Q5

a) density of states $D(E) = \frac{\text{no. of states b/w } E \text{ \& } \Delta E}{V \times \Delta E}$

Now, we know each point in 2D K-space plot, represents an allowed energy / quantum state corresponding to various integral values of n_x and n_y positive & negative values of K_x, K_y and K_z have same energy.

Now for the derivation

Let the distance b/w any two quantum states in the K_x direction is

$$K_{x+1} - K_x = (n_x + 1)\frac{\pi}{a} - n_x\left(\frac{\pi}{a}\right) = \pi/a$$

Similarly,

$$K_{y+1} - K_y = (n_y + 1)\pi/a - n_y(\pi/a) = \pi/a$$

$$K_{z+1} - K_z = (n_z + 1)(\pi/a) - n_z(\pi/a) = \pi/a.$$

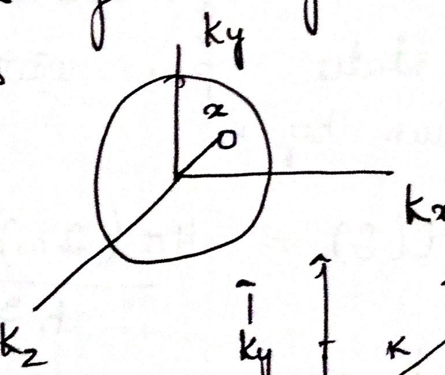
So, volume of single quantum state V_K is

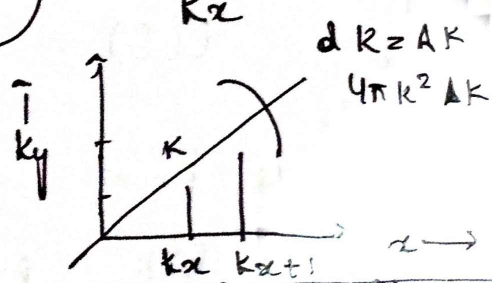
$$V_K = (\pi/a)^3$$

A volume in K space is given by $4\pi k^2 dx$.

So, density of quantum states $D(k)\,dx$ in K space is

$$D(k)\,dx = 2 \times \frac{1}{8}\left(\frac{4\pi k^2 dK}{(\pi/a)^3}\right)$$





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∴ factor 2 takes into 2 spin states allowed for each quantum status factor 1/8 takes into account that only +ve values of k_x and k_y and k_z are considered.

So,

$$D(k)\,dk = \frac{\pi k^2 dk}{(\pi/a)^3} \quad \text{----}$$

For free electron, $E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \frac{\sqrt{2mE}}{\hbar}$

$$\frac{dk}{dE} = \frac{\sqrt{2m}}{\hbar} \times \frac{1}{2} E^{-1/2}$$

$$\Rightarrow dk = \frac{1}{\hbar}\sqrt{\frac{m}{2E}}\,dE$$

Now,

$$D(E)\,dE = \frac{a^3}{\pi^2}\left(\frac{2mE}{\hbar^2}\right)\frac{1}{\hbar}\sqrt{\frac{m}{2E}}\,dE$$

$$\Rightarrow D(E)\,dE = \frac{4\pi a^3}{h^3}(2m)^{3/2}\cdot\sqrt{E}\,dE$$

$E \rightarrow$ total no. of quantum state.

quantum state per unit volume per unit energy is given by,

$$D(E) = \frac{4\pi(2m)^{3/2}}{h^3}\cdot\sqrt{E}$$

Now, we know.

$$E - E_c = \frac{\hbar^2 k^2}{2m^*_n} \quad \& \quad E - E_v = \frac{-\hbar^2 k^2}{2m^*_p}$$

