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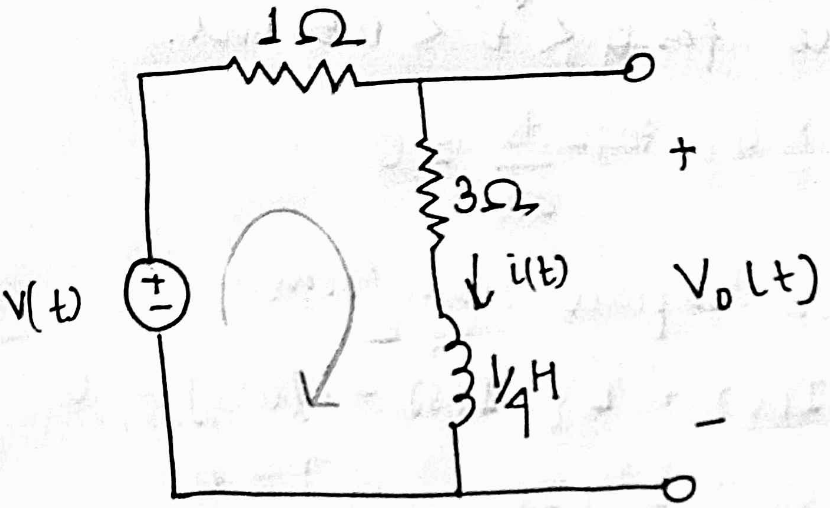
NT Assignment-2

Q1. consider given circuit. Find $V_0(t)$ if $i(0) = 2$ A and $v(t) = 0$



$$i(t) = i(0)\, e^{-t/\tau}$$

$$R_{eq} = 3\Omega + 1\Omega = 4\Omega$$

$$\tau = L/R_{eq} = \frac{1/4}{4} = 1/16$$

$$i(t) = 2e^{-16t}$$

$$V_0(t) = 3i + L\frac{di}{dt} = 6e^{-16t} + \frac{1}{4}(-16)2e^{-16t}$$

$$V_0(t) = -2e^{-16t}\, u(t) \text{ volts.}$$

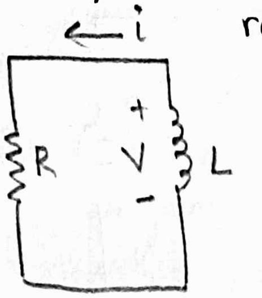
Q2. $v(t) = 20e^{-10^3 t}$ V , $t > 0$

$i(t) = 4e^{-10^3 t}$ mA , $t > 0$

$i(0) = 4$ mA.

a) Find R, L, & τ.

b) Calculate the energy dissipated in resistance for $0 < t < 0.5$ ms.



a) $R i_{(t)} + L\frac{di}{dt} = 0$

Use Laplace transform.

$RI(s) + L[sI(s) - i(0^+)] = 0$

$\Rightarrow RI(s) + L[sI(s) - 4] = 0$

$\Rightarrow I(s)[R + sL] = 4L$

$\Rightarrow I(s) = \frac{4L/L}{R/L + s}$

$I(s) = \frac{4}{R/L + s}$

Using inverse Laplace

$i(t) = 4e^{-R/L\, t}$ mA.

From comparing eqn's we get.

$\frac{R}{L} = \frac{1}{\tau} = 10^3 \Rightarrow \boxed{\tau = 1\text{ ms}}$

We know.

