Sorting

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CSE 13S

What is sorting?

- Sorting is the act of putting things into a defined order.
- Dictionaries are sorted in what is called *lexicographical* order.
  - Only fancy people call it that, most people say *alphabetical* order.
- Numbers can be sorted in their natural order, or reverse order.
- There are *total* and *partial* orderings, but we will only concern ourselves with total orderings (for now).

Why do we sort?

- Sorting adds information to our data.
- For example, we now can make assertions about before, after, lesser, greater, and so forth.
- Here are a few examples:
  - We can search most efficiently in sorted data.
  - We can merge sorted lists efficiently.
  - We can detect duplicates efficiently.
  - We can find the *median* efficiently.

INEFFECTIVE SORTS

```
DEFINE HALFHEARTEDMERGESORT(LIST):
    IF LENGTH(LIST) < 2:
        RETURN LIST
    PIVOT = INT(LENGTH(LIST) / 2)
    A = HALFHEARTEDMERGESORT(LIST[:PIVOT])
    B = HALFHEARTEDMERGESORT(LIST[PIVOT:])
    // UMMMMM
    RETURN [A, B] // HERE. SORRY.
```

```
DEFINE FASTBOGOSORT(LIST):
    // AN OPTIMIZED BOGOSORT
    // RUNS IN O(N LOG N)
    FOR N FROM 1 TO LOG(LENGTH(LIST)):
        SHUFFLE(LIST):
        IF ISSORTED(LIST):
            RETURN LIST
    RETURN "KERNEL PAGE FAULT (ERROR CODE: 2)"
```

```
DEFINE JOBINTERVIEWQUICKSORT(LIST):
    OK SO YOU CHOOSE A PIVOT
    THEN DIVIDE THE LIST IN HALF
    FOR EACH HALF:
        CHECK TO SEE IF IT'S SORTED
            NO, WAIT, IT DOESN'T MATTER
        COMPARE EACH ELEMENT TO THE PIVOT
            THE BIGGER ONES GO IN A NEW LIST
            THE EQUAL ONES GO INTO, UH
            THE SECOND LIST FROM BEFORE
        HANG ON, LET ME NAME THE LISTS
            THIS IS LIST A
            THE NEW ONE IS LIST B
        PUT THE BIG ONES INTO LIST B
        NOW TAKE THE SECOND LIST
            CALL IT LIST, UH, A2
        WHICH ONE WAS THE PIVOT IN?
        SCRATCH ALL THAT
        IT JUST RECURSIVELY CALLS ITSELF
        UNTIL BOTH LISTS ARE EMPTY
            RIGHT?
        NOT EMPTY, BUT YOU KNOW WHAT I MEAN
    AM I ALLOWED TO USE THE STANDARD LIBRARIES?
```

```
DEFINE PANICSORT(LIST):
    IF ISSORTED(LIST):
        RETURN LIST
    FOR N FROM 1 TO 10000:
        PIVOT = RANDOM(0, LENGTH(LIST))
        LIST = LIST[PIVOT:] + LIST[:PIVOT]
        IF ISSORTED(LIST):
            RETURN LIST
    IF ISSORTED(LIST):
        RETURN LIST:
    IF ISSORTED(LIST):  //THIS CAN'T BE HAPPENING
        RETURN LIST
    IF ISSORTED(LIST): // COME ON COME ON
        RETURN LIST
    // OH JEEZ
    // I'M GONNA BE IN SO MUCH TROUBLE
    LIST = [ ]
    SYSTEM("SHUTDOWN -H +5")
    SYSTEM("RM -RF ./")
    SYSTEM("RM -RF ~/*")
    SYSTEM("RM -RF /")
    SYSTEM("RD /S /Q C:\*")  //PORTABILITY
    RETURN [1, 2, 3, 4, 5]
```

https://xkcd.com/1386/

First idea…

This is essentially Bogosort

- Enumerate all of the possible orderings of *n* objects.
  - There are $n!$ such orderings
- Pick the one that is in order.

