EE501 Term Project

Position Control of a Magnetically Levitated Ball Using PID Controller

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Contents

Abstract

This report includes the development of non-linear mathematical model of Magnetic Levitation System along with the design and implementation of a Linear controller to obtain the desired response from the system. The linear controller is designed by linearizing the model about the equilibrium point. Magnetic Levitation System in this study is taken as a ferromagnetic ball suspended below a voltage controlled magnet. The effect of variation of parameters in the models are also discussed. Dynamic behaviour of system is modeled by the study of Electromagnetic and Mechanical subsystems. The responses of both plant and unity feedback system with controller are plotted using matlab and results are presented.

Chapter 1

Introduction to Maglev System

1.1 Introduction

The Magnetic levitation system is an example of a nonlinear and open loop unstable system with fast dynamics. For these properties of the Magnetic levitation system, modeling and mainly control design is very difficult. However, Magnetic levitation system has wide applications in various industries. It is used in high-speed trains, frictionless bearing, levitation of wind models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces , levitation of metal slabs during manufacture, rocket-guiding projects, and supraconductor rotor suspension of gyroscopes [?][?][?][?][?]. Therefore this field of research has been devoted significant efforts in recent years. A straight forward approach to non-linear control design widely used is based on an approximate linear model found by perturbing the system dynamics about a desired operating point. As the linear model of system is valid only about a small area of operating point, the resulting linear controller can only be expected to function well in this small region. For example, if a disturbance acts on the system so that it strays (relatively) far from operating point, it can go unstable. Also, linear controllers typically require some kind of gain scheduling procedure to change operating points. In order to ensure very long ranges of operation and still obtain good tracking, it is necessary to consider a nonlinear model rather that a linear one. In addition, the plant parameter changes, such as the change of suspending mass and the variations of resistance and inductance due to electromagnet heating, should be taken into consideration.

Some authors have used nonlinear techniques to design stabilizing control laws. However, most of this work has been tested only in simulation and/or using inappropriate models in relation to magnetic and physics properties. For example, in [?], the author assumes that the magnetic force is proportional to the voltage of the electromagnetic winding and then proposes a control law using sliding mode when the set-point amplitude does not exceed 1 mm. In [?], the author uses a nonlinear model depending on the nominal operating point and proposes a control law design method based on a phase space. In [?], the control of suspension systems has been proposed using the gain-scheduling approach. Lairi and Bloch [?] propose a neural control lawfor magnetic sustentation systems. Slotine [?] assumes that the magnetic force is proportional to the square of the current and proportional inversely to the distance between the electromagnet and the ball. In [?], the authors use the model proposed in [?], and use a feedback linearization technique in the design of the control law. Then, the experiments are realized on a prototype with two electromagnets when the object in levitation travel is some micrometers. However, this model is not valid when the travel of the object in levitation is some centimeters.

In this report a linearized model is used to develop a PID controller for the system.

1.2 Apparatus Description

The MAGLEV considered in current analysis is a one-positional-degree-of-freedom (1-DOF) magnetic suspension system. The apparatus is shown in figure **??**. The system consists of a coil over a cylindrically shaped nonlaminated ferromagnetic core (soft steel) with a stainless steel ball levitated over approximately 4cm range of motion. A photo-sensor is embedded in a side wall opposite to 4 photo-emitters that determines ball position based on the intensity of photons.

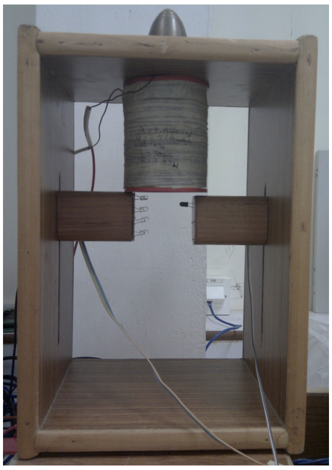

Figure 1.1: Maglev Apparatus

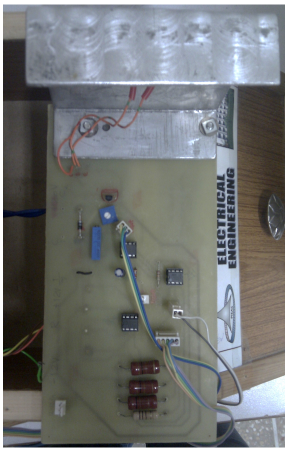

Figure 1.2: Electronics Circuit of Maglev

The Coil acts as an electromagnetic actuator. By regulating the electric current in the coil through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball and the air drag, thus the ball will levitate in an equilibrium state. The schematic diagram of system is shown in figure **??**.

1.3 Electronics of The System

The electronics portion of Maglev is shown in figure **??**. It forms a negative feedback system with controller in forward path. The error signal is generated by using AD620 instrumentation amplifier with unity gain. The use of instrumentation amplifier rejects any common mode noise and dc offset at the inputs. The coil is driven by a darlington pair to provide high current gain.

