Q4 a) Since the equation $(x/a)^{2/3} + (y/b)^{2/3} + (z/c)^{2/3} = 1$ doesnot change by putting $-x$ for x, $-y$ for y and $-z$ for z.

So, equation is symmetrical in all eight octants.

Volume = 8 x Volume of portion of solid lying in tue octant.

Volume of small element situated at any point $(x, y, z) = dx\, dy\, dz$

$\therefore$ Volume of solid in tue octant

$$= \iint dx\, dy\, dz$$

$$(x/a)^{2/3} + (y/b)^{2/3} + (z/c)^{2/3} \leq 1$$

Now Put $(x/a)^{2/3} = u$

$$(y/b)^{2/3} = v$$

$$(z/c)^{2/3} = w$$

ie. $x = au^{3/2}$, $y = bv^{2/3}$, $z = cw^{3/2}$

$dx = 3/_2\, au^{1/2}\, du$, $dy = 3/_2\, bv^{1/2}\, dv$, $dz = 3/_2\, cw^{1/2}\, dw$

= Volume in positive octant

$$\iiint 27/_8\, abc\, u^{(3/2)-1}\, v^{(3/2)-1}\, w^{(3/2)-1}\, du\, dv\, dw$$

where $u + v + w \leq 1$

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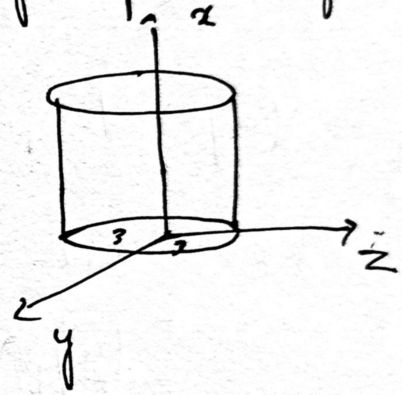
$$\frac{27}{8}\, abc\, [\gamma(3/2)]^3 = \frac{27}{8}\, abc \cdot \frac{\left(\frac{1}{2}\cdot\sqrt{\pi}\right)^3}{\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}$$

$$\frac{27}{8}\, abc \cdot \frac{\pi}{8} \cdot \frac{32}{35\cdot 27} = \frac{\pi abc}{8} \cdot \frac{4}{35}$$

Hence volume $= 8 \cdot \dfrac{\pi abc}{8} \cdot \dfrac{4}{35} = \dfrac{4\pi abc}{35}$

(b) $A = 2x^2 y\hat{i} - y^2\hat{j} + 4xz^2 k = \vec{F}$

Region in first octant bounded by cylinder $y^2+z^2=9$ and Plane ~~y~~ $x=2$



div $F = 4xy - 2y + 8xz$

From divergence theorem

$$I = \iiint_T (4xy - 2y + 8xz)\, dx\, dy\, dz$$

Let $y = r\cos\theta$, $z = r\sin\theta$, $|J| = r$

So, $0 \le \theta \le 2\pi$, $0 \le r \le 3$

$dx\, dy\, dz = r\, dr\, d\theta\, dx$

$$I = \int_0^2 \int_{\theta=0}^{2\pi} \int_{r=0}^{r=3} (4xr\cos\theta - 2r\cos\theta + 8r^2\cos\theta\sin\theta)\, r\, dr\, d\theta\, dx$$

$$= \int_0^2 \int_0^{2\pi} \left[\frac{4xr^3}{3}\cos\theta - \frac{2r^3}{3}\cos\theta + \frac{8r^4}{4}\cos\theta\sin\theta\right]_0^3 d\theta\, dx$$

$$\int_0^2\int_0^{2\pi} \left(36x\cos\theta - 18\cos\theta + (81\times 2)\cos\theta\sin\theta\right) d\theta\, dx$$

$$= \int_0^2 \left(\left[36x\sin\theta - 18\sin\theta\right]_0^{2\pi} + 324 \right) dx$$

$$= \underline{\underline{108}} \quad \text{(Ans)}.$$

~~Thus~~ ~~done~~

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