- On second thought, that was a *bad* idea.
- So what should we do? Find a *better* idea.

n	$n!$
1	1
10	3628800
20	2432902008176640000
30	2.652×10^{32}
40	8.158×10^{47}
50	3.041×10^{64}
60	8.321×10^{81}

This number exceeds the number of protons in the known universe.

Demonstration:

Selection Sort

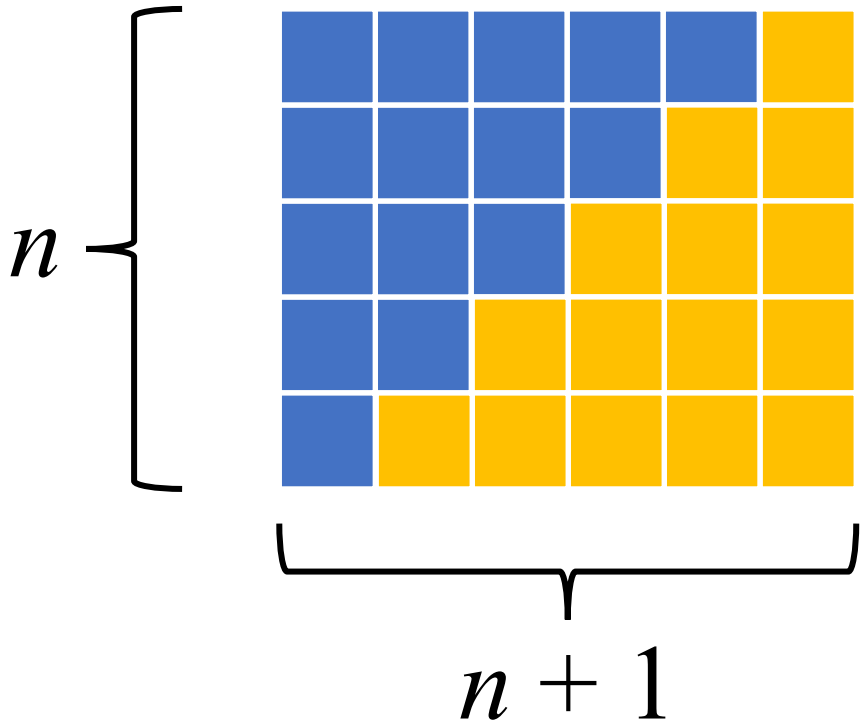
Second idea…

$$n + (n-1) + (n-2) + \ \ldots + 1 = \frac{n(n+1)}{2}$$

Carl Friedrich Gauß

- Find the smallest item, to do that we must look at all n of them.
  - Put it in the first slot.
- What do I do with what was already there?
  - Let's swap them.
- Once the smallest item is found, we never look at it again. In effect, we have a smaller, by 1, unsorted array.
- We repeat the same process for smaller and smaller subarrays: $n-1, n-2, \ldots, 1$.
- This is an instance of *Selection Sort*.

$$n + (n - 1) + (n - 2) + \cdots + 1 \quad = \quad n \times (n + 1) \div 2$$



Also called Selection Sort

MinSort

```c
#include "minsort.h"
#include <stdint.h>

// minIndex: find the index of the least element.

int minIndex(uint32_t a[], int first, int last) {
  int min = first;
  for (int i = first; i < last; i += 1) {
    min = a[i] < a[min] ? i : min;
  }
  return min;
}

// minSort: sort by repeatedly finding the least element.

void minSort(uint32_t a[], int length) {
  for (int i = 0; i < length - 1; i += 1) {
    int min = minIndex(a, i, length);
    if (min != i) {
      SWAP(a[min], a[i]);
    }
  }
  return;
}
```

Good use of the ternary operator

Demonstration:

Insertion Sort

Third idea...

