Introduction to the Betting Exchange

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Abstract

A basic introduction is made to trading in a betting exchange. Beyond analysing the central system of equations for trading in its matrix form, an equation is provided aiming at a dynamic configuration of the proportion between returns. Also, various approaches are presented, denoting the possibility for a trader to manage the profits based on his expectation of the event.

Contents

1 Betting Exchange

An online betting exchange is a web service where users can, among other things, trade contracts with each other about the outcome of future random events. The pay-off of these contracts can either be some fixed amount of money or nothing at all. The main innovation of this system, compared to traditional bookmakers, lies in providing a method to set fixed odds against an outcome — known as *laying* — and invite other users to bet in favour of the outcome — know as *backing*. This type of freedom had previously been reserved only to bookmakers.

The betting exchange concept, envisioned and created by Betfair® in 2000, has revolutionized sports and race wagering, attracting the attention of sports bodies, major competitors and governments, who seem uncertain about how to deal with this revolutionary transaction system, as well as customers globally, who are attracted by the far superior value proposition offered. [1] Smaller companies exist but Betfair® is considered to have a virtual monopoly.

In facilitating betting as a neutral intermediary the company responsible for maintaining the exchange generates revenue by taking a commission from the winner of the contract. This means that the company is only interested in maximizing the amount of money transacted between users and that it has no vested interest in the outcome of the events. The commission charged is calculated as a percentage of net winnings for each customer on each event, or market. The final value that the customer will get can be calculated with the *house function*, H, which is formulated in the following manner:

$$H(x) = \begin{cases} x & \text{if x} \leq 0 \\ x \times (1-h) & \text{if x} > 0 \end{cases} \tag{1}$$

where h is the house percentage (normally 0.05).

The neutral position makes customers, whose betting activities have traditionally been "restricted" by bookmakers (normally successful users that won too much money), able to place bets only limited by market liquidity. [2]

2 Decimal Odds and Probabilities

Traditional odds in favour of an event, O_t, is the ratio of the probability that an event will happen to the probability that it will not happen. For example, the traditional odds that a randomly chosen day of the week is a Sunday are one to six, which is written 1/6 or 1:6.

Decimal Odds, O_d, are simpler to use than traditional ones and are the most common form of odds quoted in countries outside the UK. Unlike the traditional interpretation, the customer stake is included as part of his total return, $O_t = O_d + 1$, relating more closely to the concept of probability. In the previous example, each day of the week has decimal odds of 7.0.

The implied probability of an outcome described by decimal odds, equals 1 divided by its odds:

$$\text{Probability} = \frac{1}{O_d} = \frac{1}{O_t - 1}$$

which concludes that everyday there is a $\frac{1}{7} = 14.3\%$ chance of being Sunday.

This means that when a customer makes a bet, he is actually making a financial commitment about his expectations on the outcome of an event through the implied probability of the bet. As we will see in appendix A, the expected value of bets depend on a relation between the real probability of the event and the implicit probability of the bet.

3 Betting Terminology

Various terms are nowadays well established to characterize the way betting exchange's customers build their sets of bets.

First of all, clear distinctions arise related to the number of betted events. If the set only backs and/or lays one event, than it is said that the costumer is *Hedging*. On the contrary, if more than one event is betted upon, than its called *Dutching*.

A particular case of *Hedging* is when the costumer only closes one bet. This sort of action is known as *Speculating* since the costumer's (colloquially called a *punter*) transactions are based on hints. For instance, a *punter* will easily close a back bet on a single event of a soccer game and wait until the end to see the outcome.

Dutching also has a special case called *Surebeting*. This happens when the events betted upon are collectively exhaustive (all the possible events).

Number o Betted Events	Terminology
Single	*Hedging* of *Speculating*
Multiple	*Dutching* or *Surebeting*

The closing of bets can be further distinguished based on its timing, denoting two well known fashions: *Arbitrage* and *Trading*.

In economics and finance, *Arbitrage* is the practice of taking advantage of a price difference between two or more markets: striking a combination of matching deals that capitalize upon the imbalance, the profit being the difference between the market prices.

