📒 COMPREHENSIVE SUMMARY: CSE242 CRYPTOGRAPHY & SECURITY

PART 1: RAINBOW TABLE ATTACKS & HASHING

1. Rainbow Table Attacks

- **Definition**: A type of attack that uses precomputed tables (rainbow tables) of hash values to crack password hashes without brute-forcing.
- **How it works**:
  - Precompute hash chains for common passwords using a specific hash function (e.g., SHA-1).
  - Compare stolen hashes from a database against the rainbow table.
  - When a match is found, the plaintext password is revealed.
- **Real-world examples**:
  - LinkedIn (2012): 6.5M hashed passwords cracked.
  - Adobe (2013): 150M encrypted passwords exposed.
  - Ubuntu Forums (2013): 1.8M hashed passwords compromised.
- **Protection**:
  - **Password hygiene**: Long, complex, unique passwords.
  - **Hash salting**: Adding random salt before hashing makes rainbow tables ineffective.
  - **Strong encryption**: Use AES, RSA.
  - **Multi-factor authentication (MFA)**.
  - **Regular software updates**.

2. Hash Functions in Cryptography

- **Definition**: Mathematical algorithms that convert input into fixed-size hash (digest).
- **Properties**:
  - Deterministic
  - Fixed-length output
  - Pre-image resistance
  - Collision resistance
  - Avalanche effect
  - Fast computation

- **Applications**:
  - Data integrity verification
  - Digital signatures
  - Blockchain
  - Password storage
- **Common hash functions**:
  - **MD5** (128-bit, insecure)
  - **SHA-1** (160-bit, deprecated)
  - **SHA-256** (256-bit, widely used)
  - **SHA-3** (variable, quantum-resistant)
  - **BLAKE2/3** (fast and secure)

3. HMAC (Hash-Based Message Authentication Code)

- **Definition**: Combines cryptographic hash function with secret key for authentication.
- **Purpose**: Verify data integrity and authenticity.
- **Formula**: `HMAC(K, M) = H((K ⊕ opad) || H((K ⊕ ipad) || M))`
- **Use cases**: Secure APIs, message verification.

PART 2: SYMMETRIC CRYPTOGRAPHY & AES

1. Symmetric-Key Cryptography

- **Definition**: Same key for encryption and decryption.
- **Types**:
  - **Block ciphers**: Encrypt fixed-size blocks (AES, DES, Blowfish).
  - **Stream ciphers**: Encrypt data bit-by-bit (RC4).
- **Modes of operation**:
  - **ECB (Electronic Codebook)**: Insecure, deterministic.
  - **CBC (Cipher Block Chaining)**: Uses IV, sequential.
  - **CTR (Counter)**: Parallelizable, no padding.
  - **CFB/OFB**: Stream cipher modes.

2. DES (Data Encryption Standard)

- **Block size**: 64 bits
- **Key size**: 56 bits (weak)
- **Rounds**: 16
- **Feistel structure**
- **Weaknesses**: Small key size, vulnerable to brute force.
- **Triple DES (3DES)**: Applies DES three times with 2/3 keys for stronger security.

3. AES (Advanced Encryption Standard)

- **Block size**: 128 bits
- **Key sizes**: 128, 192, 256 bits
- **Rounds**: 10, 12, 14 (depending on key size)
- **SP network** (Substitution-Permutation)
- **Strong, fast, widely adopted** (replaced DES).
- **Security**: Resists known attacks, quantum-vulnerable (Grover's algorithm).

PART 3: ASYMMETRIC CRYPTOGRAPHY

1. RSA (Rivest-Shamir-Adleman)

- **Based on**: Factoring large primes (hard problem).
- **Key generation**:
  - Choose primes p, q
  - Compute $n = p \times q$
  - Compute $\varphi(n) = (p-1)(q-1)$
  - Choose e (public exponent) coprime to $\varphi(n)$
  - Compute d (private exponent) where $d \times e \equiv 1 \bmod \varphi(n)$
- **Public key**: (e, n)
- **Private key**: (d, n)
- **Encryption**: $c = m^e \bmod n$
- **Decryption**: $m = c^d \bmod n$
- **Use cases**: SSL/TLS, digital signatures, secure email.

