Freelance Project

Robust Gust Load Reduction of an Aircraft using $H\infty$ Control

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Chapter 1

H∞ Problem Formulation

This chapter describes problem formulation using mixed sensitivity functions to design a robust H∞ controller. Guidelines for choosing suitable weighting functions for the sensitivity functions are given followed by steps to design the controller using Matlab. The steps are then applied to design a controller for gust load reduction of an aircraft.

1.1 Mixed Sensitivity Problem

The system to be controlled (P), in closed-loop connection is depited in fig1.1.

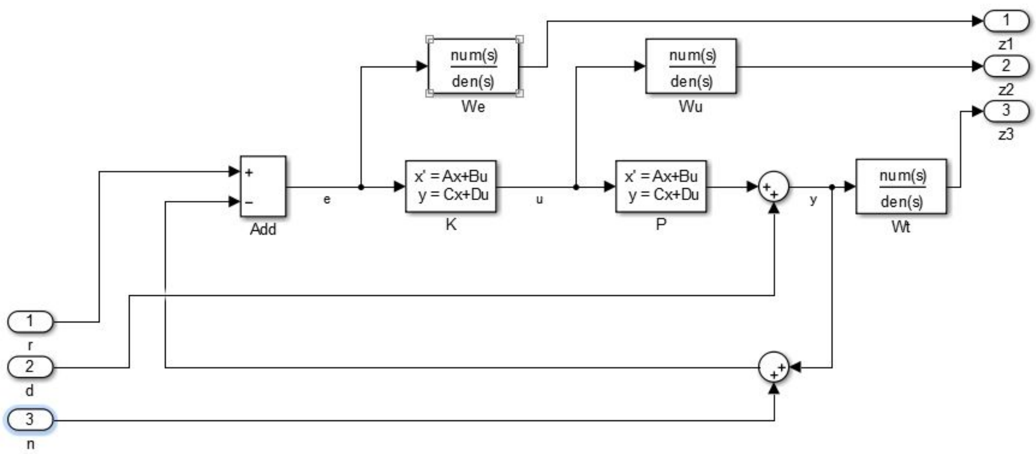

Figure 1.1: Close-loop system (mixed sensitivity)

Where, r is reference input, d is output disturbance and n is measurement noise. K is controller and W_e, W_u and W_t represent weighting functions. z_1, z_2 and z_3 are the weighted version of the signals that need to be controlled. The plant (P) is represented by the transfer function,

$$P = C(sI - A)^{-1}B + D \tag{1.1}$$

The sensitivity and complementary sensitivity function of feedback system is,

$$\begin{aligned} S &= (1 + KP)^{-1} && (1.2) \\ T &= KP(1 - KP)^{-1} = 1 - S && (1.3) \end{aligned}$$

It can be shown that:

- S is the transfer function from r to e, d to y and d to e. Its frequency response should be high pass to reduce the error signal and attenuate the disturbance which is usually low frequency signal.

- K*S is the transfer function from r to u, n to u and d to u. It should be a low pass filter with controlled gain to prevent saturation of the plant and to attenuate the effect of measurement noise which is usually high pass.

- T is transfer function from r to y, n to e and n to y. It should be a low pass filter to allow tracking of low frequency reference signal and to attenuate high frequency noise.

- Naturally, it is required to reject the disturbances, measurements noise and attenuate error signal for all the band of frequencies, but it is not possible since all of these transfer functions cannot modified independently. Any value of K cannot simultaneously convert them into all frequency attenuators. Hence, specific frequency bands based on the behaviour of signals are chosen. Sometimes, a trade off is made between these functions. It may happen that there is steady state error at output at the cost of low energy consumption, disturbance and noise rejection. In this case gain can be applied to reference signal without effecting over all performance of system.

