Department of Computer Science, University College London

# COMP0043 Numerical Methods for Finance

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**Exercises for Section 3 Random numbers**

1. Matlab's high-quality standard uniform random number generator `rand` implements the MT19937 variant of the Mersenne twister, which has the period $2^{19937}-1$ and produces floating point values in the closed interval $[2^{-53}, 1-2^{-53}]$, i.e. it does not produce an exact 0 and an exact 1. See Makoto Matsumoto, Takuji Nishimura, Mersenne twister: a 623-dimensionally equidistributed uniform pseudo-random number generator, ACM Transactions on Modeling and Computer Simulation, 8 (1) 3–30, DOI 10.1145/272991.272995.
   (a) Implement a linear congruential random number generator following the instructions in Seydel's Course Notes, Section 2.1 A, page 202.
   (b) Test it producing
      i. the empirical probability density function $f_U(x)$, i.e. a histogram normalised to 1 of the values $U_n$ returned by the RNG,
      ii. a scatter plot in the $(U_n, U_{n-1})$ unit square,
      iii. a scatter plot in the $(U_n, U_{n-1}, U_{n-2})$ unit cube

      with the good parameters of page 203 and with the pathologic parameters of pages 208–210, Seydel's Course Notes, Section 2.1 A.
   (c) Do the same three tests with Matlab's `rand`.

2. If $U$ is a uniform random number on $[-\pi/2, \pi/2]$, the probability density function (PDF) of $X = \cos U$ was worked out in COMP0045_slides.pdf, page 86,

$$f_X(x) = \frac{2}{\pi\sqrt{1-y^2}}.$$

   Check this result numerically: sample an appropriate quantity of uniform random numbers, take their cosine, build a histogram normalised to 1 and overplot it with the analytical PDF $f_X(x)$.

3. Starting from the linear congruential uniform random number generator that you wrote in Question 1 above, implement the Fibonacci generator described in Seydel's Course Notes, Section 2.1 B, pages 211–212, and reproduce the scatter plot on page 213.

4. There are several methods to obtain standard normal random numbers starting from standard uniform random numbers. Which do you know? Implement
   (a) the Box-Muller method, see COMP0045_slides.pdf, page 90, or Seydel's Course Notes, Section 2.3, pages 224–225;
   (b) the rejection method using the Laplace (or double exponential) distribution as the majorant function, following the instructions on Seydel's Course Notes, Section 2.1 C, pages 221–223.