Sensor-Management for Multitarget Filters via Minimization of Posterior Dispersion

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This paper presents a new sensor management method for multitarget filtering, that is designed based on maximizing a measure of confidence in accuracy of the multitarget state estimate. Confidence of estimation is quantified by optimal subpattern assignment-based dispersion of the multitarget posterior about its statistical mean. Implementation of the algorithm for generic multitarget filters is presented. Simulation studies with labeled multi-Bernoulli filter demonstrate excellent performance in challenging sensor control scenarios.

Manuscript received February 3, 2016; revised July 21, 2016 and February 21, 2017; released for publication April 9, 2017. Date of publication June 28, 2017; date of current version December 5, 2017.

DOI. No. 10.1109/TAES.2017.2718280

Refereeing of this contribution was handled by P. Braca.

This work was supported in part by the ARC Discovery Project under Grant DP130104404 and in part by the National Natural Science Foundation of China under Grant 61175030.

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I. INTRODUCTION

Sensor management in multitarget filters is usually employed to control or select one or more sensors with the aim of acquiring the *best* measurements for operational purposes [12], [20]. In general, sensor management is based on maximizing the expected performance of the multitarget tracking system in future time steps. Commonly, future performance of the tracking system is evaluated and optimized via an objective function. The employed objective function plays a crucial role in the decision-making process at the heart of any sensor management solution.

In our recently published work in this domain [9], we introduced a sensor management method with "posterior expected error of cardinality and states" (PEECS) chosen as the cost function. The expert reviews expressed significant interest in understanding the relationship between the PEECS objective function and "optimal subpattern assignment" (OSPA) metric [27]. The main question was: How does variance reduction in PEECS result in better estimation and consequently error reduction in terms of OSPA metric? This prompted us to develop a multitarget tracking task-based objective function for sensor management that is directly oriented to minimize the OSPA metric. Although the use of any multitarget tracking performance metric as an objective function for the sensor management problem may look trivial, the main difficulty is the evaluation of a such objective function in absence of groundtruth. This issue has not yet been tackled in the literature. As such, we will introduce a new objective function, whose optimization would minimize the dispersion of the multitarget posterior, quantified based on the OSPA distance.

The proposed objective function can be applied in conjunction with various multitarget filters such as probability hypothesis density (PHD) [18], cardinalized PHD (CPHD) [30], MeMBer [18], CB-MeMBer [31], labeled multi-Bernoulli (LMB) [21], Vo-Vo filter (also called GLMB) [29] and marginalized δ-GLMB [4] filters. The sequential Monte Carlo (SMC) implementation of the devised cost function in conjunction with the LMB filter is also presented. The utility and advantages of the proposed approach are demonstrated through simulation studies.

II. RELATED WORKS

In multitarget tracking applications, sensor management is usually used for two purposes: sensor control (in applications with controllable sensors) or sensor selection (usually in sensor networks). The aim of sensor control is to direct sensor(s) toward the right targets at the right times [17], so that *the best measurements* are acquired. Similarly, in sensor selection, the aim is to pick one or more sensor nodes that are expected to acquire *the best measurements*. The multitarget filtering problem generally involves stochastic variations in the number of targets, measurement/process noise and detections, and can be formulated in the form of an optimal stochastic control problem. In this context, the sensor management problem is usually tackled via formulating the stochastic process as a partially observable Markov decision process (POMDP) [19].

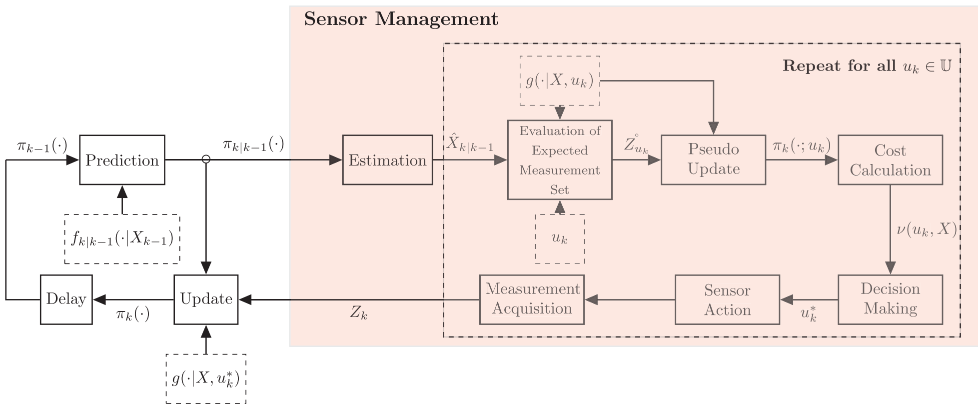

Fig. 1. General schematic block diagram for sensor management within a multi-target filter, in POMDP framework.

Devising a proper objective function is at the core of formulating sensor management techniques in a POMDP framework. There are two approaches in defining an objective function. In *information-driven* methods, the *information content* of predicted and/or updated distributions is utilized to build the criterion for goodness. This is usually quantified via a *divergence* function such as Rényi divergence [11], [14], [23]–[25] and Cauchy–Schwarz divergence [10]. In these methods, statistical expectation of the chosen divergence function is selected as a *reward* function, which is maximized to solve the sensor management problem.

In the second approach, called *task-driven*, sensor management methods are designed with a direct focus on the expected performance of the multitarget filter as the measure of goodness. A *good measurement* is assumed to be the one that reduces the *dispersion* of the multitarget filter posterior. The smaller dispersion is taken to convey *more confidence* in the estimation process. Examples of such cost functions include estimated target cardinality variance [5], [11] and PEECS [6], [9]. Also, a third approach has been emerging in past few years. In this approach, the notion of operational risk is used as a utility function for performing sensor management [2], [13], [17].

