Optimal Operation of Residential Energy Hubs in Smart Grids

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***Abstract*—This paper presents mathematical optimization models of residential energy hubs which can be readily incorporated into automated decision making technologies in Smart Grids, and can be solved efficiently in a real-time frame to optimally control all major residential energy loads, storage and production components while properly considering the customer preferences and comfort level. Novel mathematical models for major household demand, i.e., fridge, freezer, dishwasher, washer and dryer, stove, water heater, hot tub, and pool pumps are formulated. Also, mathematical models of other components of a residential energy system including lighting, heating, and air-conditioning are developed, and generic models for solar PV panels and energy storage/generation devices are proposed. The developed mathematical models result in Mixed Integer Linear Programming (MILP) optimization problems with the objective functions of minimizing energy consumption, total cost of electricity and gas, emissions, peak load, and/or any combination of these objectives, while considering end-user preferences. Several realistic case studies are carried out to examine the performance of the mathematical model, and experimental tests are carried out to find practical procedures to determine the parameters of the model. The application of the proposed model to a real household in Ontario, Canada is presented for various objective functions. The simulation results show that savings of up to 20% on energy costs and 50% on peak demand can be achieved, while maintaining the household owner's desired comfort levels.**

***Index Terms*—Smart Grids, residential energy hubs, demand response, mathematical modeling, optimization.**

I. Nomenclature

Indices

i	Index of devices
j	Index of days
t	Index of time interval
z	Index of zone

Superscripts

chd	Charge
dch	Discharge
max	Maximum value
min	Minimum value
mst	Maximum successive time

Sets

$\mathcal{A}$	Set of devices; $\mathcal{A} = \{ac, esd, dry, dw, fr, ht, il, pv, pmp, stv, wr\}$
$\mathcal{T}$	Set of indices in the scheduling horizon
$\mathcal{T}_i$	$\mathcal{T}_i \subseteq \mathcal{T}$ is the set of periods in which device i may operate; $\mathcal{T}_i = \{t \in \mathcal{T} : \mathcal{E}_i \leq t \leq \mathcal{L}_i\}$

Variables

$e_i(t)$	Energy storage level of device i at t
$l_z(t)$	Integer variable denoting illumination level produced by the lighting system of zone z at time t
$s_i(t)$	State of device i at time t, binary; ON/OFF
$\theta_{in}(t)$	Inside temperature of the house at time t
$\theta_{fr}(t)$	Inside temperature of the fridge at time t
$\theta_{wh}(t)$	Water temperature at time t
$u_i(t)$	Binary variable denoting start up of device i at time t: $u_i(t) = \begin{cases} 1 & \text{start up} \\ 0 & \text{otherwise} \end{cases}$
$v_i(t)$	Binary variable denoting shut down of device i at time t: $d_i(t) = \begin{cases} 1 & \text{shut down} \\ 0 & \text{otherwise} \end{cases}$

Parameters

$A(t)$	Activity Level at time t
$A_{fr}(t)$	Activity Level of fridge at time t
Φ_i	Cooling/Warming effect of an ON state of device i on corresponding variable ($^\circ$C/interval)
Ψ_i	Cooling/Warming effect of an OFF state of device i on corresponding variable ($^\circ$C /interval)
$\underline{C}_i(t)$	Charged power into device i at time interval t
$\overline{C}_i$	Discharged power from device i during one time interval
D_i	Minimum down time of device i
$\Delta_{wr,dry}$	Maximum allowed time gap between operation of washer and dryer
E	Energy
$\mathcal{E}_i$	Earliest operation time of device i
$I_z^{min}(t)$	Minimum required zonal illumination at time t
$I_z^{out}(t)$	Outdoor illumination level of a given zone in the house at time t
J	Objective function
K_t	Coefficient of price elasticity
$\mathcal{L}_i$	Latest operation time of device i
M	Large positive number
O_i^{mst}	Maximum successive operation time of device i
O_i^{rt}	Required operation time of device i
Ω_i	Cooling/Warming effect of activity level on corresponding variable of device i ($^\circ$C/unit of $A(t)$)

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$P^{max}(t)$	Allowed peak load of the energy hub at time t
P_i	Rated power of device i
Q_i	Heat rate of of device i
R_c, R_g	Rate of CO_2 emissions from coal/gas-fired plants, respectively; 1.0201/0.5148 tonne/MWh [1]
τ	Time interval duration
$\Theta_{out}(t)$	Forecasted outdoor temperature at time interval t
U_i	Minimum up time of device i
Υ_i	Effect of inside and outside temperature difference on the inside temperature corresponding to device i
$W(t)$	Average hourly hot water usage at time t

II. INTRODUCTION

SMART GRIDS are envisioned to support large penetrations of distributed demand-side resources coupled with system-wide demand response (DR) driven by economic and reliability signals. Utilities are also looking at Demand Side Management (DSM) and DR services to better manage their networks. DR programs induce customers to reduce loads during periods of critical grid conditions or periods of high energy costs; in return for reducing load, customers pay less. In other words, DSM and DR programs reward both utilities and customers. The role of DSM and DR will be significant and these should reach their full potential with the development of Smart Grids.