$i(t) = \frac{V(t)}{R} \Rightarrow \boxed{R = 5\text{ K}\Omega}$

and so $\frac{R}{L} = 10^3$

$\therefore L = \frac{5 \times 10^3}{10^3} = 5$ H

$$\boxed{\therefore L = 5\text{H} , R = 5\text{ K}\Omega , \tau = 1\text{ ms}}$$

b) Energy dissipated in the resistor is

$$\mathcal{E} = \int_0^t P\,dt = \int_0^t 80 \times 10^{-3} e^{-2\times 10^3} dt$$

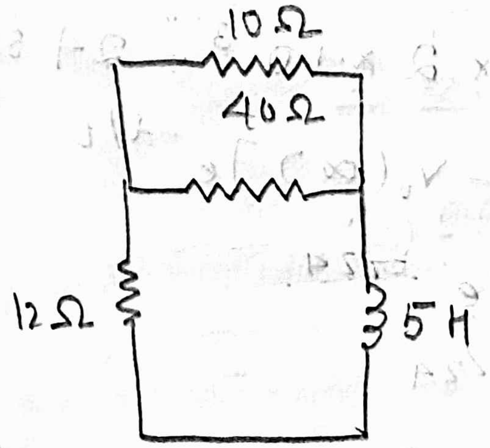
$$= \left[\frac{-80 \times 10^{-3}}{2 \times 10^3} e^{-2\times 10^3 t} \right]_0^{0.5 \times 10^{-3}}$$

$$= 40(1 - e^{-1})\,\mu J$$

$$= 25.28\,\mu J$$

(Ans)

3Q Find the time constant of circuit.



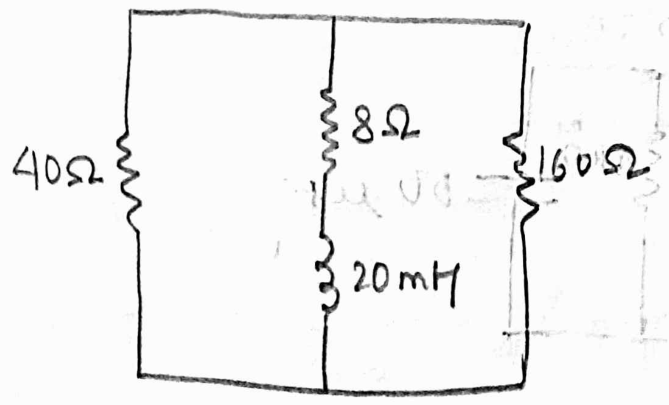
⟨a⟩

$R_{eq} = \frac{10 \times 40}{40+10} + 12$

$= 8 + 12 = 20\ \Omega$.

$L = 5\ H$.

$\tau = \frac{L}{R} = \frac{5}{20} = 0.25\ s$

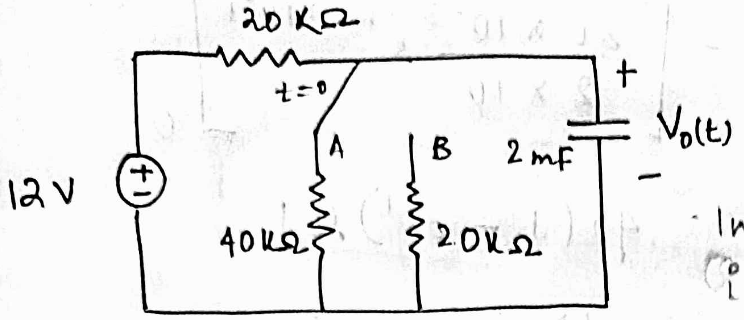


⟨b⟩

$R_{eq} = \frac{40 \times 160}{200} + 8 = 40\ \Omega$

$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{40} = 0.5\ ms$

4Q Assuming the switch in fig has been in position A for a long time and is moved to position B at $t = 0$, find $V_o(t)$ for $t \geq 0$.



Initially, $i = \frac{12}{20+40} = 0.2 \times 10^{-3}$ A.

so $V = 12 - (20 \times 10^3 \times 0.2 \times 10^{-3})$

$= 12 - 4 = 8$ V.

so

$Q_0 = CV = 8 \times 2 \times 10^{-3} = 16 \times 10^{-3}$ C

When the switch is at position B, the circuit reaches steady state, By voltage division.

