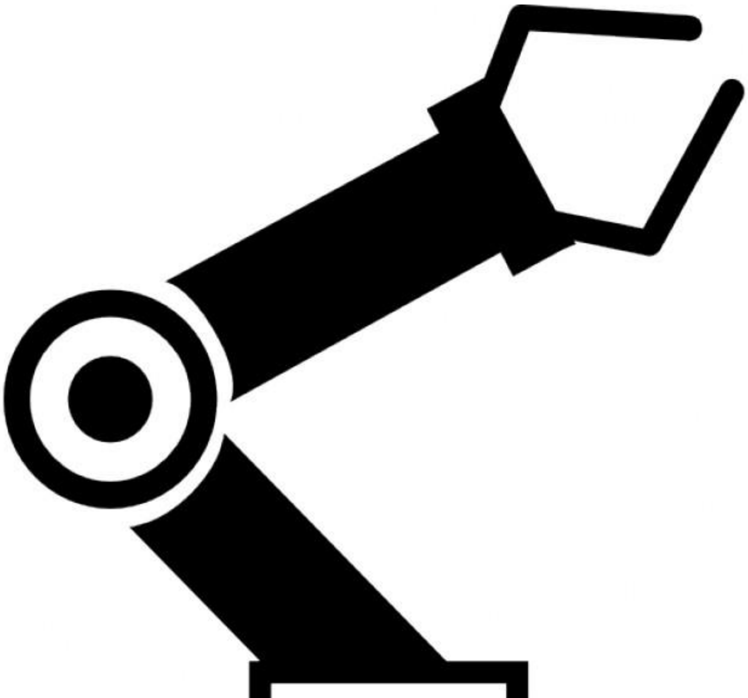
EE587 Introduction to Robotics

1st Term Project Report



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Project Definition

Consider 2 seahorses robot that carry a stick together stack to their wrapped tails.

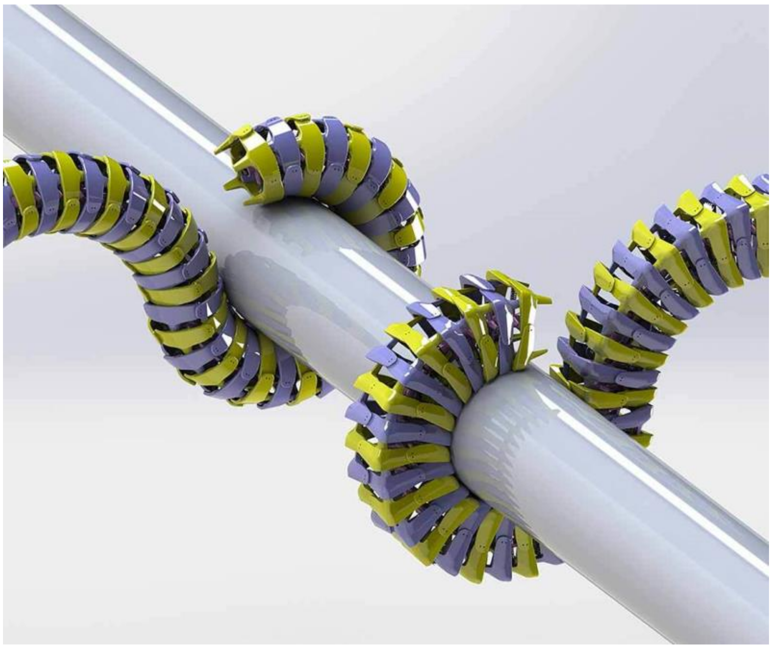

Figure 1 3D model of the tails of 2 seahorses

Assumptions

We Seahorses are of particular interest to robot researchers due to their unusual skeletal structure, which scientists believe could aid in the design of bots that are tough and strong while also being flexible enough to perform tasks in real-world settings.

For this issue there are 2 types based on searching through references and on the internet:

1. First one as shown in the figure 1 has too many joints and small links if we would like to simplify it and if we would like to analysis it on that format it will need expensive method and expensive experimental lab to work on those felexible small joints which has been simulated the body of real seahorse.
2. The second is based on the question' data combining with sea horse toys which is I think suit to this issue as explained in the question:
   - The second modelfor the first format of the assumption I assumed that the tail can be gripper which will be so easy therefore I changed to three joints which are twisted to the stick therefore both seahorses could carry it perfectly.
   - We can understand for doing a task "together" we need common links or joints in robots, on the otherhand it will help us to analysis one seahorse and then conclude the result for both of them.

- Fins of seahorses is shown with 2 other coupled revolute joints, beside this revolute joint other 2 joints one revolute and the other prismatic will shape the body and head of each seahorse.

Even though the water friction is a required for the thrust, the water friction due to bandeon and other components are negligible. Also the water around is uniform and stationary, i.e no disturbing forces just fluidic force.