Information technology and a new generation of energy meters, typically referred to as "Smart Meters", which not only provide energy consumption readings but can also provide additional information on usage and have two-way communication capabilities, are two key developments that improve the effectiveness and capability of Energy Management Systems (EMSs). With these developments, both utilities and customers can have access to two-way communication infrastructures, control devices, and visual interfaces that allow them to send, retrieve, visualize, process and/or control their energy needs. These technologies, referred to as Advanced Metering Infrastructure (AMI), render automated operational decision making technologies feasible in Smart Grids. In this context, mathematical modeling for these processes, which is the main concern of the present work, would play a critical role.

In any electric energy system, the customers' objective is to minimize their energy cost, whereas utilities are not only concerned about the cost, but also other issues such as load shape, peak load, quality of service, etc. In this context, a two-tier hierarchical scheme is used in Energy Hub Management System (EHMS) in order to distinguish between the different objectives of the customers and the utility [2]. Therefore, at the lower level, i.e., micro hub level, the objective is to optimize the energy consumption from the customer's point of view, whereas at the macro hub level, i.e., a group of micro hubs controlled and scheduled together, the objective is to optimize the energy consumption from the utility's point of view. Figure 1 shows the overall picture of the macro hub and micro hub interactions in an EHMS, and the associated data and information exchanges between them. In view of the above discussions, the main objective of this research is to develop mathematical models of residential micro hubs which can be readily incorporated into EHMS.

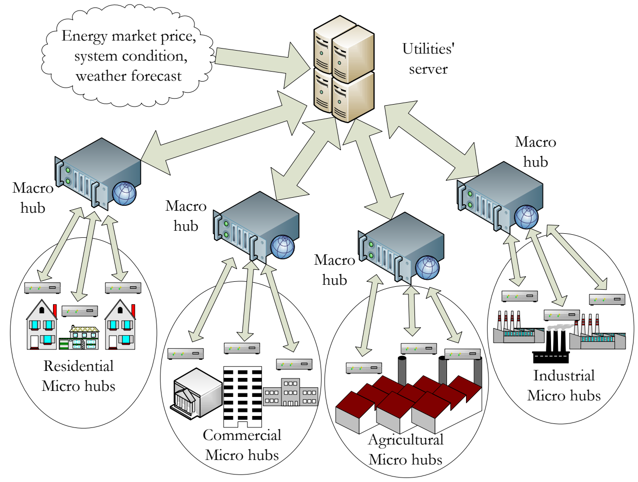

Fig. 1. Overall picture of an EHMS.

In [3], an approach based on multi-pass dynamic programming is proposed for Direct Load Control (DLC) of air conditioners to determine the required amount of load to be controlled at each time stage in order to maximize cost savings and peak load reduction. In [4] and [5], the authors present physically based load models to implement and assess DLC on air-conditioning loads. In [6], the authors present a Mixed Integer Linear Programming (MILP) model to analyze the influence of price signals on household power demand; some major appliances are modeled to minimize energy costs and maximize comfort by rescheduling the use of appliances. A particle swarm optimization based method is proposed in [7] for coordinated scheduling of available residential distributed energy resources to maximize net benefits from smart home electrical energy services. A method based on game theory is proposed in [8] for incentive-based energy consumption scheduling in the context of smart grids. In [9], an optimal and automatic residential load scheduling framework is presented which attempts to achieve a trade-off between minimizing the electricity payment and minimizing waiting time for the operation of each appliance in households considering real-time pricing combined with block rates. The ongoing research project discussed in [10] proposes to use a system of learning (both by machine and occupant) for automatically responding to electricity price signals to optimize the cost and thermal comfort.

From a detailed review of the technical literature, it is clear that most of the existent work only considers particular appliances/devices (e.g., HVAC and water heaters); are designed for one particular objective (e.g., peak demand reduction or energy consumption minimization); do not properly take into account end-users preferences and comfort levels; and/or are not appropriate for real-time applications. The current paper tries to address these shortcomings by proposing a novel mathematical optimization models for a typical residential energy hub that can be readily incorporated into automated decision making technologies, such as Home Automation

Systems (HAS) and EMSs, in the context of Smart Grids. The proposed models can be solved efficiently in a real-time frame to optimally control all major residential energy loads, storage and production components while properly considering the customer preferences and comfort level, and hence can facilitate the integration of residential customers into Smart Grids. These multi-period scheduling optimization models consider different objective functions such as minimization of demand, total cost of electricity and gas, emissions and peak load over the scheduling horizon while considering end-user preferences.