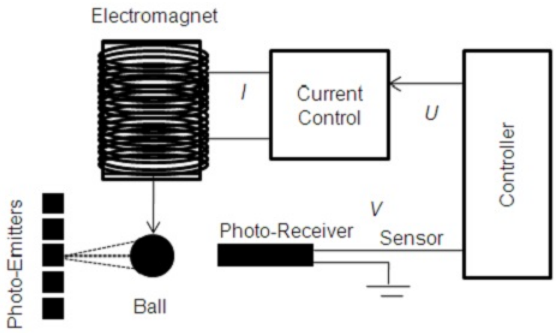

Figure 1.3: Schematic Diagram of System

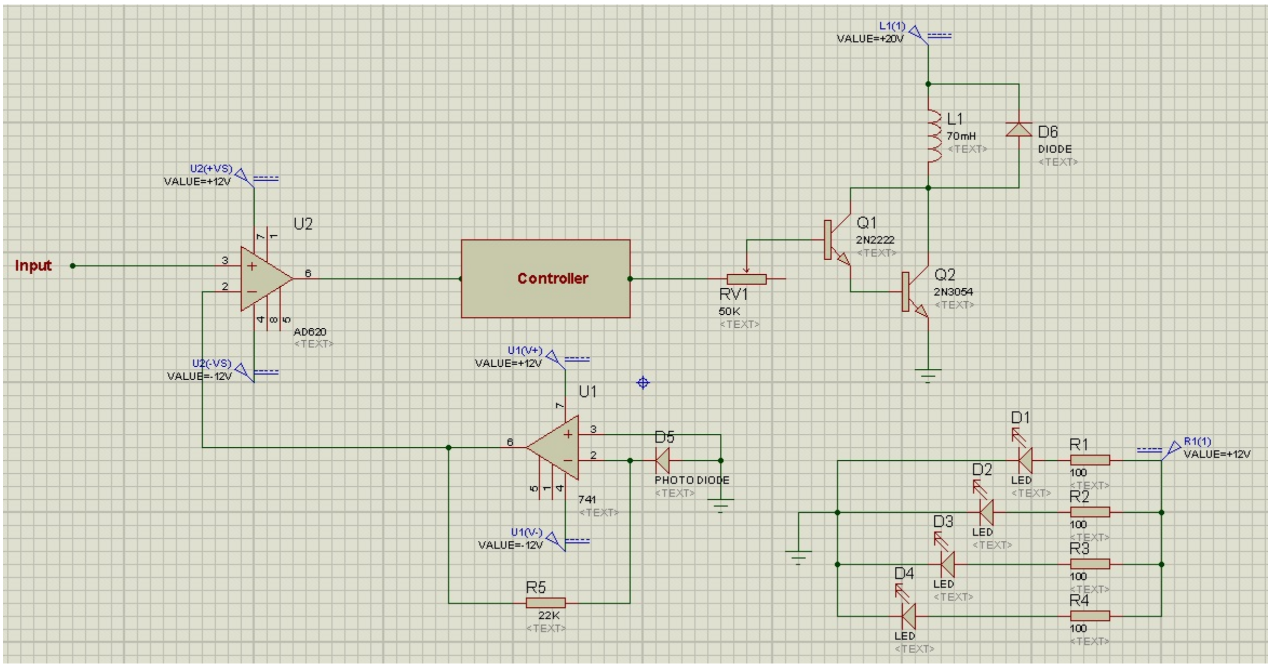

Figure 1.4: Schematic Diagram of Electronics Circuit

It is formed by 2N2222 which is a general purpose NPN BJT and 2N3305 which is an NPN power BJT. The coil is supplied by a +20V supply unlike rest of the circuit which is supplied by a supply of 12V. A free wheeling diode is connected in parallel with the coil to provide a path for reverse current. 4 photo-emitters and a photo-sensor is used to detect the position of the ball. The photo-sensor is connected to subtractor through an amplifier for isolation. The schematic diagram is shown in figure **??**.

Chapter 2

Mathematical Modelling of Maglev System

The free body diagram of Maglev is shown in figure **??**. x denotes the gap between coil and centre of the ball. $x = 0$ when the ball is in contact with the coil and it increases in the downward direction.

2.1 Modelling of Electromagnetic System

This part involves modelling of the electric circuit that drives the current through electromagnetic coil. From figure **??** the driver circuit consists of darlington pair that operates either in linear region or in cut off region if the voltage at input is too small to turn it on. The operation in cut off region is required since to increase x the coil must lose its current. This system can be approximated by an RL circuit since its simpler to model and any deviation can be adjusted by varying controller gain.

$$\begin{aligned} u &= V_R + V_L \\ u &= iR + \frac{dL(x)i}{dt} \end{aligned} \tag{2.1}$$

Where,
u=applied voltage
i=current in the coil of electromagnet
R=coil's resistance
L=coil's inductance

2.2 Modelling of Mechanical System

There are two forces acting on the ball, one is due to the mass of the ball while other is the magnetic force due to the attraction of the coil. The drag of air can be ignored since the ball moves small distance and it keeps the model simpler. The net force on the ball will be,

$$\begin{aligned} f_{net} &= f_g - f_{em} \\ m\ddot{x} &= mg - C(\frac{i}{x})^2[\textbf{?}] \end{aligned} \tag{2.2}$$

Where,
m=mass of the ball

$\ddot{x}$=acceleration of the ball
g=gravitational acceleration
i=current in the coil of electromagnet
C=magnetic force constant

It is clear that equation **??** is a non-linear function of i and x.