2. ElGamal Encryption

- **Based on**: Discrete logarithm problem.
- **Key generation**:
  - Choose large prime p, generator g
  - Private key: random integer x
  - Public key: $h = g^x \bmod p$
- **Encryption**:
  - Choose random k
  - $c_1 = g^k \bmod p$
  - $c_2 = m \times h^k \bmod p$
- **Decryption**:
  - $s = c_1^x \bmod p$
  - $m = c_2 \times s^{-1} \bmod p$
- **Features**: Probabilistic encryption (different ciphertexts for same plaintext).

3. RSA vs ElGamal Comparison

Feature	RSA	ElGamal
Basis	Factoring	Discrete logarithm
Encryption type	Deterministic	Probabilistic
Ciphertext size	Smaller	Larger (2× plaintext)
Speed	Faster	Slower
Security	Vulnerable to quantum (Shor's)	Vulnerable to quantum (Shor's)
Use cases	Digital signatures, key exchange	Secure messaging, hybrid encryption

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PART 4: DIGITAL SIGNATURES & CERTIFICATES

1. Digital Signatures

- **Purpose**: Authentication, integrity, non-repudiation.
- **How it works**:
  - **Signing**: Hash message → encrypt hash with private key.
  - **Verification**: Decrypt signature with public key → compare hashes.
- **Algorithms**:
  - **RSA**: Widely used.
  - **DSA**: NIST standard.
  - **ECDSA**: Efficient, small keys.
- **Applications**: Software signing, legal documents, secure email.

2. Digital Certificates

- **Definition**: Electronic document binding public key to identity.
- **Components**:
  - Subject info
  - Public key
  - Issuer (CA)
  - Validity period
  - Digital signature of CA
- **X.509 standard**: Format for certificates.
- **Certificate Authority (CA)**: Trusted entity issuing certificates.
- **PKI (Public Key Infrastructure)**: Framework for managing certificates.

3. Digital Signatures vs Digital Certificates

Aspect	Digital Signatures	Digital Certificates
Purpose	Authenticate message	Authenticate entity
Creation	Private key of sender	Issued by CA
Verification	Public key of sender	CA's root certificate
Usage	Sign documents, software	SSL/TLS, email, VPN

PART 5: QUANTUM THREATS & MODERN CHALLENGES

1. Quantum Threats

- **Shor's algorithm**: Breaks RSA, ElGamal (factoring/discrete log).
- **Grover's algorithm**: Reduces hash security by square root.
- **BHT algorithm**: Quantum collision attack on hashes.
- **Impact**: Current asymmetric crypto becomes insecure.

2. Post-Quantum Cryptography

- **Lattice-based**: NTRU, Kyber.
- **Code-based**: McEliece.
- **Hash-based**: SPHINCS+.
- **Multivariate**: Rainbow.
- **Goal**: Develop quantum-resistant algorithms.

🔑 CRITICAL DEFINITIONS

1. **Rainbow Table**: Precomputed table of password hashes for cracking.
2. **Hash Function**: Algorithm producing fixed-size digest from input.
3. **Salt**: Random data added to password before hashing.
4. **HMAC**: Hash-based message authentication code.
5. **Symmetric Encryption**: Same key for encryption/decryption.
6. **Asymmetric Encryption**: Public/private key pair.
7. **Digital Signature**: Cryptographic proof of message origin/integrity.
8. **Digital Certificate**: CA-signed document binding key to identity.
9. **Certificate Authority**: Trusted entity issuing certificates.
10. **PKI**: Public Key Infrastructure for managing digital certificates.

? 15 COMPREHENSIVE CRYPTOGRAPHY QUESTIONS WITH ANSWERS

Question 1: Alice wants to send a secret message to Bob using RSA. Bob's public key is (e=17, n=3233). Alice's message is the numerical value m=65. What ciphertext value c does Alice send to Bob? Show the encryption calculation: $c = m^e \bmod n$.
Answer: $c = 65^{17} \bmod 3233 = 2790$.

Question 2: Using the RSA parameters p=61 and q=53 with public exponent e=17, calculate Bob's private key d. Show all steps: first compute n and φ(n), then find d such that $d \times e \equiv 1 \bmod \varphi(n)$.
Answer:
$n = p \times q = 61 \times 53 = 3233$
$\varphi(n) = (p-1)(q-1) = 60 \times 52 = 3120$
We need d where $17 \times d \equiv 1 \bmod 3120$
Using extended Euclidean algorithm: d = 2753
So private key is (d=2753, n=3233).

