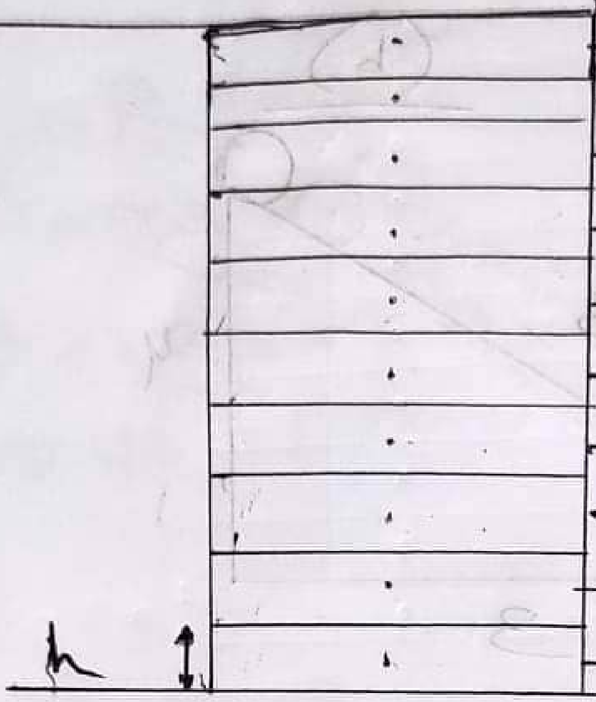


$\to mg(9h) \to mg9h = mg(10-1)h$

$\to mg(8h) \to mg8h = mg(9-1)h$

$\to mg(7h) \to mg7h = mg(8-1)h$

$\to mg(6h) \to mg6h = mg(7-1)h$

$\to mg(5h) \to mg5h = mg(6-1)h$

$\to mg(4h) \to mg4h = mg(5-1)h$

$\to mg(3h) \to mg3h = mg(4-1)h$

$\to mg(2h) \to mg2h = mg(3-1)h$

$\to mgh \to mg.1h = mg(2-1)h$

$\to mg\times 0 \to mgh = 0$

(a)

