Parzen Windows and Real Distribution Estimation

Part 1: Classification Rule Derivation

Given:
$\forall j \neq i : P(wi \mid x) \geq P(wj \mid x)$
Bayes rule: $P(wi \mid x) = P(x \mid wi) * P(wi) / P(x)$
We need to show that the classification rule can be reduced to:
$\forall j \neq i \in \{1, ..., C\} : \Sigma\phi((xik - x) / h) \geq \Sigma\phi((xjk - x) / h)$
Steps:
1. Start with Bayes Rule:
   $P(wi \mid x) = P(x \mid wi) * P(wi) / P(x)$
2. Using the given condition:
   $\forall j \neq i : P(wi \mid x) \geq P(wj \mid x)$
3. Substitute Bayes rule:
   $\forall j \neq i : P(x \mid wi) * P(wi) / P(x) \geq P(x \mid wj) * P(wj) / P(x)$
4. Simplify (since $P(x)$ is common in both):
   $\forall j \neq i : P(x \mid wi) * P(wi) \geq P(x \mid wj) * P(wj)$
5. Assume uniform priors (i.e., $P(wi) = P(wj)$ for all i, j):
   $\forall j \neq i : P(x \mid wi) \geq P(x \mid wj)$
6. Parzen Window Estimation for Likelihood:
   $P(x \mid wi) \approx 1/(ni\ h)\ \Sigma\phi((xik - x) / h)$
   Similarly,
   $P(x \mid wj) \approx 1/(nj\ h)\ \Sigma\phi((xjk - x) / h)$
7. Compare the likelihoods:
   $\forall j \neq i : 1/(ni\ h)\ \Sigma\phi((xik - x) / h) \geq 1/(nj\ h)\ \Sigma\phi((xjk - x) / h)$
8. Remove the common terms (assuming same h):
   $\forall j \neq i : \Sigma\phi((xik - x) / h) \geq \Sigma\phi((xjk - x) / h)$

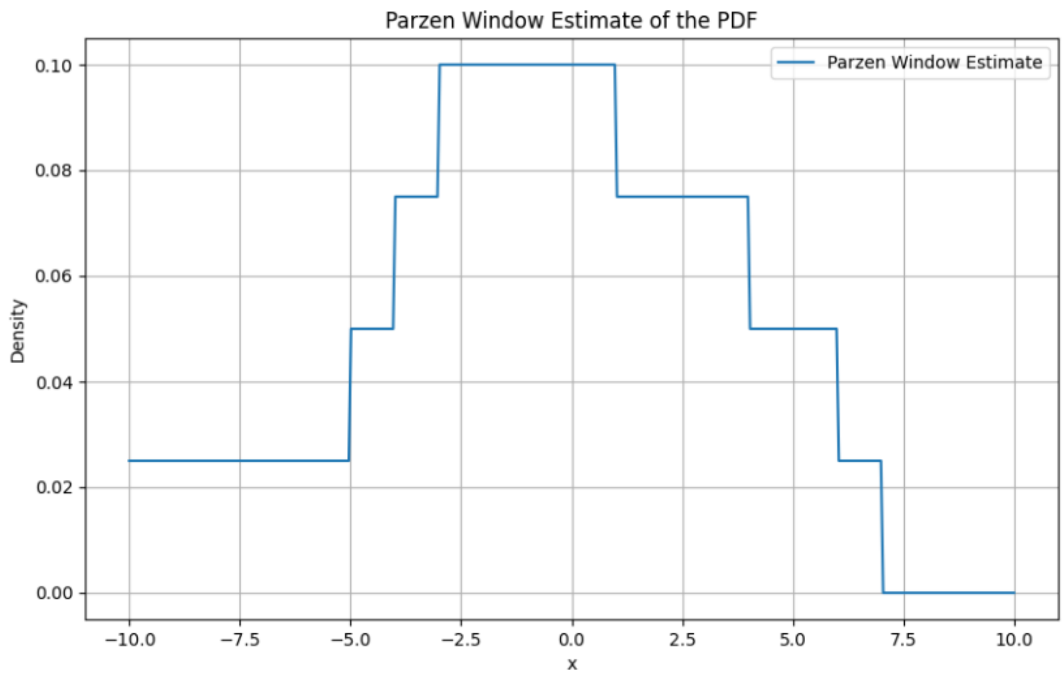
This shows that the classification rule is indeed the majority vote of neighbors as given.