Question 3: Bob receives RSA ciphertext c=2790 from Alice. Using his private key d=2753 and n=3233, what is the original message m? Show the decryption: $m = c^d \bmod n$.
Answer: $m = 2790^{2753} \bmod 3233 = 65$.

Question 4: For ElGamal encryption with parameters p=467, generator g=2, and Bob's public key h=228, Alice wants to send message m=123. She chooses random k=3. What ciphertext pair (c_1, c_2) does Alice send? Show calculations: $c_1 = g^k \bmod p$ and $c_2 = m \times h^k \bmod p$.
Answer:
$c_1 = 2^3 \bmod 467 = 8$
$c_2 = 123 \times 228^3 \bmod 467 = 123 \times (228^3 \bmod 467) \bmod 467$

First compute: 228^3 mod 467 = 228 × 228 × 228 mod 467
228 × 228 = 51984 mod 467 = 51984 - (111×467) = 51984 - 51837 = 147
147 × 228 = 33516 mod 467 = 33516 - (71×467) = 33516 - 33157 = 359
So 228^3 mod 467 = 359
Now c_2 = 123 × 359 mod 467 = 44157 mod 467 = 44157 - (94×467) = 44157 - 43898 = 259
Final ciphertext: (c_1=8, c_2=259).

Question 5: Bob receives ElGamal ciphertext (c_1=8, c_2=259) from Alice. Bob's private key is x=228 (since h = g^x mod p = 2^{228} mod 467 = 228). How does Bob decrypt to recover message m? Show: s = c_1^x mod p, then m = $c_2 \times s^{-1}$ mod p.
Answer:
First compute s = 8^{228} mod 467
This is complex, but since we know h = g^x mod p = 228, and $c_1 = g^k = 8$, then s = $(g^k)^x = (g^x)^k = h^k = 228^3$ mod 467
From previous calculation: 228^3 mod 467 = 359, so s = 359
Now find s^{-1} mod 467 (modular inverse of 359 modulo 467)
359 × 42 = 15078 mod 467 = 15078 - (32×467) = 15078 - 14944 = 134 (not 1)
Actually, 359 × 230 = 82570 mod 467 = 82570 - (176×467) = 82570 - 82192 = 378 (not 1)
Using extended Euclidean algorithm: 359 × 206 ≡ 1 mod 467
So s^{-1} = 206
Now m = $c_2 \times s^{-1}$ mod 467 = 259 × 206 mod 467 = 53354 mod 467
53354 - (114×467) = 53354 - 53238 = 116
Wait, this doesn't give 123... Let me recalculate properly.

Actually, let's solve step by step:
We have p=467, g=2, h=228, x=228, k=3, m=123
From encryption: $c_1 = g^k$ mod p = 2^3 mod 467 = 8
$h^k = 228^3$ mod 467 = 359 (calculated earlier)
$c_2 = m \times h^k$ mod p = 123 × 359 mod 467 = 44157 mod 467
44157 ÷ 467 = 94 remainder 259, so c_2 = 259

For decryption: s = c_1^x mod p = 8^{228} mod 467
But s should equal $(g^k)^x = (g^x)^k = h^k$ = 359
So s = 359
Now find s^{-1} mod 467: 359 × 206 = 73954 mod 467
73954 ÷ 467 = 158 remainder 168 (not 1)

Let me compute properly: 359 × 13 = 4667, 4667 mod 467 = 4667 - 10×467 = 4667 - 4670 = -3
We need $359 \times d \equiv 1 \mod 467$
Using Python: inverse of 359 mod 467 is 206
Check: 359 × 206 = 73954, 73954 mod 467 = 73954 - 158×467 = 73954 - 73886 = 68 (not 1)
Actually, 467 × 158 = 73886, remainder 68
Let me recalculate: 467 × 158 = 467×100=46700, 467×50=23350, 467×8=3736
46700+23350=70050, +3736=73786, not 73886
467×158 = 467×(100+50+8) = 46700 + 23350 + 3736 = 73786
73954 - 73786 = 168
So 359 × 206 = 168 mod 467, not 1

Let's find inverse properly: gcd(359, 467) = 1 (they're coprime)
Using extended Euclidean:
467 = 1×359 + 108
359 = 3×108 + 35
108 = 3×35 + 3
35 = 11×3 + 2
3 = 1×2 + 1
2 = 2×1 + 0

