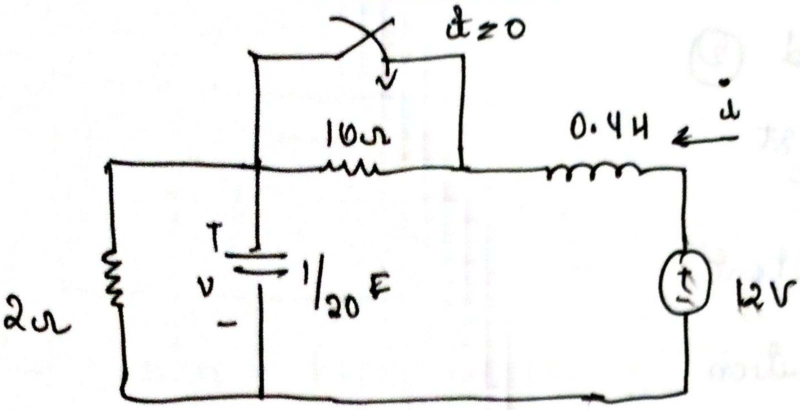
Q3

a)



(i) $i(0^+) = \frac{12}{10+2} = \frac{12}{12} = 1A.$

$v(0^+) = \frac{10}{\cancel{12}} \times \cancel{12} = 10V$

(ii) $\frac{di}{dt}(0^+) = \frac{V}{L} = \frac{10}{0.4} = \frac{100}{4}$

$= 25 \; A/s.$

$\frac{dv}{dt}(0^+) = \frac{i_c(0^+)}{C} = 0 \; V/s.$

(iii) $i(\infty) = \frac{12}{12} = 1A.$

$v(\infty) = 12V.$

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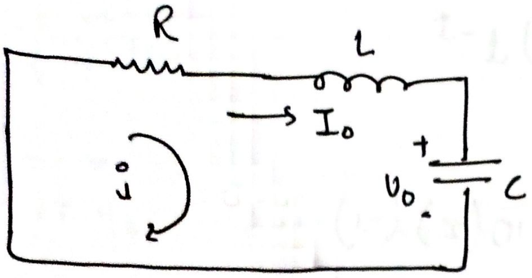
b)



Consider series RLC circuit. The circuit is being excited by energy initially stored in capacitor & inductor. The energy is represented by initial capacitor voltage V_0 & initial inductor current I_0. Thus, at $t=0$,

$$v(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0$$

$$i(0) = I_0 \quad — \text{(1)}$$

Apply KVL around the loop

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = 0$$

differentiate w.r.t t

$$\frac{d^2 i}{dt^2} + \frac{C}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

This second order differential eq. is reason for calling RLC circuit in this. To solve such a second order equation, we require 2 initial conditions such as initial value of i and its first derivative are initial values of some i & V.

$$Ri(0) + L\frac{di}{dt}(0) + V_0 = 0$$

or $$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) \quad — \text{(2)}$$

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From eq ① and ②

$$i = Ae^{st}$$

A and s → constant

By differentiation

$$As^2 e^{st} + \frac{AR}{L} s e^{st} + \frac{A}{LC} e^{st} = 0$$

$$Ae^{st}\left(s^2 + \frac{Rs}{L} + \frac{1}{LC}\right) = 0$$

so, $s^2 + \frac{Rs}{L} + \frac{1}{LC} = 0$

↓

Characteristic equation.

The two roots are →

$$s_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

A more compact way of expressing roots

$$\boxed{s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}}$$

Now s_1 & s_2 → are natural frequencies

we get.

$$i_1 = A_1 e^{s_1 t}, \quad i_2 = A_2 e^{s_2 t}$$

so, $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

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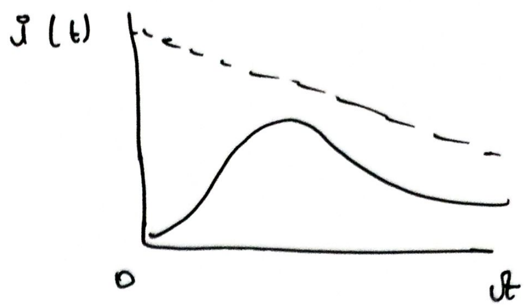
i) if $\alpha > \omega_0$

$\Rightarrow \quad C > \dfrac{4L}{R^2}$

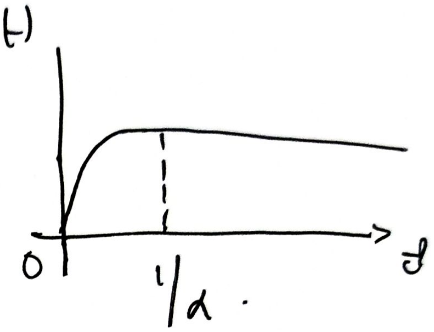
Overdamped case



ii) if $\alpha = \omega_0$,

critically damped case

$C = \dfrac{4L}{R^2}$



iii) if $\alpha < \omega_0$.

underdamped case

so, $C < \dfrac{4L}{R^2}$