$V_o(\infty) = \frac{30}{30+20}(12\text{ V}) = 7.2$ V

$R_{th} = 20 \| 30 = \frac{20 \times 30}{50} = 12$ kΩ

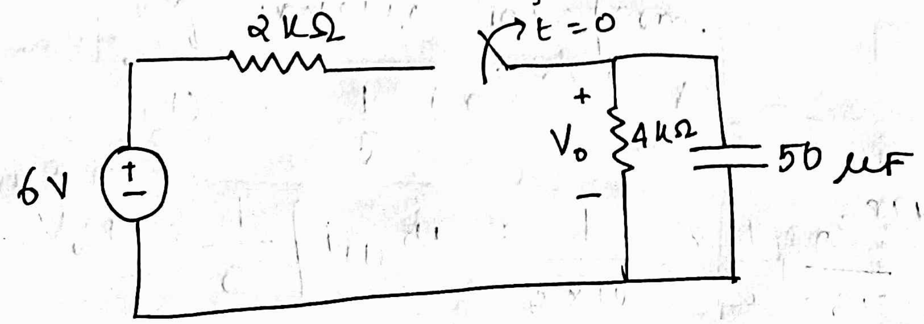
$\tau = R_{th}C = 12 \times 10^3 \times 2 \times 10^{-3} = 24$ sec.

$V_o(t) = V_o(\infty) + [V_o(0) - V_o(\infty)]e^{-t/\tau}$

$= 7.2 + (8 - 7.2)e^{-t/24}$

$= 7.2 + 0.8e^{-t/24}$

Q5. The switch in the given figure opens at $t = 0$. Find V_o for $t > 0$.



For $t < 0$, the switch is closed so that

$$V_o(0) = \frac{4}{2+4}(6) = 4\text{ V}.$$

For $t > 0$, we have source - free RC circuit

$$\tau = RC = 50 \times 10^{-6} \times 6 \times 10^{3}$$

$$= 300 \times 10^{-3}$$

$$= 3 \times 10^{2} \times 10^{-3} = 3 \times 10^{-1} = 0.3 \text{ sec}$$

So

$$V_o(t) = V_o(0)\, e^{-t/\tau}$$

$$= 4e^{-t/0.3} = 4e^{-10/3\, t}.$$

Q6. A circuit is described by $4\frac{dv}{dt} + v = 10$

a) What is the time constant of the circuit?

b) What is $v(\infty)$, the final value of v?

c) If $v(0) = 2$, find $v(t)$ for $t \geq 0$.

Let $V = V_n + V_p$, $V_p = 10$

~~so $V_n + \frac{1}{4}V_n = 0$~~ and, $V_n = 4e^{-t/4}$.

So, $V = 10 + 4e^{-0.25t}$

~~$\Rightarrow V(0) =$~~ It is given $v(0) = 2$

$\Rightarrow 10 + A = 2$

$\Rightarrow A = -8.$

so $\boxed{V = 10 - 8e^{-0.25t}}$ . . (i)

a) time constant $= \tau = \frac{100}{25} = 4$ sec.

b) $V(\infty) = 10\text{ V}$ from eq^n (i)

c) If $v(0) = 2$.

then $\boxed{v(t) = 10 - 8e^{-0.25t}\, u(t)\text{ V}}$

Q To find laplace transforms of following.

Q1. $(t^2+1)^2$

Soln. Let $f(t) = (t^2+1)^2$

Now $L\{f(t)\} = L\{(t^2+1)^2\}$

$= L\{t^4+2t^2+1\}$

$= L\{t^4\} + 2L\{t^2\} + L\{1\}$

$= \frac{4!}{s^4} + 2.\frac{2!}{s^3} + \frac{1}{s}$

(Ans)

Q2. $(\sin t + \cos t)^2$

Soln. Let $f(t) = (\sin t + \cos t)^2$

$= \sin^2 t + \cos^2 t + 2\sin t \cos t$

$= 1 + \sin 2t$ [$\because \sin 2t = 2\sin t.\cos t$]

Now $L\{f(t)\} = L\{1 + \sin 2t\}$

$= L\{1\} + L\{\sin 2t\}$

$= \frac{1}{s} + \frac{2}{s^2+4}$

$= \frac{s^2+4+2s}{s(s^2+4)}$ (Ans)

