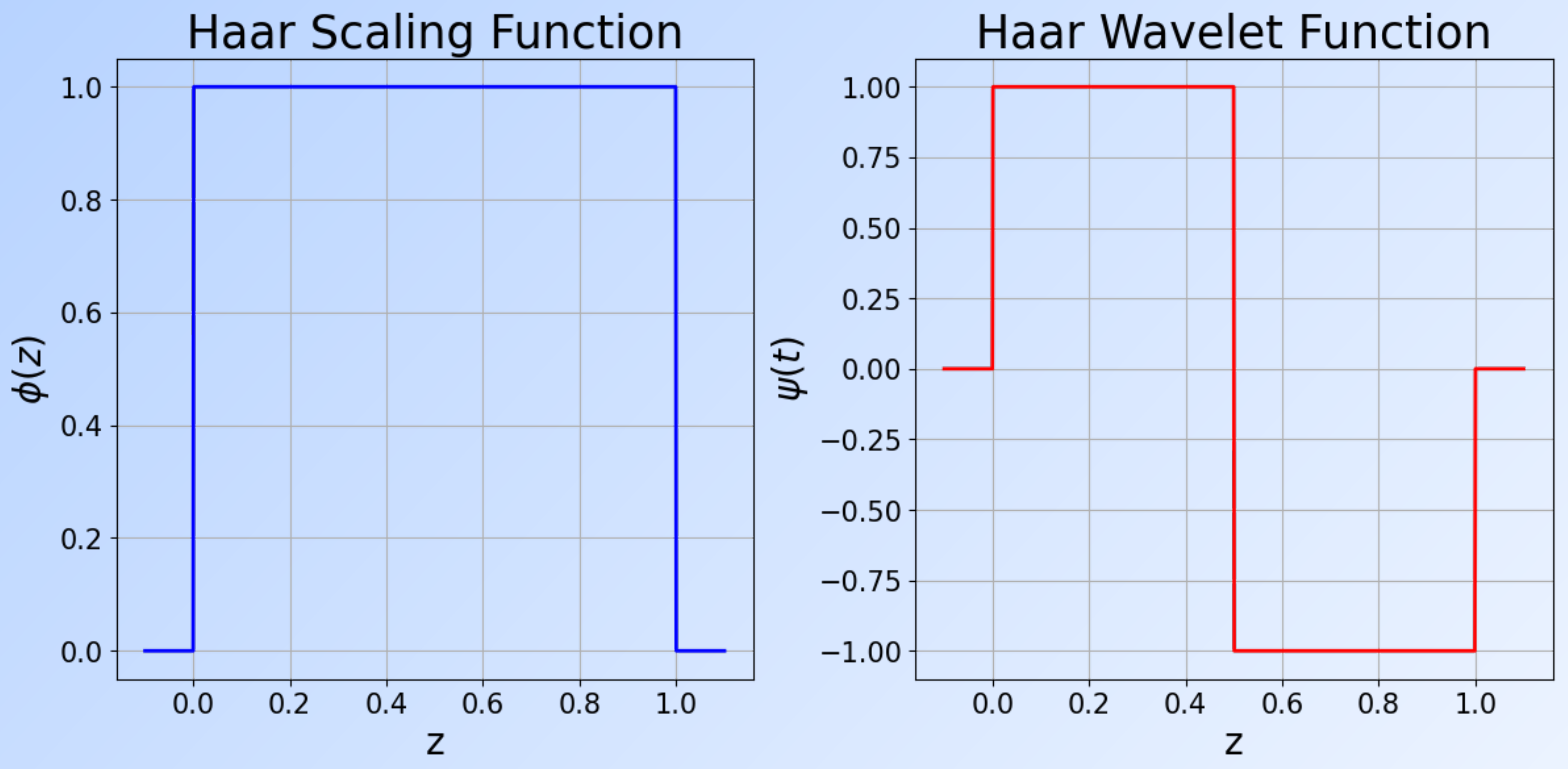
Haar Wavelets

- The simplest wavelet filters are the Haar wavelets, widely used due to their ease of implementation.