In the betting exchange context, it is possible to became an *Arbitrageur* when the implied probabilities (see 2) of all possible events sum up to more than one. For instance, in a soccer match the odds available for backing $\{\text{Home}, \text{Visitor}, \text{Draw}\}$ are $O_i = \{1.3, 4.3, 2.3\}$, respectively. This results in a total $\sum_i \frac{1}{O_i} = 1.44$ probability which now allows to take profit with a *Surebet*. *Arbitrageurs* are traditionally known to perform *surebets* on multiple bookmakers. The same situation can be performed, although highly unlikely, with hedging.

A *Trader*, someone who performs *Trading*, takes an extra risk and closes his bets at different stages when the implied probabilities offered by the market turn out to be more favourable.

Generation of Imbalance	Terminology
Immediate	*Trading*
Gradual	*Arbitrage*

4 The Back-Lay Pair

The profit/loss of back and lay bets can be represented as random variables established by a stake and an implied probability in the following manner:

$$
\begin{aligned}
Back(p_B, s_B) &= \begin{cases} s_B \times \frac{1-p_B}{p_B} & \text{if the event occurs} \\ s_B \times (-1) & \text{if it doesn't} \end{cases} \\
Lay(p_L, s_L) &= \begin{cases} s_L \times \frac{p_L-1}{p_L} & \text{if the event occurs} \\ s_L & \text{if it doesn't} \end{cases}
\end{aligned}
$$

where p_B and p_L are the implied probabilities of the bet, s_B and s_L the stakes. The expected value and variance of each variable are calculated in A.

Nowadays, in order to understand if it is possible to make profit with this pair of bets, the most common metric used is the *greenbook* which is defined has the situation of having positive profits in all markets (regardless to the distribution). Although this is an acceptable way of evaluating the established situation, a more generic metric will tell us if it is possible to make profit, which is not enough to satisfy a *greenbook*.

$$
\begin{gathered}
\text{Greenbook} \Rightarrow \text{Possibility of Proffit} \\
\text{but} \\
\text{Possibility of Proffit} \not\Rightarrow \text{Greenbook}
\end{gathered}
$$

The following matrix form is now presented where the two possible profits can be calculated with:

$$
P = E \times S \Leftrightarrow \begin{bmatrix} p \\ \bar{p} \end{bmatrix} = \begin{bmatrix} \frac{1-p_B}{p_B} & \frac{p_L-1}{p_L} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} s_B \\ s_L \end{bmatrix} \tag{2}
$$

where P is the *profit matrix*, S is the *stake matrix* and E is the *exchange matrix* built from horizontal stack of its back and lay columns ($E = [B \mid L]$).

With the exchange matrix is now possible to make an analysis about the value of its determinant:

$$
det(E) = \frac{1-p_B}{p_B} + \frac{p_L - 1}{p_L} = \frac{p_L - p_B}{p_B \times p_L}
$$

This value is only positive if $p_L - p_B > 0$ which is in fact the main objective of a trader in single event operations (in order to make profit): contradict the nature of the market by closing a back bet with lower probability than a lay bet, which is the same to say a back odd higher than a lay odd.

Unlike a *greenbook*, the determinant of the exchange matrix gives us a more general definition of when the customer will be able to make profit.

4.1 Hedging

Lets the following problem be enunciated:

Problem 1. *At a time t_0 a trader made a lay bet of 100€ (s_L) with a 3.15 odd ($p_L = 0.32$). After a while ($t > t_0$), the trader is able to make a back bet with a 5.6 odd ($p_B = 0.18$). How much should the stake s_B be?*

A problem like this can appear when using the *Lay the Draw* strategy in the beginning of a soccer game. If a strong team plays against a weaker team (colloquially the *underdog*) than the probability that the draw at 0-0 will sustain throughout the game is naturally low. Obviously, there needs to exist an exit strategy (assume the prejudice) for the eventuality that no goal at all is scored. Another tragic eventuality that works against this strategy is when the *underdog* is the first to score.

The approach described in this section aims to solve problem 1 while managing the distribution of profit/loss over all possible events in a dynamic way, allowing for various schemes to be used. Problem 1 has only two possible profits that can relate by the following proportion:

$$\beta \times H\left(s_L - s_B\right) = H\left(\frac{1-p_B}{p_B}s_B + \frac{p_L - 1}{p_L}s_L\right) \tag{3}$$

where β is a coefficient that models the proportion between profits. This coefficient is explored in section **??**.