Part 2: Parzen Window Estimation and Real Distribution

Given the samples: D = [1 -3 2 4 5 -8 0 -1 -2 -4]
Using the 0-1 window function and h = 4, we estimated the Parzen window based PDF.

We can express the Parzen window estimation of the PDF as:

$$p(x) = \frac{1}{10 \cdot 4} \sum \varphi\left(\frac{x - x_i}{h}\right)$$



(a) According to the graph, the real distribution of the data appears to be a Normal distribution because of the bell shaped curve that it has around the mean.
(b) Using MLE to estimate the distribution parameters, we find:
mu = -0.60
sigma = 3.69

Explanation:

Step-by-Step Calculation of Mean (μ) and Standard Deviation (σ)

Data:

[1, -3, 2, 4, 5, -8, 0, -1, -2, -4]

Step-by-Step Calculation for Mean (μ):
1. Number of data points (n): 10
2. Sum of data points: $\Sigma x_i = -6$
3. Mean (μ) calculation:
$\mu = \Sigma x_i / n$

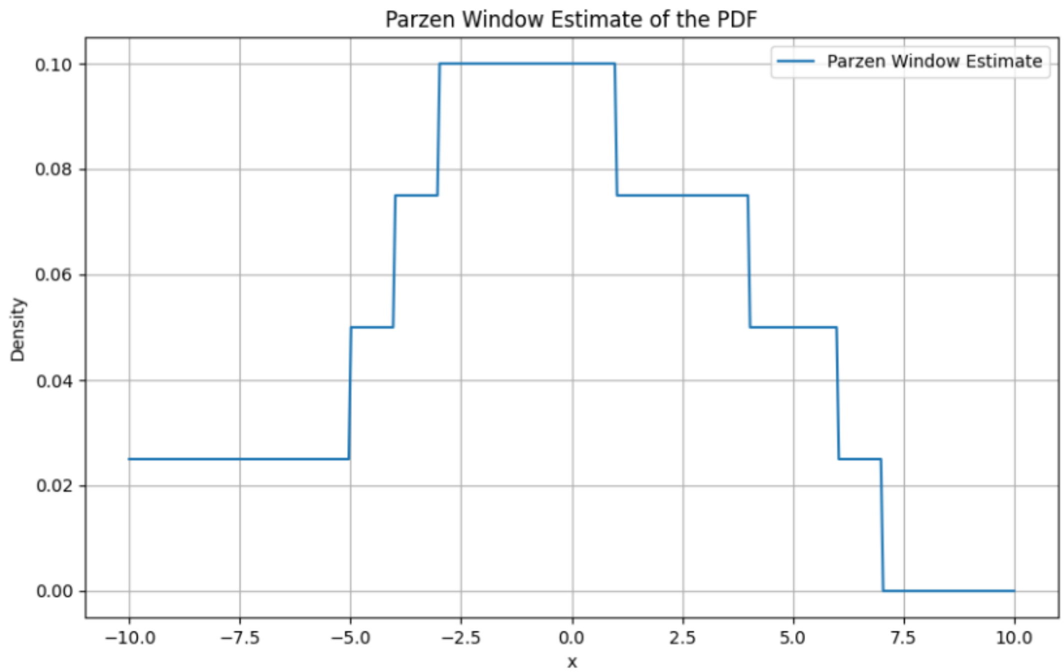
$\mu = -6 / 10$
$\mu = -0.60$

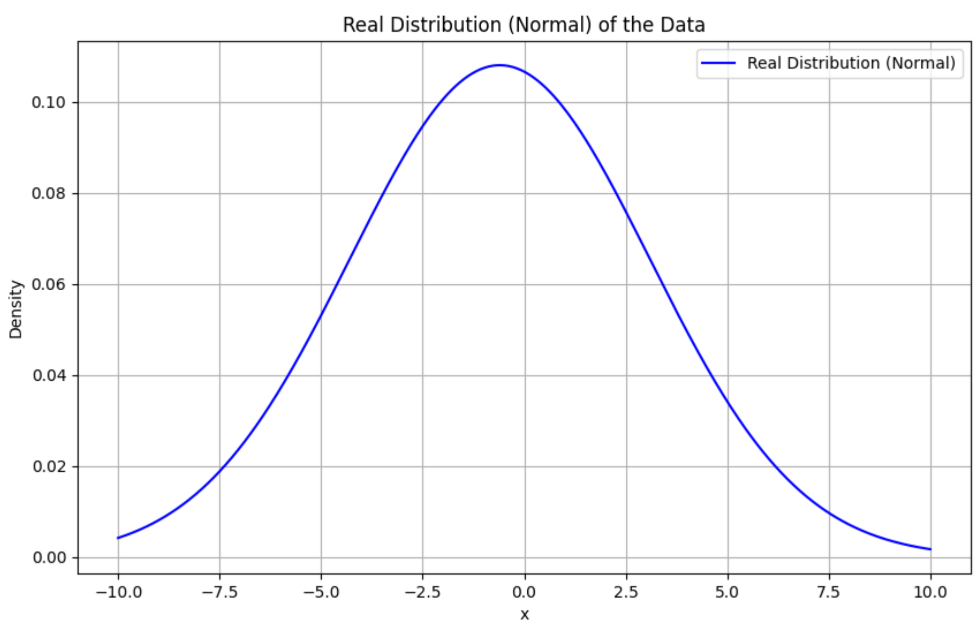
Step-by-Step Calculation for Standard Deviation (σ):

1. Mean (μ): -0.60
2. Calculate squared differences from the mean:
   $(x1 - \mu)^2 = (1 - -0.60)^2 = 2.56$