Total work done

$$W = mg(2-1)h + mg(3-1)h + mg(4-1)h + \cdots mg(10-1)h$$

$$= mgh\,(1+2+\cdots 9)$$

$$= mgh\,\frac{9(9+1)}{2}$$

$$= mgh\,\frac{9\times\cancel{10}^{5}}{\cancel{2}}$$

$$= 45mgh$$

$$= 45\times2.5\times9.8\times0.4$$

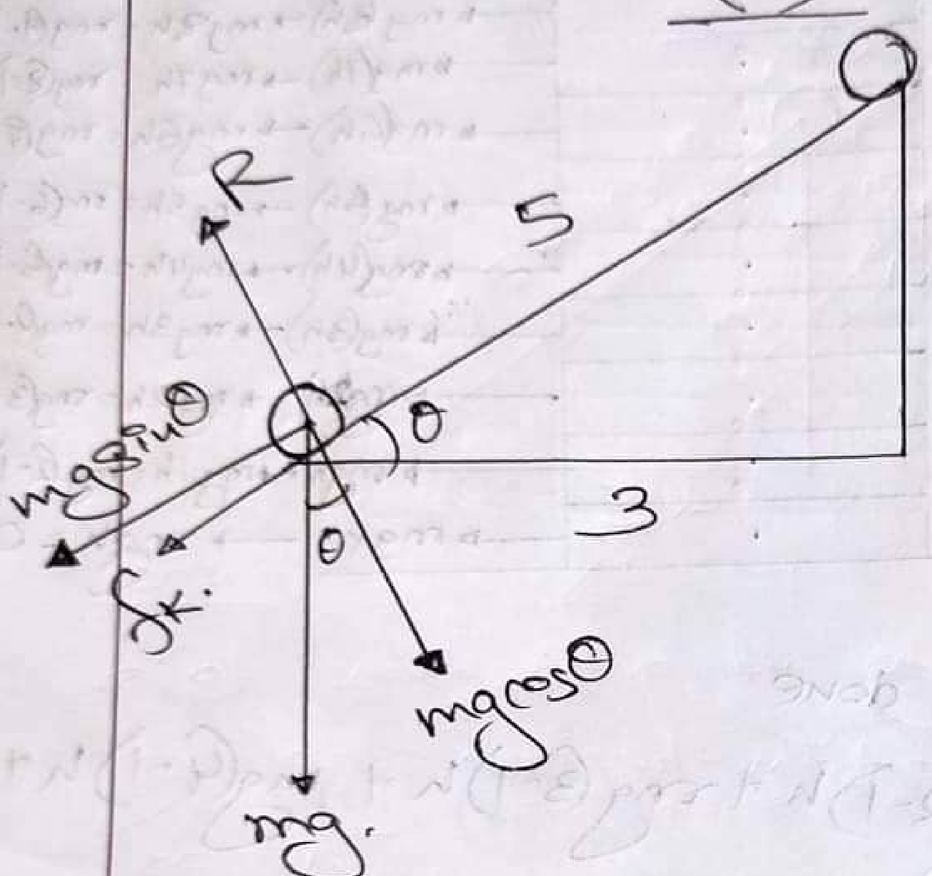
$$= 441\,J \text{ (Ans)}$$

Given,

$m = 2.5$ kg

$h = 0.4$ m

(b)



Given,

$m = 2.5$ kg.

$\mu_K = 0.2$

$g = 9.8\ ms^{-2}$.

Soln: To raise the sphere from bottom to top of the pile, work has to be done against gravitational force and frictional force.

Now,

$R = mg\cos\theta$.

$f_K = \mu_K R$

$= \mu_K mg\cos\theta$

$= 0.2 \times 2.5 \times 9.8 \times \frac{3}{5}$

$= 2.94\ N.$

Again,

$\Rightarrow mg\sin\theta$

$= 2.5 \times 9.8 \times \frac{4}{5}$

$= 19.6\ N.$

Total force required in lifting

$F = 19.6 + 2.94$

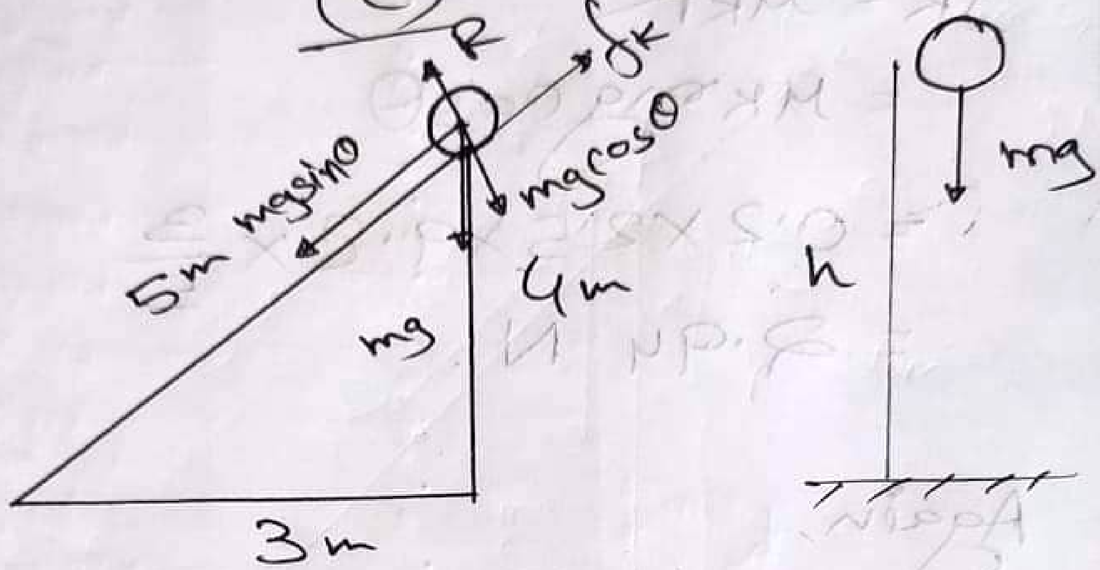
$= 22.54\ N.$

Work done, $W = FS$

$= 22.54 \times 5$

$= 112.7\ J$ (Ans).