In order to solve this equation, it is important to understand when the house commission is applied. In appendix B, a demonstration is made proving this equation can be solved has if no commission exists. This makes it possible to define the following generic relation:

Lemma 1. *In order to achieve a proportion β between the profits of a back-lay pair of bets, the proportion between stakes must be the following:*

$$s_B \times \frac{1 + p_B(\beta - 1)}{p_B} = s_L \times \frac{1 + p_L(\beta - 1)}{p_L} \tag{4}$$

where p_B and p_L are the implicit probabilities of the bets, s_B and s_L the stakes.

With this rule, it is now possible to manage the returns with the following five conditions.

1. $H(\bar{p}) = 0$
   The first condition is the easiest one. For this to happen one needs to solve $s_L - s_B = 0$ making the second stake equal to the first. In problem 1 the second stake would be $s_B = s_L = 100$€.

2. $H(p) = 0$
   Second condition can be achieved by solving $s_B \times \frac{1-p_B}{p_B} = s_L \times \frac{1-p_L}{p_L}$ which is equivalent to have $\beta = 0$. In 1 the second stake would be $s_B = 46.74$€.

3. $H(p) = H(\bar{p})$
   The third approach is solved making $\beta = 1$ in equation 4. The last needed stake can be calculated with:
   $$\frac{s_L}{p_L} = \frac{s_B}{p_B}$$
   which would give $s_B = 56.25$€ with a profit of 41.56€ in every market.

4. $\frac{H(p)}{P} = \frac{H(\bar{p})}{1-P}$
   P is the trader's expected probability.

In the fourth, we make $\beta = \frac{P}{1-P}$. This relation makes the expected profit the same in any possible situation. The difficulty here, is to set the value of P. A simple solution is to use the implicit probability that the market has established for the event at that moment: in problem 1, the odd 5.6 means an implicit probability of $P = 0.18$. which gives $s_B = 49.16$€ with $H(p) = 10.6$€ and $H(\bar{p}) = 48.29$ €.

5. $\frac{H(p)}{(1+\delta)} = \frac{H(\bar{p})}{(1-\delta)}$
   $\beta = \frac{1+\delta}{1-\delta}$ where δ is a bias operator provided in the following manner:

$$\begin{cases} \gamma(1+\delta) = H(p) \\ \gamma(1-\delta) = H(\bar{p}) \end{cases}$$

   where γ is an unknown central profit.

   An expected, and obvious, result is that when $\delta \to 0$ this relation becomes the same as if $\beta = 1$.

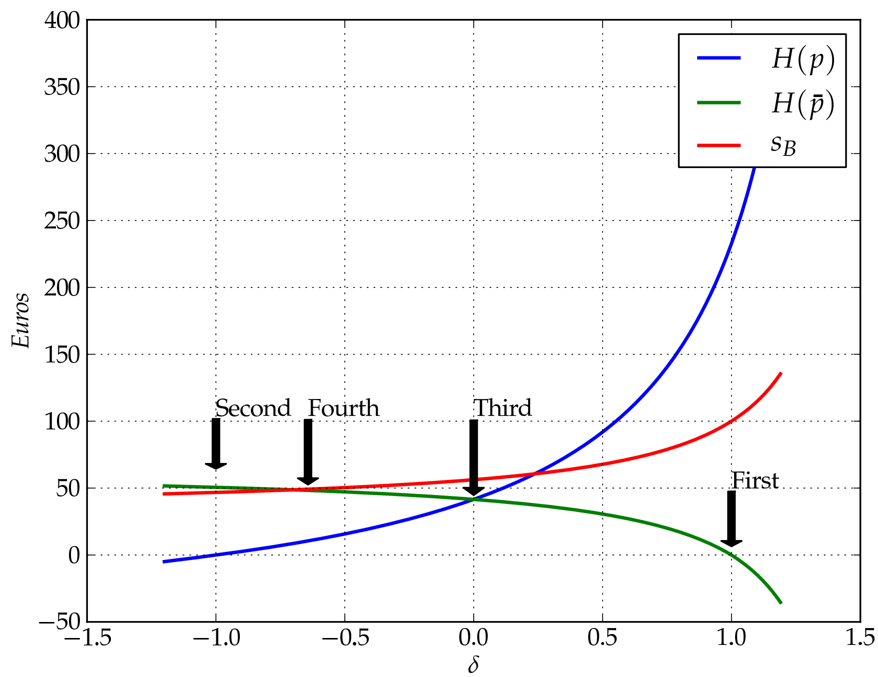

Figure 1: Relation between the profits of problem 1 and δ.

