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1. Given $f(z) = \log_e(1+z)$ where $z = x^T x$, $x \in \mathbb{R}^d$

⇒ if $x = \begin{bmatrix} x_1 \\ x_2 \\ x_d \end{bmatrix}$ then $x^T = [x_1, x_2 \cdots x_d]$

$x^T x = [x_1^2 + x_2^2 \cdots + x_d^2]$

Applying chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}(\log(1+z)) \cdot \frac{d}{dx}(x^T \cdot x)$$

$$= \frac{1}{1+z}(2x_1 + 2x_2 + \cdots + 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \cdots + x_d)$$

$$= \frac{2}{1+z} \sum_{i=0}^{d} x_i$$

Ans

2. $f(z) = e^{\frac{-z}{2}}$; where $z = g(z) \cdot g(y) = y^T \cdot S^{-1} y$, $y = h(x)$,

$h(x) = x - \mu$

$\Rightarrow$ Using Chain rule

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Here, $\frac{df}{dz} = \frac{d}{dz}\left(e^{\frac{-z}{2}}\right) = -\frac{e^{\frac{-z}{2}}}{2}$

$$\frac{dz}{dy} = \frac{d}{dy}(y^T S^{-1} y)$$

$$= \lim_{h \to 0} \frac{y(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h)\, y^{-1}(y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T \cdot S^{-1} y + y^T S^{-1} h + h S^{-1} y + h^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{h(S^T S^{-1} + S^{-1} y + h S^{-1})}{h}$$

$$= \lim_{h \to 0} (y^T S^{-1} + S^{-1} y + h S^{-1})$$

$$= y^T S^{-1} + S^{-1} y$$

$$\frac{dy}{dx} = \frac{d(x-l)}{dx} = 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{e^{-\frac{z}{2}}}{2} \left(y^T S^{-1} + S^{-1} y\right) \cdot 1$$

$$= -\frac{e^{-\frac{z}{2}}}{2} \cdot \frac{1}{S} \left(y^T + y\right)$$

Ans