Q3. $\cos^3 2t$

Soln. Let $f(t) = \cos^3 2t$

[We know $\cos 3A = 4\cos^3 A - 3\sin A$]

So $\cos 6t = \cos 3(2t)$

$= 4\cos^3 2t - 3\cos 2t$

$\Rightarrow 4\cos^3 2t = \cos 6t + 3\cos 2t$

$\Rightarrow \cos^3 2t = \frac{1}{4}(\cos 6t + 3\cos 2t)$

Now $L\{\cos^3 2t\} = \frac{1}{4} L\{\cos 6t\} + \frac{3}{4} L\{\cos 2t\}$

[by linearity property]

$$= \frac{1}{4}\left(\frac{s}{s^2+36}\right) + \frac{3}{4}\left(\frac{s}{s^2+4}\right)$$

$$= \frac{s}{4}\left[\frac{1}{s^2+36} + \frac{3}{s^2+4}\right]$$

(Ans)

Q To find Inverse Laplace Transform of following.

Q4. $\frac{s}{(s^2+a^2)^2}$

Soln.

Let $f'(s) = \frac{s}{(s^2+a^2)}$

Taking Inverse laplace transform on both sides.

$$\pounds^{-1}\{f'(s)\} = \pounds^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$$

$$= \pounds^{-1}\left\{\frac{-1}{2}\frac{d}{ds}\left(\frac{1}{s^2+a^2}\right)\right\}$$

By inverse laplace transform of derivative,

If $\pounds^{-1}\{f'(s)\} = f(t)$

then $\pounds^{-1}\left\{\frac{d}{ds} f'(s)\right\} = -t f(t)$

$$\therefore \mathcal{L}^{-1}\left\{\frac{-1}{2}\frac{d}{ds}\left(\frac{1}{s^2+a^2}\right)\right\} = \frac{-1}{2}\mathcal{L}^{-1}\left\{\frac{d}{ds}\left(\frac{1}{s^2+a^2}\right)\right\}$$

$$= \frac{-1}{2}(-1)\,t\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\}$$

$$= \frac{1}{2}\,t\,.\,\frac{1}{a}\sin at.$$

$$\boxed{\mathcal{L}^{-1}\{f(s)\} = \frac{t\sin at}{2a}.}$$

Q5. $Y(s) = \frac{1}{s(s-1)}$

Soln.

Let $y(t)$ be inverse laplace transform of $Y(s)$.

$y(0) = \lim_{s\to 0} sY(s)$

$$Y(s) = \frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$= \frac{A(s-1) + Bs}{s(s-1)}$$

So Now $A = -1$ at $s = 0$

$B = 1$ at $s = 1$.

$$Y(s) = \frac{-1}{s} + \frac{1}{s-1}$$

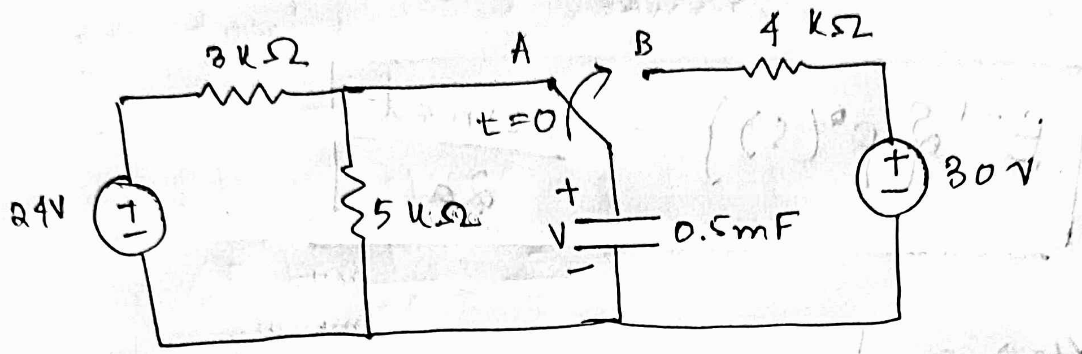
$\Downarrow$

$$\boxed{y(t) = -u(t) + e^{t}u(t)}$$ (Ans).