The rest of the paper is organized as follows: First, the modeling approach of the residential energy hubs is discussed in Section III. Then, the proposed mathematical models and some simulation results are presented and discussed in Section IV and Section V, respectively. Implementation and deployment aspects, and some experimental test results of the developed mathematical models are discussed in Section VI. Section VII highlights the main contributions of the paper.

III. Modeling Approach

The energy hub is a concept recently developed in the context of integrated energy systems with multiple energy carriers. *Hub* is defined as a center of activity; hence, *energy hub* is any location where energy system activities, namely, energy production, conversion, storage, and consumption of different energy carriers, take place [11]. In this work, households are considered as multi-carrier energy hubs with energy demand, generation, and storage capabilities.

Major household appliances consume a large portion of a house total energy demand, and some of those can be scheduled without a major effect on customer comfort while reducing energy costs and emissions. Currently, smart appliance controllers are available that allow the customer to enter daily, weekly, as well as seasonal schedules for various device operations. Also, the operation of appliances can be controlled in a house using home networking systems developed to enable remote appliance control [12]–[15]. These systems usually comprise several dedicated controllers which communicate with a central appliance controller when plugged into any electric socket in the house, and allow ON/OFF control of appliances. The user can thus program different schedules and events and implement rule-based decision making within the central appliance controller. In this context, an intelligent decision making core is proposed in this paper to optimally control residential sector energy hubs. The intelligent core is proposed to be an integral part of EMSs, based on the mathematical model of the hub.

Figure 2 presents an overview of the residential energy hub which includes various appliances, energy storage systems (e.g., batteries), energy production systems (e.g., solar photovoltaic, wind power), a smart meter and two-way communication links between these components. The proposed mathematical model and associated optimization solver will reside in the central hub controller. This controller uses the mathematical model of each component in the hub, parameter settings and external information as well as user preferences to

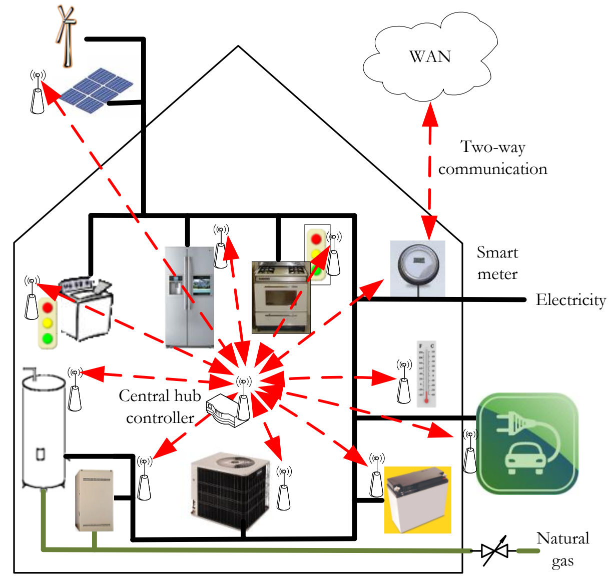

Fig. 2. Residential energy hub structure.

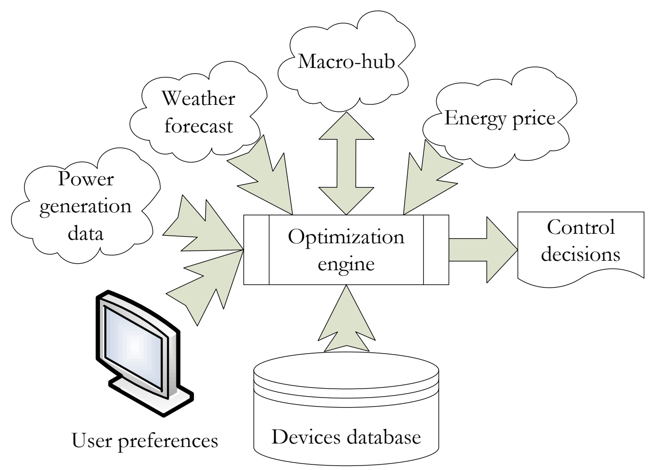

Fig. 3. Functional overview of the central hub controller.

generate the optimal operating decisions for all components in the energy hub over the scheduling horizon, as shown in Fig. 3. The device database includes all the technical characteristics of the components (e.g., rated power, storage/production level), and external information includes energy price information, weather forecast, solar radiation, and CO_2 emissions forecasts. The scheduling horizon and the length of each time interval in the optimization models can vary depending on the type of energy hub and activities which take place in the hub.