(c)



Soln:

Let work done by gravitational force for downward motion of sphere be W_1.

$$W_1 = mgh = 2.5 \times 9.8 \times 4$$
$$= 98 \text{ J}.$$

Again work done in inclined plane be W_2.

$$\text{Net force}, F_{net} = mg\sin\theta - f_K$$
$$= mg\sin\theta - \mu_K mg\cos\theta.$$
$$= mg(\sin\theta - \mu_K\cos\theta)$$

$= 2.5 \times 9.8 \left(\frac{4}{5} - 0.2 \times \frac{3}{5} \right)$

$= 2.5 \times 9.8 \times 0.68$.

$= 16.66\ N$.

$W_2 = F_{net} \times S$.

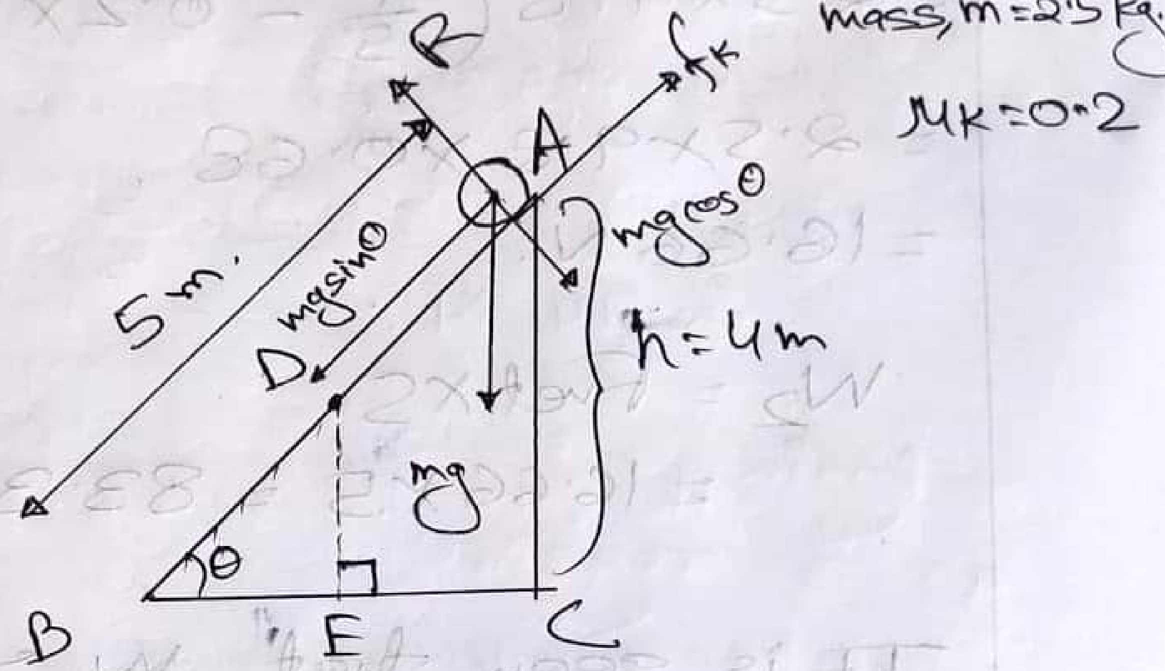
$= 16.66 \times 5 = 83.3\ J$.

It is seen that $W_1 > W_2$. So the work done will not be equal for both cases.

Logic: For vertically downward motion there is no presence of non conservative force like air resistance, fluid friction etc. But in the case of inclined plane friction is presence for which some energy is lost. So the work done will not be equal.

(d)

Soln:



Let D be the midpoint along AB.
We draw a perpendicular from D on BC, which meets at E.

In ΔABC and ΔDBE.

$$\sin\theta = \frac{4}{5}.$$

For triangle DBE,

$$\frac{DE}{BD} = \sin\theta.$$

or, $\frac{DE}{2.5} = \frac{4}{5}$

or, $DE = \frac{4}{5} \times 2.5$

$\therefore DE = 2\ m.$

At A,

Total energy, E_A = Potential energy + Kinetic energy.