- The continuous Haar wavelets are defined as follows:

$$\phi(z) = \begin{cases} 1 & \text{for } 0 \leq z < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{(Scaling function)}$$

$$\psi(t) = \begin{cases} 1 & \text{for } 0 \leq z < 0.5 \\ -1 & \text{for } 0.5 \leq z < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{(Wavelet function)}$$



Haar Wavelets

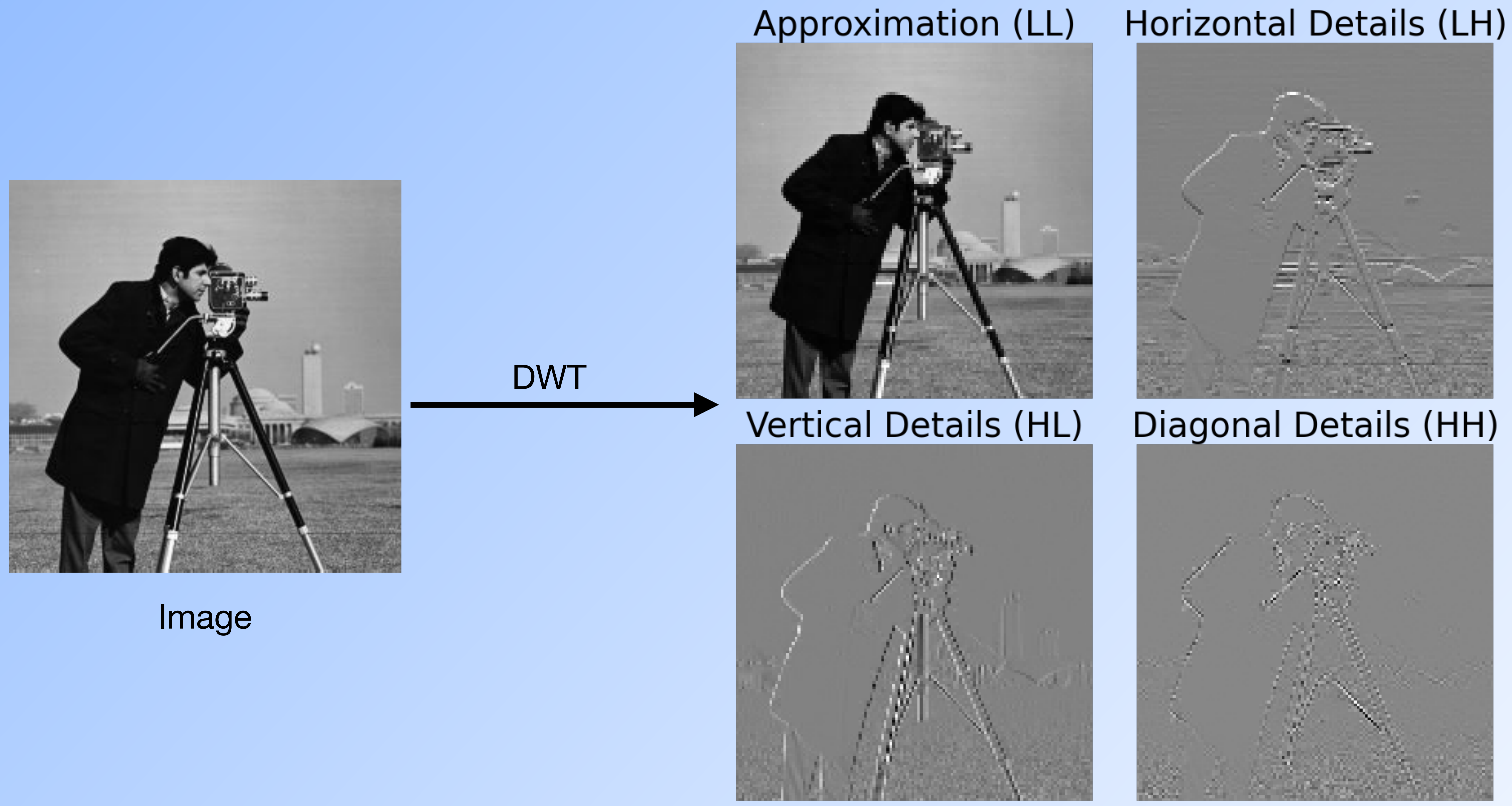
- The low pass and high pass filter coefficients used to perform DWT using Haar wavelets are derived by discretizing the corresponding continuous wavelets and are as follows:

  Low pass filter (Scaling function): $\frac{1}{\sqrt{2}}[1 \quad 1]$

  High pass filter (Wavelet function): $\frac{1}{\sqrt{2}}[1 \quad -1]$

- Each filter is first applied row-wise and then column wise to produce the filtered subbands.
- The Haar wavelets are computationally efficient.
- Haar wavelets are not effective in capturing smooth changes.
- The discontinuities in the Haar wavelets manifest as blocking artifacts, when used for compression followed by reconstruction.