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So,

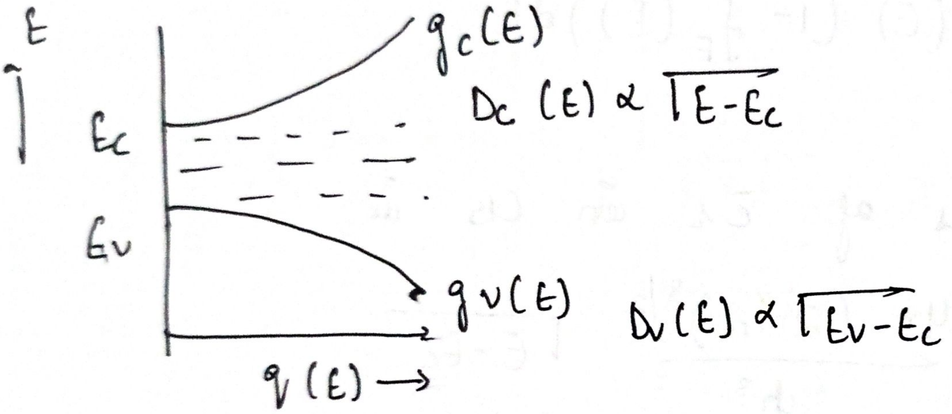
$$D_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c} \quad ; \quad E > E_c$$

Similarly,

$$D_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v-E} \quad ; \quad E < E_v$$

in the forbidden band energy levels are not allowed so,

$$D(E) = 0 \quad ; \quad E_v < E < E_c$$



if $m_n^* = m_p^*$

then $D_c(E)$ and $D_v(E)$ are symmetrical.

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(b) Total $\bar{e}$ conc. per unit volume

$$n(E) = \int_{E_c}^{\text{top of CB}} D_c(E) - f_F(E)\, dE = \int_{E_c}^{\infty} D_c(E) \cdot f_F(E)\, dE$$

Similarly, the total hole concentration per unit volume, $P(E)$ in VB is

$$P(E) = \int_{\text{bottom of VB}}^{E_v} D_v(E)\,(1 - f_F(E))\, dE$$

$$= \int_{-\infty}^{E_v} D_v(E)\,(1 - f_F(E))\, dE$$

Density of states of $\bar{e}$s in CB is

$$D_c(E) = \frac{4\pi\,(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

So,

$$n(E) = \int_{E_c}^{\infty} \frac{4\pi\,(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c} \cdot \frac{1}{1 + e^{(E - E_F)K_BT}}\, dE$$

$[\therefore\ E - E_F \gg K_BT]$

So,

$$n(E) = \int_{E_c}^{\infty} \frac{4\pi\,(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c} \cdot e^{-\frac{(E - E_F)}{K_BT}}\, d(E - E_c)$$

Let $\displaystyle \vartheta = \frac{E - E_c}{K_BT}$

$$n(E) = 4\pi\left(\frac{2m_n K_BT}{h^3}\right)^{3/2} \cdot e^{-\frac{(E_c - E_F)}{K_BT}} \int_0^{\infty} \sqrt{\vartheta}\, e^{-\vartheta}\, d\vartheta$$

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The integral is the gamma function with a value of $\sqrt{\pi}/2$

So, $$n \approx n(E) = 2\left(\frac{2\pi m_n^* K_B T}{h^2}\right)^{3/2} e^{-\frac{(E_c - E_F)}{K_B T}}$$

Similarly, we can find

holes concentration $$p = 2\left(\frac{2\pi m_p^* K_B T}{h^2}\right)^{3/2} e^{-\frac{(E_F - E_V)}{K_B T}}$$

Since $\bar{e}$ and holes conc. are equal in intrinsic semiconductor ie $n = p$

$$\Rightarrow 2\left(\frac{2\pi m_n^* K_B T}{h^2}\right)^{3/2} \cdot e^{-\frac{(E_c - E_F)}{K_B T}} = 2\left(\frac{2\pi m_p^* K_B T}{h^2}\right)^{3/2} \cdot e^{-\frac{(E_F - E_V)}{K_B T}}$$

$$\Rightarrow e^{-\frac{(E_c - E_F) + (E_F - E_V)}{K_B T}} = \left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$\Rightarrow -\frac{(E_c - E_F) + (E_F - E_V)}{K_B T} = \frac{3}{2} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$2E_F - (E_c + E_V) = \frac{3}{2} K_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_F = \frac{(E_c + E_V)}{2} + \frac{3}{4} K_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$

if $m_p^* = m_n^*$ then fermi level is exactly in center of bandgap (E_g)

$m_p^* > m_n^*$, Fermi level is slightly above centre of g.

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$m^*_p < m^*_n$ it is slightly below centre of equation