$= mgh + 0$

$= 2.5 \times 9.8 \times 4$

$= 98\ J.$

Net force on sphere,

$F_{net} = mg\sin\theta - f_k$

$= mg\sin\theta - \mu_k mg\cos\theta$

$= mg(\sin\theta - \mu_k\cos\theta)$

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A.T-Q,

$$\cancel{m}a = \cancel{m}g(\sin\theta - \mu_k\cos\theta)$$

$$\therefore a = g(\sin\theta - \mu_k\cos\theta)$$

Velocity at B.

$$V_B^2 = u^2 + 2as_1$$

$$V_B^2 = 2as$$

$$V_B = \sqrt{2as_1}$$

$$= \sqrt{2 \times g(\sin\theta - \mu_k\cos\theta) \times 2.5}$$

$$= \sqrt{2 \times 9.8\left(\frac{4}{5} - 0.2 \times \frac{3}{5}\right) \times 2.5}$$

$$= 5.772\ ms^{-1}.$$

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At B,

Total energy,

E_B = Potential energy + Kinetic energy

$= mg(DE) + \frac{1}{2} m V_B^2$

$= 2.5 \times 9.8 \times 2 + \frac{1}{2} \times 2.5 \times (5.772)^2$

$= \cancel{122}\ 49 + 41.64$

$= 90.64$ J.

~~At C,~~

~~Total en~~

Velocity at C,

$$V_c = \sqrt{2as}$$

$$= \sqrt{2 \times g(\sin\theta - \mu_k \cos\theta) \times S}$$

$$= \sqrt{2 \times 9.8\left(\frac{4}{5} - 0.2 \times \frac{3}{5}\right) \times 5}$$

$$= 8.16\ ms^{-1}.$$

At C,

Total energy,

E_C = Potential energy + Kinetic energy

$$= 0 + \frac{1}{2} m V_c^2$$

$$= \frac{1}{2} \times 2.5 \times (8.16)^2$$

$$= 83.23\ J.$$

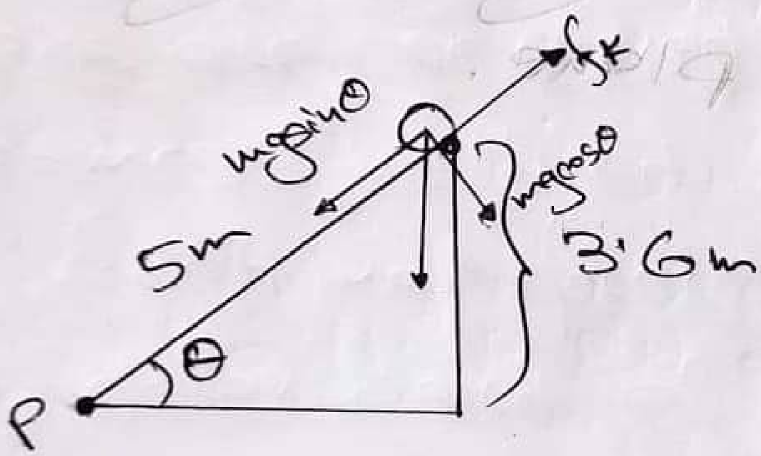
Hence, we can see that

$E_A \neq E_B \neq E_C$. Thus conservation of energy will not be applicable while the body falling over inclined plane.

(e)

When one rectangular body is removed, then height becomes

$h_1 = 9 \times 0.4 = 3.6\ m.$



$\sin\theta = \frac{3.6}{5}.$

$\cos\theta = \frac{3.47}{5}.$

$F_{net} = mg\sin\theta - f_k$

$= mg\sin\theta - \mu_k mg\cos\theta$

$= mg(\sin\theta - \mu_k\cos\theta).$

A.T.Q,

$\cancel{m}a = \cancel{m}g(\sin\theta - \mu_k\cos\theta)$

$\therefore a = g(\sin\theta - \mu_k\cos\theta).$

Velocity at P.