DH Parameters

In order to be able to generate DH parameters, we need to identify the end effector. In this case, seperating the system into two will be more appropriate because controlling one seperately cause the control of the other one as we know thses 2 seahorses carry one stick at the same time. In this system, there are 2 end-effectors if we start naming the seahorses from tail to head.

Since seahorses are identical and their centers overlaps in z direction intially, the DH parameters generated for one can be used in the other.

Since there are multiple end effectors, DH parameters will be generated for the frame which is just before the end effectors, namely the body frame for quadrotor design.

A system is a device which has 6-DOF. 1 of them are prismatic joints and 5 of them are revolute joints. Using Euler angles,

$$q = (X, \Theta)^T where\ X = (x, y, z)^T, \Theta = (\psi, \theta, \phi)^T$$

	α_i	a_i	d_i	θ_i
1	0	a_1	0	θ_1
2	90	a_2	0	θ_2
3	90	a_3	0	θ_3
4	90	a_4	0	θ_4
5	90	a_5	0	θ_5
6	90	0	d_6	0

Kinematics and Dynamics

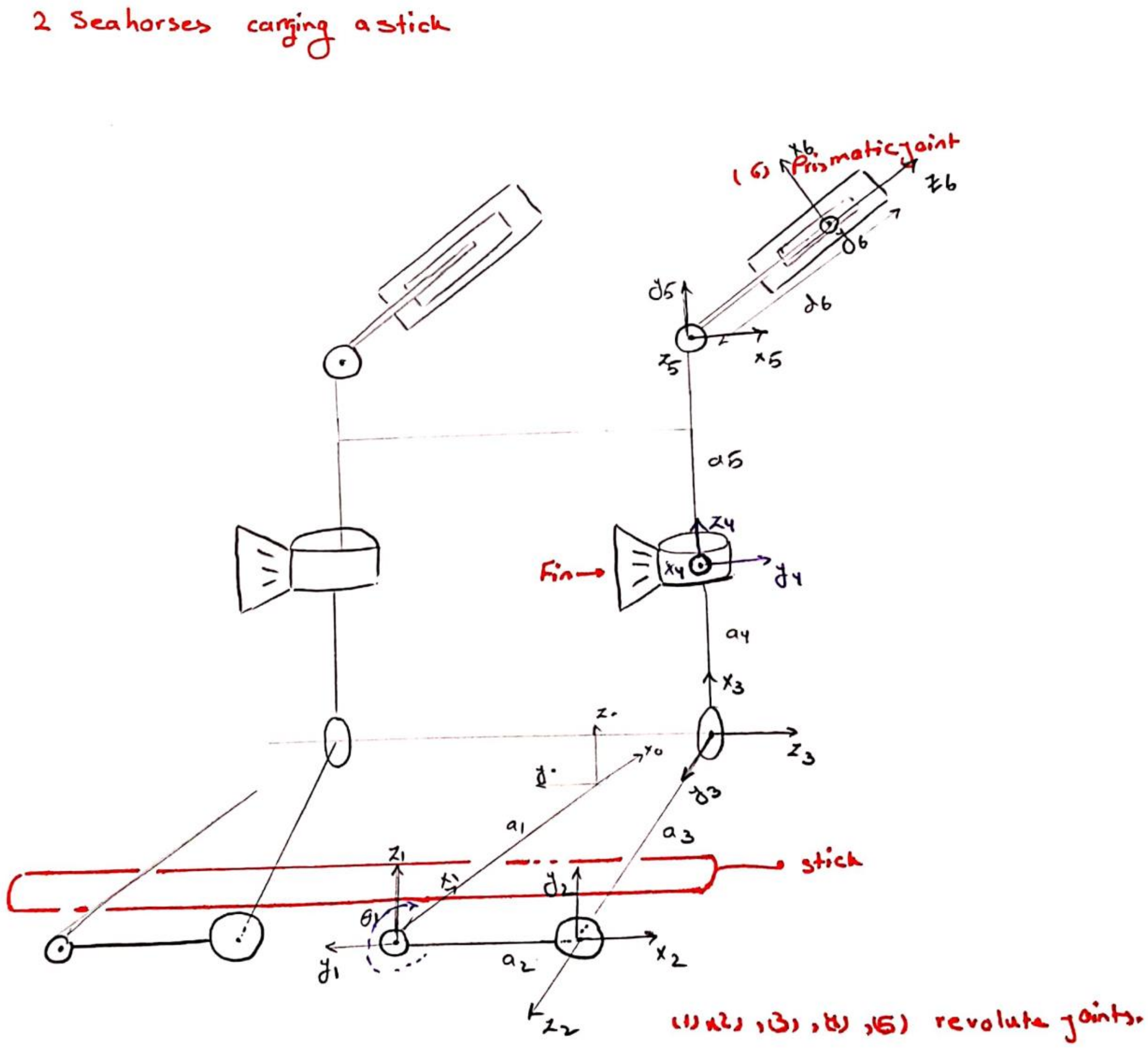

Figure 2: Planar viewof the seahorses system

A commonly used convention for selecting frames of reference in robotic applications is the Denavit-Hartenberg, or DH convention. In this convention, each homogeneous transformation Ai is represented as a product of four basic transformations:

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$A_1^0 = \begin{bmatrix} c\Theta_1 & -s\Theta_1 & 0 & a_1 c\Theta_1 \\ s\Theta_1 & c\Theta_1 & 0 & a_1 s\Theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2^1 = \begin{bmatrix} c\Theta_2 & 0 & s\Theta_2 & a_2 c\Theta_2 \\ s\Theta_2 & 0 & -c\Theta_2 & a_2 s\Theta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} c\Theta_3 & 0 & s\Theta_3 & a_3 c\Theta_3 \\ s\Theta_3 & 0 & -c\Theta_3 & a_3 s\Theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4^3 = \begin{bmatrix} c\Theta_4 & 0 & s\Theta_4 & 0 \\ s\Theta_4 & 0 & -c\Theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^0 = \begin{bmatrix} c\Theta_5 & 0 & s\Theta_5 & a_5 c\Theta_5 \\ s\Theta_5 & 0 & -c\Theta_5 & a_5 s\Theta_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad A_6^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From now on although I try to calculate everything manually it will be critical because of the complexity of the system and on the other hand for having trustable evaluation tool I wrote all af these fromula as an mfile in MATLAB and will see the result by comparing to it.

```
% DENAVIT HARTENBERG Parameteres %
clear all
close all
clc
prompt = {'Enter how many robot arms?'};
dlg_title = 'Input';
num_lines = 1;
def = {'1'};
answer = inputdlg(prompt,dlg_title,num_lines,def);
num = str2num(answer{:});
F = sym('A', [num 4]);
B=eye(4);
C = sym('C', [4 4]);
clc
for i=1:num
    prompt = {'Enter a:','Enter alfa:','Enter d:','Enter theta:'};
    dlg_title = sprintf('arm%d',i);
    num_lines = 1;
    def1 =
{sprintf('a%d',i),sprintf('alfa%d',i),sprintf('d%d',i),sprintf('t%d',i)};
    answer1 = inputdlg(prompt,dlg_title,num_lines,def1);
    F(i,1)=answer1(1,1);
    F(i,2)=answer1(2,1);
    F(i,3)=answer1(3,1);
    F(i,4)=answer1(4,1);
    C=simplify([cos(F(i,4)) -sin(F(i,4))*cos(F(i,2)) sin(F(i,4))*sin(F(i,2))
F(i,1)*cos(F(i,4));
             sin(F(i,4)) cos(F(i,4))*cos(F(i,2)) -cos(F(i,4))*sin(F(i,2))
F(i,1)*sin(F(i,4));
             0 sin(F(i,2)) cos(F(i,2)) F(i,3);
             0 0 0 1]);
    eval(sprintf('A%d = C;',i));
    B=B*C;
    eval(sprintf('A%d',i))
end
sprintf('T from Arm 0 to Arm %d is:',i)
pretty(simplify(B))
'R is Rotation Matrix'  , R=B(1:3,1:3);
pretty(R)
d=B(1:3,4);
'd is translation Matrix' , pretty(d)
```