A. Customer Preferences

The operational models of the residential energy hubs must give priority to customer's preferences, and be simple enough for successful implementation and easy interpretation of the results. The models should include normal behavior of the customer such as the desired room temperatures and the hours of operation of each device. Also, the maximum deviations

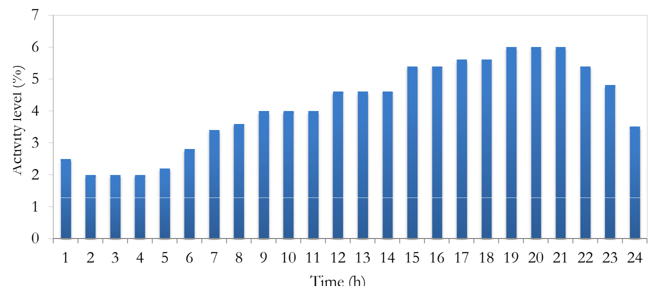

Fig. 4. The hourly activity level of a residential energy hub.

that the customer is willing to accept for each device such as maximum temperature deviations and the latest acceptable time to complete a task, should be incorporated.

B. Activity Level

In the residential sector, the occupancy of the house has a major effect on energy consumption patterns. Furthermore, energy consumption patterns differ in each house depending on the season and the day such as weekdays and weekends. To consider the effect of household occupancy on energy consumption patterns, a new index termed as the *Activity Level* is proposed here for electrical appliances. This represents the hourly activity level of a house over the scheduling horizon. To determine a reasonable value of the Activity Level of a residential energy hub, historical data of energy consumption provided by installed smart meters at each house can be used. Therefore, the measured data of the previous weeks, months, and years can be used to predict the energy consumption on a particular day and thus generate residential load profiles. These load profiles could be modified to obtain the proposed Activity Level of a house on an hourly basis.

Figure 4 shows an example of the Activity Level over a day for a single detached house; this is obtained from hourly variations of energy consumption of the household on a summer day. In this graph, the Activity Level (y axis) is normalized with respect to the total energy consumption of the day, which is assumed to be 100%. It should be noted that the Activity Level index has a different effect on each of the electrical appliances in the house. For example, the effect of the activity level on the fridge temperature is not the same as its effect on the room temperature. Thus, the Activity Level index is related to each of the energy consuming devices with an appropriate coefficient. A detailed explanation of how the Activity Level index can be determined is presented in Section VI.

C. Energy Pricing

1) Electricity: The main objective of dynamic pricing programs is to encourage the reduction of energy consumption during peak-load hours. Fixed-Rate Plan (FRP), Time-of-Use (TOU), and Real-Time (RT) pricing are three types of pricing schemes available to electricity customers in various jurisdictions. Currently, most residential sector customers are being charged based on either FRP or TOU pricing schemes. However, Hourly Ontario Electricity Price (HOEP) day-ahead forecasts are considered as RT prices in this work to examine their effect on optimal operation of residential energy hubs.

2) Natural Gas: Natural gas rates for residential customers are usually FRP, which include transportation, storage and delivery charges beside the commodity charges. Hence, this pricing scheme have been used in this study.

D. CO_2 Emissions

CO_2 emissions of the power system at each hour need to be forecasted in order to minimize carbon footprint of the customer. Coal and gas-fired generating units, which are the main sources of CO_2 emissions in the power sector, produce different amounts of CO_2. The system operators do not typically provide power generation forecasts for power plants. Therefore, the power generation from coal and gas-fired generating units needs to be forecasted in order to estimate the CO_2 emissions from the system. These forecasts are carried out using an econometric time-series model at utility servers, as discussed in detail in [16], and sent to each energy hub as required. A 24-hour ahead total system demand profile obtained from pre-dispatch data; hourly cumulative generation from coal- and gas-fired units respectively, for the past 14 days; and hourly total system demand for the past 14 days are the public data used by the time-series forecasting model to forecast the power generation from coal- and gas-fired power plants, separately. The day-ahead emissions profile is then calculated using separate rates of emissions for gas and coal fired units, and the marginal cost of CO_2 emissions is calculated using the social cost of carbon dioxide emissions.

E. Other External Inputs

Outdoor weather conditions have major impacts on energy consumption of heating and cooling systems of a household. Heat transfer through walls and solar radiations are examples of the ways that outdoor conditions can affect indoor temperature. Nowadays, accurate climate forecasts are available every few hours for next days. These forecasts are employed in this work to generate the optimal schedule for operation of residential energy hubs.

IV. Mathematical Model

The proposed general form of the optimization model for the residential sector is as follows:

$$\min \quad J = \text{Objective function} \tag{1a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{A}} P_i\, s_i(t) \le P^{max}(t), \qquad \forall t \in \mathcal{T} \tag{1b}$$

$$\text{Device } i \text{ operational constraints}, \qquad \forall i \in \mathcal{A} \tag{1c}$$

Constraint (1b) ensures that maximum power consumption at a given time does not exceed a specified value. For customers who participate in critical peak power reduction programs provided by various utilities (e.g., Peak-Saver program [17]), this constraint can be used to integrate automatic responses to the utility operator's power reduction requests during critical times for the system.