In the next section, we present a novel cost function that is directly related to the widely adopted multitarget estimation error, OSPA metric. As it was mentioned previously, the sensor management solution and algorithm presented here are applicable in most of the statistical multi-target filtering solutions, such as PHD , CPHD, multi-Bernoulli, LMB, and Vo-Vo filters.

III. SOLUTION FRAMEWORK

The most common methodology for handling a sensor management problem is to employ the POMDP framework. Examples of sensor control solutions using the POMDP framework include the works reported in [5], [6], [8], [9], [11], [15], [16], [19], [24], [25], [32], [33].

A. POMDP Framework

This paper focuses on devising a sensor management solution for multitarget tracking in the POMDP framework while instantaneous performance is considered (myopic approach). The elements of the POMDP framework are as follows.

1) X_k: A finite set of single-object states.
2) $\mathbb{U}$: A set of sensor management commands.
3) $f_{k|k-1}(X_k|X_{k-1})$: Multitarget state transition density.
4) Z: A finite set of observations.
5) $g_k(Z|X, u)$: A stochastic measurement model.
6) $\mathcal{V}(u; X)$: An objective (cost or reward) function.

Note that in principle, the action space is infinite and for the sake of simplicity, a finite set of actions or commands is considered. In the POMDP framework, the purpose of sensor management is to find the control/selection command $\overset{\star}{u} \in \mathbb{U}$ that optimizes the objective function. In stochastic filtering, where the multitarget states X_{k-1} and X_k are characterized by their distributions, the command $\overset{\star}{u}$ is commonly chosen to optimize the statistical mean of the objective function $\mathcal{V}(u; X)$ over all observations. That is

$$\overset{\star}{u} = \underset{u \in \mathbb{U}}{\operatorname{argmin}} / \underset{u \in \mathbb{U}}{\operatorname{argmax}} \left\{ \mathbb{E}_{Z(u)} \left[\mathcal{V}(u; X) \right] \right\} \tag{1}$$

where the objective function is assumed to be a cost function.

The objective function is a function of future measurements while the values of the future measurements are unknown. A common approach is to generate a set of pseudomeasurements for each hypothesized action/command. The PIMS is a widely adopted choice for such pseudomeasurements [17]. The PIMS comprises the clutter-free and noise-free measurements that are most likely to be obtained from the selected/controlled sensor(s). As it is shown in Fig. 1, in its most general form, sensor management involves pseudoupdating the multitarget distribution using the PIMS as pseudomeasurements, then computing the cost associated with the resulting posterior. In Fig. 1, the predicted density is denoted by $\pi_{k|k-1}(\cdot)$, pseudoupdated density is denoted by $\pi_k(\cdot; u_k)$, and the updated density is denoted by $\pi_k(\cdot)$.

For the results reported in this paper, an LMB filter was chosen and implemented as the multitarget filter, though the sensor management method can be used with many other filters. A desirable property of the LMB filter is that it produces track-valued estimates [21], [28], [29]. Besides this advantage, its usage in our simulations is for a fair comparison of the results with the recently published PEECS-based method [7], which is implemented with the LMB filter.

B. Labeled Multi-Bernoulli Filter

In this section, a summary of the notation and formulation of the LMB filter, as introduced in [21], is presented. Lower case letters (e.g., x and $\mathbf{x}$) denote single-target states, while upper case letters (e.g., X and $\mathbf{X}$) denote multitarget states. Bold face letters are used for *labeled* states, i.e., single- or multitarget states that include label(s).

The LMB RFS is a special case of GLMB RFS [28], [29]. Similar to the multi-Bernoulli RFS, it is completely described by its components $\pi = \{(r^{(\zeta)}, p^{(\zeta)}) : \zeta \in \Psi\}$ with index set Ψ. The LMB RFS density is given by

$$\pi(\mathbf{X}) = \Delta(\mathbf{X}) w(\mathcal{L}(\mathbf{X}))\,[p]^{\mathbf{X}} \tag{2}$$

where

$$\begin{aligned} p(x,\ell) =&\ p^{(\ell)}(x) \\ w(L) =&\ \prod_{i\in\mathbb{L}} \left(1 - r^{(i)}\right) \prod_{\ell\in L} \frac{1_{\mathbb{L}}(\ell) r^{(\ell)}}{(1 - r^{(\ell)})} \\ \Delta(\mathbf{X}) =&\ \delta_{|\mathbf{X}|}(\mathcal{L}(\mathbf{X})). \end{aligned}$$

Here, labels are denoted by ℓ, a discrete label space is denoted by $\mathbb{L}$, and a set of track labels is denoted by L. Similar to the general multitarget Bayes filter, the LMB multitarget Bayes recursion propagates multitarget posterior density at each time according to the Chapman–Kolmogorov and the Bayes rule (update step). If the multitarget posterior density is an LMB RFS with parameter set $\pi = \{(r^{(\ell)}, p^{(\ell)}) : \ell \in \mathbb{L}\}$ with state space $\mathbb{X}$ and label space $\mathbb{L}$ and the birth model is also an LMB RFS with parameter set $\pi_{\mathbf{B}} = \{(r_B^{(\ell)}, p_B^{(\ell)}) : \ell \in \mathbb{B}\}$ with state space $\mathbb{X}$ and label space $\mathbb{B}$, then the predicted multitarget density is also an LMB RFS with state space $\mathbb{X}$ and label space $\mathbb{L}_+ = \mathbb{B} \cup \mathbb{L}$ ($\mathbb{B} \cap \mathbb{L} = \emptyset$) and it is given by

$$\pi_+ = \{(r_{+,S}^{(\ell)}, p_{+,S}^{(\ell)}) : \ell \in \mathbb{L}\} \cup \{(r_B^{(\ell)}, p_B^{(\ell)}) : \ell \in \mathbb{B}\} \tag{3}$$

where $\{(r_{+,S}^{(\ell)}, p_{+,S}^{(\ell)}) : \ell \in \mathbb{L}\}$ denotes the LMB RFS of surviving targets [21].

