Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Communication Systems ENEE 3309

Course Project

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Section:3

1. Find $a_0, a_1, a_2, a_3, b_1, b_2, and\ b_3$ (you can use matlab or any other code to find numerical values of the coefficients)

```
syms n t
w0=2*pi/0.1;
T0=0.1;
n=1:3;
clc
a0=(1/T0)*(int(1,t,0,0.05)+0.5*0.05)
```

a0 =

$$\frac{3}{4}$$

```
an=(2/T0)*(int(cos(n*w0*t),t,0,0.05)+int(2*(1-t/T0)*cos(n*w0*t),t,0.05,0.1))
```

an =

$$\begin{pmatrix} -\frac{2}{\pi^2} & 0 & -\frac{2}{9\pi^2} \end{pmatrix}$$

```
bn=(2/T0)*(int(sin(n*w0*t),t,0,0.05)+int(2*(1-t/T0)*sin(n*w0*t),t,0.05,0.1))
```

bn =

$$\begin{pmatrix} \frac{1}{\pi} & \frac{1}{2\pi} & \frac{1}{3\pi} \end{pmatrix}$$

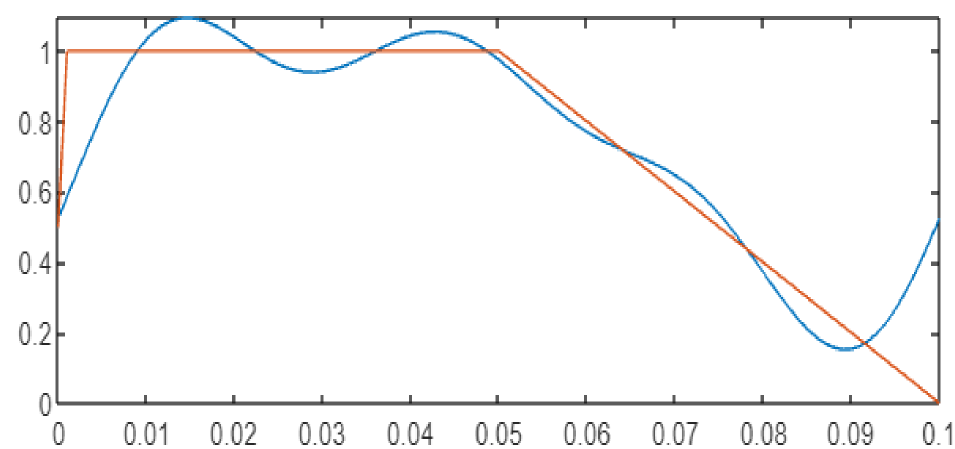
2. **Use matlab to plot $g(t)$ and $g_a(t)$ for K = 3, on the same figure for one cycle of $g(t)$.**

```

pt= 0:0.001:0.1;

g1 = heaviside(pt) - heaviside(pt-(T0/2));
y = trimf(pt,[0 (T0/2) T0]);
g2 = (heaviside(pt-(T0/2))-heaviside(pt-T0)).*y;
gt = g1 + g2;
gapt=a0;
for i =1:3
    gapt=gapt+an(i).*cos(i.*w0.*pt) + bn(i).*sin(i.*w0.*pt);
end

hold on
subplot(2,1,1);
axis([-2 0.1 0 3]);
plot(pt, gapt,pt,gt);
hold off;
```



3. **The mean square error between $g(t)$ and $g_a(t)$ is defined as**

$$MSE = \frac{1}{T_0}\left(\int_0^{T_0} (g(t) - g_a(t))^2 dt\right)$$

Find the mean square error for K=1, 2, and 3. Summarize your results in a table.

```
clc
ga1 = a0 +an(1)*cos(w0*t)+bn(1)*sin(w0*t)
ga2 = a0+an(1)*cos(w0*t)+bn(1)*sin(w0*t)+an(2)*cos(2*w0*t)+bn(2)*sin(2*w0*t)
ga3 = a0+an(1)*cos(w0*t)+bn(1)*sin(w0*t)+an(2)*cos(2*w0*t)+bn(2)*sin(2*w0*t)+an(3)*cos(3*w0*t)+ bn(3)*sin(3*w0*t)

MSE_1= (1/T0)*(int((1-ga1).^2,t,0,0.05)+int((2*(1-t/T0)-ga1).^2,t,0.05,0.1))
MSE_2= (1/T0)*(int((1-ga2).^2,t,0,0.05)+int((2*(1-t/T0)-ga2).^2,t,0.05,0.1))
MSE_3= (1/T0)*(int((1-ga3).^2,t,0,0.05)+int((2*(1-t/T0)-ga3).^2,t,0.05,0.1))
```

```
clc
ga1 = a0 +an(1)*cos(w0*t)+bn(1)*sin(w0*t)
```

