

### Assignment

a) Construct 3 equations and express it in  $AX=B$  form.

⇒ We get equations from the assignment are

$$x+y+z = 1500 \text{ — (i)}$$

$$0.2x + 0.4y + 0.3z = 460 \text{ [multiplying with 10]}$$

$$\Rightarrow 2x + 4y + 3z = 4600 \text{ — (ii)}$$

$$0.4x + 0.2y + 0.5z = 540 \text{ [multiplying with 10]}$$

$$\Rightarrow 4x + 2y + 5z = 5400 \text{ — (iii)}$$

If we put it in the format  $AX=B$  matrix we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1500 \\ 4600 \\ 5400 \end{bmatrix}$$

Here

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1500 \\ 4600 \\ 5400 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

b) Let the matrix be  $A$  and check whether it is involutory or not.

$\Rightarrow$  From (a) we get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

If matrix  $A$  is involutory then  $A^2 = I$

Now we do  $A^2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+4 & 1+4+2 & 1+3+5 \\ 2+8+12 & 2+16+6 & 2+12+15 \\ 4+4+20 & 4+8+10 & 4+6+25 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix} \neq I$$

Therefore  $A$  is not an involutory matrix

c) Determine  $\text{Adj}(A)$

$\Rightarrow$  From (a) we get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

Now, we find the co-factor matrix of  $A$

$$A_{11} = 14 \quad A_{12} = 2 \quad A_{13} = -12$$

$$A_{21} = -3 \quad A_{22} = 1 \quad A_{23} = 2$$

$$A_{31} = -1 \quad A_{32} = -1 \quad A_{33} = 2$$

$$\text{Adj}(A) = \begin{bmatrix} 14 & 2 & -12 \\ -3 & 1 & 2 \\ -1 & -1 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix} \quad (\text{Ans})$$

d) If  $A^3 + 3A = 2I_3 + 11Y$ , then determine  $Y$

From (a) we get  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$

From (b) we get  $A^2 = \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix}$

Now,

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} 7+22+28 & 7+24+22 & 9+29+35 \\ 14+88+84 & 14+96+66 & 18+116+105 \\ 28+44+140 & 28+48+110 & 36+58+175 \end{bmatrix}$$

$$= \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix}$$

Now putting all values we get

$$A^3 + 3A = 2I_3 + 11Y$$

$$\Rightarrow \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 11Y$$

$$\Rightarrow \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 6 & 12 & 9 \\ 12 & 6 & 15 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 11Y$$

$$\Rightarrow \begin{bmatrix} 60 & 56 & 76 \\ 192 & 188 & 248 \\ 224 & 192 & 284 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 11Y$$

$$\Rightarrow \begin{bmatrix} 58 & 56 & 76 \\ 192 & 186 & 248 \\ 224 & 192 & 282 \end{bmatrix} = 11Y$$

$$\Rightarrow Y = \begin{bmatrix} 58/11 & 56/11 & 76/11 \\ 192/11 & 186/11 & 248/11 \\ 224/11 & 192/11 & 282/11 \end{bmatrix} \quad (\text{Ans})$$

e) Solving the equations find out the numbers of students of Humanities, Business Studies, and Science.

⇒ From (c) we get

$$\text{Adj}(A) = \begin{bmatrix} 14 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$\text{Det}(A) = 14 + 2 - 12 = 4$$

We know

$$A^{-1} = \frac{\text{Adj}(A)}{|A|} = \frac{1}{4} \begin{bmatrix} 14 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3.5 & -0.75 & -0.25 \\ 0.5 & 0.25 & -0.25 \\ -3 & 0.5 & 0.5 \end{bmatrix}$$

Now,

$$AX = B$$

$$\Rightarrow A^{-1}AX = A^{-1}B \quad [\text{multiplying both sides by } A^{-1}]$$

$$\Rightarrow X = A^{-1}B$$

$$= \begin{bmatrix} 3.5 & -0.75 & -0.25 \\ 0.5 & 0.25 & -0.25 \\ -3 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 1500 \\ 4600 \\ 5400 \end{bmatrix}$$

$$= \begin{bmatrix} 5250 - 3450 - 1350 \\ 750 + 1150 - 1350 \\ -4500 + 2300 + 2700 \end{bmatrix} = \begin{bmatrix} 450 \\ 550 \\ 500 \end{bmatrix}$$

Therefore,

Students in humanities = 450

Students in business studies = 550

Students in science = 500 (Ans)

Ans. to the q. no. 4

Given,

No. of students in humanities =  $x$

" " " " business studies =  $y$

" " " " science =  $z$

According to,

1st condition,  $x + y + z = 1500$  --- (i)