As the predicted LMB distribution is not a conjugate prior for point measurement likelihood, the updated multitarget distribution needs to be approximated. Similar to the derivation of the multi-Bernoulli filter [31], Reuter *et al.* [21] approximated the updated RFS by an LMB with matching first moment. The predicted multitarget density is denoted as an LMB RFS with parameter set $\pi_+ = \{(r_+^{(\ell)}, p_+^{(\ell)}) : \ell \in \mathbb{L}_+\}$, and the LMB approximation of the multitarget posterior is denoted by

$$\pi(\cdot|Z) = \{(r^{(\ell)}, p^{(\ell)}(\cdot) : \ell \in \mathbb{L}_+\}. \tag{4}$$

For equations governing the propagation of labels and multi-Bernoulli parameters through the update step, refer to [21].

C. OSPA Metric

Let $X = \{x_1, \ldots, x_m\}$ and $Y = \{y_1, \ldots, y_n\}$ be finite sets of points, where m and n denotes the cardinality of sets X and Y, respectively. For $0 < p < \infty$, where p is the order parameter, $c > 0$, where c is the cut-off parameter and $m \le n$, OSPA [27] is defined as $d_p^{(c)}(X,Y) \triangleq (\frac{1}{n}(\min_{\pi\in\Pi_n} \sum_{i=1}^m d^{(c)}(x_i, y_{\pi(i)})^p + c^p(n-m)))^{\frac{1}{p}}$. In this equation, $d^{(c)}(\cdot,\cdot)$ denotes the distance between two points, subject to the maximum cut-off c and Π_n denotes the set of permutations on $\{1,\ldots,k\}$, $k \in \mathbb{N}$.

IV. OSPA-BASED COST FUNCTION

Consider a single-object filtering scenario, with the object state estimate being computed from the updated posterior in every filtering iteration. The confidence in the object state estimate is usually quantified by measuring *the dispersion of the posterior around its extracted estimate*. A straightforward measure of this dispersion is the statistical mean of *Euclidean distance* of the state from its mean. One possible option to sensor management in such a scenario would be to choose the action command that is expected to minimize this distance.

In multitarget filtering, where both the object states and their number need to be estimated, a *set distance* is needed for quantification of dispersion around the mean. In multitarget tracking literature, the OSPA [27] distance has been widely adopted as a suitable metric for set distance [1], [3], [22], [26], [27]. As the calculation of OSPA requires ground truth, it has been only used in the literature to quantify the multitarget estimation error in simulation studies involving synthetic data, where the ground truth is accurately known. In this paper, we present how OSPA metric can be used to quantify the dispersion of a multitarget distribution around its mean.

Assume that for each action command $u \in \mathbb{U}$, the predicted ideal measurement set (PIMS) has been computed and used to update the multitarget distribution. We denote the so-called *pseudoupdated* distribution by $\pi_{k,u}(\cdot)$ and its corresponding multitarget random set by $X_{k,u}$. Let us denote the OSPA distance between two sets, X and Y by $\bar{d}_p^{(c)}(X,Y)$, where p and c are the user-defined parameters of the metric [27]. Our proposed cost function, the OSPA-based dispersion of $\pi_{k,u}(\cdot)$ around its mean, is defined as

$$\mathcal{V}(u; X_{k|k-1}) = \mathbb{E}\left[\bar{d}_p^{(c)}(X_{k,u}, \bar{X}_{k,u})\right] \tag{5}$$

where the expectations are over the posterior $\pi_{k,u}$, i.e.

$$\begin{aligned} \mathcal{V}(u; X_{k|k-1}) =&\ \mathbb{E}\left[\bar{d}_p^{(c)}(X_{k,u}, \bar{X}_{k,u})\right] \\ =&\ \int \bar{d}_p^{(c)}(X, \bar{X}_{k,u}) \pi_{k,u}(X) \delta X. \end{aligned} \tag{6}$$

To calculate the above cost function, the posterior distribution can be constructed using L Monte Carlo samples, $\pi_{k,u}(X) \approx \frac{1}{L}\sum_{i=1}^L \delta_{X_{k,u}^{(i)}}(X)$, and we have

$$\mathcal{V}(u; X_{k|k-1}) \approx \frac{1}{L}\sum_{i=1}^{L} \bar{d}_p^{(c)}(X_{k,u}^{(i)}, \bar{X}_{k,u}). \tag{7}$$

Algorithm 1: OSPA-Based Sensor Management With SMC Implementation.

INPUTS: predicted multi-target distribution $\pi_{k|k-1}(\cdot)$, measurement likelihood function $g_k(\cdot|x, u)$, finite set of admissible commands $\mathbb{U}$, OSPA parameters p and c.

OUTPUT: The best action command $\overset{\star}{u}$.