A. Objective Function

Depending on end-user choice, different objective functions can be adopted to solve the optimization problem. Thus, minimization of the customer's total energy costs, total energy consumption, peak load, emissions, and/or any combination of these over the scheduling horizon are considered here as possible objective functions for the optimization model.

1) Energy Costs: The customer's total energy costs represent the cost of electricity consumption, the the revenue from selling stored/produced electricity to the power grid, and the cost of gas consumption.

2) Energy Consumption: Total energy consumption of the hub consists of the energy consumption of electrical devices, the electrical energy injected to the grid from stored/produced electricity, and the gas consumption.

3) CO_2 Emissions Costs: CO_2 emissions costs includes carbon footprint of the customer from the grid electricity usage, CO_2 emissions from gas consumption within the house, and the carbon reduction from injecting emissions free electricity (from PV arrays) to the grid.

4) Peak Load: An objective function for minimization of peak load is formulated based on total electricity demand of the energy hub at each time interval to reduce the demand of the energy hub.

5) Multi-Objective Optimization: In addition to the aforementioned individual objective functions, any combinations of these objective functions can also be used as an objective that simultaneously minimizes all of them:

$$J = \omega_1 J_1 + \omega_2 J_2 + \omega_3 J_3 + \omega_4 J_4 \tag{2}$$

where J_1, J_2, J_3, and J_4 are the the objective functions representing the customers' total energy cost, total energy consumption, total emissions cost, and peak demand charges, respectively; and $\omega_1, \omega_2, \omega_3$, and ω_4 are the weights attached to these objective functions, respectively. Note that customers in the residential sector would likely have different preferences and priorities. Thus, the purpose here is not to find Pareto optimal solutions, but rather provide all the customers with appropriate objective functions for them to decide the combinations that better fit their interests and wants. In fact, in the application and implementation of the proposed models, one of the objectives is to determine which weights the customers will choose for the different objective function components, and thus get a sense of their motivations and interests.

B. Devices' Operational Constraints

Mathematical models of major household appliances, energy storage/generation devices, and photo-voltaic (PV) solar array are presented next. These models represent the operational constraints of the residential energy hub. The time period over which device i can be in operation is specified by:

$$s_i(t) = \begin{cases} 0 \text{ or } 1 & \text{if } t \in \mathcal{T}_i \\ 0 & \text{if } t \notin \mathcal{T}_i \end{cases} \qquad \forall i \in \mathcal{A} \tag{3}$$

where the customer defines the $\mathcal{E}_i$ and the $\mathcal{L}_i$ of each device that defines $\mathcal{T}_i$ (see Nomenclature).

Devices with thermal storage capacity such as the fridge, AC, heating, and water heater should maintain their respective temperatures within user specified ranges. Therefore, the following common set of constraints are utilized for these devices:

$$\Theta_i^{min}(t) \leq \theta_i(t) \leq \Theta_i^{max}(t), \forall t \in \mathcal{T}_i,\ i \in \{ac,fr,ht,wh\} \tag{4a}$$

$$s_i(1) = \begin{cases} 1 & \text{if } \theta_i(0) > \Theta_i^{max}(1), \quad i \in \{ac, fr\} \\ 0 & \text{if } \theta_i(0) < \Theta_i^{min}(1), \quad i \in \{ac, fr\} \end{cases} \tag{4b}$$

$$s_i(1) = \begin{cases} 0 & \text{if } \theta_i(0) > \Theta_i^{max}(1), \quad i \in \{ht, wh\} \\ 1 & \text{if } \theta_i(0) < \Theta_i^{min}(1), \quad i \in \{ht, wh\} \end{cases} \tag{4c}$$

Here, (4a) ensures that the devices' temperatures are within the customer's preferred ranges, and (4b) or (4c) guarantee that if the temperature of device i at the time interval before the model initialization is more or less than the upper or lower limit, respectively, as specified by the customer, the device is ON in the first time interval. In addition to these common set of constraints, each device has specific mathematical equations to properly model its operation as discussed next.

1) Fridge: The model seeks to maintain the fridge temperature within a specified range, while taking into account technical aspects of the fridge operation as well as the customer preferences. The operational constraints of the fridge are as follows ($i = fr$):

$$\begin{aligned} \theta_{fr}(t) = \theta_{fr}(t-1) + \tau\,[\Psi_{fr}\ A_{fr}(t) \\ -\Phi_{fr}\ s_i(t) + \Omega_{fr}]\,, \qquad \forall t \in \mathcal{T} \end{aligned} \tag{5}$$

plus (4a) and (4b). Equation (5) relates the temperature of the fridge at time t to the temperature of the fridge at time $t-1$, activity level of the fridge at time t, and ON/OFF state of the fridge at time t. The effect of the activity level on the fridge temperature is modeled using Ψ_{fr}, so that as the household activity level increases there is more cooling demand on the fridge.