For using this program I use an interface that I can use the data of the D-H table information and insert all the data from the table for each link from the table as shown below:

```
A1 =

[cos(t1), -sin(t1), 0, a1*cos(t1)]
[sin(t1),  cos(t1), 0, a1*sin(t1)]
[      0,        0, 1,          0]
[      0,        0, 0,          1]

A2 =

[cos(t2), -cos(90)*sin(t2),  sin(90)*sin(t2), a2*cos(t2)]
[sin(t2),  cos(90)*cos(t2), -sin(90)*cos(t2), a2*sin(t2)]
[      0,          sin(90),          cos(90),          0]
[      0,                0,                0,          1]

A3 =

[cos(t3), -cos(90)*sin(t3),  sin(90)*sin(t3), a3*cos(t3)]
[sin(t3),  cos(90)*cos(t3), -sin(90)*cos(t3), a3*sin(t3)]
[      0,          sin(90),          cos(90),          0]
[      0,                0,                0,          1]


A4 =

[cos(t4), -cos(90)*sin(t4),  sin(90)*sin(t4), a4*cos(t4)]
[sin(t4),  cos(90)*cos(t4), -sin(90)*cos(t4), a4*sin(t4)]
[      0,          sin(90),          cos(90),          0]
[      0,                0,                0,          1]


A5 =

[cos(t5), -cos(90)*sin(t5),  sin(90)*sin(t5), a5*cos(t5)]
```

```
[sin(t5),  cos(90)*cos(t5), -sin(90)*cos(t5), a5*sin(t5)]
[      0,          sin(90),          cos(90),          0]
[      0,                0,                0,          1]


A6 =

[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d6]
[0, 0, 0,  1]


ans =

    'T from Arm 0 to Arm 6 is:'

-- --
| |  sin(t5) #1 - cos(t5) #4, sin(90) #6 + cos(90) sin(t5) #4 + cos(90) cos(t5) #1,
-- --

  cos(90) #6 - sin(90) sin(t5) #4 - sin(90) cos(t5) #1, a1 cos(t1) + a3 cos(t1 + t2) cos(t3) + a2 cos(t1)
cos(t2) - a2

                                                          2                       a4 #19 cos(t4) sin(t3)
   sin(t1) sin(t2) + a4 cos(t1 + t2) cos(t3) cos(t4) + a4 sin(90)  sin(t1 + t2) sin(t4) - ---------------------
-
                                                                                        2

     a4 #20 cos(t4) sin(t3)             3                                                    3
   - ---------------------- + d6 cos(90)  sin(90) sin(t1 + t2) - a3 cos(90) sin(t1 + t2) sin(t3) - d6 sin(90)
                2

                                     2                                  2
   cos(t1 + t2) cos(t5) sin(t3) + a5 sin(90)  cos(t1 + t2) sin(t3) sin(t5) + a5 sin(90)
```

```
                            a5 #19 cos(t4) cos(t5) sin(t3)   a5 #20 cos(t4) cos(t5) sin(t3)
 sin(t1 + t2) cos(t5) sin(t4) - ------------------------------ - ------------------------------
                                          2                              2


          3                            2                            3
 + d6 sin(90)  sin(t1 + t2) sin(t4) sin(t5) + d6 cos(90)  sin(90) cos(t1 + t2) sin(t3) + d6 cos(90)


                                            3                              3
 sin(90) sin(t1 + t2) cos(t3) - d6 cos(90) sin(90)  sin(t1 + t2) cos(t4) - d6 cos(90) sin(90)  sin(t1 +
t2) cos(t5) + a5


            2
 cos(90) sin(90)  sin(t1 + t2) sin(t5) - a4 cos(90) cos(t1 + t2) sin(t3) sin(t4) + a5


                                            2
 cos(t1 + t2) cos(t3) cos(t4) cos(t5) - a4 cos(90)  sin(t1 + t2) cos(t3) sin(t4) + d6


                                          d6 cos(90) sin(90) #19 sin(t3) sin(t4)
 cos(90) sin(90) cos(t1 + t2) cos(t3) sin(t4) - --------------------------------------
                                                        2


  d6 cos(90) sin(90) #20 sin(t3) sin(t4)          2                                     3
 - -------------------------------------- + d6 cos(90)  sin(90) cos(t1 + t2) cos(t4) sin(t3) + d6 cos(90)
                    2


                                                 3                                   3
 sin(90) sin(t1 + t2) cos(t3) cos(t4) - d6 cos(90) sin(90)  sin(t1 + t2) cos(t3) cos(t5) - d6 cos(90)
sin(90)


                                            2                                  2
 sin(t1 + t2) cos(t4) cos(t5) + a5 cos(90) sin(90)  sin(t1 + t2) cos(t3) sin(t5) + a5 cos(90) sin(90)


 sin(t1 + t2) cos(t4) sin(t5) - a5 cos(90) cos(t1 + t2) cos(t3) sin(t4) sin(t5) - a5


 cos(90) cos(t1 + t2) cos(t5) sin(t3) sin(t4) + d6 sin(90) cos(t1
```

```
                          a5 cos(90) #19 sin(t3) sin(t4) sin(t5)   a5 cos(90) #20 sin(t3) sin(t4) sin(t5)
   + t2) cos(t3) cos(t4) sin(t5) + -------------------------------------- + --------------------------------------
                                              2                                      2

     d6 sin(90) #19 cos(t4) sin(t3) sin(t5)   d6 sin(90) #20 cos(t4) sin(t3) sin(t5)
   - -------------------------------------- - --------------------------------------
                      2                                      2

             2                                       2                                        3
   - a5 cos(90)  cos(t1 + t2) cos(t4) sin(t3) sin(t5) - a5 cos(90)  sin(t1 + t2) cos(t3) cos(t5) sin(t4) - a5
cos(90)

                                           2                                          3
   sin(t1 + t2) cos(t3) cos(t4) sin(t5) + d6 cos(90)  sin(90) cos(t1 + t2) cos(t4) cos(t5) sin(t3) + d6
cos(90)

                                                  2
   sin(90) sin(t1 + t2) cos(t3) cos(t4) cos(t5) - d6 cos(90)  sin(90) sin(t1 + t2) cos(t3) sin(t4) sin(t5) +
d6

   cos(90) sin(90) cos(t1 + t2) cos(t3) cos(t5) sin(t4) - d6 cos(90) sin(90) cos(t1

                          d6 cos(90) sin(90) #19 cos(t5) sin(t3) sin(t4)
