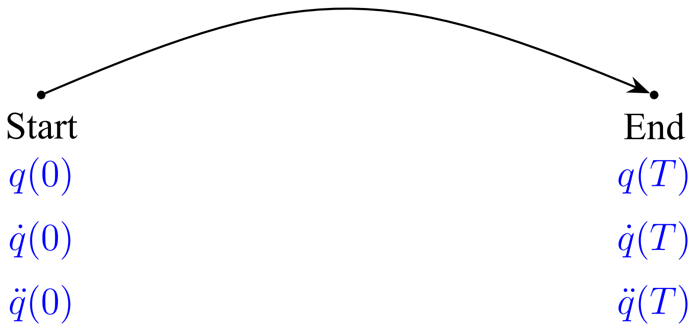
1 QUINTIC POLYNOMIAL TRAJECTORY PLANNER

Since at this stage of the project, we aim to implement a basic version of the algorithm that contributes toward the final goal of this master's thesis, we are using a **time-based, Single-Segment Quintic Polynomial Trajectory**. This method generates smooth, minimum-jerk motion using a 5^{th}-order polynomial .



According to the theory, in this method we define six boundary conditions (three at the start and three at the end).

- **Start:** $q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0, \quad \ddot{q}(0) = \ddot{q}_0$
- **End:** $q(T) = q_d, \quad \dot{q}(T) = \dot{q}_d, \quad \ddot{q}(T) = \ddot{q}_d$

By substituting these conditions into the general form of the quintic polynomial, we obtain a system of six equations:

$$\begin{aligned}
q_0 &= a_5 t_0^5 + a_4 t_0^4 + a_3 t_0^3 + a_2 t_0^2 + a_1 t_0 + a_0 \\
\dot{q}_0 &= 5a_5 t_0^4 + 4a_4 t_0^3 + 3a_3 t_0^2 + 2a_2 t_0 + a_1 \\
\ddot{q}_0 &= 20a_5 t_0^3 + 12a_4 t_0^2 + 6a_3 t_0 + 2a_2 \\
q_d &= a_5 t_1^5 + a_4 t_1^4 + a_3 t_1^3 + a_2 t_1^2 + a_1 t_1 + a_0 \\
\dot{q}_d &= 5a_5 t_1^4 + 4a_4 t_1^3 + 3a_3 t_1^2 + 2a_2 t_1 + a_1 \\
\ddot{q}_d &= 20a_5 t_1^3 + 12a_4 t_1^2 + 6a_3 t_1 + 2a_2
\end{aligned} \tag{1}$$

Then, we rewrite this system in matrix form as:

$$\boldsymbol{\xi} = \mathbf{T}^{-1}\boldsymbol{\sigma}$$

where ξ is the vector of polynomial coefficients and σ contains the boundary conditions.

$$\underbrace{\begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ q_d \\ \dot{q}_d \\ \ddot{q}_d \end{bmatrix}}_{\boldsymbol{\sigma}} = \underbrace{\begin{bmatrix} t_0^5 & t_0^4 & t_0^3 & t_0^2 & t_0 & 1 \\ 5t_0^4 & 4t_0^3 & 3t_0^2 & 2t_0 & 1 & 0 \\ 20t_0^3 & 12t_0^2 & 6t_0 & 2 & 0 & 0 \\ t_1^5 & t_1^4 & t_1^3 & t_1^2 & t_1 & 1 \\ 5t_1^4 & 4t_1^3 & 3t_1^2 & 2t_1 & 1 & 0 \\ 20t_1^3 & 12t_1^2 & 6t_1 & 2 & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \cdot \underbrace{\begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}}_{\boldsymbol{\xi}}$$

All the polynomial coefficients are calculated individually for each joint and substituted into the equations.

$$\begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ q_1 \\ \dot{q}_1 \\ \ddot{q}_1 \end{bmatrix} = T \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$q(t) = a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \tag{2}$$

$$\dot{q}(t) = 5a_5t^4 + 4a_4t^3 + 3a_3t^2 + 2a_2t + a_1 \tag{3}$$

$$\ddot{q}(t) = 20a_5t^3 + 12a_4t^2 + 6a_3t + 2a_2 \tag{4}$$