RL and RC Network:

Q6.

Q1. The switch in the figure has been in position A for a long time. At $t=0$, the switch moves to B. Determine $v(t)$ for $t>0$ and calculate its value at $t=1s$ & $4s$.



for $t<0$, the switch is at position A. Since v is the same as the voltage across the 5-kΩ resistor, the voltage across the capacitor just before $t=0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15\text{ V}.$$

$$v(0^-) = v(0) = v(0^+) = 15\text{ V}.$$

For $t>0$, $R_{th} = 4\text{k}\Omega$ &

$$\tau = R_{th}C = 4\times10^3 \times 0.5\times10^{-3} = 2s.$$

Since capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30\text{V}$. Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= 30 + (15-30)e^{-t/2} = (30 - 15e^{-0.5t})\text{V}$$

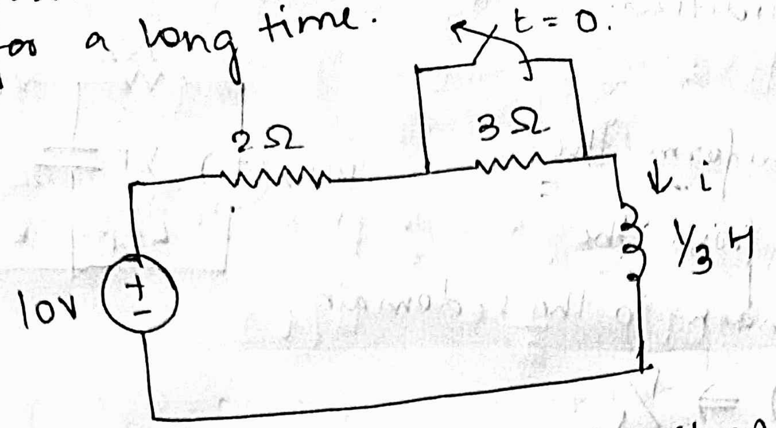
At $t=1$,

$$v(1) = 30 - 15e^{-0.5} = 20.902\text{ V}$$

At $t=4$,

$$v(4) = 30 - 15e^{-2} = 27.97\text{ V}.$$

Q2. Find $i(t)$ in the circuit in the figure. Assume that the switch has been closed for a long time.



When $t<0$, 3-Ω resistor is short-circuited, & the inductor acts like a short circuit. The current through the inductor at $t=0^-$

$$i(0^-) = \frac{10}{2} = 5\ A.$$

Since the inductor current cannot change instantaneously,

$$i(0) = i(0^+) = i(0^-) = 5\ A.$$

When $t>0$, the switch is open. The 2-Ω & 3-Ω resistor are in series, so that

$$i(\infty) = \frac{10}{2+3} = 2\ A.$$

The Thevenin resistance across the inductor terminals is $R_{th} = 2+3 = 5\ \Omega$.

For the time constant,

$$\tau = \frac{L}{R_{th}} = \frac{1/3}{5} = \frac{1}{15}\ s$$

Thus,

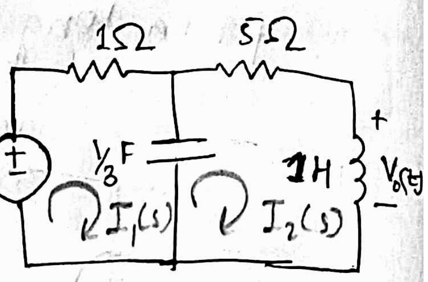
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= 2 + (5-2)e^{-15t} = 2 + 3e^{-15t}\ A, \quad t>0.$$

for $t>0$, KVL must be satisfied.

$$10 = 5i + L\frac{di}{dt}$$

$$5i + L\frac{di}{dt} = [10 + 15e^{-15t}] + \left[\frac{1}{3}(3)(-15)e^{-15t}\right] = 10.$$

3Q Find $V_0(t)$ in the circuit in fig, assuming zero initial condition.