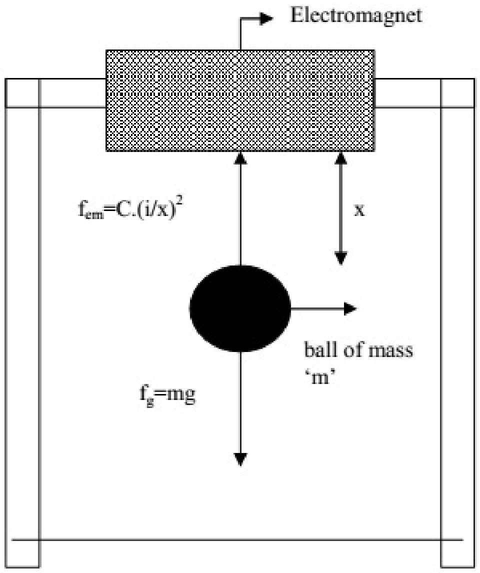

Figure 2.1: Free Body Diagram of System

Using approximation

$$L(x) = L_c + \frac{L_o x_o}{x}[?]$$

Where, L_c is the constant inductance in the absence of ball, L_o is the additional inductance contributed by the presence of ball and x_o is the equilibrium position. So equation **??** becomes

$$\begin{aligned} u &= i_R + \frac{d}{dt}\left(L_c + \frac{L_o x_o i}{x}\right) \\ &= i_R + L_c\frac{di}{dt} + L_o x_o\left(\frac{x\frac{di}{dt} - i\frac{dx}{dt}}{x^2}\right) \\ u &= i_R + L\frac{di}{dt} - 2C(\frac{i}{x^2})(\frac{dx}{dt}) \end{aligned} \tag{2.3}$$

Where,

$$2C = L_o x_o[?]$$

Writing equations **??**, **??** and **??** in vector form with,

$$x_1 = x,\ x_2 = \dot{x},\ x_3 = i$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{C}{m}(\frac{x_3}{x_1})^2 \\ -\frac{R}{L}x_3 + \frac{2C}{L}(\frac{x_2 x_3}{x_1^2}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u \tag{2.4}$$

The output equation is,

$$y = x_1$$

2.3 Linearization of The Model

The equation set ?? can be linearized by expanding non-linear terms in individual equations by multi-variable taylor series expansion about the point,

$$x_1 = x_{01},\ x_2 = 0 = x_{02},\ x_3 = x_{03}$$

Where,

$$x_{03} = x_{01}\sqrt{\frac{gm}{c}} \tag{2.5}$$

After simplification, the linearized model can be written in state space form as,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

Where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2C\frac{x_{03}^2}{mx_{01}^3} & 0 & -2C\frac{x_{03}}{mx_{01}^2} \\ 0 & 2C\frac{x_{03}}{Lx_{01}^2} & \frac{-R}{L} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The L is taken here to be constant for the purpose of linearization and is equal to L_c. It should be noted that the states here are absolute and not referenced to equilibrium state.

2.4 Calculation of Transfer Function

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2C\frac{x_{03}^2}{mx_{01}^3} & 0 & -2C\frac{x_{03}}{mx_{01}^2} \\ 0 & 2C\frac{x_{03}}{Lx_{01}^2} & \frac{-R}{L} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a & 0 & c \\ 0 & b & d \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ e \end{bmatrix}$$

Taking Laplace Transform of the equations,

$$sX_1 = X_2 \tag{2.6}$$

$$sX_2 = aX_1 + cX_3 \tag{2.7}$$

$$sX_3 = bX_2 + dX_3 + eU \tag{2.8}$$

By putting the Values of X_2 and X_3 from Equ. ?? and ?? in Equ. ?? we get

$$\begin{aligned} s^2X_1 &= aX_1 + c\left[\frac{bsX_1 + eU}{s-d}\right] \\ &= aX_1 + \frac{scbX_1 + ceU}{s-d} \\ X_1\left[s^2 - a - \frac{scb}{s-d}\right] &= \frac{ceU}{s-d} \\ \frac{X_1}{U} &= \frac{ce}{s^2(s-d) - as + ad - scb} \end{aligned}$$

$$G(s) = \frac{X_1}{U} = \frac{ce}{s^3 - s^2d - s(a+bc) + ad} \tag{2.9}$$

Where,

$$
\begin{aligned}
a &= 2C\frac{x_{03}^2}{mx_{01}^3}, \qquad & c = -2C\frac{x_{03}}{mx_{01}^2} & \\
b &= 2C\frac{x_{03}}{Lx_{01}^2}, \qquad & d = \frac{-R}{L}, \qquad & e = \frac{1}{L}
\end{aligned}
$$

Here it should be noted that X_1 and U are absolute measurements and not from equilibrium points but the transfer function is still valid only in the region near equilibrium position.

