

$$\int \frac{2}{x^2-1} dx$$

$$\text{let } x = \sec \theta$$

$$dx = \sec \theta \tan \theta d\theta$$

$$\int \frac{2}{\sec^2 \theta - 1} \sec \theta \tan \theta d\theta$$

$$\int \frac{2 \sec \theta \tan \theta}{\tan^2 \theta} d\theta$$

$$2 \int \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} d\theta = 2 \int \csc \theta d\theta$$

$$= 2 \int \csc \theta \cdot \frac{\csc \theta + \cot \theta}{\csc \theta + \cot \theta} d\theta$$

$$= -2 \int \frac{(\csc^2 \theta + \csc \theta \cot \theta)}{\csc \theta + \cot \theta} d\theta$$

$$\boxed{\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c}$$

$$= -2 \ln |\csc \theta + \cot \theta| + c$$

$$= -2 \ln \left| \frac{x}{\sqrt{x^2-1}} + \frac{1}{\sqrt{x^2-1}} \right| + c$$

$$= -2 \ln \left| \frac{x+1}{\sqrt{x^2-1}} \right| + c$$

$$= \ln \left( \frac{\sqrt{x^2-1}}{x+1} \right)^2 + c = \ln(x^2-1) - 2 \ln(x+1) + c$$