References

[1] Mark Davies, Leyland Pitt, Daniel Shapiro, and Richard Watson. Betfair.com: Five technology forces revolutionize worldwide wagering. *European Management Journal*, 23:533–541, 2005.

[2] Des Laffey. Entrepreneurship and innovation in the uk betting industry: The rise of person-to-person betting. *European Management Journal*, 23:351–359, 2005.

A Expected Value and Variance

First we need to suppose a value to the real probability that the event will happen, P. Normally a simple solution is to use the implicit probability that the market has established for the event at that moment. More complex approaches can be made with the retrieval of probabilistic information from external sources to the exchange or with the estimation of a trend of the market.

The expected value of a bet is colloquially called the value of the bet.

A.1 Back

The expected value of a back bet with real probability P is:

$$E\left[Back(p_B, s_B)\right] = \mu_B = P \times s_B \frac{1-p_B}{p_B} - (1-P) \times (s_B) = s_B \left(\frac{P}{p_B} - 1 \right)$$

The variance is:

$$Var\left[Back(p_B, s_B)\right] = P \left(s_B \frac{1-p_B}{p_B} - \mu_B \right)^2 + (1-P)\left(-s_B - \mu_B\right)^2$$

A.2 Lay

The expected value of a lay bet with probability P is:

$$E\left[Lay(p_L, s_L)\right] = \mu_L = P \times s_L \frac{p_L - 1}{p_L} + (1-P) \times (s_L) = s_L \left(1 - \frac{P}{p_L} \right)$$

The variance is:

$$Var\left[Lay(p_L, s_L)\right] = P \left(s_L \frac{p_L - 1}{p_L} - \mu_L \right)^2 + (1-P)\left(s_L - \mu_L\right)^2$$

B Back-Lay Commission Simplification

Observing the behaviour of H in equation 3, the following conditions are easily noted:

$$\begin{cases} C_1 : s_L > s_B \\ C_2 : \frac{1-p_B}{p_B} s_B > \frac{1-p_L}{p_L} s_L \end{cases}$$

Equation 3 can now be rewritten in the following way:

$$\alpha \times \beta \times (s_L - s_B) = \frac{1-p_B}{p_B} s_B + \frac{p_L - 1}{p_L} s_L \tag{5}$$

where α is

$$\begin{cases} (1-h) & \text{if } C_1 \wedge \bar{C_2} \\ \frac{1}{1-h} & \text{if } \bar{C_1} \wedge C_2 \\ 1 & \text{if } (C_1 \wedge C_2) \vee (\bar{C_1} \wedge \bar{C_2}) \end{cases}$$

The problem now is that C_1 and C_2 depend on the values of the stakes, which are the values we are trying to model. This said, both conditions will be solved in order to the implicit probabilities. Equation 5 can be simplified to the following form:

$$s_L \times \left(\alpha\beta + \frac{1-p_L}{p_L} \right) = s_B \times \left(\alpha\beta + \frac{1-p_B}{p_B} \right) \Leftrightarrow s_L = s_B \frac{\alpha\beta + \frac{1-p_B}{p_B}}{\alpha\beta + \frac{1-p_L}{p_L}}$$

The first condition becomes:

$$s_L > s_B \Leftrightarrow s_B \frac{\alpha\beta + \frac{1-p_B}{p_B}}{\alpha\beta + \frac{1-p_L}{p_L}} > s_B \Leftrightarrow \alpha\beta + \frac{1-p_B}{p_B} > \alpha\beta + \frac{1-p_L}{p_L} \Leftrightarrow p_L > p_B$$

And the second:

$$\frac{1-p_B}{p_B} s_B > \frac{1-p_L}{p_L} s_L \Leftrightarrow \frac{1-p_B}{p_B}\left(\alpha\beta + \frac{1-p_L}{p_L}\right) > \frac{1-p_L}{p_L}\left(\alpha\beta + \frac{1-p_B}{p_B}\right) \Leftrightarrow$$

$$\Leftrightarrow \alpha\beta \times \frac{1-p_B}{p_B} > \alpha\beta \times \frac{1-p_L}{p_L} \Leftrightarrow p_L > p_B$$

Concluding that $C_1 \Leftrightarrow C_2$ which tells us that $\alpha = 1$.