   $(x2 - \mu)^2 = (-3 - -0.60)^2 = 5.76$
   $(x3 - \mu)^2 = (2 - -0.60)^2 = 6.76$
   $(x4 - \mu)^2 = (4 - -0.60)^2 = 21.16$
   $(x5 - \mu)^2 = (5 - -0.60)^2 = 31.36$
   $(x6 - \mu)^2 = (-8 - -0.60)^2 = 54.76$
   $(x7 - \mu)^2 = (0 - -0.60)^2 = 0.36$
   $(x8 - \mu)^2 = (-1 - -0.60)^2 = 0.16$
   $(x9 - \mu)^2 = (-2 - -0.60)^2 = 1.96$
   $(x10 - \mu)^2 = (-4 - -0.60)^2 = 11.56$
3. Sum of squared differences: $\Sigma(x_i - \mu)^2 = 136.40$
4. Variance (σ^2) calculation:
   $\sigma^2 = \Sigma(x_i - \mu)^2 / n$
   $\sigma^2 = 136.40 / 10$
   $\sigma^2 = 13.64$
5. Standard Deviation (σ) calculation:
   $\sigma = \sqrt{\sigma^2}$
   $\sigma = \sqrt{13.64}$
   $\sigma = 3.69$

The PDF of the normal distribution is calculated using these parameters.

Comparison between the 2 graphs:


Parzen Window Estimate of the PDF
Parzen Window Estimate
Density
0.10
0.08
0.06
0.04
0.02
0.00
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x


Real Distribution (Normal) of the Data
Real Distribution (Normal)
Density
0.10
0.08
0.06
0.04
0.02
0.00
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x