Carleton UNIVERSITY

PROJECT REPORT

OPTIMIZATION FOR ENGINEERING APPLICATIONS—SYSC 5004W

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Sensor selection in magnetic tracking based on convex optimisation

Introduction

Sensor positioning largely influences the performance of any measurement system. In most case of biomedical systems, the optimal use of magnetic sensors will lead to better efficiency with low cost. This paper proposes the use of convex optimisation to determine the optimal positioning of sensors in a magnetic tracking system.

1. The process exploits the performance metrics obtained from the Fisher Information Matrix (FIM) by formulating an optimisation problem with integer variables.
2. A convex optimisation problem is formed by relaxing the constraints of the integer variable.

Measurement methodology

The system consists of one transmitting coil with know positions and orientation given by $Pt = [xt, yt, zt, \widehat{mxt}, \widehat{myt}, \widehat{mzt}]^T$ and Nr receiving coils given by $Pr = [\ xr\ , yr\ , zr\ , \widehat{mxr}\ , \widehat{myr}\ , \widehat{mzr}]^T$. The induced voltage in the in the receiving coils is given by

$$V = -\alpha \frac{\mu_o}{4\pi}\left(\frac{3(\widehat{mt}.\bar{R})(\widehat{mr}.\bar{R})}{R^5} - \frac{(\widehat{mt}.\widehat{mr})}{R^3}\right)$$

Where $\widehat{mr} = [\widehat{mxr}\ , \widehat{myr}\ , \widehat{mzr}]^T$ and $\widehat{mt} = [\widehat{mxt}\ , \widehat{myt}\ , \widehat{mzt}]^T$ are the unit vectors corresponding to the receiving coils and the transmitter coil respectively. $\bar{R}$ is the vector of length R given by $\bar{R} = [\overline{rr} - \overline{rt}]$, where $\overline{rr}, \overline{rt}$ are the normalised position vector of receiving and transmitting coil(s) respectively. The normalising distance is given by d (here we consider the z length between the receiving coil and transmitting coil as we consider xy planar sensor array parallel to the transmitter). And $Nr = 1,2, \ldots k$.

Linearization of variables

In order to obtain optimised sensor positions we find the derivative of the induced voltage given above partially with respect to the transmitter and then linearized. The gradient of the induced voltage at a k^{th} receiver coil is given as below.

$$\nabla Vk_{trans} = \left[\frac{\partial V}{\partial xt}, \frac{\partial V}{\partial yt}, \frac{\partial V}{\partial zt}, \frac{\partial V}{\partial mxt}, \frac{\partial V}{\partial myt}, \frac{\partial V}{\partial mzt}\right]^T$$

$$= -\alpha \frac{\mu_o}{4\pi}\left(\frac{15(\widehat{mt}.\bar{R})(\widehat{mr}.\bar{R})\bar{R}}{R^7} - \frac{3(\widehat{mt}.\bar{R})\widehat{mr} + (\widehat{mr}.\bar{R})\widehat{mt} + (\widehat{mt}.\widehat{mr})\bar{R} + 3(\widehat{mt}.\bar{R})\bar{R}}{R^5} - \frac{\widehat{mr}}{R^3}\right)$$

Fisher Information Matrix

A performance measure is needed in order to compare the different sensor layouts and find the optimal sensor position, here we consider the FIM to express the performance measurements. We know the transmitter has 6 degrees of freedom as it is given by $Pt = [xt, yt, zt, \widehat{mxt}, \widehat{myt}, \widehat{mzt}]^T$, therefore we consider $\breve{p} \in \mathbb{R}^6$ to be the parameters that needs to be estimated. The signals measured by Nr sensors $\acute{V}(P_o) \in \mathbb{R}^{Nr}$ is modeled as true signal added with additive gaussian noise as $\acute{V}(P_o) = V(P_o) + n$, where $n \in \mathcal{N}(0, \sigma^2 I)$ and $n \in \mathbb{R}^{Nr}$ are independent and identically distributed. It denotes a multivariant gaussian distribution with mean and covariance matrix C. The voltage measured is expanded in Taylor series around P, where the deviation is given as

$$\delta P \ = \ \nabla_p V(P_o) \ = \begin{bmatrix} \nabla_p V_1(P_o)^T \\ \nabla_p V_2(P_o)^T \\ \vdots \\ \nabla_p V_{N^r}(P_o)^T \end{bmatrix} = \ A$$

By neglecting the higher order terms and equating the above equation with measured signal we can formulate the equation as $\delta P \ = \ -A^{-1}n$. Using multivariant normal distribution under affine transformations i.e., If $If\ X \in \ \mathcal{N}(\mu, \Sigma)$ then $Y \ = \ c + BX \ \in \mathcal{N}(c + B\mu, B\Sigma B^T)$ where $X, \mu \in \ \mathbb{R}^m, \Sigma \in \ \mathbb{R}^{mxm}\ , Y, c \ \in \ \mathbb{R}^n \ and\ B \ \in \ \mathbb{R}^{nxm}$, we obtain

$$\delta P \in \mathcal{N}(0, A^{-1}\sigma^2 I (A^{-1})^T) \ = \ \mathcal{N}(0, \sigma^2 (A^T A)^{-1})$$

Therefore, the FIM, say $\mathrm{M} = A^T A \ \in \ \mathbb{R}^{pxp}$ is equal to the sum of individual sensors FIMs.