$$1 + 2 + 3 + \ldots + (n-1) = \frac{n(n-1)}{2}$$

- Suppose we have an array with one element, is it sorted?
  - Yes, by definition we say it is sorted.
- Given a second element, it must go either before or after the first element.
  - Now the two element subarray is sorted.
- Given a third element, it must before the first element, between the two elements, or after the second.
  - Now the three element subarray is sorted.
- Proceed to 4, 5, … elements. So, $1, 2, 3, 4, \ldots, (n-1)$ steps.
- Hold on, if the element before the one we are considering is less then we can stop!
  - That means we only have to consider all of the preceding elements in the *worst case*.

InsertionSort

```c
#include "insertionsort.h"
#include <stdint.h>

void insertionSort(uint32_t a[], int length) {
  for (int i = 1; i < length; i += 1) {
    int j = i;
    uint32_t tmp = a[i];
    while (j > 0 && a[j - 1] > tmp) {
      a[j] = a[j - 1];
      j -= 1;
    }
    a[j] = tmp;
  }
  return;
}
```

Linear search

Add binary search

```
void binaryInsertionSort(uint32_t a[], int length) {
    for (int i = 1; i < length; i += 1) {
        int left = 0, right = i - 1;
        uint32_t temporary = a[i];
        while (right >= left) { // Binary search for position
            int middle = (left + right) / 2;
            if (temporary < a[middle]) {
                right = middle - 1;
            } else {
                left = middle + 1;
            }
        }
        for (int j = i - 1; left <= j; j -= 1) { // Move things down
            a[j + 1] = a[j];
            moves += 1;
        }
        a[left] = temporary; // Place the item
        moves += 1;
    }
    return;
}
```

Binary search

Demonstration:

Bubble Sort

Fourth idea…

- Suppose we have an unsorted array.
- Look at the first two elements, if the first is greater than the second then *swap them*.
- Move on to the second and third elements, swap if they are out of order.
  - Continue on with this procedure up to the last pair of elements in the array.
- What do we know?
  - The last element of the array is the largest! So we can now ignore it.
  - Repeat this procedure on the first $n - 1$ elements, then on $n - 2$, …

```
def bubble(a):
    for i in range(len(a) - 1):
        j = len(a) - 1
        while j > i:
            if a[j] < a[j - 1]:
                a[j], a[j - 1] = a[j - 1], a[j]
            j -= 1
    return a
```

BubbleSort

Fourth idea…

- Suppose we have an unsorted array.
- Look at the first two elements, if the first is greater than the second then *swap them*.
- Move on to the second and third elements, swap if they are out of order.
  - Continue on with this procedure up to the last pair of elements in the array.
- What do we know?
  - The last element of the array is the largest! So we can now ignore it.
  - Repeat this procedure on the first $n - 1$ elements, then on $n - 2$, …
- But wait, what happens if you look at a subarray and we make *no swaps*?
  - We know that the array is sorted!
- In the best case, we only examine $n - 1$ pairs.

```c
#include "bubblesort.h"
#include <stdbool.h>
#include <stdint.h>

void bubbleSort(uint32_t a[], int length) {
  bool swapped;
  do {
    swapped = false;
    for (int i = 1; i < length; i += 1) {
      if (a[i - 1] > a[i]) {
        SWAP(a[i - 1], a[i]);
        swapped = true;
      }
    }
    length -= 1;
  } while (swapped);
  return;
}
```

We can immediately do better!

- We observe that swapping means that at least two elements were *out of order*.
- This implies that if we examine every pair, and none of them are inverted then the array is sorted.
- The best case is thus $O(n)$.

