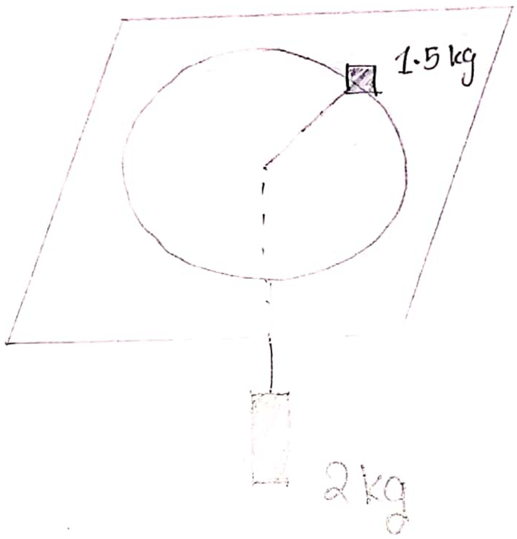
Title: Newtonian Mechanics



a) Write the Newton's equation for the 1.5 kg body rotating with uniform speed.

Ans: According to Newton's second law for rotational motion, the rate of change of angular momentum is directly proportional to the net torque.

$$\therefore \frac{d\vec{L}}{dt} \propto \vec{\tau}$$

$$\Rightarrow \frac{d}{dt}(I\vec{\omega}) \propto \vec{\tau}$$

$$\Rightarrow I\frac{d\vec{\omega}}{dt} = k\vec{\tau}$$

Here k is the proportionality constant.

$$\Rightarrow I\vec{\alpha} = k\vec{\tau}$$

In S.I. unit, $k=1$

$$\therefore \vec{\tau} = I\vec{\alpha} = mr^2\vec{\alpha}$$

For, 1.5 kg body, $m = 1.5$ kg

$$\therefore \vec{\tau} = 1.5r^2\vec{\alpha}$$

Given, the 1.5 kg body is rotating with uniform speed. Therefore,

$$\vec{\omega} = \text{constant}$$

$$\Rightarrow \frac{d\vec{\omega}}{dt} = \vec{0}$$

$$\Rightarrow \vec{\alpha} = \vec{0}$$

$$\therefore \text{Net torque} = I(\vec{0}) = \vec{0}.$$

b) What is the amount of work done for the uniform rotation of the 1.5 kg body?

Ans: Centripetal force acts on a rotating body. And the angle between the centripetal force and displacement is 90°.

We know, $W = Fs\cos\theta$

Therefore, work done by centripetal force $= Fs\cos 90$

$= Fs \times 0 = 0.$

Again, there is friction on the surface on which the 1.5 kg body is rotating. So work has to be done to keep the body rotating with uniform speed.

Let, the 1.5 kg body is rotating with an uniform speed of $v_0\ ms^{-1}$.

Since the body ~~has no ac~~ is rotating with uniform speed, it has no acceleration along the tangent.