$$\mathrm{M} = A^T A \ = \ \Sigma_{i=1}^{Nr} M_i \ = \begin{bmatrix} \left(\frac{\partial V_i}{\partial xt}\right)^2 & \cdots & \left(\frac{\partial V_i}{\partial xt}\right)\left(\frac{\partial V_i}{\partial mzt}\right) \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial V_i}{\partial mzt}\right)\left(\frac{\partial V_i}{\partial xt}\right) & \cdots & \left(\frac{\partial V_i}{\partial mzt}\right)^2 \end{bmatrix}$$

Optimization problem

Maximizing the FIM corresponds to minimizing the errors in estimated parameters according to Cramer-Rao inequality i.e. $cov\ P \succcurlyeq \ M^{-1}$ which gives the lower bound for the covariance of P. We resolve to the D-Optimal design criteria to find M^{-1}. D- Optimal design is a scalarization technique used to minimize the determinant of error covariance matrix. Using D optimal design, a design problem can be given in the form

$$Minimize_{\lambda} \quad log\ det\left(\Sigma_{i=1}^{p} \lambda_i v_i {v_i}^T\right)^{-1}$$
$$Subject\ to \quad \lambda_i \geq \ 0, \qquad \Sigma\,\lambda \ = \ 1$$

Here we wish to find the optimal sensor position when the transmitter is with in the domain Ωp, thus the sensor selection problem is defined as minimizing the function $\Psi(M) = -log\ det(M)$.

$$Minimize_{\bar{r}_k} \quad \Psi(M(p; \bar{r}_1, .., \bar{r}_{Nr})) \ = -log\ det(M(p; \bar{r}_1, .., \bar{r}_{Nr}))$$
$$Subject\ to \quad p \in \Omega p, \qquad k \ = \ 1, \ldots Nr$$

Relaxation to convex problem

To make the above problem into a convex optimisation problem as described by D optimal design problem we specify a finite number of J allowed sensor positions. The problem is slightly modified by introducing $\lambda_j = \frac{w_j}{N^{rec}}$ where w_j denotes if a particular sensor is selected (wj=1) or not, N^{rec} denotes the total number of measurements performed by a sensor j, and then relaxing the constrains on the weights so they become real numbers. Now the problem becomes as

$$Minimize_{\lambda_j} \quad -log\ det\left(\sum_{j=1}^{J} \lambda_j\, M_j(p)\right)$$

$$Subject\ to \quad p \in \Omega p$$

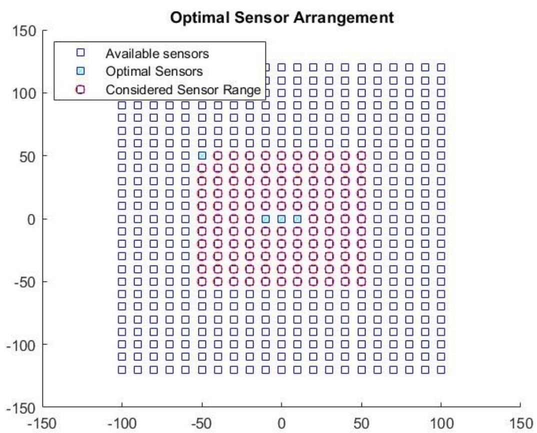
$$\lambda_j > 0, \quad j = 1, \ldots \mathrm{J}$$

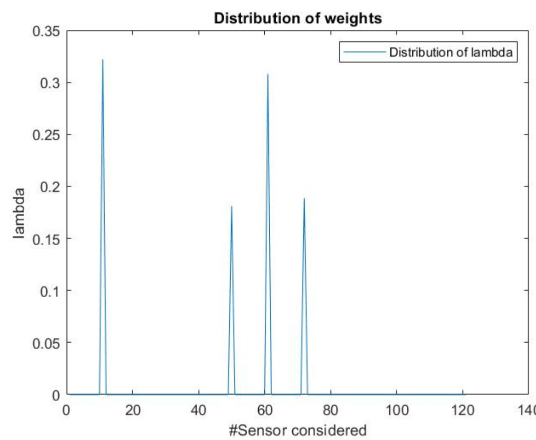
$$\sum_j \lambda_j = 1$$

Simulation

Single stationary transmitter and Receiver – XY planar array considered in Cartesian grid.

Domain, Ωp	x ∈ [−50, 50]mm, y ∈ [−50, 50]mm, z ∈ [100, 200] mm, θ ∈ [70◦, 110◦], ϕ ∈ [70◦, 110◦]
Transmitter position and orientation	pt = [0 cm, 0 cm, 10 cm, 90◦, 90◦]
Receiver array positions and orientation (XY planar)	|xr| ≤ 100 mm, |yr| ≤ 120mm, with a sensor cell size of 10mm in each direction. The receiver array is considered parallel to the transmitter. Therefore, Total number of receiving sensors = 525 Number of allowed receiving sensor in Ωp = 121





Conclusion

The sensor selection problem was formulated as an optimisation problem with integer variables which are then relaxed using convex optimisation as a convex optimisation problem with real variables. The convex problem is simulated for a planar sensor array and found that for a transmitter placed parallel to the sensor plane the optimal sensor configuration is a rectangle with sensor at corners and at midpoints of the long side.

Reference

[1] Talcoth, Oskar & Risting, Gustav & Rylander, Thomas. (2016). Convex optimization of measurement allocation for magnetic tracking systems. Optimization and Engineering. 18. 10.1007/s11081-016-9342-1.