   + t2) sin(t3) sin(t4) sin(t5) - ----------------------------------------------
                                                2

     d6 cos(90) sin(90) #20 cos(t5) sin(t3) sin(t4) --
   - ----------------------------------------------  |,
                         2                    --

  --
  |  cos(t5) #5 - sin(t5) #2, sin(90) #7 - cos(90) sin(t5) #5 - cos(90) cos(t5) #2,
  --

  cos(90) #7 + sin(90) cos(t5) #2 + sin(90) sin(t5) #5, a1 sin(t1) + a3 sin(t1 + t2) cos(t3) + a2 cos(t1)
sin(t2) + a2
```

```
                       2                                      a4 #21 cos(t4) sin(t3)
  cos(t2) sin(t1) - a4 sin(90)  cos(t1 + t2) sin(t4) + a4 sin(t1 + t2) cos(t3) cos(t4) + ----------------------
-
                                                                      2

     a4 #22 cos(t4) sin(t3)            3
  + ---------------------- - d6 cos(90)  sin(90) cos(t1 + t2) + a3 cos(90) cos(t1 + t2) sin(t3) + a5
             2

                                a5 #21 cos(t4) cos(t5) sin(t3)   a5 #22 cos(t4) cos(t5) sin(t3)
  sin(t1 + t2) cos(t3) cos(t4) cos(t5) + ------------------------------ + ------------------------------
                                             2                                2

            3                              3                               2
  - d6 sin(90)  sin(t1 + t2) cos(t5) sin(t3) - d6 sin(90)  cos(t1 + t2) sin(t4) sin(t5) + a5 sin(90)

                                 3                                   3
  sin(t1 + t2) sin(t3) sin(t5) - d6 cos(90)  sin(90) cos(t1 + t2) cos(t3) + d6 cos(90) sin(90)  cos(t1 +
t2) cos(t4) + d6

              3                              2                        2
  cos(90) sin(90)  cos(t1 + t2) cos(t5) - a5 cos(90) sin(90)  cos(t1 + t2) sin(t5) + d6 cos(90)

                                                              2
  sin(90) sin(t1 + t2) sin(t3) - a4 cos(90) sin(t1 + t2) sin(t3) sin(t4) + a4 cos(90)  cos(t1 + t2) cos(t3)
sin(t4) - a5

       2
  sin(90)  cos(t1 + t2) cos(t5) sin(t4) + d6 cos(90) sin(90) sin(t1

                   d6 cos(90) sin(90) #21 sin(t3) sin(t4)   d6 cos(90) sin(90) #22 sin(t3) sin(t4)
  + t2) cos(t3) sin(t4) + -------------------------------------- + --------------------------------------
                                      2                              2

            3                                  3                                 3
```

```
   - d6 cos(90)  sin(90) cos(t1 + t2) cos(t3) cos(t4) + d6 cos(90) sin(90)  cos(t1 + t2) cos(t3) cos(t5)
 + d6 cos(90) sin(90)

                                            2                                        2
   cos(t1 + t2) cos(t4) cos(t5) - a5 cos(90) sin(90)  cos(t1 + t2) cos(t3) sin(t5) - a5 cos(90) sin(90)

                                      2
   cos(t1 + t2) cos(t4) sin(t5) + d6 cos(90)  sin(90) sin(t1 + t2) cos(t4) sin(t3) - a5

   cos(90) sin(t1 + t2) cos(t3) sin(t4) sin(t5) - a5 cos(90) sin(t1

                              a5 cos(90) #21 sin(t3) sin(t4) sin(t5)   a5 cos(90) #22 sin(t3) sin(t4) sin(t5)
   + t2) cos(t5) sin(t3) sin(t4) - -------------------------------------- - --------------------------------------
                                              2                                      2

                                            d6 sin(90) #21 cos(t4) sin(t3) sin(t5)
   + d6 sin(90) sin(t1 + t2) cos(t3) cos(t4) sin(t5) + --------------------------------------
                                                              2

     d6 sin(90) #22 cos(t4) sin(t3) sin(t5)             2                                        3
   + -------------------------------------- + a5 cos(90)  cos(t1 + t2) cos(t3) cos(t5) sin(t4) + a5 cos(90)
                      2

                                            2
   cos(t1 + t2) cos(t3) cos(t4) sin(t5) - a5 cos(90)  sin(t1 + t2) cos(t4) sin(t3) sin(t5) - d6

                                                        3
   cos(90) sin(90) sin(t1 + t2) sin(t3) sin(t4) sin(t5) - d6 cos(90)  sin(90) cos(t1 + t2) cos(t3) cos(t4)
 cos(t5) + d6

          2                                              2
   cos(90)  sin(90) cos(t1 + t2) cos(t3) sin(t4) sin(t5) + d6 cos(90)  sin(90) sin(t1 + t2) cos(t4) cos(t5)
 sin(t3) + d6

                                                  d6 cos(90) sin(90) #21 cos(t5) sin(t3) sin(t4)
   cos(90) sin(90) sin(t1 + t2) cos(t3) cos(t5) sin(t4) + ----------------------------------------------
```

```
                                                  2

   d6 cos(90) sin(90) #22 cos(t5) sin(t3) sin(t4) --
 + ---------------------------------------------  |,
                        2                      --

 [cos(t5) #9 - sin(t5) #8, - sin(90) #10 - cos(90) sin(t5) #9 - cos(90) cos(t5) #8,

 #3, d6 #3 + a3 sin(90) sin(t3) + a4 sin(t4) #17 + a5 cos(t5) #9 - a5 sin(t5) #8 + a4 sin(90) cos(t4)
sin(t3)],

          --
 [0, 0, 0, 1]  |
          --

where

          2
  #1 == sin(90)  #13 + cos(90) sin(t4) #12 - cos(90) cos(t4) #11

          2
  #2 == sin(90)  #16 + cos(90) sin(t4) #15 + cos(90) cos(t4) #14

  #3 == sin(90) sin(t5) #9 - cos(90) #10 + sin(90) cos(t5) #8

  #4 == cos(t4) #12 + sin(t4) #11

  #5 == cos(t4) #15 - sin(t4) #14

  #6 == sin(90) cos(t4) #11 + cos(90) sin(90) #13 - sin(90) sin(t4) #12

  #7 == sin(90) sin(t4) #15 - cos(90) sin(90) #16 + sin(90) cos(t4) #14

  #8 == sin(90) #18 - cos(90) cos(t4) #17 + cos(90) sin(90) sin(t3) sin(t4)

  #9 == sin(t4) #17 + sin(90) cos(t4) sin(t3)
```