We first transform the circuit from the time domain to the s-domain.

$$u(t) \Rightarrow 1/s$$

$$1\,H \Rightarrow sL = s$$

$$\tfrac{1}{3}F \Rightarrow \frac{1}{sC} = \frac{3}{s}$$

The resulting s-domain circuit in the fig. Applying mesh analysis.

For mesh 1,

$$\frac{1}{s} = \left(1 + \frac{3}{s}\right) I_1 - \frac{3}{s} I_2 \quad \cdots \text{(i)}$$

For mesh 2,

$$0 = -\frac{3}{s} I_1 + \left(s + 5 + \frac{3}{s}\right) I_2 .$$

or $$I_1 = \frac{1}{3}\left(s^2 + 5s + 3\right) I_2 \quad \cdots \text{(ii)}$$

From (i) & (ii)

$$\frac{1}{s} = \left(1 + \frac{3}{s}\right)\frac{1}{3}\left(s^2 + 5s + 3\right) I_2 - \frac{3}{s} I_2$$

Multiplying through by 3s gives

$$3 = \left(s^3 + 8s^2 + 18s\right) I_2 \Rightarrow I_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

$$V_0(s) = s I_2 = \frac{3}{s^2 + 8s + 18} = \frac{3}{\sqrt{2}} \frac{\sqrt{2}}{(s+4)^2 + (\sqrt{2})^2}$$

Taking the inverse transform yields

$$V_0(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin\sqrt{2}t \ \text{V}, \quad t \geq 0$$

4Q The output of a linear system is ~~$y(t) = 10e^{-t}\cos 4u$~~ $y(t) = 10e^{-t}\cos 4t\,u(t)$ when the input is $x(t) = e^{-1}u(t)$. Find the transfer function of the system and its impulse response.

If $x(t) = e^{-1}u(t)$ and $y(t) = 10e^{-t}\cos 4t\,u(t)$,

then

$$X(s) = \frac{1}{s+1} \quad \text{and} \quad Y(s) = \frac{10(s+1)}{(s+1)^2+4^2}$$

Hence,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10}{(s+1)^2+16} = \frac{10}{s^2+2s+17}$$

$$\Rightarrow H(s) = \frac{10}{4}\left(\frac{4}{(s+1)^2+4^2}\right)$$

so now

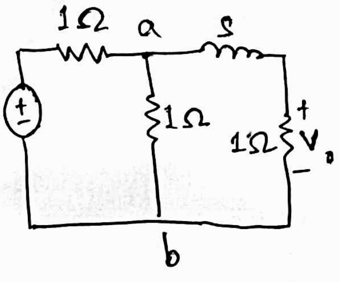
$$\boxed{h(t) = 2.5e^{-t}\sin 4t}$$

(Ans)

impulse response.

5Q For the s-domain circuit in given figure. Find a) the transfer function $H(s) = V_o/V_i$

b) the impulse response.



a) Using voltage division,

$$V_o = \frac{1}{s+1} V_{ab} \quad \ldots (i)$$

but

$$V_{ab} = \frac{1 \,\|\, (s+1)}{1 + 1\,\|\,(s+1)} V_i = \frac{(s+1)/(s+2)}{1+(s+1)/(s+2)} V_i$$

or

$$V_{ab} = \frac{s+1}{2s+3} V_i \quad \ldots (ii)$$

Substituting (ii) in (i) gives.

$$V_o = \frac{V_i}{2s+3}$$

Thus, impulse response is

$$H(s) = \frac{1}{2}\left(\frac{1}{s+\frac{3}{2}}\right) \qquad \left[\because H(s) = \frac{V_o}{V_i}\right]$$

b) Its inverse L.T gives impulse response.

$$h(t) = \frac{1}{2} e^{-3t/2} \cdot u(t)$$