The effect of the ON state on fridge temperature reduction is represented by Φ_{fr}, and the warming effect of the OFF state of the fridge is modeled by Ω_{fr}. The latter is to address the thermal leakage because of difference in temperatures of the fridge and the kitchen. The parameters Φ_{fr}, Ψ_{fr}, and Ω_{fr} can be measured or estimated from simple performance tests as discussed in Section VI. The same model with different coefficients and parameter settings can be used to model a freezer.

2) Air Conditioning (AC) and Heating: In addition to residential thermal loss, activity level of household, ambient temperature, and the maximum temperature deviation that the customer is willing to tolerate are included in modeling of AC and heating systems of a house to capture most of the aspects of the customer preferences. Operational constraints developed for modeling the heating system in a house are similar to the operational constraints of the AC. Therefore, the AC and heating system constraints are presented using a common set of equations, as follows ($i = ac/ht$):

$$s_{ac}(t) + s_{ht}(t) \leq 1, \qquad \forall t \in \mathcal{T}_i \tag{6a}$$

$$\theta_{in}(t) = \theta_{in}(t-1) + \tau \left[\Psi_{ac}\ A(t) - \Phi_{ac} s_i(t) + \Upsilon_{ac}(\Theta_{out}(t) - \theta_{in}(t))\right], \qquad \forall t \in \mathcal{T}, i = ac \tag{6b}$$

$$\theta_{in}(t) = \theta_{in}(t-1) + \tau \left[\Psi_{ht}\ A(t) + \Phi_{ht} s_i(t) - \Upsilon_{ht}(\theta_{in}(t) - \Theta_{out}(t))\right], \qquad \forall t \in \mathcal{T}, i = ht \tag{6c}$$

plus (4a) and (4b) or (4c) for the AC or heating, respectively. Constraint (6a) ensures that the heating and AC do not operate at the same time. Equations (6b) and (6c) represent the dynamics of indoor temperature for the AC and the heating system, respectively. These equations state that the indoor temperature at time t is a function of the indoor temperature at time $t-1$, household activity level at time t, ON/OFF state of the AC (heating system) at time t, and the outdoor and indoor temperature difference. The cooling (warming) effect of an ON state of the AC (the heating system) on indoor temperature is represented by Φ_{ac} (Φ_{ht}). The effect of the Activity Level on indoor temperature increase is modeled by Ψ_{ac} (Ψ_{ht}), and Υ_{ac} (Υ_{ht}) represents the effect of outdoor and indoor temperature difference on indoor temperature. This model captures the normal temperature and the maximum temperature deviation that the customer is willing to accept.

3) Water Heater: An average hourly hot water usage pattern can be considered for each individual house. The Building Energy Efficiency Standards of PG&E recommend an hourly schedule for average daily hot water usage of residential customers [18], where it is observed that there are significant variations in hot water consumption patterns between weekdays and weekends, recommending that these be respected in the schedules. Thus, this issue has been considered in the development of models for water heater. The procedure to calculate the hot water usage is explained in detail in [18].

The operational constraints of the water heater are represented by ($i = wh$):

$$\theta_{wh}(t) = \theta_{wh}(t-1) + \tau \left[\Phi_{wh} s_i(t) - \Psi_{wh} W(t) - \Omega_{wh}\right], \qquad \forall t \in \mathcal{T} \tag{7}$$

plus (4a) and (4c). Constraint (7) states that water heater temperature at a time interval t is a function of the water temperature at the previous time interval, the average hot water usage, and the ON/OFF state of the water heater at time interval t.

The operational constraints of the water heater can also be used for a hot tub water heater. The only difference between these models is in their parameter settings such as average hot water usage, temperature settings, operational time, and associated coefficients that may have different values.

4) Dishwasher, Washer, and Dryer: The proposed operational model for the dishwasher, Washer and Dryer is as follows ($i \in \{dw, wr, dry\}$):

$$u_i(t) - v_i(t) = s_i(t) - s_i(t-1), \qquad \forall t \in \mathcal{T}_i \tag{8a}$$

$$u_i(t) + v_i(t) \leq 1, \qquad \forall t \in \mathcal{T}_i \tag{8b}$$

$$\sum_{t \in \mathcal{T}_i} s_i(k) = O_i^{rt}, \qquad \forall t \in \mathcal{T}_i \tag{8c}$$

$$\sum_{k=t}^{t+O_i^{mst}} s_i(k) \leq O_i^{mst} + M(1 - u_i(t)), \qquad \forall t \in \mathcal{T}_i \tag{8d}$$

$$\sum_{k=t-U_i+1}^{t} u_i(t) \leq s_i(t), \qquad \forall t \in [\mathcal{E}_i + U_i + 1, \mathcal{L}_i] \tag{8e}$$

$$\sum_{k=t-D_i+1}^{t} v_i(t) \leq 1 - s_i(t), \qquad \forall t \in [\mathcal{E}_i + D_i + 1, \mathcal{L}_i] \tag{8f}$$

In this model, in addition to the time periods over which these devices can be in operation, which are specified by the customer's $\mathcal{E}_i$ and $\mathcal{L}_i$ settings, additional constraints on the required operation time, maximum successive operation time, minimum up time, and minimum down time of these devices are specified by the end-user, and modeled by (8c) to (8f), respectively.