The sensitivity function S depends on frequency and in ideal case should be zero. In practical applications, for certain range of frequencies, an upper bound on peak value of S is specified. H∞ is actually the peak amplitude of frequency response of a transfer function and can be represented as,

$$\|S\|_\infty = \sup_{\omega \in R} |S(j\omega)| \tag{1.4}$$

In order to reduce the dependence and represent the desired frequency response, a weight function $We(j\omega)$ is used as,

$$\|W_e S\|_\infty = \sup_{\omega \in R} |W_e(j\omega) S(j\omega)| \tag{1.5}$$

The problem of H_∞ controller synthesis can be formulated as: Given generalized plant, exogenous inputs, outputs and control specifications, find all admissible controllers K such that the H_∞ norm of transfer matrix from exogenous inputs to outputs is minimized, subjected to constraint that all K's stabilize the plant P. In practical applications it is sufficient to find a controller such that,

$$\|W_e(j\omega) S(\omega)\|_\infty < \gamma \tag{1.6}$$

and this problem is called H_∞ suboptimal problem. In design procedure it is appropriate to specify more than one performance index, expressed mathematically as,

$$\left\| \begin{matrix} W_e S \\ W_u K S \\ W_t T \end{matrix} \right\|_\infty < \gamma \tag{1.7}$$

The equation 4.1 defines problem of mixed sensitivity and it means a good tracking, limitation of control signal energy and insensitive to disturbance and noise. It can be seen that the inverse of weighting functions set design requirements for sensitivity functions i.e. $S < \frac{\gamma}{W_e}, KS < \frac{\gamma}{W_u}, T < \frac{\gamma}{W_t}$. This is the problem which is solved here. For given real number (γ), there exists an admissible controller, $K_{adm} = Y.X^{-1}$, if and only if the following conditions are met:

$$H_\infty \in dom(Ric) \ and \ X_\infty = Ric(H_\infty) \geq 0 \tag{1.8}$$

$$J_\infty \in dom(Ric) \ and \ Y_\infty = Ric(J_\infty) \geq 0 \tag{1.9}$$

$$\rho(X_\infty Y_\infty) < \gamma^2 \tag{1.10}$$

Where, H_∞, J_∞ are given in [1] and Ric refers to algebric riccati equation. If conditions are met then,

$$K_{adm} = \begin{bmatrix} A_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \tag{1.11}$$

Where, $A_\infty, Z_\infty, L_\infty \ and \ F_\infty$ are given in [1].

Chapter 2

Selection of Weighting Functions

Since S should be high pass, $\frac{1}{W_e}$ should also be high pass to set a bound on S. Then, W_e will be a low pass filter, which can be chosen as follows:

$$W_e = r_e \frac{\frac{s}{M_e} + \omega_{be}}{s + \omega_{be}\epsilon_e} \tag{2.1}$$

Where, M_e represents pass band peak, ω_{be} is the cutoff frequency, ϵ_e is the stop band attenuation and r_e is the additional gain for the magnitude. It should be noted that all these parameters represent the response of $\frac{1}{W_e}$ not W_e. Now, KS should be low pass, this means $\frac{1}{W_u}$ will also be low pass and W_e will be a high pass filter, which can be selected as follows:

$$W_u = r_u \frac{s + \frac{\omega_{bu}}{M_u}}{\epsilon_u + s\omega_{bu}} \tag{2.2}$$

Where, M_u represents pass band peak, ω_{bu} is the cutoff frequency, ϵ_u is the stop band attenuation and r_u is the additional gain for the magnitude of $\frac{1}{W_u}$. The weighting function W_t can be chosen in similar fashion as W_u. Following points should be noted while choosing these parameters,

- ω_b should be chosen after the frequency analysis of the signal attenuated by the corresponding sensitivity function. For example, if d lies in the band from 0 to 10Hz, then ω_{be} should be greater than 10.
- M should be chosen based on how much amplification is required for the signals that are allowed to pass. Usually, its value is 1 or slightly greater than 1. M_t can be adjusted to make steady state error equal to 0.
- ϵ should be chosen based on the required attenuation on the signal to be rejected. For example, ϵ_{be} will decide attenuation of disturbance signal and should be slightly greater than 0.
- r adds an offset to the frequency response of a weighting function. It can be used for tuning of weighting functions and is usually equal to 1. It can also be used for setting priorities in case of more than 1 performance index, where, greater value of r shows greater priority.