$$\#10 == \cos(90)\ \#18 - \sin^2(90)\ \sin(t3)\ \sin(t4) + \sin(90)\ \cos(t4)\ \#17$$

$$\#11 == \cos(90)\ \cos(t1 + t2)\ \sin(t3) - \sin^2(90)\ \sin(t1 + t2) + \cos^2(90)\ \sin(t1 + t2)\ \cos(t3)$$

$$\#12 == \frac{\#19\ \sin(t3)}{2} + \frac{\#20\ \sin(t3)}{2} - \cos(t1 + t2)\ \cos(t3)$$

$$\#13 == \cos(90)\ \sin(t1 + t2) + \cos(t1 + t2)\ \sin(t3) + \cos(90)\ \sin(t1 + t2)\ \cos(t3)$$

$$\#14 == \sin^2(90)\ \cos(t1 + t2) + \cos(90)\ \sin(t1 + t2)\ \sin(t3) - \cos^2(90)\ \cos(t1 + t2)\ \cos(t3)$$

$$\#15 == \sin(t1 + t2)\ \cos(t3) + \frac{\#21\ \sin(t3)}{2} + \frac{\#22\ \sin(t3)}{2}$$

$$\#16 == \cos(90)\ \cos(t1 + t2) - \sin(t1 + t2)\ \sin(t3) + \cos(90)\ \cos(t1 + t2)\ \cos(t3)$$

$$\#17 == \cos(90)\ \sin(90) + \cos(90)\ \sin(90)\ \cos(t3)$$

$$\#18 == \sin^2(90)\ \cos(t3) - \cos^2(90)$$

$$\#19 == \sin(t1 + t2 - 90)$$

$$\#20 == \sin(t1 + t2 + 90)$$

$$\#21 == \cos(t1 + t2 - 90)$$

$$\#22 == \cos(t1 + t2 + 90)$$

ans =

'R is Rotation Matrix'

```
[[sin(t5) #1 - cos(t5) #4, sin(90) #6 + cos(90) sin(t5) #4 + cos(90) cos(t5) #1,

  cos(90) #6 - sin(90) sin(t5) #4 - sin(90) cos(t5) #1],

  [cos(t5) #3 - sin(t5) #2, sin(90) #5 - cos(90) sin(t5) #3 - cos(90) cos(t5) #2,

  cos(90) #5 + sin(90) sin(t5) #3 + sin(90) cos(t5) #2],

  [cos(t5) #8 - sin(t5) #7, - sin(90) #9 - cos(90) sin(t5) #8 - cos(90) cos(t5) #7,

  sin(90) sin(t5) #8 - cos(90) #9 + sin(90) cos(t5) #7]]
```