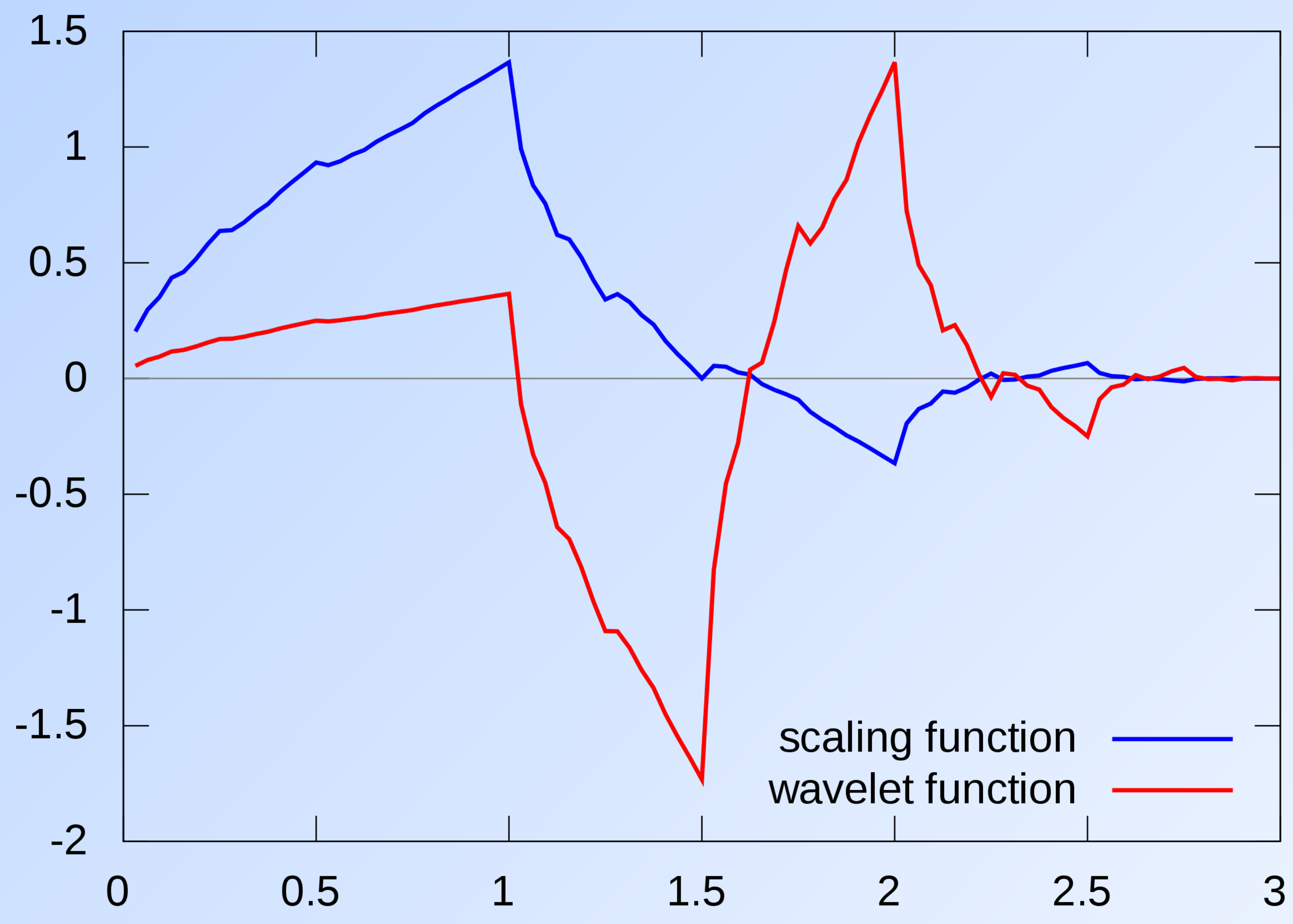
Wavelet Decomposition



DWT Using Haar Wavelets

Daubuchies Wavelets

- Daubechies wavelets are a family of wavelets characterized by their smoothness, compact support and orthogonality.
- Daubechies wavelets are denoted by dbN, where N is the number of vanishing moments of the filter.
- The db2 filter is given by:
  - Low pass (scaling) coefficients:
    [0.48296, 0.8365, 0.22414, -0.12940]
  - High pass (wavelet) coefficients:
    [-0.12940, -0.22414, 0.8365, -0.48296]



db2 Wavelets

Applications of Wavelet Decomposition

- Image Compression (e.g. in JPEG 2000)
- Noise Reduction
- Image Fusion
- Texture Analysis