Chapter 3

LFT of Closed Loop System

After choosing appropriate weighting functions, the next step to design $H\infty$ controller involves representation of the closed loop system as shown in fig3.1.

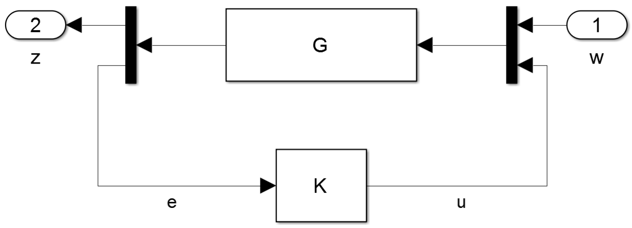

Figure 3.1: LFT of closed loop system

For mixed sensitivity problem, $z = [z_1\ z_2\ z_3]^T, w = [r\ d\ n]^T$. It can be seen that z is a vector of weighted versions of the signal that need to controlled, w is the vector of exogenous inputs acting on the system, y is the input to the controller and u is the output of the controller which is the control signal. The transfer matrix T_{zw} such that $[z \quad w]^T = T_{zw}[w \quad u]^T$ can be formed from Fig. 1.1,

$$
\begin{aligned}
z_1 &= W_e r - W_e d - W_e n - W_e P u & (3.1)\\
z_2 &= W_u u & (3.2)\\
z_3 &= W_t d + W_t P n; & (3.3)\\
e &= r - d - n - P u & (3.4)
\end{aligned}
$$

So,

$$
T_{zw} = \begin{bmatrix} W_e & -W_e & -W_e & -W_e P \\ 0 & 0 & 0 & W_u \\ 0 & W_t & 0 & W_t P \\ 1 & -1 & -1 & -P \end{bmatrix} \tag{3.5}
$$

The minimal realization of T_{zw} which is equal to G is obtained using Matlab. Once the above steps are completed, every thing is ready to achieve the $H\infty$ control solution using Matlab. Following command can be used for this purpose:

$$
[K, T_{zw}, \gamma] = hinfsyn(G, n_m, n_{con} min, max, tol) \tag{3.6}
$$

Where, n_m is the dimension of e, n_{con} is the dimension of u, min is the minimum value for γ, max is the maximum value for γ and tol is the relative error tolerance for γ. If the command gives error, max or weighting functions should be adjusted.

Chapter 4

Controller Design for Gust Load Reduction

This section applies the $H\infty$ control method described above to the aircraft model to minimize the effect of disturbance (gust of wind). The closed loop system used for design purpose is shown in Fig. 4.1. K represents the controller to be designed, P is the linearized aircraft model, d shows the gust of wind affecting the system, W_u and W_y are the weighting functions for the signals that need to be controlled which are u and y, respectively. It can be seen that e is not controlled, since it is just negative version of y.