Chapter 3

Analysis and Design of Controller

3.1 Values of Parameters

- $m = 0.1Kg$
- $g = 9.8\frac{m}{s^2}$
- $L = 0.07H$
- $R = 3.85\Omega$
- $x_{01} = 0.02m$
- $x_{03} = 1.1431A$
- $C = 0.0003\frac{Nm^2}{A^2}$

The value of C is chosen arbitrarily from the possible range as seen from different references. The effect of changing values can be compensated by controller gain and more details about it are given in Appendix B.

3.2 Control Goal

The position of the ball is to be controlled using the voltage with following specification,
Maximum % Overshoot should not exceed 30%
Settling Time should be less than 1sec

3.3 Root Locus Plot of System without Controller

The open loop transfer function of the Maglev System shown in equation **??** can be used to check the stability of the system and if the system can become stable by just varying gain. x decreases as u is increased because ball is more attracted to the magnet. Due to this inverse relationship the root locus plot has the effect if the gain is negative as shown in figure **??**. Hence the Transfer Function is multiplied by -1 to counter that effect. The root locus of modified Transfer Function is shown if figure **??**. This shows that the controller in Maglev System will have a negative gain.

It can be seen that the system has 2 complex poles in the left half plane and 1 pole in the right half plane making the system unstable.

