SDE Modelling Cheat Sheet

Arithmetic Brownian Motion (ABM)

SDE

$$dX_t = \mu\, dt + \sigma\, dW_t.$$

When to Use

- Modelling spreads or differences.
- Short horizons where negative values are acceptable.
- Simple linear mispricing dynamics.

Finance Interpretation

- Gaussian increments with constant volatility.
- Can be negative, unsuitable for asset prices.

Typical Use Cases

- Pair trading spreads.
- Commodity forward residuals.

Brownian Bridge

Definition

$$X_t \mid X_0, X_T \quad \text{with fixed } X_T.$$

When to Use

- Simulation conditioned on an endpoint.
- Variance reduction.

Finance Interpretation

- Conditional Brownian motion.
- Variance shrinks as t approaches T.

Typical Use Cases

- Barrier options.
- Exposure modelling.

CIR / Feller Square Root Process

SDE

$$dV_t = \kappa(\theta - V_t)\, dt + \sigma\sqrt{V_t}\, dW_t.$$

When to Use

- Variable must remain positive.
- Mean-reverting volatility or rates.

Finance Interpretation

- Skew and heavy tails from square-root term.
- Feller condition ensures $V_t \geq 0$.

Typical Use Cases

- Heston variance.
- Short rate modelling.

Geometric Brownian Motion (GBM)

SDE

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t.$$

When to Use

- Prices must remain positive.
- Proportional volatility.

Finance Interpretation

- Black–Scholes model.
- Lognormal returns with constant volatility.

Typical Use Cases

- Baseline equity modelling.
- Analytic option pricing.

Heston Stochastic Volatility Model

System

$$dS_t = \mu S_t \, dt + \sqrt{V_t} \, S_t \, dW_t^{(1)}, \quad dV_t = \kappa(\theta - V_t) \, dt + \sigma \sqrt{V_t} \, dW_t^{(2)}.$$

When to Use

- Need stochastic, mean-reverting volatility.
- Desire realistic smile and skew.

Finance Interpretation

- Captures volatility clustering.
- Leverage effect allowed via correlation.

Typical Use Cases

- Equity and FX options.
- Volatility derivatives.

Ornstein–Uhlenbeck (OU)

SDE

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma dW_t.$$

When to Use

- Mean-reverting variable that may be negative.
- Gaussian linear dynamics.

Finance Interpretation

- Stationary distribution.
- Not suitable for positive-only variables.

Typical Use Cases

- Pair trading.
- Vasicek interest rate model.

Mean-Reverting GBM (Log-OU)

SDE

$$dS_t = \kappa(\theta - \ln S_t)\,S_t\,dt + \sigma S_t\,dW_t.$$

When to Use

- Positive, mean-reverting prices.
- Commodities or FX with fundamental levels.

Finance Interpretation

- Mean reversion in log space.
- Suitable for commodity spot dynamics.

Typical Use Cases

- Energy markets.
- FX equilibrium modelling.

Variance Gamma (VG)

Definition

$$X_t = \theta G_t + \sigma W_{G_t}, \qquad G_t \sim \text{Gamma process}.$$

When to Use

- Need jumps, heavy tails, skew.
- Short-horizon equity return realism.

Finance Interpretation

- Pure jump, infinite activity.
- No instantaneous volatility.

Typical Use Cases

- Equity index modelling.
- Option pricing with skew and kurtosis.

Gamma Process

Definition

$$G_t \text{ is non-negative, increasing, pure jump.}$$

When to Use

- Modelling cumulative activity.
- Time-change processes.

Finance Interpretation

- Activity clock in Lévy models.
- Operational loss accumulation.

Typical Use Cases

- Variance Gamma.
- Operational risk.