If a customer wants to coordinate the operation of the washer and the dryer such that the dryer operates after the washer completes its operation, but a large time gap between the operation of the two appliances is not desired, the following set of constraints needs to be considered:

$$s_{dry}(t) \leq \sum_{k=1}^{\Delta_{wr,dry}} s_{wr}(t-k), \qquad \forall t \in \mathcal{T} \tag{9a}$$

$$s_{dry}(t) + s_{wr}(t) \leq 1, \qquad \forall t \in \mathcal{T} \tag{9b}$$

$$\sum_{t \in \mathcal{T}_{dry}} s_{dry}(t) = \sum_{t \in \mathcal{T}_{wr}} s_{wr}(t) \tag{9c}$$

Constraints (9a) to (9c) ensure that the dryer is scheduled after the washer without exceeding the maximum allowed time gap within the scheduling horizon.

5) Stove: The operation of the stove depends on the household habits and hence direct control of the stove in not reasonable. Therefore, it is proposed to advise the customer on the "preferred" operation times of the stove. The proposed operational model of the stove is as follows ($i = stv$):

$$u_i(t) \geq s_i(t) - s_i(t-1), \qquad \forall t \in \mathcal{T}_i \tag{10a}$$

$$\sum_{t \in \mathcal{T}_i} s_i(k) = O_i^{rt}, \qquad \forall t \in \mathcal{T}_i \tag{10b}$$

$$\sum_{k=t-U_i+1}^{t} u_i(t) \leq s_i(t), \qquad \forall t \in [\mathcal{E}_i + U_i + 1, \mathcal{L}_i] \tag{10c}$$

$$\sum_{k=t}^{t+O_i^{mst}} s_i(k) \leq O_i^{mst} + M(1 - u_i(t)), \qquad \forall t \in \mathcal{T}_i \tag{10d}$$

Here, the required operation time, minimum up time, and maximum successive operation time of the stove are parameter settings specified by the end-user, and are modeled by (10b), (10c) and (10d), respectively.

6) Pool pump: Pool pumps are used to maintain the quality of water in swimming pools by circulating the water through the filtering and chemical treatment systems. Therefore, by operating the pool pump for particular hours a day, the pumping system keeps the water relatively clean, and free of bacteria. The operational model of the pool pump is as follows

($i = pmp$):

$$\sum_{t \in \mathcal{T}_i} s_i(k) = O_i^{rt}, \qquad \forall t \in \mathcal{T}_i \quad (11a)$$

$$u_i(t) \geq s_i(t) - s_i(t-1), \qquad \forall t \in \mathcal{T}_i \quad (11b)$$

$$\sum_{k=t-U_i+1}^{t} u_i(t) \leq s_i(t), \qquad \forall t \in [\mathcal{E}_i + U_i + 1, \mathcal{L}_i] \quad (11c)$$

$$\sum_{k=t-D_i+1}^{t} u_i(t) \leq 1 - s_i(t - D_i), \quad \forall t \in [\mathcal{E}_i + D_i + 1, \mathcal{L}_i] \quad (11d)$$

$$\sum_{k=t}^{t+O_i^{mst}} s_i(k) \leq O_i^{mst} + M(1 - u_i(t)), \qquad \forall t \in \mathcal{T}_i \quad (11e)$$

Constraint (11a) ensures that the pool pump operates for the required time over the scheduling horizon; (11c) and (11d) model the minimum up-time and down-time requirements of the pool pump; and (11e) ensures that the maximum number of successive operation time intervals of the pool pump is not more than a pre-set value.

7) Energy Storage Device: A modern household is expected to be equipped with some form of Energy Storage/production Device (ESD), such as batteries and plug-in hybrid electric vehicles (PHEVs). To develop the model of the ESD for a residential hub, it is assumed that the amount of energy charged into the ESD at each time interval is known. The generic model of the ESD is given by ($i = esd$):

$$e_{esd}(t) = e_{esd}(t-1) + \tau \left[C_{esd}(t) - s_i(t)\, \overline{C}_{esd} \right], \qquad \forall t \in \mathcal{T} \quad (12a)$$

$$E_{esd}^{min} \leq e_{esd}(t) \leq E_{esd}^{max}, \qquad \forall t \in \mathcal{T}_i \quad (12b)$$

$$u_i(t) \geq s_i(t) - s_i(t-1), \qquad \forall t \in \mathcal{T}_i \quad (12c)$$

$$\sum_{k=t-U_i+1}^{t} u_i(t) \leq s_i(t), \qquad \forall t \in [\mathcal{E}_i + U_i + 1, \mathcal{L}_i] \quad (12d)$$

$$\sum_{k=t-D_i+1}^{t} u_i(t) \leq 1 - s_i(t - D_i), \quad \forall t \in [\mathcal{E}_i + D_i + 1, \mathcal{L}_i] \quad (12e)$$