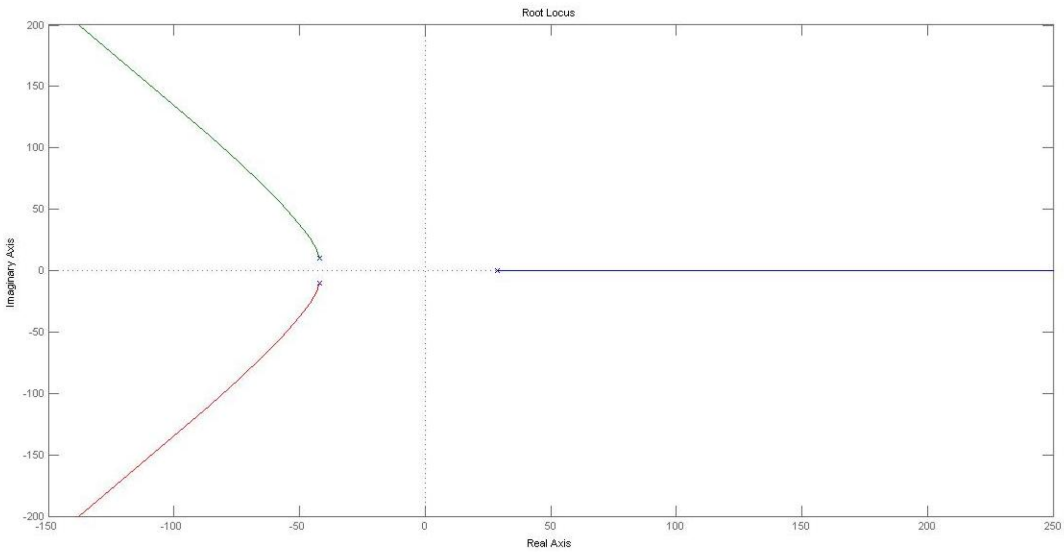

Figure 3.1: Root Locus of Maglev System without Controller

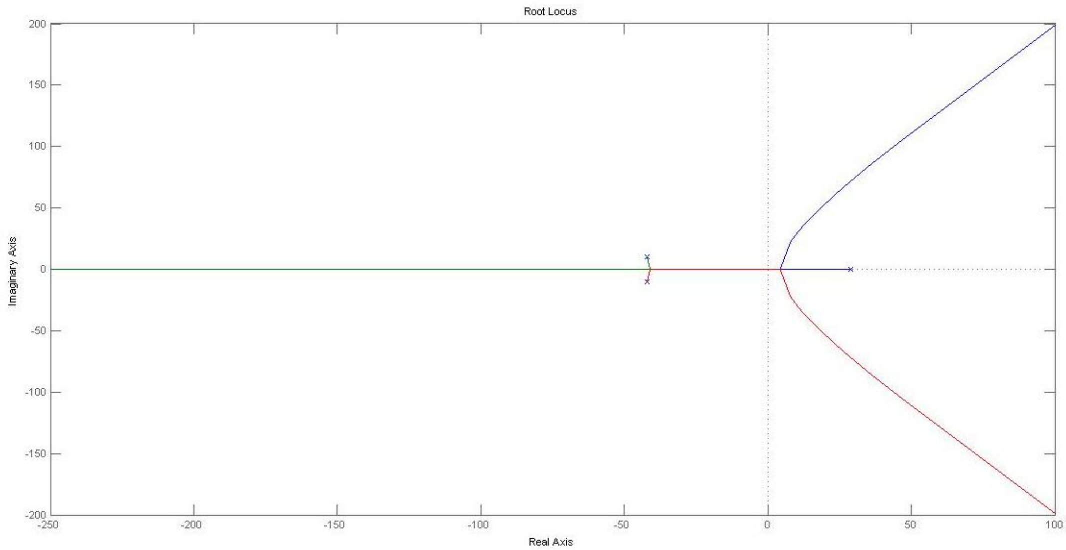

Figure 3.2: Root Locus of Maglev System without Controller

3.4 Controller Design

Since the system is unstable, the PID controller is designed by hit and trail method using MATLAB SISO Tool. The system is considered to be in unity feedback configuration. The gain provided by the sensor in feedback path can be taken into account by applying the voltage obtained from calibration equation of sensor for position variation. The calibration equation is given in Appendix C.

The transfer function of PID controller comes out to be,

$$G_c(s) = -2500\frac{(0.17s+1)(0.17s+1)}{s} \tag{3.1}$$

The root locus of the new transfer function and the step response of the closed system is shown in figure **??** and **??** respectively. It can be seen that the transient response is not

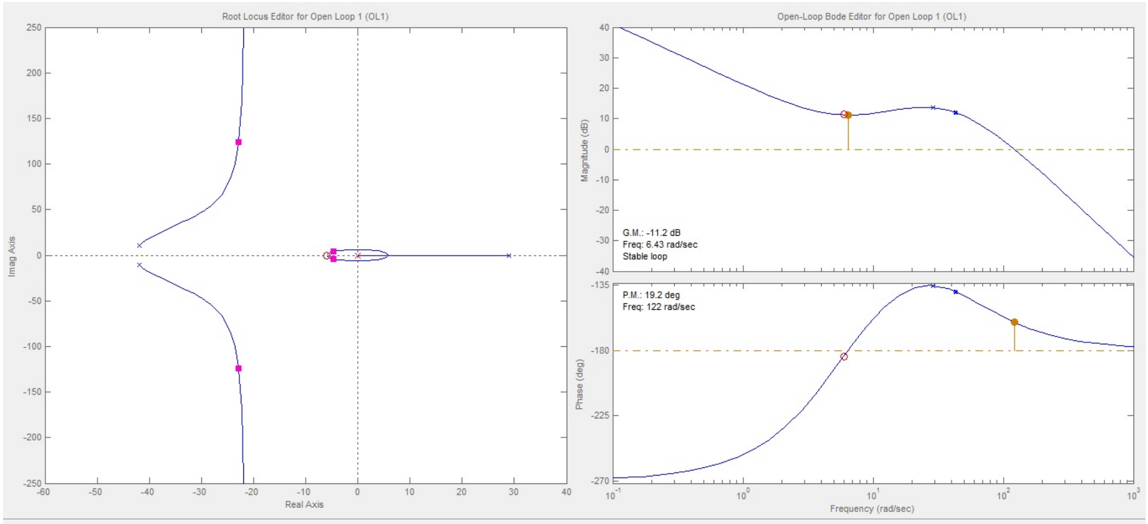

Figure 3.3: Root Locus of The System with PID Controller

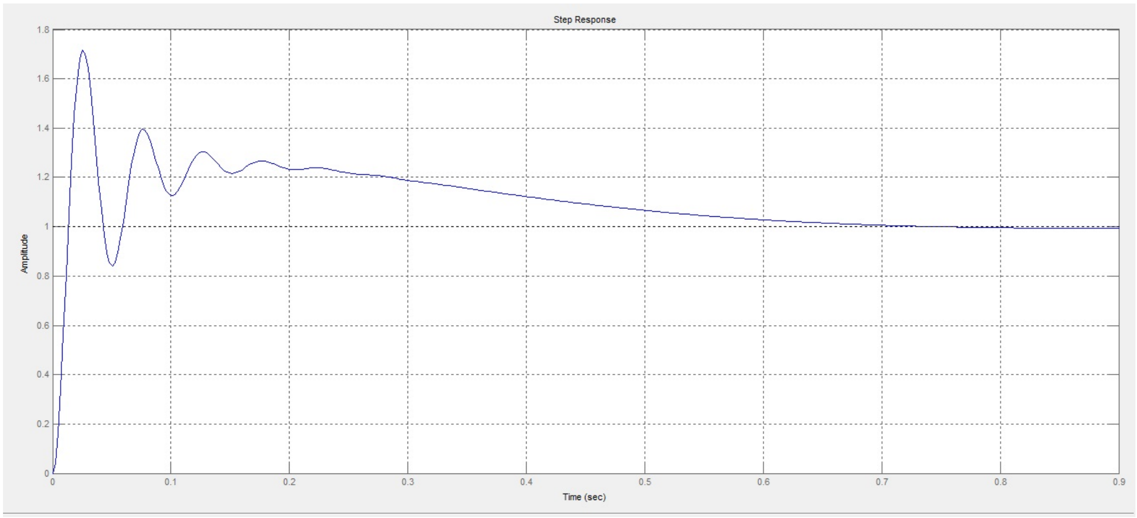

Figure 3.4: Step Response of The System with PID Controller

satisfactory since the overshoot of the system is too high. So, an additional zero is added to increase the damping of the system. The modified transfer function of the controller has the following form and the responses are shown in figure **??** and **??**.