$\hat{X}_{k|k-1} \leftarrow \text{MOE}(\pi_{k|k-1})$ ▷ Estimating target states from prediction

$\hat{M}_{k|k-1} \leftarrow |\hat{X}_{k|k-1}|$ ▷ Estimate cardinality from prediction

for all $u \in \mathbb{U}$ **do**,

  $\mathring{Z} \leftarrow \varnothing$. ▷ Constructing PIMS...

  for $\ell = 1, \ldots, \hat{M}_{k|k-1}$ **do**

    $\xi \leftarrow \underset{z}{\text{argmax}}\, g(z|\hat{x}^{(\ell)}_{k|k-1}, u)$

    $\mathring{Z} \leftarrow \mathring{Z} \cup \{\xi\}$

  end for

  $\pi_{k,u} \leftarrow$ Updated density using $\mathring{Z}$ as measurement set. ▷ Pseudo update

  $\bar{X}_{k,u} \leftarrow \text{MOE}(\pi_{k,u})$

  $\mathbf{X}_{k,u} \leftarrow \text{MC}(\pi_{k,u}, L)$ ▷ Draw Monte Carlo samples from $\pi_{k,u}$.

  $\mathcal{V}(u) \leftarrow \frac{1}{L}\sum_{X \in \mathbf{X}_{k,u}} \bar{d}^{(c)}_p(X, \bar{X}_{k,u})$ ▷ Compute the cost

end for

$\overset{\star}{u} \leftarrow \text{argmin}_u\, \mathcal{V}(u)$ ▷ Decision making for sensor management

Algorithm 2: Obtaining Multiobject Estimates of a Multi-Bernoulli Distribution.

INPUTS: probabilities of existence $\mathbf{r} = \{r^{(1)}, \ldots, r^{(M)}\}$, and Bernoulli component particles and their weights $\mathbf{P} = \left\{\{(w^{(i,j)}, x^{(i,j)})\}_{j=1}^{L^{(i)}}\right\}_{i=1}^{M}$.

OUTPUTS: The multi-object estimates of the given multi-Bernoulli distribution.

function MOE($\mathbf{r}$, P)

  $\hat{M} \leftarrow \lceil \sum_{i=1}^{M} r^{(i)} \rceil$ ▷ Cardinality estimation, note that $\hat{M} = |\hat{X}|$.

  $[\mathbf{r}, I] \leftarrow -\text{ SORT }(-\mathbf{r})$ ▷ Sort r's in descending order and save the indices.

  $\hat{X} \leftarrow \varnothing$

  for $i = 1, \ldots, \hat{M}$ **do**

    $x \leftarrow \sum_{j=1}^{L^{(I_i)}} w^{(I_i,j)} x^{(I_i,j)}$

    $\hat{X} \leftarrow \hat{X} \cup \{x\}$

  end for

  return $\hat{X}$

end function

The pseudocode of a general OSPA-based sensor management method is presented in Algorithm 1. Step-by-step algorithms for functions MOE $(\pi_{k,u})$ and MC $(\pi_{k,u}, L)$, which return the multiobject estimates and L samples for a multi-Bernoulli distribution, are outlined in Algorithms 2 and 3.

V. LMB IMPLEMENTATION

The sensor management solution presented in Algorithm 1 can be used with any Bayesian multitarget filter. To demonstrate its performance, the implementation of the algorithm with an LMB filter [21] is presented. Consider an LMB prior (obtained after prediction step of the filter) with components $\pi = \{(r^{(\ell)}, p^{(\ell)}) : \ell \in \Psi\}$. The prediction step in the LMB filter is identical to the original multi-Bernoulli filter [21]. For the purpose of sensor management, the label information can be discarded, and Algorithm 1 be implemented with the *unlabeled* version of the predicted LMB distribution as its input distribution $\pi_{k|k-1}(\cdot)$.

Algorithm 3: Monte Carlo Sampling of a Multi-Bernoulli Distribution With Given Parameters and Particles.

INPUTS: probabilities of existence $\mathbf{r} = \{r^{(1)}, \ldots, r^{(M)}\}$, Bernoulli component particles and their weights $\mathbf{P} = \left\{\{(w^{(i,j)}, x^{(i,j)})\}_{j=1}^{L^{(i)}}\right\}_{i=1}^{M}$, and number of Monte Carlo set samples L.

OUTPUTS: A set $\mathbf{X}$ comprised of L sets, each being a Monte Carlo sample of the multi-Bernoulli distribution, in the form of $X_\ell = \{x_{\ell,1}, \ldots, x_{\ell,n_\ell}\}$.

function MC($\mathbf{r}$, P, L)

  $\mathbf{X} \leftarrow \varnothing$

  for $\ell = 1, \ldots, L$ **do**

    $X_\ell \leftarrow \varnothing$

    for $i = 1, \ldots, M$ **do**

      $u \sim \mathcal{U}(0, 1)$. ▷ Generate a random number in [0,1].

      if $u < r_i$ **then**

        $v \sim \mathcal{U}(0, 1)$.

        $j \leftarrow \lceil L_{\max} v \rceil$. ▷ Index j randomly generated in $[1, L_{\max}]$.

        $X_\ell \leftarrow X_\ell \cup \{x_{ij}\}$.

      end if

    end for

    $\mathbf{X} \leftarrow \mathbf{X} \cup \{X_\ell\}$ ▷ The set of MC sets is gradually completed.

  end for

  return X

end function

Full implementation of Algorithm 1 needs the two functions, MOE and MC to be implemented. The construct and design of these functions depend on the multiobject filter used in the application. Let us denote the predicted (unlabeled) multi-Bernoulli density by $\{r^{(i)}_{k|k-1}, p^{(i)}_{k|k-1}\}_{i=1}^{M_{k|k-1}}$ in which each density is approximated by weighted particles. The MOE estimate of the multitarget is computed as outlined in Algorithm 2 which is based on the method devised in [31]. First, the EAP estimate of cardinality is computed. Then, the probabilities of existence are sorted and the state estimate for each ith object is calculated.

