Random experiment :

A random experiment is such an experiment which can be repeated any number of times under some identical conditions. In any random experiment, the outcome of any particular trial should not be known beforehand. But all possible outcomes should be known in advance.

Sample space :

The set or collection of all possible outcomes of a random experiment is defined as the sample space of that random experiment. It is generally denoted by 'S'.

Event :

Any possible outcome or a set of possible outcomes of a random experiment is called an event.

Sure Event :

An event whose occurrance is a must in any random experiment is known as a sure event. The probability of a sure event is one.

Impossible event :

An event whose occurance is quite impossible in a random experiment is called an impossible event. The probability of an impossible event is zero.

Uncertain event :

When the outcomes of an event may or may not happen, then this type of event is called uncertain event.

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- **Mutually exclusive event :**

If there is no common element within two or more than two events, these events are called mutually exclusive event. In other words, if the happening of any of the events excludes the happening of all the other, then the events would be termed as mutually exclusive event.

Example :
In the experiment with throwing an unbiased die, the sample space, $S = \{1, 2, 3, 4, 5, 6\}$. The event of getting even number on top of die, $A: \{2, 4, 6\}$ and event of the obtained odd number on top of die, $B: \{1, 3, 5\}$. Since there is no common element between A and B, they are mutually exclusive events.

- **Non-mutually exclusive event :**

When two or more events have common elements in a random experiment, then they are called non-mutually exclusive event. That is, if two events A and B are such that $A \cap B \neq \phi$ and $P(A \cap B) \neq 0$, they are called non mutually exclusive event.

Example:

If two events are $A=\{2,4,6\}$ and $B=\{3,6\}$ then $A\cap B=\{6\}$ [i.e $A\cap B \neq \phi$]. So, A and B are non mutually exclusive events.

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Given,

$P(A)=\frac{1}{3}$

$P(B)=\frac{3}{4}$

$P(A\cup B)=\frac{5}{6}$

We know, if A and B are two non mutually exclusive events,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

or, $\frac{5}{6}=\frac{1}{3}+\frac{3}{4}-P(A\cap B)$

or, $P(A\cap B)=\frac{1}{3}+\frac{3}{4}-\frac{5}{6}$

or, $P(A\cap B)=\frac{4+9-10}{12}=\frac{13-10}{12}=\frac{3}{12}=\frac{1}{4}$

$\therefore P(A\cap B)=\frac{1}{4}$

Again, we know,

If A and B are two independent events,

$P(A\cap B)=P(A)\times P(B)$

or, $\quad '' \quad =\frac{1}{\cancel{3}}\times\frac{\cancel{3}}{4}$

or, $P(A\cap B)=\frac{1}{4}$, which is equal to the above found value.

Since both the values are equal, that means A and B are independent events.

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Here,

$P(A) = \frac{1}{3}$

$P(B) = \frac{3}{4}$

$P(A \cup B) = \frac{5}{6}$

Previously we found $P(A \cap B) = \frac{1}{4}$

Now,

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$$

We need to find $P(\bar{A}/B)$ and $P(A/\bar{B})$

We know, $P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)}$

and, $P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{\cancel{3}\ 4-3}{12} = \frac{1}{12}$$

So, $P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{3} = \frac{2}{3}$

$$P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{1}{\cancel{12}_{3}} \times \frac{\cancel{4}}{1} = \frac{1}{3}$$

$\therefore$ Required values are $P(\bar{A}/B) = \frac{2}{3}$ and $P(A/\bar{B}) = \frac{1}{3}$

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