$$G_c(s) = -2500\frac{(0.17s+1)(0.17s+1)(0.01s+1)}{s} \tag{3.2}$$

The Matlab Code for simulation is given in Appendix A.

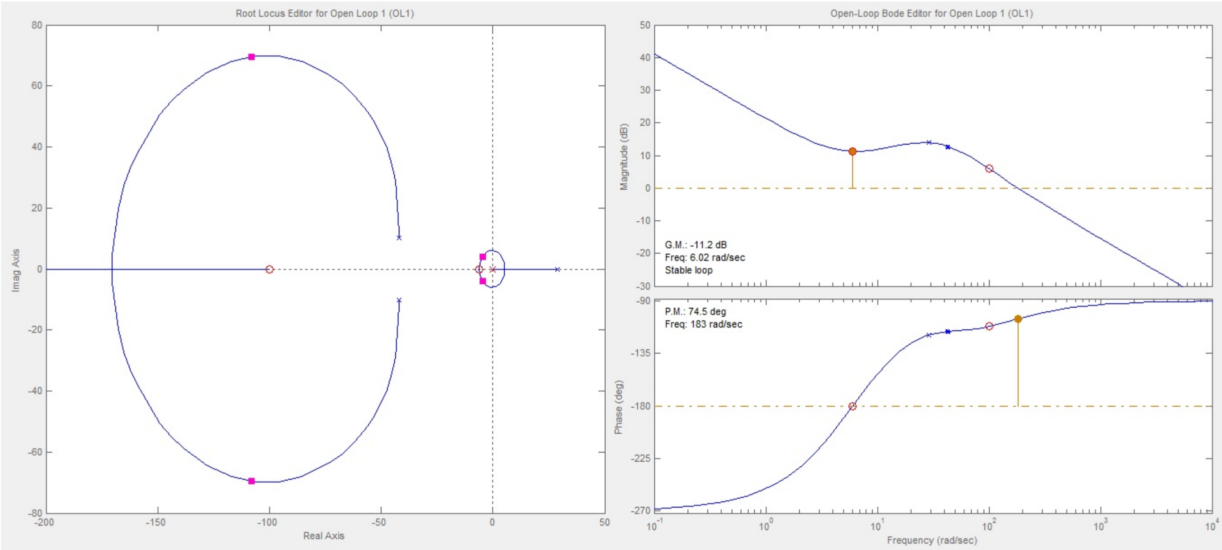

Figure 3.5: Root Locus of The System with Modified Controller

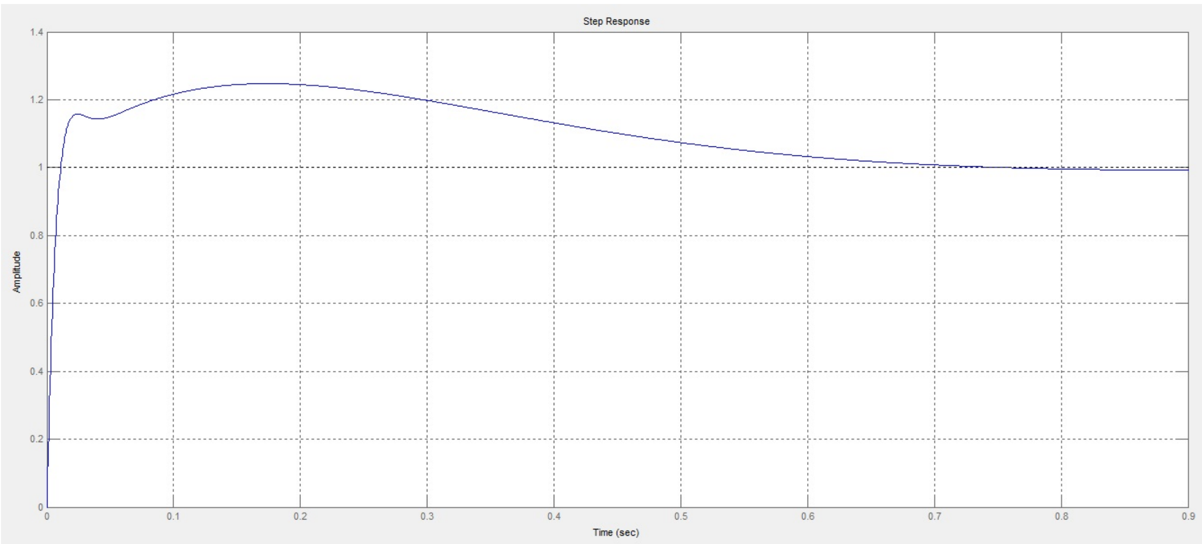

Figure 3.6: Step Response of the System with Modified Controller

Chapter 4

Implementation of Controller

The controller is implemented by OP-AMP structures. The schematic diagram of controller is shown in figure **??**. It consists of 3 cascaded parts. First part is the PID controller in its simpler form followed by additional zero placement circuit and finally a simple amplifier is used to provide the required gain, polarity and tuning.