Working backwards:
1 = 3 - 1×2
= 3 - 1×(35 - 11×3) = 12×3 - 1×35
= 12×(108 - 3×35) - 1×35 = 12×108 - 37×35
= 12×108 - 37×(359 - 3×108) = 123×108 - 37×359
= 123×(467 - 1×359) - 37×359 = 123×467 - 160×359

So $-160 \times 359 \equiv 1 \mod 467$
-160 mod 467 = 467 - 160 = 307
Thus $307 \times 359 \equiv 1 \mod 467$
Check: 307 × 359 = 110213, 110213 ÷ 467 = 236 remainder 1 ✓
So $s^{-1} = 307$

Now $m = c_2 \times s^{-1} \mod p$ = 259 × 307 mod 467
259 × 307 = 79513
79513 ÷ 467 = 170 remainder 123 ✓
So m = 123.

Question 6: In a Diffie-Hellman key exchange, Alice and Bob agree on public parameters p=23 and g=5. Alice chooses private key a=6, Bob chooses private key b=15. What shared secret K do they compute? Show: Alice sends $A = g^a \bmod p$, Bob sends $B = g^b \bmod p$, then $K = B^a \bmod p = A^b \bmod p$.

Answer:

Alice computes $A = 5^6 \bmod 23 = 15625 \bmod 23$

$23 \times 679 = 15617$, remainder 8, so A = 8

Bob computes $B = 5^{15} \bmod 23$

First compute $5^{15} \bmod 23$ step by step:

$5^2 = 25 \bmod 23 = 2$

$5^4 = (5^2)^2 = 2^2 = 4$

$5^8 = (5^4)^2 = 4^2 = 16$

$5^{15} = 5^8 \times 5^4 \times 5^2 \times 5^1 = 16 \times 4 \times 2 \times 5 = 16\times4=64,\ 64\times2=128,\ 128\times5=640$

640 mod 23: $23\times27=621$, remainder 19, so B = 19

Shared secret:

Alice computes $K = B^a \bmod p = 19^6 \bmod 23$

Bob computes $K = A^b \bmod p = 8^{15} \bmod 23$

Compute $19^6 \bmod 23$:

$19^2 = 361 \bmod 23$: $23\times15=345$, remainder 16

$19^4 = (19^2)^2 = 16^2 = 256 \bmod 23$: $23\times11=253$, remainder 3

$19^6 = 19^4 \times 19^2 = 3 \times 16 = 48 \bmod 23$: $23\times2=46$, remainder 2

So shared secret K = 2.

Question 7: If an attacker eavesdrops on the Diffie-Hellman exchange in Question 6 and sees A=8 and B=19 (with p=23, g=5), can they compute the shared secret K=2 without knowing a=6 or b=15? What attack could they attempt?

Answer: Yes, they could attempt to solve the discrete logarithm problem: find a such that $5^a \equiv 8 \bmod 23$, or find b such that $5^b \equiv 19 \bmod 23$. For small p=23, they can brute force:

$5^1 \bmod 23=5$, $5^2=2$, $5^3=10$, $5^4=4$, $5^5=20$, $5^6=8$ ✓ so a=6

$5^{15} \bmod 23=19$ ✓ so b=15

Then compute $K = 19^6 \bmod 23 = 8^{15} \bmod 23 = 2$.

Question 8: Compare RSA and ElGamal encryption: For the same security level (2048-bit modulus), why does ElGamal produce ciphertexts twice as long as RSA?
Answer: RSA encrypts a message m to a single ciphertext $c = m^e \bmod n$ (same size as n). ElGamal encrypts to a pair (c_1, c_2) where both c_1 and c_2 are the same size as the modulus p. So if RSA ciphertext is 2048 bits, ElGamal ciphertext is 4096 bits (2048×2).

Question 9: In RSA, why must p and q be large primes? What happens if p=3, q=11 (n=33)? Show why this is insecure by factoring n=33 and decrypting without d.
Answer: If n=33, anyone can factor it as 3×11. Then compute $\varphi(n)=(3-1)(11-1)=20$. Given public key e, they can compute $d = e^{-1} \bmod 20$. Example: if e=3, then d=7 since $3\times7=21\equiv1 \bmod 20$. So security relies on factoring n being computationally infeasible.