2nd " "  $x \cdot 20\% + y \cdot 40\% + z \cdot 30\% = 460$

or,  $x \cdot \frac{1}{5} + y \cdot \frac{2}{5} + z \cdot \frac{3}{10} = 460$

$\therefore 2x + 4y + 3z = 4600$  --- (ii)

3rd condition,  $x \cdot 40\% + y \cdot 20\% + z \cdot 50\% = 540$

or,  $x \cdot \frac{2}{5} + y \cdot \frac{1}{5} + z \cdot \frac{1}{2} = 540$

$\therefore 4x + 2y + 5z = 5400$  --- (iii)

Now,

$$x + y + z = 1500$$

$$2x + 4y + 3z = 4600$$

$$4x + 2y + 5z = 5400$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1500 \\ 4600 \\ 5400 \end{bmatrix}$$

$$\Rightarrow AX = B \quad \text{--- (1)}$$

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1500 \\ 4600 \\ 5400 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore AX = B$$

Ans. to the q-no-b

Involuntary Matrix: Any A matrix can be called involuntary matrix if  $A^2 = A \cdot A = I$ .

From 'a' we get.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+4 & 1+4+2 & 1+3+5 \\ 2+8+12 & 2+16+6 & 2+12+15 \\ 4+8+20 & 4+8+10 & 4+6+25 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix} \neq I_3$$

$\therefore A$  is not involuntary matrix.

Ans. to the q.no. 1

From 'a' we get,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

Now,

$$A_{11} = (-1)^{1+1} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = 20 - 6 = 14$$

$$A_{12} = (-1)^{1+2} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = -1(10 - 12) = 2$$

$$A_{13} = (-1)^{1+3} \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} = 4 - 16 = -12$$

$$A_{21} = (-1)^{2+1} \begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix} = -1(5 - 2) = -3$$

$$A_{22} = (-1)^{2+2} \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix} = 5 - 4 = 1$$

$$A_{23} = (-1)^{2+3} \begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} = -1(2 - 4) = 2$$

$$A_{31} = (-1)^{3+1} \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix} = 3 - 4 = -1$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1(3-2) = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4-2 = 2$$

$$\therefore \text{Adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -12 \\ -3 & 1 & 2 \\ -1 & -1 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix}$$

Ans. to the q-no-d

From 'a'

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

From 'b'

$$A^2 = \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix}$$

Now,

$$A^3 = A^2 \cdot A = \begin{bmatrix} 7 & 7 & 9 \\ 22 & 24 & 29 \\ 28 & 22 & 35 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7+14+36 & 7+28+18 & 7+21+45 \\ 22+48+116 & 22+96+58 & 22+72+145 \\ 28+44+140 & 28+88+70 & 28+66+175 \end{bmatrix}$$

$$= \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix}$$

$\therefore A^3$

Given,

$$A^3 + 3A - 2I_3 = 11Y$$

$$\text{or, } \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 2 \\ 4 & 2 & 5 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11Y$$

$$\text{or, } \begin{bmatrix} 57 & 53 & 73 \\ 186 & 176 & 239 \\ 212 & 186 & 269 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 6 & 12 & 6 \\ 12 & 6 & 15 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 11Y$$

$$\text{or, } \begin{bmatrix} 57+3-2 & 53+3-0 & 73+3-0 \\ 186+6-0 & 176+12-2 & 239+6-0 \\ 212+12-0 & 186+6-0 & 269+15-2 \end{bmatrix} = 11Y$$

$$\text{or, } \begin{bmatrix} 58 & 56 & 76 \\ 192 & 186 & 245 \\ 224 & 192 & 282 \end{bmatrix} = 11Y$$

$$\text{or, } Y = \frac{1}{11} \begin{bmatrix} 58 & 56 & 76 \\ 192 & 186 & 248 \\ 224 & 192 & 282 \end{bmatrix}$$

Ans: to the q.no.e

from 'a'  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}$

$\text{Adj}(A) = \begin{bmatrix} 14 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix}$

Now,

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 3 \\ 4 & 2 & 5 \end{vmatrix} = 1(20-6) - 1(10-12) + 1(4-16) \\ &= 14 + 2 - 12 \\ &= 4 \end{aligned}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{4} \begin{bmatrix} 14 & -3 & -1 \\ 2 & 1 & -1 \\ -12 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} & -\frac{3}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -3 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

We get.

$$AX = B$$

$$\therefore X = A^{-1}B$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{3}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -3 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1500 \\ 1600 \\ 5400 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5250 - 3450 - 1350 \\ 750 + 1150 - 1350 \\ -4500 + 2300 + 2700 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 450 \\ 550 \\ 500 \end{bmatrix}$$

$\therefore$  No. of students in humanities = 450

" " " " Business studies = 550

" " " " Science = 500