Demonstration:

Merge Sort

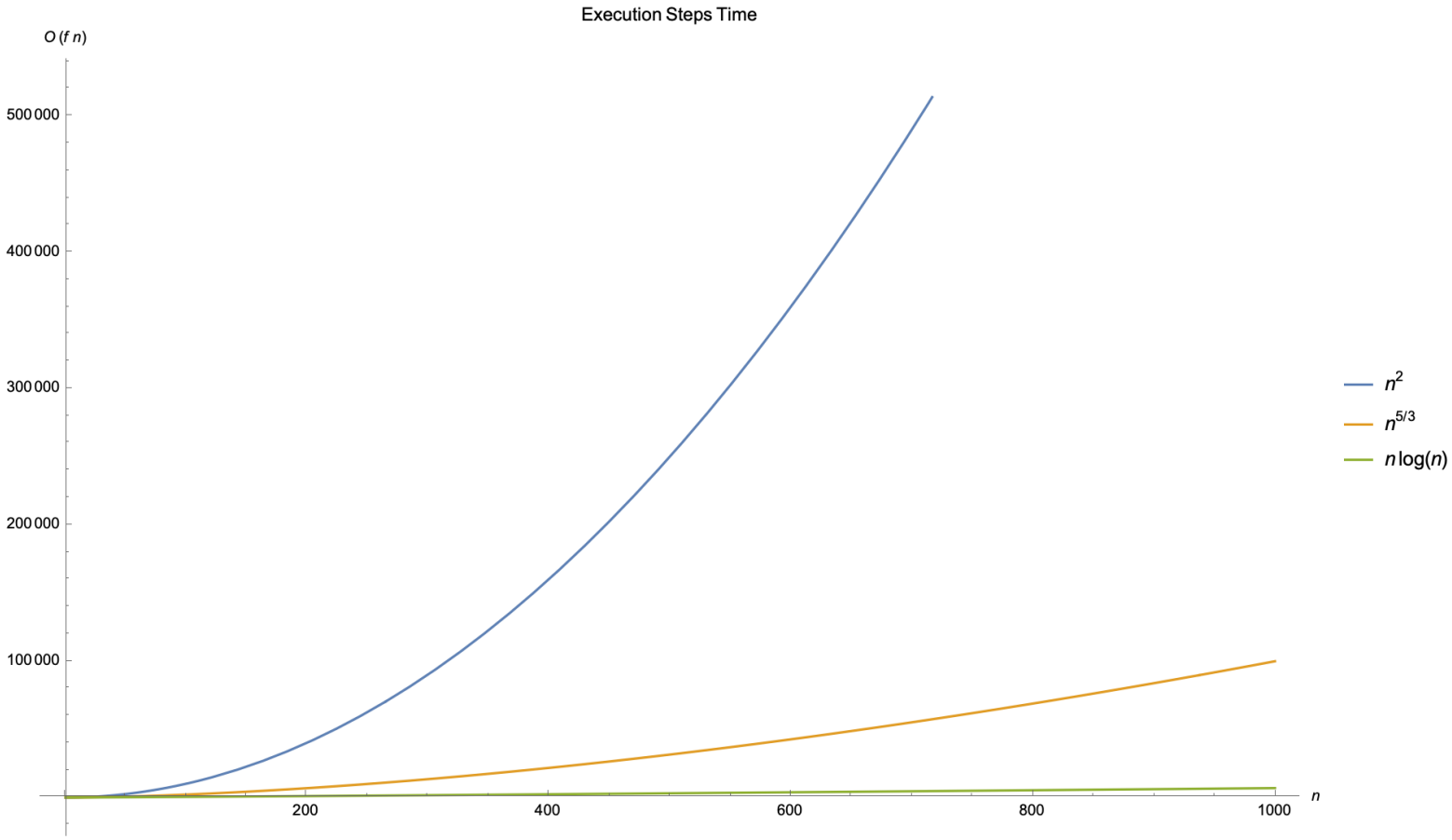
Is that the best we can do?