The function MC in Algorithm 1 is to return Monte Carlo samples of the pseudoupdated multitarget density. When the unlabeled predicted multi-Bernoulli distribution is pseudoupdated (using the PIMS), the resulting posterior is still an unlabeled multi-Bernoulli. Sampling this distribution is straightforward if its particles are *resampled* in the sense that the resampled particles all share the same weights. Algorithm 3 shows the pseudocode suggested for implementation of Monte Carlo sampling of a multi-Bernoulli distribution. Let us denote the input distribution parameters by $\{r^{(i)}_k, p^{(i)}_k(\cdot)\}_{i=1}^{M_k}$, where the distribution of each Bernoulli component $p^{(i)}(\cdot)$ is approximated by $L^{(i)}$ particles, i.e., $p^{(i)}(x) \approx \sum_{j=1}^{L^{(i)}} w^{(i,j)}\, \delta_{x^{(i,j)}}(x)$. After resampling, each Bernoulli component is approximated by $L_{\max}$ particles with identical weights $w^{(i,j)} = 1/L_{\max}$. To construct each Monte Carlo sample set, we sample from M uniform distributions in [0,1] and compare the sample with the probabilities of existence to determine if the corresponding Bernoulli component is represented in the sample set. We then sample the single Bernoulli distribution (by randomly choosing one of its resampled particles) of those Bernoulli components which are determined to exist in the sample set.

Algorithm 4: The LMB Multitarget Filtering Recursion With OSPA-Based Sensor Management.

INPUTS: time k, dynamic model $f_{k|k-1}(\cdot|x_{k-1})$, LMB birth model parameters, prior LMB parameters and particles from time $k-1$, detection probability $p_D(\cdot)$, measurement likelihood function $g_k(\cdot|x,u)$, and clutter intensity $\nu(\cdot)$ and its integral λ_c, current sensor states or sensor node choices, finite set of admissible commands $\mathbb{U}$, number of resampled particles $L_{\max}$, number of Monte Carlo set samples L, and OSPA parameters p and c.

OUTPUT: The best action (selection or control) command $\overset{\star}{u}$ and updated multi-Bernoulli parameters.

Prediction:

Compute the predicted LMB component parameters and particles, based on the given prior and birth parameters and particles and dynamic model and survival probability. ▷ Refer to [21] for equations.

$\mathbf{r}_{k|k-1} \leftarrow \{r^{(1)}_{k|k-1}, \ldots, r^{(M_{k|k-1})}\}$

$\mathbf{P}_{k|k-1} \leftarrow \left\{\{(w^{(i,j)}_{k|k-1}, x^{(i,j)}_{k|k-1})\}_{j=1}^{L^{(i)}_{k|k-1}}\right\}_{i=1}^{M_{k|k-1}}$

Pre-estimation:

$\hat{X}_{k|k-1} \leftarrow \mathrm{MOE}(\mathbf{r}_{k|k-1}, \mathbf{P}_{k|k-1})$

$\hat{M}_{k|k-1} \leftarrow |\hat{X}_{k|k-1}|$

Sensor Management:

for all $u \in \mathbb{U}$ **do**,

$\quad \mathring{Z} \leftarrow \varnothing$. ▷ Constructing the PIMS ...

$\quad$ **for** $i = 1, \ldots, \hat{M}_{k|k-1}$ **do**

$\quad\quad \xi \leftarrow \underset{z}{\operatorname{argmax}}\, g(z|\hat{x}^{(i)}_{k|k-1}, u)$

$\quad\quad \mathring{Z} \leftarrow \mathring{Z} \cup \{\xi\}$

$\quad$ **end for** ▷ PIMS constructed.

▷ Update for sensor control:

$\quad$ Update the unlabelled multi-Bernoulli with $\mathring{Z}$ as the measurement set. ▷ Refer to [31] for update equations.

$\quad$ Resample $L_{\max}$ particles from each updated Bernoulli component.

▷ Note that updated multi-Bernoulli parameters are dependent on u.

$\quad \mathbf{r}_{k,u} \leftarrow \{r^{(i)}_{k,u}\}_{i=1}^{M_{k,u}}$ ▷ Existence probabilities

$\quad \mathbf{P}_{k,u} \leftarrow \left\{\{x^{(i,j)}_{k,u}\}_{j=1}^{L_{\max}}\right\}_{i=1}^{M_{k,u}}$ ▷ Resampled Particles

$\quad \bar{X}_{k,u} \leftarrow \mathrm{MOE}(\mathbf{r}_{k,u}, \mathbf{P}_{k,u})$ ▷ Compute the mean of the updated posterior

$\quad \mathbf{X}_{k,u} \leftarrow \mathrm{MC}(\mathbf{r}_{k,u}, P_{k,u}, L)$ ▷ Monte-Carlo sample the updated posterior

$\quad \mathcal{V}(u) \leftarrow \sum_{X \in \mathbf{X}_{k,u}} \frac{1}{L} \bar{d}^{(c)}_p(X, \bar{X}_{k,u})$ ▷ Compute dispersion of posterior around its mean.

end for

$\overset{\star}{u} \leftarrow \operatorname{argmin}_u \hat{\mathcal{V}}(u; X_k)$ ▷ Sensor management concluded.

Measurement:

Apply the action command $\overset{\star}{u}$ and read the actual measurement set Z_k accordingly.

Update:

Update the LMB distribution parameters ad particles using the actual measurement set Z_k. ▷ Refer [21] for update equations.

Algorithm 4 shows the complete pseudocode for multitarget tracking using an LMB filter with OSPA-based sensor management. It is important to note that unlabeled multi-Bernoulli distributions are only calculated within the sensor management routine. In each iteration of multitarget filtering, once the best sensor action command $\overset{\star}{u}$ is found and sensors are managed accordingly, an actual measurement set from sensors is acquired and used to update the predicted LMB distribution, resulting in an updated LMB posterior. In our simulations, for the pseudoupdate operations needed by Algorithm 4, we have used the update formula of cardinality-balanced multi-Bernoulli filter [31], as well as the update formula of the LMB filter [21] for the final LMB posterior.