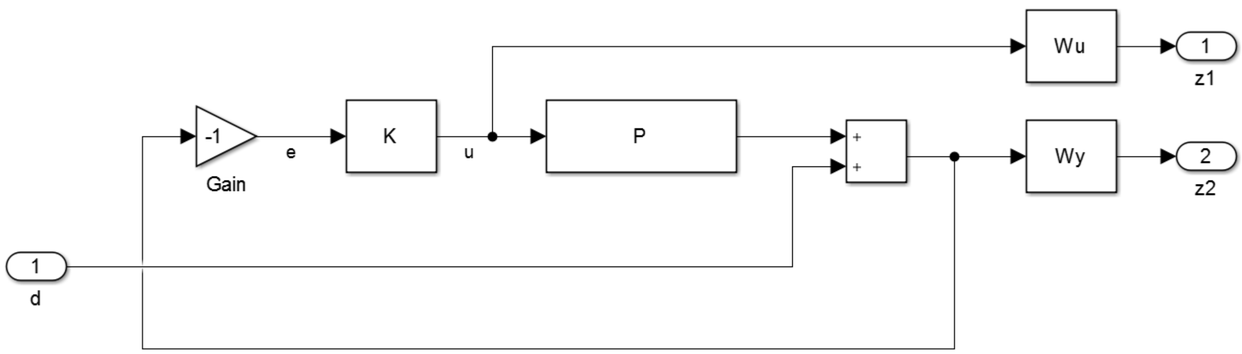

Figure 4.1: Closed loop system with aircraft model

This model has been found to represent the disturbance effects more accurately on the aircraft. Previously, when the design and simulation were performed by modeling disturbance in the input to the plant, the designed controller made the system unstable. Some important points about the model are:

- There are different inputs to the plant but for this case 4 inputs are used namely *elevators*, *symail*, *left spoiler* and *right spoiler*.
- The output chosen to be controlled is A_z, the acceleration in z-direction. But it has been found that choosing A_z poses difficulty in designing the controller. To solve this problem, A_z is divided into its linear and rotational part. Now, there are two outputs which are fed back, i.e, A_{zlin} and A_{zrot}.
- The plant P used for design is taken as a SISO model. This means that separate controllers

are designed for each input. Each input is fed by one of the outputs, A_{zlin} is fed to *elevator* input and A_{zrot} is fed to rest of the inputs.

- Keeping in view all the above points, four linearized plant models are obtained. So, total four controllers will be designed in this case.

The design steps for one of the controllers is given below which can also be applied to rest of the cases. The $H\infty$ problem formulated for this case can be represented as,

$$\left\| \begin{matrix} W_y S \\ W_u K S \end{matrix} \right\|_\infty < \gamma \tag{4.1}$$

Where, $S = (1 + KP)^{-1}$ is the transfer function from d to y and KS is the transfer function from d to u. For the input as *elevators* and output as A_{zrot}, the weighting functions are chosen as follows:

$$W_u = r_u \frac{s + \frac{\omega_{bu}}{M_u}}{\epsilon_u + s\omega_{bu}} \tag{4.2}$$

Where, $r_u = 1$, $M_u = 50$, $\epsilon_u = 0.1$ and $\omega_{bu} = 100$ and,

$$W_y = r_y \frac{\frac{s}{M_y} + \omega_{by}}{s + \omega_{by}\epsilon_e} \tag{4.3}$$

Where, $r_u = 1$, $M_u = 50$, $\epsilon_u = 0.1$ and $\omega_{bu} = 100$. The weighting functions are chosen based on the open loop response of A_{zrot}, under the disturbance whose frequency components are shown in Fig. 4.2. The functions are then tuned to give an appropriate response.