Question 10: For ElGamal with p=23, g=5, private key x=13, what is the public key h? If message m=7 is encrypted with random k=3, what is the ciphertext (c_1, c_2)?
Answer:
Public key $h = g^x \bmod p = 5^{13} \bmod 23$
$5^2=2$, $5^4=4$, $5^8=16$, $5^{13}=5^8\times5^4\times5^1=16\times4\times5=320 \bmod 23$
23×13=299, remainder 21, so h=21

Encryption: choose k=3
$c_1 = g^k \bmod p = 5^3 \bmod 23 = 125 \bmod 23 = 125 - 5\times23 = 125-115=10$
$h^k = 21^3 \bmod 23 = 9261 \bmod 23$: 23×402=9246, remainder 15
$c_2 = m \times h^k \bmod p = 7 \times 15 \bmod 23 = 105 \bmod 23 = 105 - 4\times23 = 105-92=13$
Ciphertext: $(c_1=10, c_2=13)$.

Question 11: Decrypt the ElGamal ciphertext from Question 10: $(c_1=10, c_2=13)$ with private key x=13, p=23. Show: $s = c_1^x \bmod p$, then $m = c_2 \times s^{-1} \bmod p$.
Answer:
$s = c_1^x \bmod p = 10^{13} \bmod 23$
Compute $10^{13} \bmod 23$:
$10^2=100 \bmod 23=8$, $10^4=64 \bmod 23=64-2\times23=64-46=18$
$10^8=18^2=324 \bmod 23=324-14\times23=324-322=2$

$10^{13}=10^8\times10^4\times10^1=2\times18\times10=360 \bmod 23=360-15\times23=360-345=15$
So $s=15$

Find $s^{-1} \bmod 23$: inverse of 15 mod 23
$15\times3=45 \bmod 23=45-2\times23=45-46=-1$, so $15\times(-3)\equiv1 \bmod 23$
$-3 \bmod 23=20$, so $s^{-1}=20$

$m = c_2 \times s^{-1} \bmod p = 13 \times 20 \bmod 23 = 260 \bmod 23$
$23\times11=253$, remainder 7 ✓
So $m=7$.

Question 12: In Diffie-Hellman, what is a man-in-the-middle attack? How can Alice and Bob prevent it?
Answer: An attacker sits between Alice and Bob, intercepting A and B, and replaces them with their own values. They establish separate keys with Alice and Bob. Prevention: authenticate the exchanged values using digital signatures or a PKI with certificates.

Question 13: Why is ElGamal encryption called "probabilistic" while RSA is "deterministic"? Give an example: encrypt m=10 twice with ElGamal (using different random k values) and show different ciphertexts.
Answer: ElGamal uses random k each time, so same m gives different (c_1, c_2). Example with $p=23$, $g=5$, $h=21$, $m=10$:
With $k=3$: $c_1=5^3 \bmod 23=10$, $h^k=21^3 \bmod 23=15$, $c_2=10\times15 \bmod 23=150 \bmod 23=150-6\times23=150-138=12 \rightarrow (10,12)$
With $k=4$: $c_1=5^4 \bmod 23=4$, $h^k=21^4 \bmod 23=441^2 \bmod 23$, $441 \bmod 23=441-19\times23=441-437=4$, so $4^2=16$, $c_2=10\times16 \bmod 23=160 \bmod 23=160-6\times23=160-138=22 \rightarrow (4,22)$
Different ciphertexts for same plaintext.

Question 14: RSA vulnerability: if Alice encrypts the same message m to two different recipients with public keys (e_1, n_1) and (e_2, n_2), and $e_1=e_2=3$, how can an attacker recover m without factoring? Assume $m^3 < n_1\times n_2$.

Answer: Chinese Remainder Theorem attack. Attacker sees $c_1 = m^3 \bmod n_1$ and $c_2 = m^3 \bmod n_2$. Since $m^3 < n_1 \times n_2$, they can compute m^3 exactly via CRT, then take cube root to get m.

Question 15: In ElGamal, why must the random k be different for each encryption and never reused? Show what happens if same k is used for two messages m_1 and m_2.
Answer: If same k is used:
For m_1: (c_1, c_2) where $c_1=g^k$, $c_2=m_1 \times h^k$
For m_2: (c_1', c_2') where $c_1'=g^k$ (same), $c_2'=m_2 \times h^k$
Then attacker can compute: $c_2/c_2' = (m_1 \times h^k)/(m_2 \times h^k) = m_1/m_2 \bmod p$
If m_1 is known, m_2 is revealed: $m_2 = c_2' \times m_1 / c_2 \bmod p$.