ga1 =

$$\frac{\sin(20\pi t)}{\pi} - \frac{2\cos(20\pi t)}{\pi^2} + \frac{3}{4}$$

```
ga2 = a0+an(1)*cos(w0*t)+bn(1)*sin(w0*t)+an(2)*cos(2*w0*t)+bn(2)*sin(2*w0*t)
```

ga2 =

$$\frac{\sin(20\pi t)}{\pi} - \frac{2\cos(20\pi t)}{\pi^2} + \frac{\sin(40\pi t)}{2\pi} + \frac{3}{4}$$

```
ga3 = a0+an(1)*cos(w0*t)+bn(1)*sin(w0*t)+an(2)*cos(2*w0*t)+bn(2)*sin(2*w0*t)+an(3)*cos(3*w0*t)+ bn(3)*sin(3*w0*t)
```

ga3 =

$$\frac{\sin(20\pi t)}{\pi} - \frac{2\cos(60\pi t)}{9\pi^2} - \frac{2\cos(20\pi t)}{\pi^2} + \frac{\sin(40\pi t)}{2\pi} + \frac{\sin(60\pi t)}{3\pi} + \frac{3}{4}$$

MSE for k =1 & 2 & 3

```
MSE_1= (1/T0)*(int((1-ga1).^2,t,0,0.05)+int((2*(1-t/T0)-ga1).^2,t,0.05,0.1))
```

MSE_1 =

$$\frac{5}{48}-\frac{10\left(\frac{\pi^2}{40}+\frac{3}{10}\right)}{\pi^4}-\frac{10\left(\frac{\pi^2}{40}-\frac{1}{10}\right)}{\pi^4}$$

```
MSE_2= (1/T0)*(int((1-ga2).^2,t,0,0.05)+int((2*(1-t/T0)-ga2).^2,t,0.05,0.1))
```

MSE_2 =

$$\frac{5}{48}-\frac{10\left(\frac{3\pi^2}{160}+\frac{1}{30}\right)}{\pi^4}-\frac{10\left(\frac{7\pi^2}{160}+\frac{1}{6}\right)}{\pi^4}$$

```
MSE_3= (1/T0)*(int((1-ga3).^2,t,0,0.05)+int((2*(1-t/T0)-ga3).^2,t,0.05,0.1))
```

MSE_3 =

$$\frac{5}{48}-\frac{10\left(\frac{31\pi^2}{1440}+\frac{47}{2025}\right)}{\pi^4}-\frac{10\left(\frac{67\pi^2}{1440}+\frac{121}{675}\right)}{\pi^4}$$

$$MSE\text{-}1 = \frac{5}{48} - \frac{10\left(\frac{\pi^2}{40} + \frac{3}{10}\right)}{\pi^4} - \frac{10\left(\frac{\pi^2}{40} - \frac{1}{10}\right)}{\pi^4}$$

$$= \frac{5}{48} - \frac{5{,}46}{97{,}2} - \frac{1{,}46}{97{,}2}$$

$$= 0{,}104 - 0{,}056 - 0{,}015$$

$$= 0{,}033$$

$$MSE\text{-}2 = \frac{5}{48} - \frac{10\left(\frac{3\pi^2}{160} + \frac{1}{30}\right)}{\pi^4} - \frac{10\left(\frac{7\pi^2}{160} + \frac{1}{6}\right)}{\pi^4}$$

$$= 0{,}104 - \frac{2{,}18}{97{,}2} - \frac{5{,}98}{97{,}2}$$

$$= 0{,}104 - 0{,}0224 - 0{,}0615$$

$$= 0{,}0201$$

$$MSE\text{-}3 = \frac{5}{48} - \frac{10\left(\frac{31\pi^2}{1440} + \frac{47}{2025}\right)}{\pi^4} - \frac{10\left(\frac{67\pi^2}{1440} + \frac{121}{675}\right)}{\pi^4}$$

$$= \frac{5}{48} - \frac{2{,}35}{97{,}2} - \frac{6{,}38}{97{,}2}$$

$$= 0{,}104 - 0{,}024 - 0{,}0656$$

$$= 0{,}0144$$

Q3

$$MSE = \frac{1}{T_0}\left(\int_0^{T_0} (g(t) - g_a(t))^2 \, dt\right)$$

$$g_a(t) = a_0 + \sum_{n=1}^{K} (a_n \cos n w_0 t + b_n \sin n w_0 t)$$

$$g_{a1} = a_0 + a(1) * \cos(w_0 t) + b(1) \sin w_0 t$$

$$g_{a2} = a_0 + \sum_{n=1}^{2} (a_n \cos(n w_0 t) + b_n \sin(n w_0 t))$$

$$g_{a2} = a_0 + a_n(1) \cos(w_0 t) + b_n(1) \sin(w_0 t) + a_n(2) \cos(2 w_0 t) + b_n(2) \sin(2 w_0 t)$$