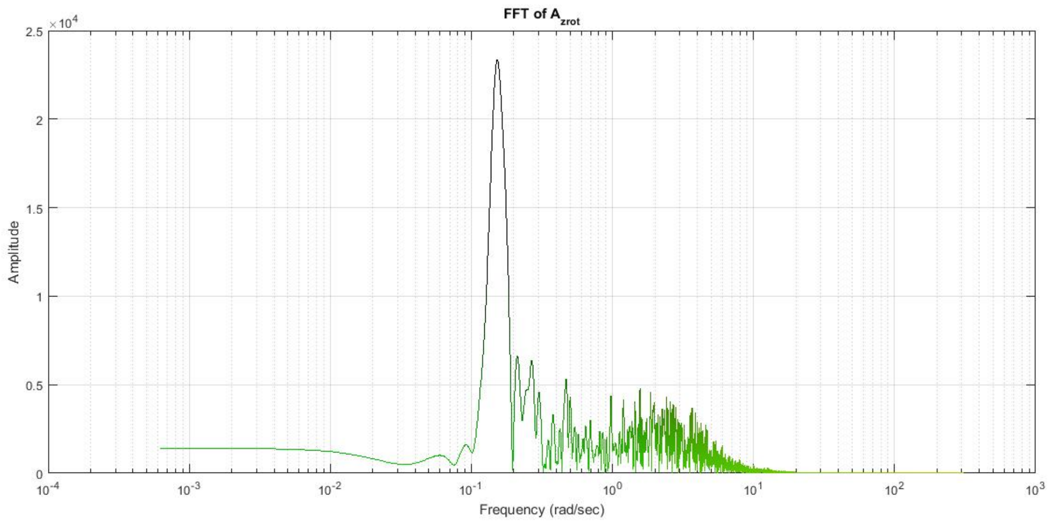

Figure 4.2: FFT of A_{zrot} in open loop with wind disturbance

The transfer matrix T_{zw} for the system can be given as,

$$T_{zw} = \begin{bmatrix} 0 & W_u \\ W_y & W_y P \\ -1 & -P \end{bmatrix} \tag{4.4}$$

After obtaining T_{zw}, Matlab is used to obatin the $H\infty$ controller as follows:

```
Gt=ss(Tzw);
G=minreal(Gt);
[K, CL, GAM]=hinfsyn(G, 1, 1, 0, 1e6, 0.0001);
```

Similarly, using these steps all four controllers can be designed. The results of closed loop system (using non-linear aircraft model) with the controller designed above are shown in Fig. 4.3.

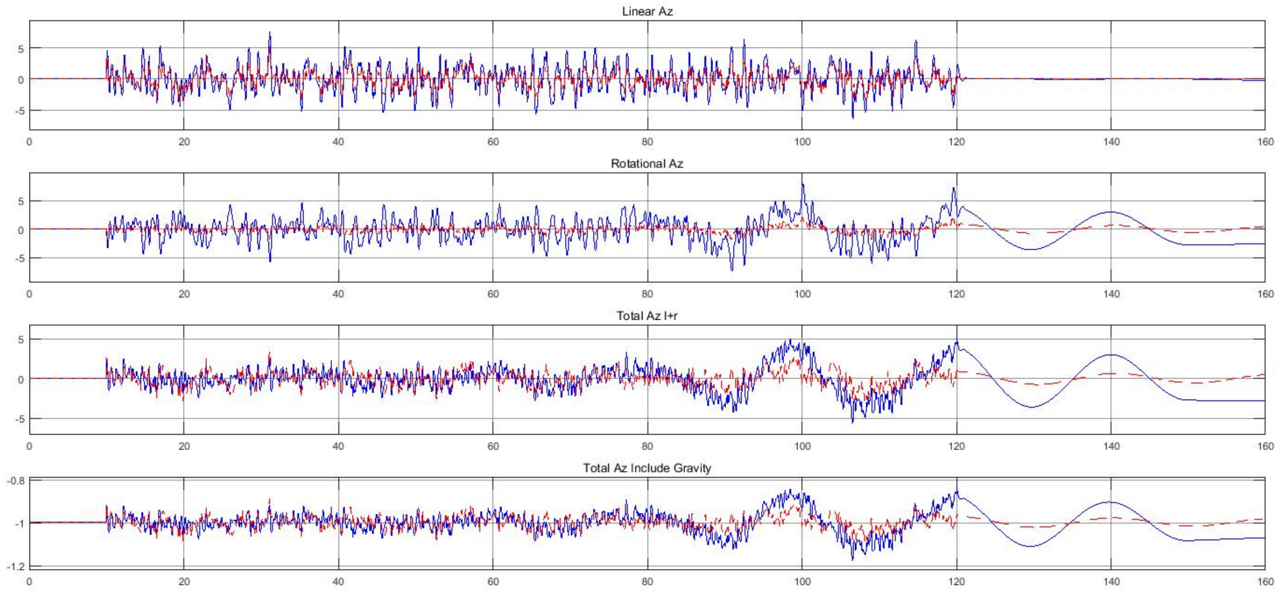

Figure 4.3: Simulation results for the designed controller

Bibliography

[1] Kemin Zhou and John Comstock Doyle. *Essentials of robust control*, volume 180. Prentice hall Upper Saddle River, NJ, 1998.