$$V_1 = \sqrt{2as}$$

$$= \sqrt{2 \times g(\sin\theta - \mu_k \cos\theta) \times s}$$

$$= \sqrt{2 \times 9.8\left(\frac{3.6}{5} - 0.2 \times \frac{3.47}{5}\right) \times 5}$$

$$= 7.54\ ms^{-1}.$$

Again if another body is removed,

$$h_2 = 8 \times 0.4 = 3.2.$$

$$\sin\theta = \frac{3.2}{5}$$

$$\cos\theta = \frac{3.84}{5}.$$

$$V_2 = \sqrt{2as}$$

$$= \sqrt{2 \times 9.8\left(\frac{3.2}{5} - 0.2 \times \frac{3.84}{5}\right) \times 5}$$

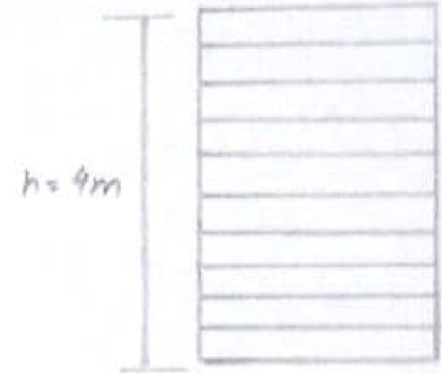
$$= 6.904\ ms^{-1}.$$

From the above calculation, we may come to a conclusion that if one block is removed then its speed decreases. The more blocks are removed, more it becomes inclined. As a result, the frictional force increases and downward effective force decreases. This is why speed changes.

Answer to the Question Number (a)

Given,

Mass of each rectangular body, $m = 2.5\,kg$



Height of each rectangular body, $d = 0.4\,m$

Number of rectangular body, $n = 10$

Now,

Mechanical energy spent to make the ~~pip~~ pile,

$$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10}$$

$$= mg \times 0 + mgd + mg2d + mg3d + mg4d + mg5d + mg6d + mg7d + mg8d + mg9d$$

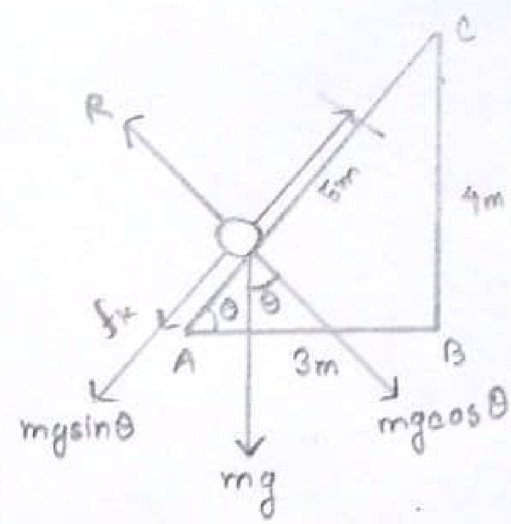
$$= mg(d + 2d + 3d + 4d + 5d + 6d + 7d + 8d + 9d)$$

$$= 2.5 \times 9.8 \times (45 \times 0.4)$$

$$= 441 \text{ joule.}$$

$\therefore$ The mechanical energy spent is 441 joule

Answer to the Question Number (b)



