Ans. to the Q.no-A

Given,

$z_1 = -1 + i$

$$\frac{z_1}{3+4i} = m + in$$

or, $$\frac{-1+i}{3+4i} = m + in$$

or, $$m + in = \frac{(-1+i)(3-4i)}{(3+4i)(3-4i)}$$ [Multiplying the numerator and denominator by $(3-4i)$]

or, $$m + in = \frac{(-1+i)(3-4i)}{9-16i^2}$$

or, $$m + in = \frac{-3+4i+3i-4i^2}{9+16}$$

or, $$m + in = \frac{1+7i}{25}$$

or, $$m + in = \frac{1}{25} + \frac{7}{25}i$$

So, $m = \frac{1}{25}$, $n = \frac{7}{25}$

Now,

$$m^4 - m^2n^2 + n^4 = \left(\frac{1}{25}\right)^4 - \left(\frac{1}{25}\right)^2 \times \left(\frac{7}{25}\right)^2 + \left(\frac{7}{25}\right)^4$$

$$= \frac{1}{25^4} - \frac{7^2}{25^4} + \frac{7^4}{25^4}$$

$$= \frac{1-7^2+7^4}{25^4} = \frac{2353}{390625}$$ (Ans)

Ans. to the Q.no-B

Given,

$z_1 = -1 + i$

$\therefore \bar{z_1} = -1 - i$ [conjugate of z_1]

Now, modulus, $r = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$

Argument, $\theta = -\pi + \tan^{-1}\left(\frac{-1}{-1}\right)$ [since it lies in the 3rd quadrant.]

$= -\pi + \tan^{-1}(1)$

$= -\pi + \frac{\pi}{4}$

$= -\frac{3\pi}{4}$

Now, expressing in polar form:

$\bar{z_1} = r\cos\theta + i r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

$= \sqrt{2}\left(\cos\frac{3\pi}{4} \cancel{+ i s} - i\sin\frac{3\pi}{4}\right)$

[$\because \sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$]

Ans: to the Q.no-c

Given,

$$z_1 = -1 + i$$

Let,

$$x^2 = -1 + i = z_1$$

$$\therefore x = \sqrt{z_1}$$

Now,

$$x^2 = -1 + i$$

$$or,\ x^2 = \frac{1}{2}(-2 + 2i)$$

$$or,\ '' = \frac{1}{2}\left(-2 + 2\cdot i\sqrt{(\sqrt{2}+1)} \times \sqrt{(\sqrt{2}-1)}\right)$$

$$or,\ '' = \frac{1}{2}\left(\sqrt{2} - 1 - \sqrt{2} - 1 + 2\cdot i\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}\right)$$

$$or,\ '' = \frac{1}{2}\left\{\left(\sqrt{\sqrt{2}-1}\right)^2 + i^2\left(\sqrt{\sqrt{2}+1}\right)^2 + 2\cdot\sqrt{(\sqrt{2}-1)}\cdot i\sqrt{(\sqrt{2}+1)}\right\}$$

$$or,\ x^2 = \frac{1}{2}\left\{\left(\sqrt{\sqrt{2}-1}\right) + \left(i\sqrt{\sqrt{2}+1}\right)\right\}^2$$

$$or,\ x = \pm\frac{1}{\sqrt{2}}\left(\sqrt{\sqrt{2}-1} + i\sqrt{\sqrt{2}+1}\right)$$

$$So,\ \sqrt{z_1} = \pm\frac{1}{\sqrt{2}}\left\{(\sqrt{2}-1)^{\frac{1}{2}} + i(\sqrt{2}+1)^{\frac{1}{2}}\right\}$$

Ans. to the Q.no-D

Given,

$z = x + iy$

and $z = p + p^{-1}$

$p = 3(\cos\theta + i\sin\theta)$

$\therefore z = 3(\cos\theta + i\sin\theta) + \dfrac{1}{3(\cos\theta + i\sin\theta)}$

or, $\quad = 3(\cos\theta + i\sin\theta) + \dfrac{(\cos\theta - i\sin\theta)}{3(\cos^2\theta - i^2\sin^2\theta)}$

or, $\quad = 3(\cos\theta + i\sin\theta) + \dfrac{(\cos\theta - i\sin\theta)}{3(\cos^2\theta + \sin^2\theta)}$

or, $\quad = 3(\cos\theta + i\sin\theta) + \frac{1}{3}(\cos\theta - i\sin\theta)$

or, $\quad = 3\cos\theta + \frac{1}{3}\cos\theta + 3 \cdot i\sin\theta - \frac{1}{3} \cdot i\sin\theta$

or, $z = \frac{10}{3}\cos\theta + \frac{8}{3} i\sin\theta$

Thus, $\frac{10}{3}\cos\theta + \frac{8}{3} i \cdot \sin\theta = x + iy$

$\therefore x = \frac{10}{3}\cos\theta$

$y = \frac{8}{3}\sin\theta$

Now,

$L.H.S = \dfrac{9x^2}{100} + \dfrac{9y^2}{64} = \dfrac{\cancel{9} \times \frac{10^2}{\cancel{3^2}}\cos^2\theta}{\cancel{100}} + \dfrac{\cancel{9} \times \frac{8^2}{\cancel{3^2}}\sin^2\theta}{\cancel{64}}$

$= \cos^2\theta + \sin^2\theta$

$= 1 = R.H.S \qquad \therefore L.H.S = R.H.S$

$\therefore \dfrac{9x^2}{100} + \dfrac{9y^2}{64} = 1$ [proved]

Ans. to the Q.no-E

Given,

$z_1 = -1 + i$

$\therefore \overline{z_1} = -1 - i$

$\therefore \frac{1}{2}(z_1 + \overline{z_1}) = \frac{1}{2}(-1 + \cancel{i} - 1 - \cancel{i}) = \frac{1}{2}(-2) = -1$

Now, $\frac{1}{2}(z_1 + \overline{z_1}) = a$

or, $a = -1$

or, $\sqrt[6]{a} = \sqrt[6]{-1}$

Let, $x = \sqrt[6]{-1} = \sqrt[6]{a}$

$\therefore x^6 = -1$

or, $x^6 + 1 = 0$

or, $x^6 - i^2 = 0$

or, $(x^3)^2 - (i)^2 = 0$

or, $(x^3 - i)(x^3 + i) = 0$

Either $x^3 - i = 0$ or $x^3 + i = 0$

If $x^3 - i = 0$

$x^3 + i^3 = 0$

or, $(x+i)(x^2 + i^2 - xi) = 0$

~~or~~, $x + i = 0$ or $x^2 + i^2 - xi = 0$

$x = -i$

$x = \frac{i \pm \sqrt{i^2 - 4 \times 1 \times i^2}}{2}$

$= \frac{i \pm \sqrt{-1 + 4}}{2}$

$= \frac{i \pm \sqrt{3}}{2}$

If $x^3+i=0$

$x^3-i^3=0$

$(x-i)(x^2+i^2+xi)=0$

$x-i=0$ or $x^2+i^2+xi=0$

$x=i$

$$x=\frac{-i\pm\sqrt{i^2-4\times1\times i^2}}{2}$$

$$=\frac{-i\pm\sqrt{-1+4}}{2}$$

$$=\frac{-i\pm\sqrt{3}}{2}$$

So, the required value of $\sqrt[6]{a}$

$\sqrt[6]{a}=\pm i,\ \frac{i\pm\sqrt{3}}{2},\ \frac{-i\pm\sqrt{3}}{2}$ (Ans)