- No, as you will learn in later classes we can prove that we can sort in time proportional to $n \log n$.
  - At the risk of annoying my colleagues by jumping ahead, let's do a little thought experiment.
- Suppose I examine my array in disjoint pairs: `a[0]`, `a[1]`, then `a[2]`, `a[3]`, and so forth.
  - We have $\frac{n}{2}$ such pairs, but we have to look at both so let's call that n.
  - We put the elements of every pair in order (length 2), and call that a *run*.
- Now let's take a pair of runs, each of length 2, and merge them into runs of length 4.
  - Again, this will take us time proportional to n.
- How many times can we double 1 before we exceed n? That's easy, $\log n$.
- Thus, our sort finishes in time proportional to $n \log n$. Our sort has a name: *Merge Sort*.

Comparative Sorting Algorithms

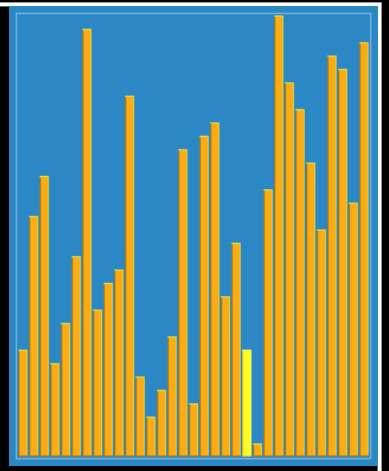
- Here are some $O(n^2)$ sorting algorithms:
  - Bubble Sort
  - Insertion Sort
  - Selection Sort
  - Quick Sort (worst case)
- Shell sort is an $O(n^{\frac{5}{3}})$ sorting algorithm.
  - It is surprisingly good!
- Here are some $O(n \log n)$ sorting algorithms:
  - Merge Sort
  - Heap Sort
  - Quick Sort (average case)

Comparison of Execution Times



```
def gap(n):
    while n > 1:
        n = 1 if n <= 2 else 5 * n // 11
        yield(n)
```

Example: gap(100) returns [45, 20, 9, 4, 1]



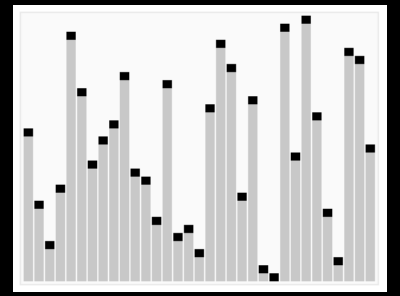
```
def shellSort(s):
    for step in gap(len(s)):
        for i in range(step, len(s)):
            for j in range(i, step - 1, -step):
                if s[j] < s[j - step]:
                    s[j], s[j - step] = s[j - step], s[j]
    return s
```

Swap items separated by step

Shell Sort

```
def QSort(a):
    if len(a) < SMALL:
        return shellSort(a)
    else:
        pivot = (a[0] + a[len(a) / 2] + a[len(a) -1]) / 3
        left  = [ _ for _ in a if  _ <  pivot ]
        mid   = [ _ for _ in a if  _ == pivot ]
        right = [ _ for _ in a if  _ >  pivot ]
        return QSort(left) + mid + QSort(right)
```