where

```
  #1 == sin(90) #15 + cos(90) sin(t4) #14 - cos(90) cos(t4) #13

  #2 == sin(90) #12 + cos(90) sin(t4) #11 + cos(90) cos(t4) #10

  #3 == cos(t4) #11 - sin(t4) #10

  #4 == cos(t4) #14 + sin(t4) #13

  #5 == sin(90) sin(t4) #11 - cos(90) #12 + sin(90) cos(t4) #10

  #6 == cos(90) #15 - sin(90) sin(t4) #14 + sin(90) cos(t4) #13

  #7 == sin(90) #17 - cos(90) cos(t4) #16 + cos(90) sin(90) sin(t3) sin(t4)

  #8 == sin(t4) #16 + sin(90) cos(t4) sin(t3)
```

```
  #9 == cos(90) #17 - sin(90)  sin(t3) sin(t4) + sin(90) cos(t4) #16

  #10 == sin(90) #20 + cos(90) sin(t3) #19 - cos(90) cos(t3) #18

  #11 == sin(t3) #18 + cos(t3) #19

  #12 == cos(90) #20 + sin(90) cos(t3) #18 - sin(90) sin(t3) #19

  #13 == cos(90) sin(t3) #22 - sin(90) #23 + cos(90) cos(t3) #21

  #14 == sin(t3) #21 - cos(t3) #22

  #15 == cos(90) #23 + sin(90) cos(t3) #21 + sin(90) sin(t3) #22

  #16 == cos(90) sin(90) + cos(90) sin(90) cos(t3)

                    2                  2
  #17 == sin(90)  cos(t3) - cos(90)

  #18 == cos(90) cos(t1) cos(t2) - cos(90) sin(t1) sin(t2)

  #19 == cos(t1) sin(t2) + cos(t2) sin(t1)

  #20 == sin(90) cos(t1) cos(t2) - sin(90) sin(t1) sin(t2)

  #21 == cos(90) cos(t1) sin(t2) + cos(90) cos(t2) sin(t1)

  #22 == cos(t1) cos(t2) - sin(t1) sin(t2)

  #23 == sin(90) cos(t1) sin(t2) + sin(90) cos(t2) sin(t1)


ans =

    'd is translation Matrix'
```

[[a1 cos(t1) - d6 (sin(90) sin(t5) #6 - cos(90) (cos(90) #7 - sin(90) sin(t4) #14 + sin(90) cos(t4) #13) + sin(90) cos(t5) #1)

- a3 sin(t3) #19 + a3 cos(t3) #20 - a5 cos(t5) #6 + a2 cos(t1) cos(t2) - a4 cos(t4) #14 - a2 sin(t1) sin(t2) - a4

sin(t4) #13 + a5 sin(t5) #1],

[a1 sin(t1) + d6 (cos(90) (sin(90) sin(t4) #12 - cos(90) #8 + sin(90) cos(t4) #11) + sin(90) sin(t5) #5 + sin(90) cos(t5) #2)

+ a3 sin(t3) #16 + a3 cos(t3) #17 + a5 cos(t5) #5 + a4 cos(t4) #12 + a2 cos(t1) sin(t2) + a2 cos(t2) sin(t1) - a4

sin(t4) #11 - a5 sin(t5) #2],

2

[d6 (sin(90) sin(t5) #4 - cos(90) (cos(90) #9 - sin(90) sin(t3) sin(t4) + sin(90) cos(t4) #10) + sin(90) cos(t5) #3)

+ a3 sin(90) sin(t3) + a4 sin(t4) #10 + a5 cos(t5) #4 - a5 sin(t5) #3 + a4 sin(90) cos(t4) sin(t3)]]

where

#1 == sin(90) #7 + cos(90) sin(t4) #14 - cos(90) cos(t4) #13

#2 == sin(90) #8 + cos(90) sin(t4) #12 + cos(90) cos(t4) #11

#3 == sin(90) #9 - cos(90) cos(t4) #10 + cos(90) sin(90) sin(t3) sin(t4)

#4 == sin(t4) #10 + sin(90) cos(t4) sin(t3)

#5 == cos(t4) #12 - sin(t4) #11

#6 == cos(t4) #14 + sin(t4) #13

#7 == cos(90) #18 + sin(90) cos(t3) #19 + sin(90) sin(t3) #20

#8 == cos(90) #15 + sin(90) cos(t3) #16 - sin(90) sin(t3) #17

#9 == $\sin(90)^2 \cos(t3) - \cos(90)^2$

#10 == cos(90) sin(90) + cos(90) sin(90) cos(t3)

#11 == sin(90) #15 + cos(90) sin(t3) #17 - cos(90) cos(t3) #16

#12 == sin(t3) #16 + cos(t3) #17

#13 == cos(90) sin(t3) #20 - sin(90) #18 + cos(90) cos(t3) #19

#14 == sin(t3) #19 - cos(t3) #20

#15 == sin(90) cos(t1) cos(t2) - sin(90) sin(t1) sin(t2)

