Question 4 – PAC, VC dimension, Bias vs Variance

Section 1

A circle (r,c) is defined by its center c and its radius r. Look at the following classifiers family:

$$\mathcal{H} = \{h_{r,c}:\ r \in \mathbb{R}, c \in \mathbb{R}^2\} \text{ where } h_{r,c}(x) = 1 \text{ iff x inside the circle (r,c)}$$

Find the VCdim of this class with full proof.

Section 2

Consider a training set $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ where $x_i \in \{0,1\}^3$. In other words, each sample has 3 Boolean features $\{X_1, X_2, X_3\}$. You are also given the classification rule $Y = (X_1 \wedge X_2) \vee (\neg X_1 \wedge \neg X_2)$.

We try to learn the function $f: X \to Y$ using a "depth 1 decision trees". A "depth-1 decision tree" is a tree with two leaves, all distance 1 from the root.

Analyze this problem and decide the appropriate sample complexity formula. Justify your answer.

Section 3

Dana was given a hard classification problem and she decided to use SVM with polynomial kernel with d=2,10,20. For each degree, she tried 15 to 85 training samples, with jumps of 5 (15, 20, ..).

The following graphs describe the train and test error for each d separately. However, she forgot which graph belongs to which d, and for each graph, what line is the train and the test.

Your task is to match each graph to the correct d and mark which lines are the test and the train. No explanation required.