Department of Computer Science, University College London

COMP0043 Numerical Methods for Finance

Prof. Guido Germano

Exercises for Section 2 Fundamental probability distributions

1. You have been provided scripts that plot the probability density function (PDF) and the cumulative distribution function (CDF) of the exponential, normal, lognormal and non-central χ^2 distributions. While the CDF can be computed analytically for some distributions like the exponential one, this is not possible for others like the normal.

   Modify the script normal.m so that the CDF is found besides MATLAB's built-in command `cdf` also with the approximation

   $$F_X(x) = \int_{-\infty}^{x} f_X(x')dx' \approx \sum_{i=0}^{n} w_i f_X(x_i)\Delta x.$$

   (a) In the simplest case where the integration weights are $\mathbf{w} = (0, 1, \ldots, 1)$, this can be done with `cumsum`. Show that this reproduces approximately the result of `cdf`.

   (b) Repeat (b) using a `for` loop and `sum` rather than `cumsum`.

   (c) If the first and the last integration weights are 1/2 while the others are 1 (trapezoidal rule), this can be done substituting `sum` with `trapz`. Show that this reproduces better the result of `cdf`.

   (d) Repeat (c) using a `for` loop and neither `sum` or `trapz`.

   (e) Repeat (c) without a `for` loop, applying a correction vector to `cumsum`.

   (f) Output the first 10 and the central 10 elements of the result vectors of (a)–(e).

   (g) Plot the CDF obtained with `cdf`, `cumsum` and `trapz`.

   Numerical integration is also called quadrature. For further reading, see Chapter 4 of Numerical Recipes, especially Sections 4.0–4.2 which include the trapezoidal rule, and Chapter 11 of Tools from Stochastic Analysis for Mathematical Finance: A Gentle Introduction.

2. Overplot the PDF of the noncentral chi-squared distribution which we obtained directly from the `pdf` library function computing it from a modified Bessel function of the first kind $I_\alpha(x)$, Eq. (41); use a library function for the latter.

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   Numerical integration is also called quadrature. For further reading, see Chapter 4 of Numerical Recipes, especially Sections 4.0–4.2 with the weights for the closed, open and semi-open Newton-Cotes formulas which include the trapezoidal and Simpson's rule, and Chapter 11 of Tools from Stochastic Analysis for Mathematical Finance: A Gentle Introduction.

2. Overplot the PDF of the noncentral chi-squared distribution which we obtained directly from the `pdf` library function computing it from

   (a) The definition from the sum of k independent standard normal random variables with mean μ_i and standard deviation $\sigma = 1$, Eq. (38) in Slides 2.

   (b) The modified Bessel function of the first kind $I_\alpha(x)$, Eq. (40) in Slides 2; use a library function for this.

2. In `ncchisq.m` we plotted the PDF of the noncentral chi-squared distribution with k degrees of freedom and noncentrality parameter λ using the `pdf` library function.

   (a) Overplot the PDF with the expression based on the modified Bessel function of the first kind $I_\alpha(x)$, Eq. (38) in Slides 2,

$$f_{\chi'^2_{k,\lambda}}(x) = \frac{1}{2} e^{-\frac{x+\lambda}{2}} \left(\frac{x}{\lambda}\right)^{\frac{k}{4}-\frac{1}{2}} I_{\frac{k}{2}-1}(\sqrt{\lambda x}).$$

   Use a library function to compute $I_\alpha(x)$. Keep the values of k and λ in `ncchisq.m`.

   (b) Sample the distribution of χ'^2 using its quantile function provided by `icdf`.

   (c) Sample the distribution using the dedicated random number generator `ncx2rnd`.

   (d) Sample the distribution from the definition, Eqs. (36) and (37) in Slides 2,

$$\chi'^2(k,\lambda) = \sum_{i=1}^{k} N_i^2(\mu_i, 1) = \sum_{i=1}^{k} (Z_i + \mu_i)^2$$

$$\lambda = \sum_{i=1}^{k} \mu_i^2.$$

   (e) Compare the CPU times of the three sampling methods. Which limitation has the fastest?