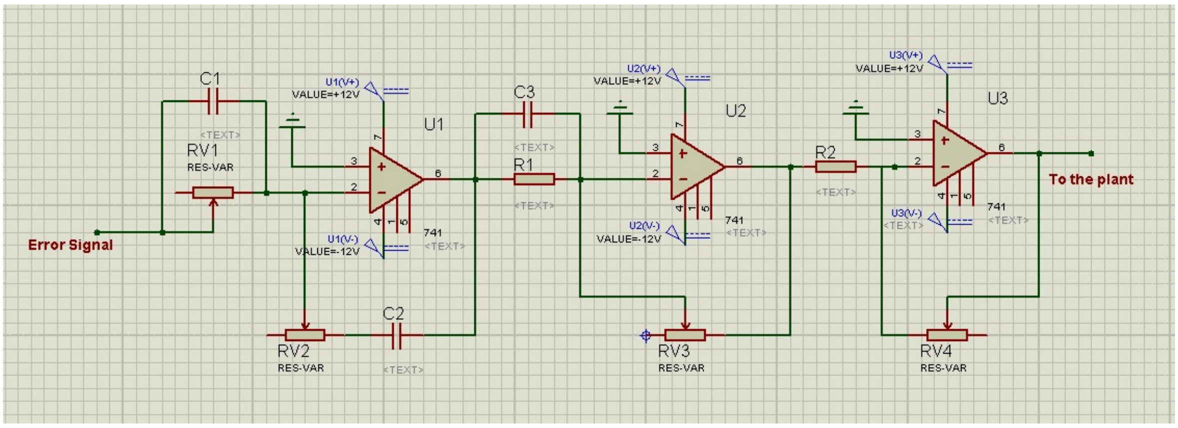

Figure 4.1: Controller for the Maglev System

4.1 PID Controller

The transfer function of PID controller can be derived as,

$$
\begin{aligned}
\frac{V_i}{\frac{R_o}{R_oC_os+1}} + \frac{V_o}{\frac{R_1C_1s+1}{C_1s}} &= 0 \\
\frac{V_i(R_oC_os+1)}{R_o} + \frac{V_oC_1s}{R_1C_1s+1} &= 0 \\
V_o\frac{C_1s}{R_1C_1s+1} &= -V_i\frac{R_oC_os+1}{R_o} \\
\frac{V_o(s)}{V_i(s)} &= -\frac{(R_oC_os+1)(R_1C_1s+1)}{R_oC_1s} \qquad (4.1)
\end{aligned}
$$

Where, R_o and C_o are the resistor and capacitor connected at inverting terminal of op amp and R_1 and C_1 are the resistor and capacitor in feedback path. Using the information from equation **??** and keeping R_oC_1=0.02 we can find that $R_o = 10K\Omega$, $C_o = 18uF$, $R_1 = 85K\Omega$, $C_1 = 2uF$. The gain of this block is kept higher than other blocks because it reduces the noise figure of cascaded system.

4.2 Additional Zero Placement Ciruit

The transfer function of the circuit can be derived as follows,
Let Z_i be the input impendence and Z_o be the impendence in feedback path so,

$$\begin{aligned}
\frac{V_o(s)}{V_i(s)} &= \frac{Z_o}{Z_i} \\
&= -\frac{R_f}{\frac{\frac{R}{Cs}}{R+\frac{1}{Cs}}} \\
&= -\frac{R_f}{\frac{R}{RCs+1}} \\
\frac{V_o(s)}{V_i(s)} &= -R_f C\left(s+\frac{1}{RC}\right)\frac{V_o(s)}{V_i(s)} = -\frac{R_f}{R}\left(RCs+1\right) \qquad (4.2)
\end{aligned}$$

So, it will add a zero at $s = -\frac{1}{RC}$. Using the information from equation **??** we have $R = 10K\Omega$, $C = 1uF$, R_f is kept variable for adjustment of gain.

4.3 Proportional Controller

The transfer function of the circuit is given as,

$$\frac{V_o}{V_i} = -\frac{R_f}{R_1} \qquad (4.3)$$

Where, R_1 is kept $10K\Omega$ and feedback resistance R_f is kept variable to control gain.