VI. SIMULATION RESULTS

The performance of the proposed sensor management method was evaluated and compared with the recently published PEECS-based method [7], [9] in two case studies, which involved sensor control for tracking up to 15 targets that born and die in various times within $k \in [1, 40]$. In the first case study, each single target state $x = [\mathrm{x}\ \mathrm{y}\ \dot{\mathrm{x}}\ \dot{\mathrm{y}}]^\top$ comprised position and velocity. Each target state evolved according to the constant velocity model with the model parameters copied from [9] for the sake of fair comparison. The birth RFS was a multi-Bernoulli with 16 components whose probabilities of existence were all 0.03 and densities were Gaussian with their means chosen in various points within the target range. In generating the ground truth, the velocities were chosen small-valued so that pseudostationary targets are realized in points approximately around a circle. The rationale behind this choice is explained later in this section.

The observation model consisted of noisy bearing and range measurements, $z_k = [\arctan(\mathrm{x}_k/\mathrm{y}_k)\ (\mathrm{x}_k^2 + \mathrm{y}_k^2)^{1/2}]^\top + \zeta_k$, where $\zeta_k \sim \mathcal{N}(\cdot; 0, R_k)$ is the measurement noise with covariance $R_k = \mathrm{diag}(\sigma_\theta^2, \sigma_r^2)$ in which the scales of range and bearing noise are $\sigma_r = \sigma_0 + \eta_r ||X - u||^2$ and $\sigma_\theta = \theta_0 + \eta_\theta ||\mathrm{X} - \mathrm{u}||$, where $\sigma_0 = 1$ m, $\eta_r = 5 \times 10^{-5}\ \mathrm{m}^{-1}$, $\theta_0 = \pi/180$ rad, $\eta_\theta = 1 \times 10^{-5}\ \mathrm{m}^{-1}$. Measurements were synthetically generated as sets, each containing target-generated point measurements and possibly clutter and misses. In each set, every target could be detected with the distance-dependent detection probability

$$p_D(\mathbf{x}_s, \mathbf{x}_t) = \begin{cases} 0.99, & \text{if } ||\mathbf{x}_t - \mathbf{x}_s|| \leq R_0 \\ \max\{0, 0.99 - \mathfrak{h}(||\mathbf{x}_t - \mathbf{x}_s|| - R_0)\} & \text{otherwise} \end{cases}$$

where $\mathbf{x}_s, \mathbf{x}_t$ denote the sensor and target location vectors, respectively, $R_0 = 400$ m and $\mathfrak{h} = 25 \times 10^{-5}\ \mathrm{m}^{-1}$. Clutter measurements were generated based on the uniform Poisson model in the surveillance area $[-\pi/2, \pi/2]\ \mathrm{rad} \times [0, 2000]$ m, with the intensity $\lambda_c = 1.6 \times 10^{-3}\ (\mathrm{rad\ m})^{-1}$. The sensor initial position was at [0, 0]. The set of admissible control command is computed as it is explained in [11], [24], and [25]. Targets entered the scene with different initial positions, velocities, and turning rates.

As the accuracy of observations depends directly on the sensor-target distance, the sensor control routine is expected to lead to the sensor being gradually guided toward the middle of the pseudostationary targets, and remain in vicinity of the targets. In this sense, choosing pseudostationary targets distributed around a circle made it straightforward to visibly evaluate the performance of sensor control. Fig. 2 presents the results of OSPA-based versus PEECS-based

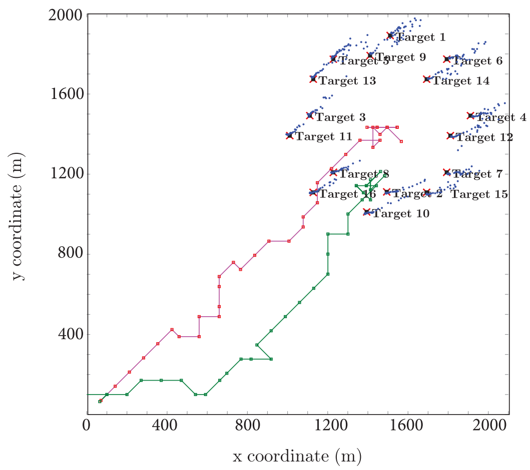

Fig. 2. Sensor locations and target tracks returned by the LMB filter equipped with sensor control, in case study 1. Red line-squares: Sensor tracks with OSPA-based sensor control. Green line-squares: Sensor tracks with PEECS-based sensor control. Black solid squares: Ground-truth location of pseudo-stationary targets. Black crosses: Target tracks with OSPA-based method. Blue circles: Target locations returned by the PEECS-based method.

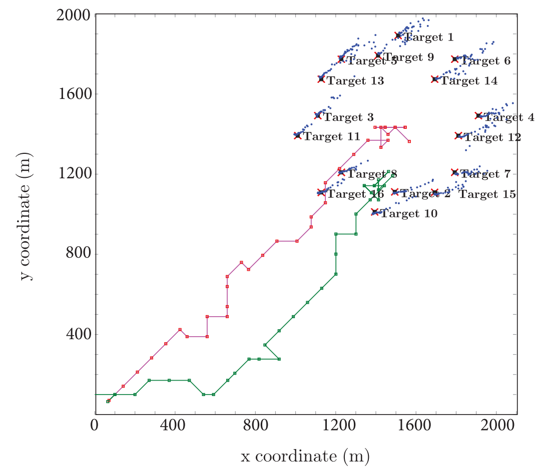

Fig. 4. Sensor locations and target tracks returned by the LMB filter equipped with sensor control, using range-only measurements in case study 2. Red line-squares: Sensor tracks with OSPA-based sensor control. Green line-squares: Sensor tracks with PEECS-based sensor control. Black solid lines: Ground-truth target tracks. Black crosses: Target tracks with OSPA-based method. Blue circles: Target tracks with PEECS-based method.