$$g_{a3} = a_0 + \sum_{n=1}^{3} (a_n \cos(n w_0 t) + b_n \sin(n w_0 t))$$

$$g_{a3} = a_0 + a_n(1) \cos(w_0 t) + b_n(1) \sin(w_0 t) + a_n(2) \cos(2 w_0 t) + b_n(2) \sin(2 w_0 t) + a_n(3) \cos(3 w_0 t) + b_n(3) \sin(3 w_0 t)$$

$$MSE(1) = \frac{1}{T_0}\left(\int_0^{\frac{T_0}{2}} (1 - g_{a1})^2 + \int_{\frac{T_0}{2}}^{T_0} (2 * ((1-t)/T_0)) - g_{a1})^2\right)$$

$$MSE(2) = \frac{1}{T_0} \int_0^{\frac{T_0}{2}} (1 - g_{a2})^2 + \int_{\frac{T_0}{2}}^{T_0} (2 * ((1-t)/T_0) - g_{a2})^2$$

$$MSE(3) = \frac{1}{T_0} \int_0^{\frac{T_0}{2}} (1 - g_{a3})^2 + \int_{\frac{T_0}{2}}^{T_0} (2 * ((1-t)/T_0) - g_{a3})^2$$

Table:

K	MSE $\rightarrow$ final resalt from matlab
1	0,033
2	0,0201
3	0,0144

4. If $g_a(t)$ (when K = 3) is multiplied by the carrier $c(t) = 10\cos 2\pi(200)t$ followed by an ideal bandpass filter to generate the single sideband signal $s(t)$, find s(t) and its spectrum.

Q4

$$m(t) \longrightarrow \otimes \xrightarrow{V(t)} \boxed{B.P.F} \longrightarrow S(t)$$

$$\uparrow c(t)$$

$c(t) = 10\cos 2\pi(200)t \rightarrow f_c = 200\,Hz$

$m(t) = g_a(t)$ (when $K=3$)

$$m(t) = a_0 + \sum_{n=1}^{3} \left(a_n \cos(n w_0 t) + b_n \sin(n w_0 t)\right)$$

$$m(t) = a_0 + a_n(1)\cos(w_0 t) + b_n(1)\sin(w_0 t) + a_n(2)\cos(2w_0 t) + b_n(2)\sin(2w_0 t) + a_n(3)\cos(3w_0 t) + b_n(3)\sin(3w_0 t)$$

$V(t) = m(t) * c(t)$

$$S(t) = 10\cos(2\pi(200)t)\left[a_0 + a_n(1)\cos(w_0 t) + b_n(1)\sin(w_0 t) + a_n(2)\cos(2w_0 t) + b_n(2)\sin(2W_0 t) + a_n(3)\cos(3W_0 t) + b_n(3)\sin(3w_0 t)\right]$$

$$V(t) = a_0 10\cos(2\pi(200)t) + a_n(1)\cos(w_0 t) * 10\cos(2\pi(200)t) + b_n(1)\sin(w_0 t) * 10\cos(2\pi(200)t) + 10\,a_n(2)\cos(2\pi(200)t) * \cos(2W_0 t) + 10\,b_n(2)\cos(2\pi(200)t)\sin(2w_0 t) + 10\,a_n(3)\cos(3w_0 t)\cos(2\pi(200)t) + 10\,b_n(3)\cos(2\pi(200)t)\sin(3W_0 t)$$

$$V(t) = \frac{30}{4}\cos(2\pi(200)t) + \frac{-20}{\pi^2}\cos(2\pi(10)t)\cos(2\pi(200)t) + \frac{10}{\pi}\sin(2\pi(10)t)\cos(2\pi(200)t) + \frac{10}{2\pi}\cos(2\pi(200)t)\sin(2\pi(20)t) + \frac{-20}{9\pi^2}\cos(2\pi(30)t)\cos(2\pi(200)t) + \frac{10}{3\pi}\cos(2\pi(200)t)\sin(2\pi(30)t)$$

The spectrum:

after the B.P.F

$$S(t) = \frac{-10}{\pi^2}\cos(2\pi(210)t) + \frac{-10}{2\pi j}\cos(2\pi(210)t) + \frac{-10}{4\pi j}\cos(2\pi(220)t)$$

$$+ \frac{-10}{9\pi^2}\cos(2\pi(230)t) + \frac{-10}{6\pi j}\cos(2\pi(230)t)$$

BW=30Hz