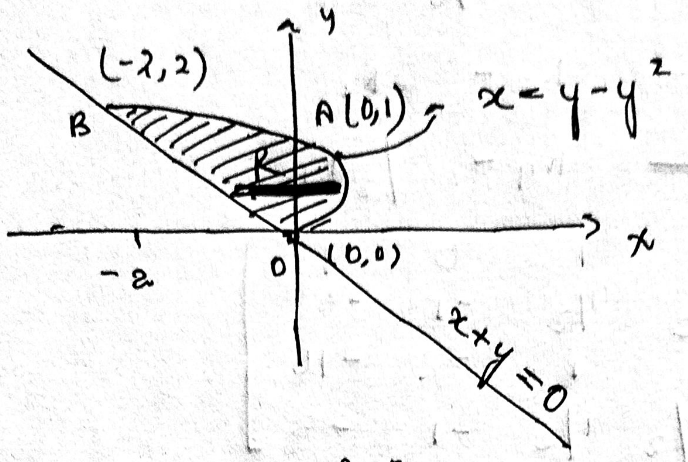
Q2 a) The plane region R (fig 6.1) is OAB with O(0,0), A(0,1), B(-2,2) since the points of intersection of parabola $x = y - y^2$ and $x + y = 0$ are $(-2,2)$ and $(0,0)$.



M contained in R is

Total mass,

$$M = \iint_R \sigma \, dA$$

$$= \int_0^2 \int_{x=-y}^{y-y^2} (x+y) \, dx \, dy$$

$$\int_0^2 \left[\left(\frac{x^2}{2} + xy \right) \right]_{-y}^{y-y^2} dy$$

$$\left[\left(\frac{y^2}{10} - \frac{y^4}{2} + \frac{2y^3}{3} \right) \right]_0^2$$

So, $M = 8/15$

For centroid, (x_2, y_2)

$$x_c = \frac{1}{M} \iint_R x \, f(x,y) \, dx \, dy$$

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$$\frac{15}{8}\int_{0}^{2}\int_{-y}^{y-y^2} x\,(x+y)\,dx\,dy$$

$$\frac{15}{8}\int_{0}^{2}\left[\frac{x^3}{3}+\frac{x^2 y}{2}\right]_{-y}^{y-y^2} dy$$

$$\frac{15}{8}\int_{0}^{2}\frac{y^3}{6}\left[4-12y+9y^2-2y^3\right] dy$$

$$\frac{1}{6}\left[\frac{4y^4}{4}-\frac{12y^5}{5}+\frac{9y^6}{6}-\frac{2y^7}{7}\right]_0^2 \times \frac{15}{8}$$

$$= -3/7$$

$$y_c = \frac{1}{M}\iint y\, f(x,y)\,dx\,dy$$

$$\frac{15}{8}\int^{2}\int_{-y}^{y-y^2} y\,(x+y)\,dx\,dy$$

$$\frac{15}{8}\int_{0}^{2}\left[\frac{yx^2}{2}+y^2x\right]_{-y}^{y-y^2} dy$$

$$\frac{15}{16}\left[\frac{y^6}{6}-\frac{8y^5}{5}+\frac{5}{2}\frac{y^4}{4}\right]=\frac{11}{8}$$

$$I_x = \int_{0}^{2}\int_{-y}^{y-y^2} y^2\,(x+y)\,dx\,dy$$

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$$\int_0^2 \left[\left(\frac{y^2x^2}{2} + y^3x \right) \right]_{-y}^{y-y^2} dy$$

$$\left[\frac{y^7}{14} - \frac{2y^6}{6} + \frac{2y^5}{5} \right]_0^2 = \frac{64}{105}$$

$$I_y = \int_0^2 \int_{-y}^{4-y^2} x^2 (x+y)\, dx\, dy$$

$$\int_0^2 \int_{-y}^{y-y^2} x^2 (x+y)\, dx\, dy$$

$$\int_0^2 \left[\frac{x^4}{4} + \frac{x^3}{3} y \right]_{-y}^{y-y^2} dy$$

$$= \frac{1}{12} \left[\frac{3y^9}{9} - \frac{16y^3}{8} + \frac{30y^7}{7} - \frac{24y^6}{6} + \frac{8y^5}{3} \right]_0^2$$

$$= \frac{256}{105}$$

$$I_o = I_x + I_y = \frac{64}{105} + \frac{256}{105} = \frac{64}{21}$$

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b) Given $\iint_R (x^2+y^2)\, dx\, dy$

Interval +ve quadrant b/w $x^2-y^2=a$

$x^2-y^2=b$, $2xy=c$

and $2xy=d$

where $0<a<b$, $0<c<d$

Using method of changing of variables

Let $u=x^2-y^2$

$v=2xy$

By Jacobian transformation,

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2x & -2y \\ 2y & 2x \end{vmatrix} = 4(x^2+y^2)$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{4(x^2+y^2)}$$

In above eq^n $\Rightarrow$ $x^2-y^2=a \Rightarrow u=a$

$x^2-y^2=b \Rightarrow u=b$

$2xy=c \Rightarrow v=c$

$2xy=d \Rightarrow v-d$

$$\iint_R (x^2+y^2)\, dx\, dy = \int_{v=c}^{d}\int_{u=a}^{b} (x^2+y^2) \times |J|\, du\, dv$$

$$= \frac{1}{4}\int_c^d (b-a)\, dv = \Big[(b-a)v\Big]_c^d \times 1/4$$

$$= \frac{1}{4}(b-a)(c-d) \text{ Ans.}$$

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