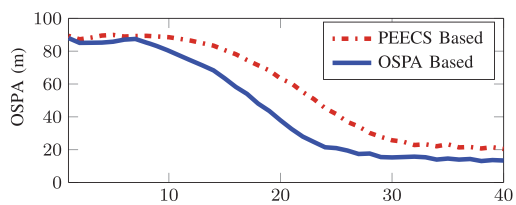

Fig. 3. OSPA errors returned by OSPA-based sensor control versus the PEECS-based method in case study 1.

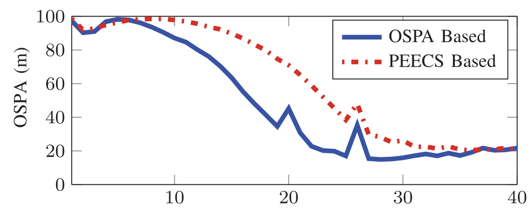

Fig. 5. OSPA errors returned by OSPA-based sensor control versus the PEECS-based method in case study 2.

sensor control in this scenario. It demonstrates how with both methods, the sensor moves from the initial position and as expected, after a few steps, remains among the targets. But the proposed OSPA-based method drives the sensor closer to the center of the circle. Hence, it leads to lower OSPA errors compared to the PEECS-based sensor control method—see Fig. 3.

In the second case study, the targets are not pseudostationary and maneuver with relatively large turn rates. Each single target state $x = [\bar{x}^\top\ \omega]^\top$ comprised position and velocity, denoted by $\bar{x} = [\mathrm{x}\ \mathrm{y}\ \dot{\mathrm{x}}\ \dot{\mathrm{y}}]^\top$ and turning rate, denoted by ω. The target dynamics follows a constant turn model [7]. In this case, the observations only include range, with the detection probability and noise parameters being sensor-target dependent similar to the first case study. Again, for the sake of fair comparison, motion and measurement parameters are chosen the same as in [9].

Fig. 4 shows the targets' movements and the tracks returned by the LMB filter with OSPA-based sensor control versus the PEECS -based sensor control. It clearly demonstrates that the OSPA-based sensor control method proposed in this paper guides the sensor toward the center of the targets, while the PEECS-based method leads to sensor locations that are relatively distant from the center. As a result, the estimated target tracks by the proposed method are observed to closely match the ground-truth tracks, while the tracks returned by the LMB filter with PEECS-based sensor control significantly deviate from the true tracks. This deviation can be quantified in terms of the OSPA error, as shown in Fig. 5, which demonstrates significant improvement in terms of tracking error when the proposed sensor control method is used. Note that in this scenario we included birth and death process which caused two peaks on the graph at time 20 and 25.

VII. CONCLUSION

A new sensor management method for multitarget filtering was presented. In the proposed method, sensor actions are selected to maximize the confidence in multitarget state estimation. The dispersion of the multitarget posterior about its statistical mean was employed as an intuitive choice for measuring confidence in estimation. We proposed to use the well-known OSPA metric as a suitable distance for quantifying the dispersion of posterior about its mean. We examined the performance of the proposed OSPA-based method in comparison with the recently developed PEECS-based sensor control. Both methods have same approach and aim to minimize the dispersion of the estimation for a number of targets and locations. The results demonstrate that the proposed method guides the sensor successfully producing excellent tracking results (in terms of OSPA errors) and outperforms the recently published PEECS-based sensor control method. It should also be noted that performances of these methods somewhat depend on the chosen values of their user-defined thresholds. The selection of an appropriate cut-off parameter for the proposed OSPA-based method is easier as explained in [27].

REFERENCES

[1] M. Beard and S. Arulampalam
Performance of PHD and CPHD filtering versus JIPDA for bearings-only multi-target tracking
In *Proc. 15th Int. Conf. Inf. Fusion* Jul. 2012, pp. 542–549.

[2] F. Bolderheij, F. G. J. Absil, and P. van Genderen
A risk-based object-oriented approach to sensor management
In *Proc. 7th Int. Conf. Inf. Fusion* 2005, pp. 1–8, doi: 10.1109/ICIF.2005.1591909.

[3] D. Crouse, P. Willett, and Y. Bar-Shalom
Developing a real-time track display that operators do not hate
IEEE Trans. Signal Process. vol. 59, no. 7, pp. 3441–3447, Jul. 2011.

[4] C. Fantacci, B.-T. Vo, F. Papi, and B.-N. Vo
The marginalized δ-GLMB filter
Jan. 2015. [Online]. Available: https://arxiv.org/pdf/1501.00926.pdf

[5] A. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar
Multi-Bernoulli sensor control for multi-target tracking
In *Proc. IEEE 8th Int. Conf. Intell. Sensors Sensor Netw. Inf. Process.* 2013, pp. 312–317.

[6] A. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar
Robust multi-Bernoulli sensor selection for multi-target tracking in sensor networks
IEEE Signal Process. Lett. vol. 20, no. 12, pp. 1167–1170, Dec. 2013.

[7] A. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar
Sensor control for multi-object tracking using labeled multi-Bernoulli filter
In *Proc. Int. Conf. Inf. Fusion* Jul. 2014, pp. 1–8.