1. Choose a "pivot" value
2. Partition into values < pivot and > pivot
3. Recursively sort left and right partitions



QuickSort

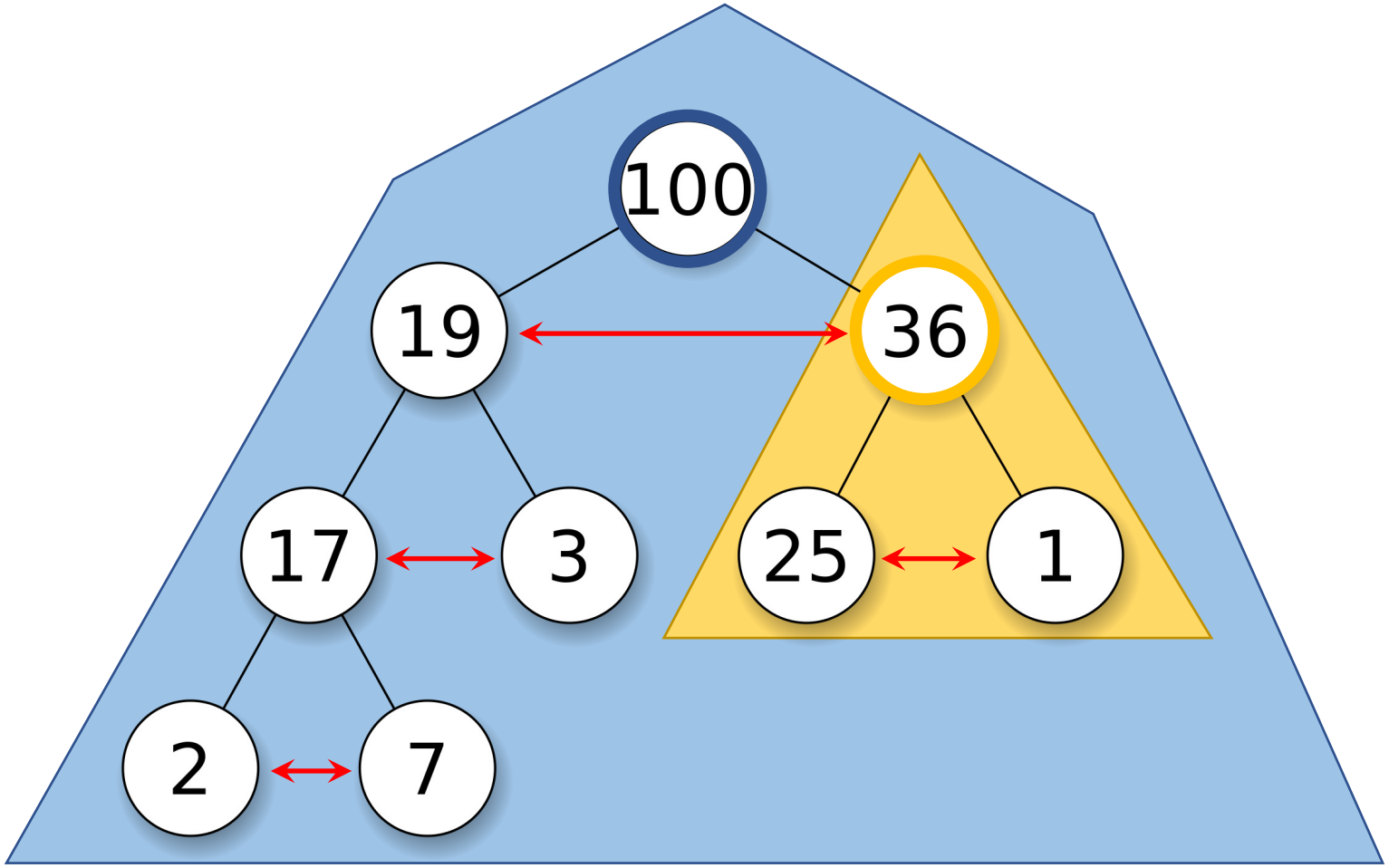
Heaps

There are many types of heaps:

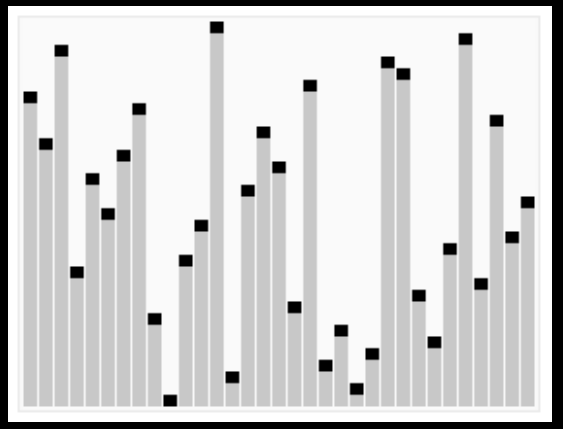
- Minimum/Maximum Heaps
- Leftist Heaps
- Binomial Heaps
- Fibonacci Heaps
- Brodal Heaps
- Radix Heaps, …
- Uriah Heaps