Given,

Mass of sphere, $m = 2.5kg$

Coefficient of friction of inclined plane, $\mu_K = 0.2$

Acceleration due to gravity, $g = 9.78 \, ms^{-2}$

We know,

Frictional force, $f_K = \mu_K R$

$= \mu_K mg\cos\theta$

$= 0.2 \times 2.5 \times 9.78 \times \cos\theta$

Here,

$$\cos\theta = \frac{AB}{AC}$$

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{5^2 - 4^2} = 3m$$

$$\therefore \cos\theta = \frac{3}{5}$$

Now,

Work done to raise the sphere, $W = Fs$

$= (mg\sin\theta + f_K)s$

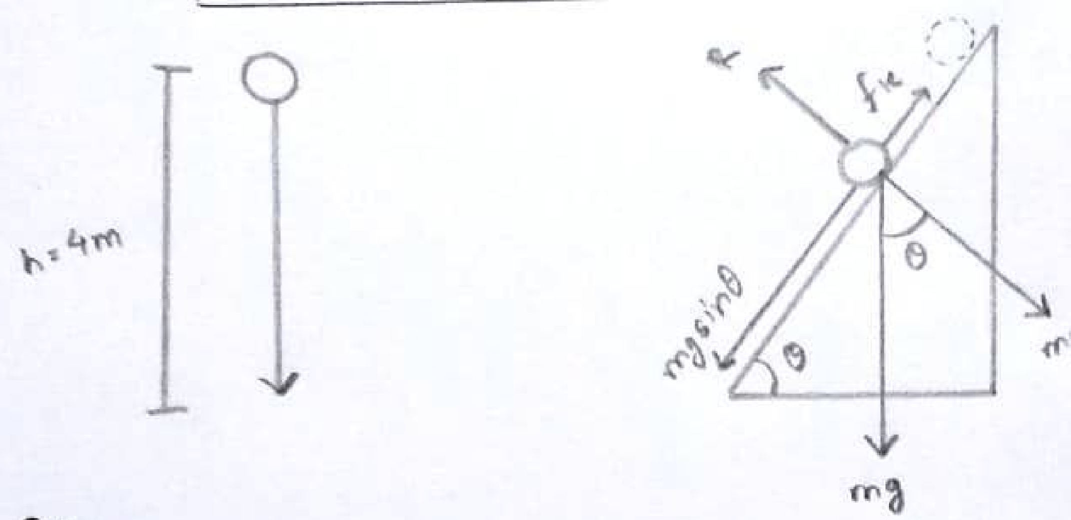
$= (mg\sin\theta + \mu_K mg\cos\theta)s$

$$= \left(2.5 \times 9.78 \times \frac{4}{5} + 0.2 \times 2.5 \times 9.78 \times \frac{3}{5}\right) \times 5$$

$$= 112.47 \text{ joule}$$

$\therefore$ The amount of work done is 112.47 joule.

Answer to the Question Number (c)



Given,

Mass of the sphere, $m = 2.5\ kg$

Acceleration due to gravity, $g = 9.78\ ms^{-2}$

Coefficient of kinetic friction, $\mu_k = 0.2$

When the sphere is dropped vertically downwards from top,

Amount of work, $W_1 = mgh$

$$= 2.5 \times 9.78 \times 4$$

$$= 97.8 \text{ joule}$$

Again, when the sphere is released over the inclined plane from top position,

Amount of work, $W_2 = Fs$

$$= (mg\sin\theta - f_K) \cdot s$$

$$= (mg\sin\theta - \mu_K mg\cos\theta) \cdot s$$

$$= mgs(\sin\theta - \mu_K \cos\theta)$$

$$= 2.5 \times 9.78 \times 5 \times \left(\frac{4}{5} - 0.2 \times \frac{3}{5}\right)$$

$$= 83.13 \text{ joule}$$

Since, $W_1 > W_2$ i.e $W_1 \neq W_2$. So, work in both cases will not be equal.

Answer to the Question Number (d)