[8] A. Gostar, R. Hoseinnezhad, A. Bab-Hadiashar, and B.-T. Vo
Control of sensor with unknown clutter and detection profile using multi-Bernoulli filter
In *Proc. Int. Conf. Inf. Fusion* 2013, pp. 1021–1028.

[9] A. K. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar
Multi-Bernoulli sensor-control via minimization of expected estimation errors
IEEE Trans. Aerosp. Electron. Syst. vol. 51, no. 3, pp. 1762–1773, Jul. 2015.

[10] H. G. Hoang, B.-N. Vo, B.-T. Vo, and R. Mahler
The Cauchy-Schwarz divergence for Poisson point processes
IEEE Trans. Inf. Theory vol. 61, no. 8, pp. 4475–4485, Aug. 2015.

[11] H. G. Hoang and B. T. Vo
Sensor management for multi-target tracking via multi-Bernoulli filtering
Automatica vol. 50, no. 4, pp. 1135–1142, 2014.

[12] F. Katsilieris
Sensor management for surveillance and tracking: An operational perspective
Ph.D. thesis, TU Delft, Delft Univ. Technol., Delft, The Netherlands, 2015.

[13] F. Katsilieris, H. Driessen, and A. Yarovoy
Threat-based sensor management for target tracking
IEEE Trans. Aerosp. Electron. Syst. vol. 51, no. 4, pp. 2772–2785, Oct. 2015.

[14] C. Kreucher, K. Kastella, and A. O. Hero
Multitarget sensor management using alpha divergence measures
In *Proc. IEEE Conf. Inf. Process. Sensor Netw.* 2003, pp. 209–222.

[15] R. Mahler
Global Posterior Densities for Sensor Management Proc. SPIE vol. 3365, pp. 252–263, 1998.

[16] R. Mahler
Objective functions for Bayesian control-theoretic sensor management, 1: Multitarget first-moment approximation
In *Proc. IEEE Aerosp. Conf.* 2003, vol. 4, pp. 1905–1923.

[17] R. Mahler
Multitarget Sensor Management of Dispersed Mobile Sensors (Series on Computers and Operations Research). Singapore: World Scientific, 2004.

[18] R. Mahler
Advances in Statistical Multisource-Multitarget Information Fusion. Norwood, MA, USA: Artech House, 2014.

[19] R. Mahler and T. R. Zajic
Probabilistic objective functions for sensor management
Proc. SPIE vol. 5429, pp. 233–244, 2004.

[20] M. Mallick, V. Krishnamurthy, and B. Vo
Integrated Tracking, Classification, and Sensor Management: Theory and Applications. Hoboken, NJ, USA: Wiley, 2012.

[21] S. Reuter, B.-T. Vo, B.-N. Vo, and K. Dietmayer
The labeled multi-Bernoulli filter
IEEE Trans. Signal Process. vol. 62, no. 12, pp. 3246–3260, Jun. 2014.

[22] S. Reuter, B.-T. Vo, B. Wilking, D. Meissner, and K. Dietmayer
Divergence detectors for the δ-generalized labeled multi-bernoulli filter
In *Proc. Workshop Sensor Data Fusion, Trends, Solutions, Appl.* Oct. 2013, pp. 1–6.

[23] B. Ristic
Particle Filters for Random Set Models. New York, NY, USA: Springer, 2013.

[24] B. Ristic and B.-N. Vo
Sensor control for multi-object state-space estimation using random finite sets
Automatica vol. 46, no. 11, pp. 1812–1818, 2010.

[25] B. Ristic, B.-N. Vo, and D. Clark
A note on the reward function for PHD filters with sensor control
IEEE Trans. Aerosp. Electron. Syst. vol. 47, no. 2, pp. 1521–1529, Apr. 2011.

[26] B. Ristic, B.-N. Vo, D. Clark, and B.-T. Vo
A metric for performance evaluation of multi-target tracking algorithms
IEEE Trans. Signal Process. vol. 59, no. 7, pp. 3452–3457, Jul. 2011.

[27] D. Schuhmacher, B.-T. Vo, and B.-N. Vo
A consistent metric for performance evaluation in multi-object filtering
IEEE Trans. Signal Process. vol. 56, no. 8, pp. 3447–3457, Aug. 2008.

[28] B.-N. Vo, B.-T. Vo, and D. Phung
Labeled random finite sets and the Bayes multi-target tracking filter
IEEE Trans. Signal Process. vol. 62, no. 24, pp. 6554–6567, Dec. 2014.

[29] B.-T. Vo and B.-N. Vo
Labeled random finite sets and multi-object conjugate priors
IEEE Trans. Signal Process. vol. 61, no. 13, pp. 3460–3475, Jul. 2013.

[30] B.-T. Vo, B.-N. Vo, and A. Cantoni
Analytic implementations of the cardinalized probability hypothesis density filter
IEEE Trans. Signal Process. vol. 55, no. 7, pp. 3553–3567, Jul. 2007.

[31] B.-T. Vo, B.-N. Vo, and A. Cantoni
The cardinality balanced multi-target multi-Bernoulli filter and its implementations
IEEE Trans. Signal Process. vol. 57, no. 2, pp. 409–423, Feb. 2009.

[32] J. Witkoskie, W. Kuklinski, S. Theophanis, and M. Otero
Random set tracker experiment on a road constrained network with resource management
In *Proc. 9th Int. Conf. Inf. Fusion.* Jul. 2006, pp. 1–7.

[33] A. Zatezalo, A. El-Fallah, R. Mahler, R. K. Mehra, and K. Pham
Joint search and sensor management for geosynchronous satellites, " *Proc. SPIE* vol. 6968, 2008, Art. no. 69680O.

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