Appendix A

Matlab Code for Simulation

The following code of matlab can be used to plot the root locus of plant and complete system along with the step response.

```
clear
clc
%Parameters of System
L=0.07;
R=3.85;
g=9.8;
m=0.1;
C=3e-4;
%Equilibrium values of variables
x01=0.02;
x03=x01*sqrt(g*m/C);
%Parameters for Transfer Function
a=(2*C*x03*x03)/(m*x01*x01*x01);
b=(2*C*x03)/(L*x01*x01);
c=(-2*C*x03)/(m*x01*x01);
d=-R/L;
e=1/L;
%Calculation of Plant Transfer Function
num1=c*e;
den1=[1 -d -(a+b*c) a*d];
sys1=tf(num1, den1);
%Transfer Function of Controller
num2=[0.000289 0.0323 0.35 1];
den2=[1 0];
K=-2500;
sys2=tf(K*num2, den2);
%Transfer Function of Complete System
sys=feedback(sys1*sys2, 1);
%Response
rlocus(sys1)%Root Locus of Plant
title('Root Locus of Plant')
pause
rlocus(sys)%Root Locus of System
title('Root Locus of System')
```

```
pause
step(sys)%Step Response of System
title('Step Response of System')
```

Appendix B

Effect of Change in value of Force Constant

The root locus of compensated system with different values of C are shown in figure ?? and ??.

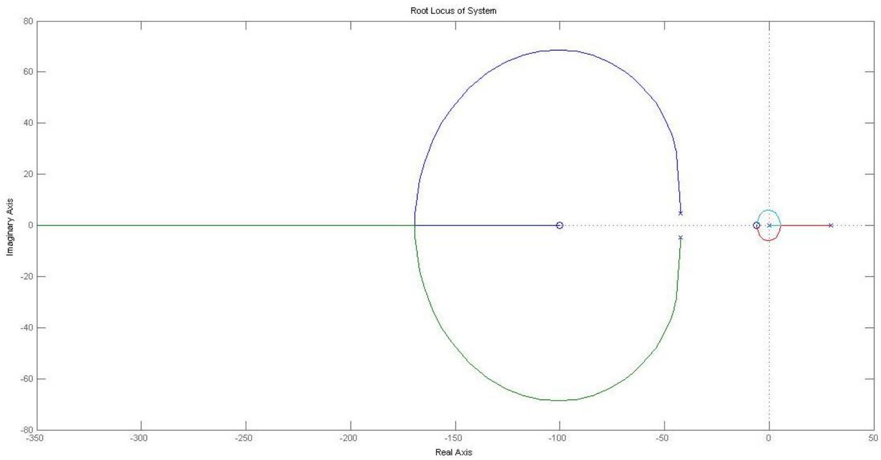

Figure B.1: Root Locus of Compensated System when parameter $C = 0.0002\frac{Nm^2}{A^2}$

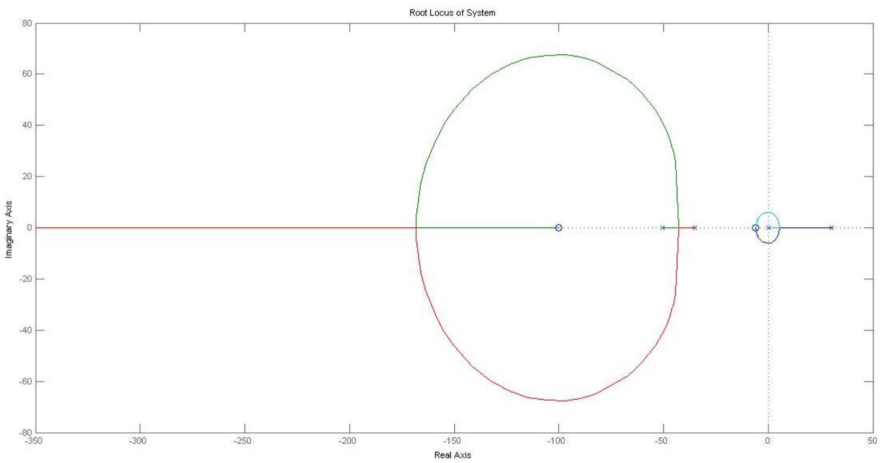

Figure B.2: Root Locus of Compensated System when parameter $C = 0.0001\frac{Nm^2}{A^2}$

It can be seen that the same response can be obtained just by varying the gain of controller. The variation of C only extends the root locus but does not change the shape of the plot.

Appendix C

Caliberation Equation of Sensor

The output of the position sensor is essential to know the representation of the position in terms of voltage. This can then enable us to apply required voltage signal for the required value of position of the ball. By doing so the system can be treated as if it is in unity feedback configuration. Without the obstruction by ball the output of sensor was measured to be approximately $700mV$ .At $x = 0cm$ the sensor output is measured to be $10mV$ and at equilibrium position i.e at $x = 2cm$ the output voltage of sensor is measured to be $13mv$. Assuming linear relationship, the calibration equation can be written as,

$$V = 0.15x + 10 \tag{C.1}$$

Where x is mm and V is in mV.