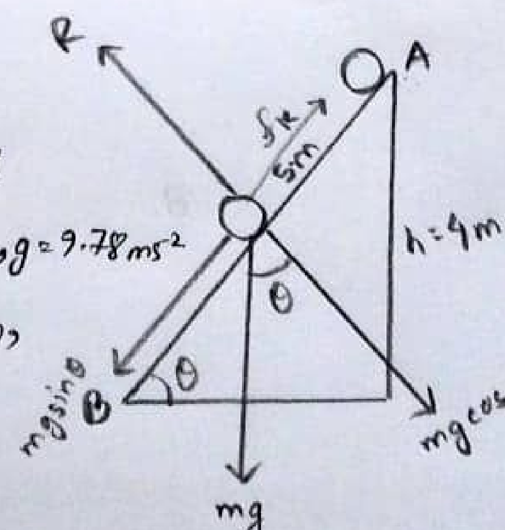
Given,

Mass of sphere, $m = 2.5$ kg

Acceleration due to gravity, $g = 9.78 ms^{-2}$

Coefficient of kinetic friction,

$\mu_K = 0.2$



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At A point,

Velocity, $v_A = 0 ms^{-1}$

$\therefore$ kinetic energy, $E_{k_A} = \frac{1}{2} mv^2$

$= 0$ joule

Potential energy, $E_{P_A} = mgh$

$= 2.5 \times 9.78 \times 4$

$= 97.8$ joule

$\therefore$ Total energy at A point, $E_A = E_{P_A} + E_{k_A}$

$= 97.8$ joule

At B point,

Velocity, $V_B^2 = V_A^2 + 2 \times a \times s$

Here,

$F = mg\sin\theta - f_k$

$\Rightarrow ma = mg\sin\theta - \mu_k mg\cos\theta$

$\Rightarrow a = g(\sin\theta - \mu_k \cos\theta)$

$= 9.78\left(\frac{4}{5} - 0.2 \times \frac{3}{5}\right)$

$= 6.6504 \ ms^{-2}$

$\therefore\ V_B^2 = 0^2 + 2 \times 6.6504 \times 5$

$= 66.504\ m^2s^{-2}$

now,

kinetic energy, $E_{K_B} = \frac{1}{2} m \times V_B^2$

$= \frac{1}{2} \times 2.5 \times 66.504$

$= 83.13$ joule

Potential energy, $E_{P_B} = mgh$

$= mg \times 0$

$= 0$ joule

Frictional force, $f_K = \mu_K mg\cos\theta$

$\therefore$ Work done due to friction, $W_{\circ} = f_K s$

~~= (mgsinθ~~

$= \mu_K mg\cos\theta \times s$

$= 0.2 \times 2.5 \times 9.78 \times \frac{3}{5} \times 5$

$= 14.67$ joule

$\therefore$ Total energy at B, $E_B = E_{P_B} + E_{K_B} + E_{f_K}$

$= (0 + 83.13 + 14.67)$

$= 97.8$ ~~jot~~ joule

$\therefore\ E_A = E_B$.

So, the law of conservation of energy is applicable while the body is falling over the inclined plane.

Answer to the Question Number (e)

Given,

Mass of sphere, $m = 2.5\ kg$

Acceleration, $a = g(\sin\theta - \mu_k \cos\theta)$ [from 'd']

Length of inclined plane, $s = 5m$

Height of each rectangular body, $l = 0.4m$

Number of rectangular body, $n = 10$

$\therefore$ Total height, $h = 10 \times 0.4 = 4m$

Coefficient of kinetic friction, $\mu_k = 0.2$

Acceleration due to gravity, $g = 9.78\ ms^{-2}$

We know,

$$v^2 = v_0^2 + 2as$$

For initial stage,

$$\theta = \sin^{-1}\left(\frac{h}{5}\right)$$

$$= \sin^{-1}\left(\frac{4}{5}\right)$$

$$= 53.1301^\circ$$

$$\therefore V_{B_2} = \sqrt{2 \times 9.78 \times (\sin 39.7918^\circ - 0.2 \times \cos 39.7918^\circ) \times 5}$$

$$= 6.8965\ ms^{-1}$$

Removing 3 rectangular bodies,

$$h_3 = (4 - 0.4 \times 3) = 2.8m$$

$$\therefore \theta_3 = \sin^{-1}\left(\frac{h_3}{5}\right)$$

$$= \sin^{-1}\left(\frac{2.8}{5}\right) = 34.0557^\circ$$

$$\therefore V_{B_3} = \sqrt{2 \times 9.78 \times (\sin 34.0557^\circ - 0.2 \times \cos 34.0557^\circ) \times 5}$$

$$= 6.2099\ ms^{-1}$$

Therefore, we can see that $V_B > V_{B_1} > V_{B_2} > V_{B_3}$ ...... $V_{B_{10}}$. We can say that by removing each of the rectangular body one by one and in every case releasing the same sphere from the top of the inclined plane, the velocity of the sphere keeps decreasing as the angle of the inclined plane with the horizontal keeps decreasing.