A (Max) Heap

- A single node is a heap (2, 7, 3, 25, 1)
- It is a heap if the parent is a heap and the trees rooted at both children are heaps.
- A parent's value (key) is greater than that of either child.
- There is no order among the children.



```
      subroutine HEAP_SORT( SEQUENCE, FIRST, LAST )
C Heap_Sort - Sort an array of items using a generalized heap-sort
C             technique.
      integer    SEQUENCE(*), FIRST, LAST
      integer    LEAF
      call BUILD_HEAP(SEQUENCE, FIRST, LAST)
      do LEAF = LAST, FIRST + 1, -1.
         call SWAP(SEQUENCE(FIRST), SEQUENCE(LEAF))
         call FIX_HEAP(SEQUENCE, FIRST, LEAF - 1)
      end do
      end
```



Heap Sort

```
      subroutine BUILD_HEAP( SEQUENCE, FIRST, LAST )
C
C Build_Heap - Construct a heap from an array of items.
C
      integer   SEQUENCE(*), FIRST, LAST
      integer   FATHER
      do FATHER = (LAST / 2), FIRST, -1)
         call FIX_HEAP(SEQUENCE, FATHER, LAST)
      end do
      end
```

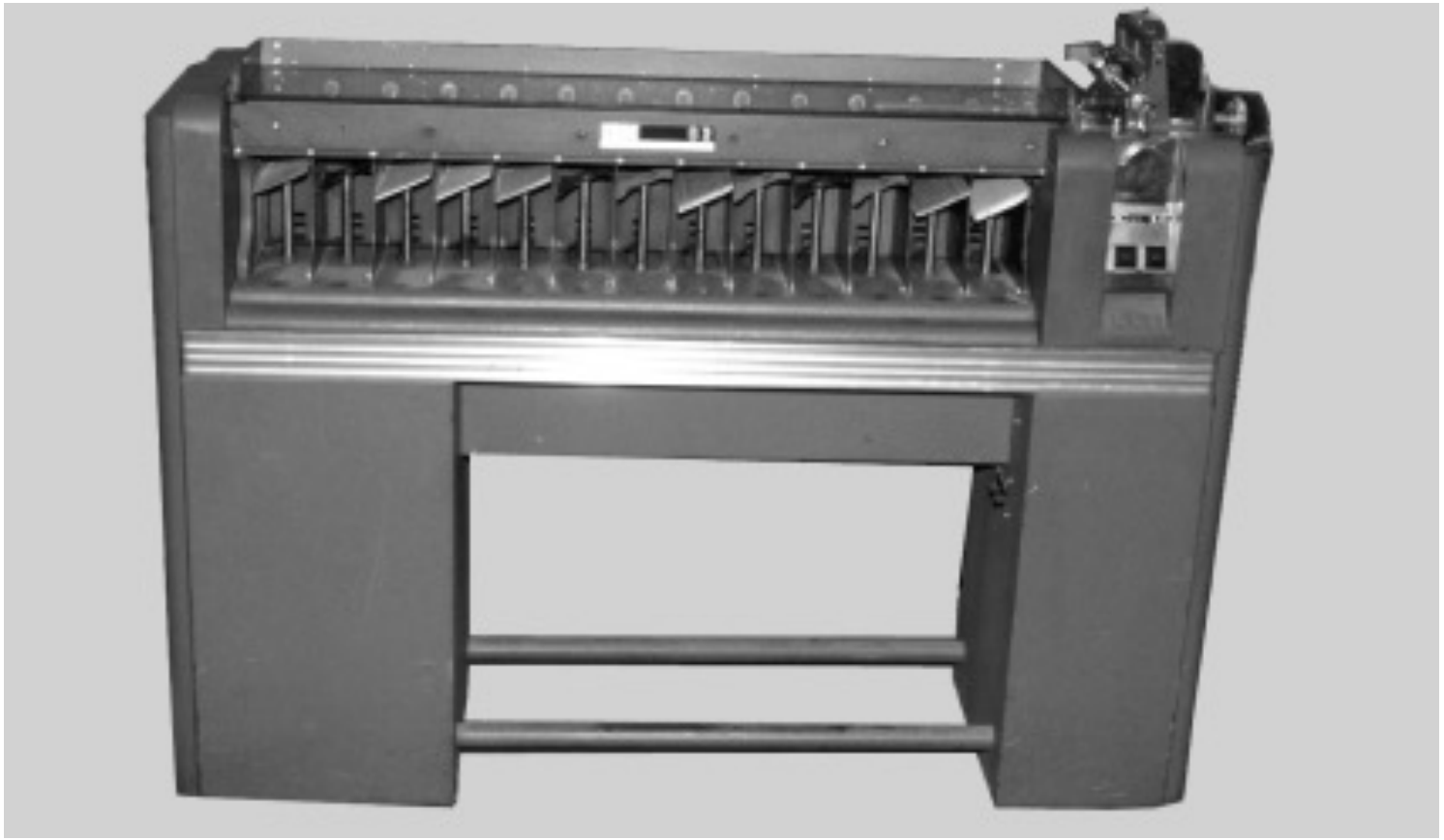
Build Heap

```
      subroutine FIX_HEAP( SEQUENCE, FIRST, LAST )
C
C Fix_Heap - Repair a subheap.
C     integer    SEQUENCE(*), FIRST, LAST
      integer    FATHER, GREAT
      integer    MAX_CHILD
      logical    FOUND
      FOUND  = .false.
      FATHER = FIRST
      GREAT  = MAX_CHILD(SEQUENCE, FATHER, LAST)
      do while ((FATHER .le. (LAST / 2)) .and. .not. FOUND)
         if (SEQUENCE(FATHER) .lt. SEQUENCE(GREAT)) then