#16 == cos(90) cos(t1) cos(t2) - cos(90) sin(t1) sin(t2)

#17 == cos(t1) sin(t2) + cos(t2) sin(t1)

#18 == sin(90) cos(t1) sin(t2) + sin(90) cos(t2) sin(t1)

#19 == cos(90) cos(t1) sin(t2) + cos(90) cos(t2) sin(t1)

#20 == cos(t1) cos(t2) - sin(t1) sin(t2)

Z'Y'X' Euler Rotation matrix is:

$$R_{z'y'x'} = \begin{bmatrix} c_\theta\, c_\psi & c_\psi s_\theta\, s_\phi - c_\phi s_\psi & s_\phi s_\psi + c_\phi c_\psi s_\theta \\ c_\theta\, s_\psi & c_\phi c_\psi + s_\theta s_\phi s_\psi & c_\phi s_{\theta s_\psi} - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}$$

And due to prismatic joints at the origin of the fixed frame.

The homogeneous transformation matrix of body frame(origin is at the center) to fixed frame will be:

$$T_B^0 = \begin{bmatrix} c_\theta\, c_\psi & c_\psi s_\theta\, s_\phi \; - c_\phi s_\psi & s_\phi s_\psi \; + \; c_\phi c_\psi s_\theta & d1 \\ c_\theta\, s_\psi & c_\phi c_\psi \; + \; s_\theta s_\phi s_\psi & c_\phi s_{\theta s_\psi} - c_\psi s_\phi & d2 \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

However, this does not give us the pozition and orientation of the end effectors, this transformation gives us the body frame which has a origin at the center of the fixed frame. For the end effectors, we need to translate the frame in the ± x and ± y with respect to the body frame.

So by post-multiplying the T with Trans $_{\pm Xb}$ and Trans $_{\pm Yb}$, we can obtain the resulting end-effector frames.

$$\mathrm{Trans}_{\pm \mathrm{XB}} = \begin{bmatrix} 1 & 0 & 0 & \pm \mathrm{L_q} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{\pm \mathrm{YB}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \pm \mathrm{L_q} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_B \; and \; Y_B \; are \; the \; axes \; with \; respect \; to \; the \; body \; frame.$$

$$T_{M_{1l}}^0 = T_B^0 * \mathrm{Trans}_{+\mathrm{XB}}$$

$$T_{M_{1u}}^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & +L_b \\ 0 & 0 & 0 & 1 \end{bmatrix} * T_B^0 * \mathrm{Trans}_{+\mathrm{XB}}$$

As stated above, since this is 6-DOF device its joint matrix will be :

$$\boldsymbol{q} = (X, \Theta)^T where\ X = (x, y, z)^T\ wrt\ fixed\ frame, \Theta = (\psi, \theta, \phi)^T$$

ψ is the yaw angle around the z-axis, θ is the pitch angle around the y-axis and φ is the roll angle around the x-axis.

Since controlling the seahorses is easier than directly trying to control the end effectors, I will generate the dynamics for the system. By doing so, only the potential energy term will differ from the dynamics of a single seahorse.

The Lagrangian in this system consists of 3 parameters:

$$L(\mathbf{q}, \boldsymbol{q}') = K_{\text{trans}} + K_{\text{rot}} - U$$

$$K_{\text{trans}} = \frac{\text{m}}{2} * X'^T * X'$$

$$K_{rot} = \frac{1}{2} * \Theta'^T * I * \Theta'$$

$$U = mgz + U_{spring1} + U_{spring2} + U_{spring3} + U_{spring4}$$

where m is the mass of the body and I is the inertia matrix.

For the rotational kinetic energy:

$$\Theta = W(q) * q'\ where\ W\ is\ the\ rotational\ part\ of\ the\ jacobian.$$

This equation is valid since rotation on a body is independent from the X.

$$W(q) = \begin{bmatrix} -s_\theta & 0 & 1 \\ c_\theta s_\phi & c_\phi & 0 \\ c_\theta c_\phi & s_\phi & 0 \end{bmatrix}$$

Which results in

$$\Theta = \begin{bmatrix} \phi' - \psi' s_\theta \\ \theta' c_\phi + \psi' c_\theta s_\phi \\ \psi' c_\theta c_\phi - \theta' s_\phi \end{bmatrix}$$

And since the body is modeled as thin rods:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Substituting the relation above in the rotational kinetic energy formulation:

$$K_{rot} = \frac{1}{2} * q'^T * W^T * I * W * q'$$

Lagrange equation with respect to q:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q'}\right) - \left(\frac{\partial L}{\partial q}\right) = \begin{pmatrix} F_X \\ \tau \end{pmatrix}$$

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