$\therefore$ Net force along the tangent $= 0$

$\Rightarrow$ Applied force $-$ frictional force $= 0$

$\Rightarrow$ Applied force $=$ frictional force

$= \mu mg = 0.2 \times 1.5 \times 9.8$ N

$= 2.94$ N

$\therefore$ Work done by applied force

$= \mu mgs \cos 0^\circ$

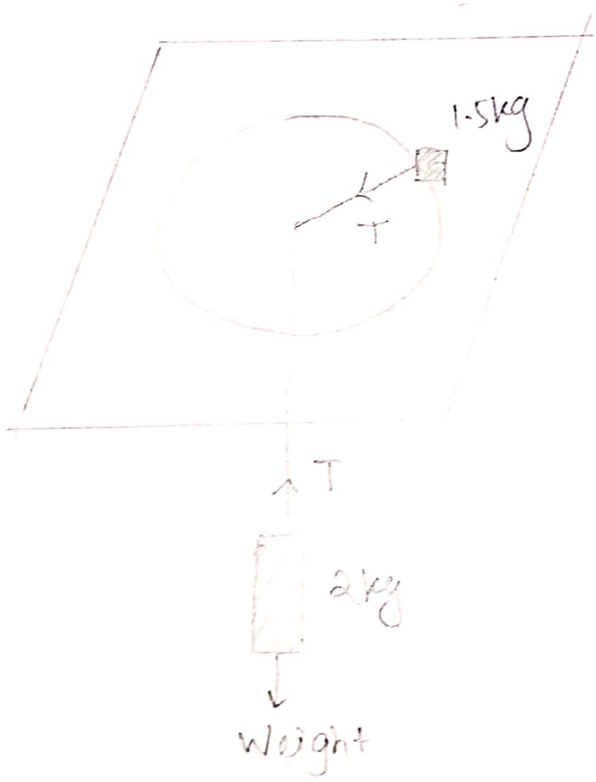
$= \mu mgr\theta = \mu mgr\omega t$

$= \mu mg v_0 t = 2.94 v_0 t$

$\therefore$ Work done by applied force in t seconds $= 2.94 v_0 t$ Joule.

c) What should be the speed of the body 1.5 kg to hold the 2 kg body at rest?

Ans:



If the 2 kg body stays at rest, net force on it will be zero.

From the figure, net force $= T - W = 0$

$\Rightarrow T - 2g = 0$

$\Rightarrow T = 2g$ N

$= 19.6$ N

The tension of the string will provide necessary centripetal force.

$$\therefore T = \frac{mv^2}{r}$$

$$\Rightarrow 2g = \frac{1.5v^2}{r}$$

$$\Rightarrow v^2 = \frac{2gr}{1.5} = 13.067r$$

$$\Rightarrow v = \sqrt{13.067r} = 3.61\sqrt{r}$$

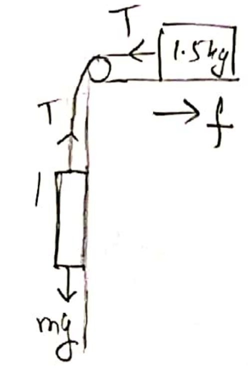
$$\therefore v = 3.61\sqrt{r}\ ms^{-1}.$$

d) Draw the velocity-time graph of the 2kg body when speed of 1.5 kg decreases gradually due to friction.

Ans: When speed of 2kg body decreases gradually due to friction it will not be able to hold the

2 kg body at rest. So, the whole system will start moving and the radius of orbit of 1.5 kg will also decrease. As a result, acceleration of the system will keep increasing.

After a definite interval of time, velocity of 1.5 kg body along the tangent will be zero. Then the two bodies will fall with constant acceleration like assignment-1.

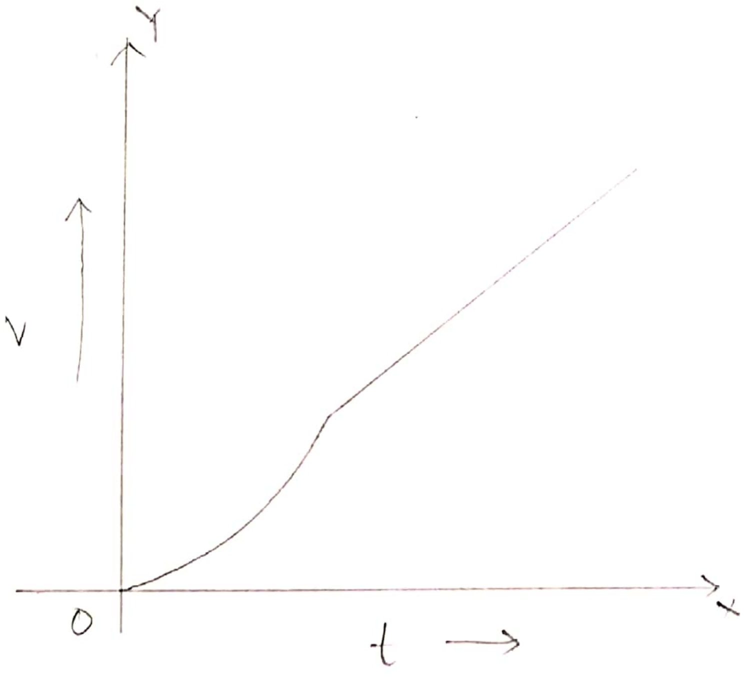


From assignment-1,

$$2a = 19.6 - T$$
$$1.5a = T - 2.94$$
$$3.5a = 16.66$$

$\therefore a = 4.76\ ms^{-2}$

So, at first the velocity versus time graph will be a curve due to variable acceleration. Then it will be a straight line due to constant acceleration.



This would be the estimated velocity-time graph of the 2 kg body.