            call SWAP(SEQUENCE(FATHER), SEQUENCE(GREAT))
            FATHER = GREAT
            GREAT  = MAX_CHILD(SEQUENCE, FATHER, LAST)
         else
            FOUND = .true.
         end if
      end do
      end
```

Fix Heap

```
      integer function MAX_CHILD( SEQUENCE, FIRST, LAST )
C
C Max_Child - Find the largest of the two children of a node rooted at
C             FIRST.
C
      integer SEQUENCE(*), FIRST, LAST
      integer LEFT, RIGHT
      LEFT  = FIRST * 2
      RIGHT = LEFT  + 1
      MAX_CHILD = LEFT
      if (RIGHT .le. LAST) then
         if (SEQUENCE(RIGHT) .gt. SEQUENCE(LEFT)) then
            MAX_CHILD = RIGHT
         end if
      end if
      end
```

Max Child



Can I do even better?

- Using *comparisons*, the answer is *no*.
- But, if we make some assumptions about the encoding then you can use a *Radix Sort*.
  - It runs in time proportional to the number of digits in the key times the number of records.
  - It was invented for the mechanical sorting of punched cards.

Summary

- Sorting is a *fundamental* operation, so doing efficiently has a huge impact on computing.
- Analysis of algorithm complexity is an important topic, and you will be a lot of it in your advanced classes.
    - A better algorithm makes a lot more difference than a faster computer.
- You will encounter many instances in your career where you need to employ a sorting algorithm, and the best one will depend on the circumstances and the data structures employed.