Constraint (12a) relates the energy storage level of the ESD at time interval t to that at time $t-1$, and the energy charge and discharge at time interval t. Constraint (12b) ensures that the energy storage level is never less than a specified minimum value. The minimum up-time and down-time requirements of the ESD are modeled by (12c)-(12e).

8) PV array: The mathematical model of a PV system is as follows ($i = pv$):

$$C_{pv}(t) = \begin{cases} P_{pv}^{chd} & \text{if } P_{pv}(t) \geq P_{pv}^{chd} \\ P_{pv} & \text{if } P_{pv} \leq P_{pv}^{chd} \end{cases} \quad (13a)$$

$$e_{pv}(t) = e_{pv}(t-1) + \tau \left[s_{pv}^{chd}(t) C_{pv}(t) - s_{pv}^{dch}(t) \overline{C}_{pv} \right], \qquad \forall t \in \mathcal{T} \quad (13b)$$

$$E_{pv}^{min} \leq e_{pv}(t) \leq E_{pv}^{max}, \qquad \forall t \in \mathcal{T}_i \quad (13c)$$

$$s_{pv}^{dch}(t) + s_{pv}^{chd}(t) \leq 1, \qquad \forall t \in \mathcal{T} \quad (13d)$$

Constraint (13a) simulates the constant current battery charger operation which is normally used to charge the PV systems batteries. For simplicity, it is assumed that the battery voltage is constant during the discharging/charging operations; thus, a constant current battery charging is assumed to be a constant power charging process. Constraint (13b) shows the effect of the charge/discharge decisions on the battery storage level. Constraint (13c) is used to protect the battery against deep discharging and over charging, and equation (13d) reflects the fact that the converter does not operate in charge and discharge mode simultaneously; it is assumed that the conversion efficiency is 100%. Notice that the proposed model for PV and energy storage devices can be readily modified to represent other renewable energy resources such as wind generation in a residential energy hub, by appropriately defining the output power $C_i(t)$ of the given resource, which is assumed to be a forecasted parameter.

9) Lighting: The lighting load of a house depends on the activity level and/or the house occupancy, and it is modeled using a illumination level index. It is assumed that the lighting load of the house can be divided into several zones and the minimum required illumination can be provided through the lighting system and outdoor illumination (sunshine). The following constraints represent the lighting load of a zone z in the house ($i = li$):

$$l_z(t) + I_z^{out}(t) \geq (1 + K_t) I_z^{min}(t), \qquad \forall t \in \mathcal{T}_i \qquad (14)$$

This constraint ensures that the total zonal illumination (from the lighting system and outdoor sunshine) is more than a minimum required level. The effect of the house occupancy on the lighting load is considered in the minimum required illumination level for each zone, and it is assumed that residential customers tend to reduce illumination during peak-price hours. Thus, this "price elasticity" of the lighting load is modeled by a linear function K_t, $0 \leq K_t \leq 1$, such that during peak hours K_t is equal to 0, which corresponds to the householder using the minimum required illumination; and during off-peak hours K_t is equal to 1, which means the household consumes more lighting than the minimum required illumination. The minimum required zonal illumination and outdoor illumination at time interval t are assumed to be exogenous inputs to the model.

V. SIMULATIONS

Extensive studies have been conducted to examine the performance of the developed mathematical model for a residential energy hub, of which the most relevant ones are presented in this paper. In these case studies, the mathematical models are run for a real residential customer in Ontario, Canada, where parameters and device ratings of the household are suitably chosen, and realistic data inputs for outside temperatures, illumination levels, and solar PV panel generation are used. In these simulations, a 24 hours scheduling horizon with time intervals of 15 minutes has been used, with the exception of the fridge which requires a shorter time interval to keep the temperature within the pre-defined ranges due to its thermodynamic characteristics; however, the same time interval may be used for all the devices to simplify the handling of the model, as it is in fact being done in the actual implementation of the models proposed here. Table I presents

TABLE I
SUMMARY OF CASE STUDIES.

Case	Description
0	Maximize customer comfort such that the temperature deviation from the set points is minimized while all other user defined constraints on operation of the devices are met. Two scenarios are considered: one with programmable thermostat (PT), and the other with fixed temperature settings (FTS).
1	Minimize total cost of energy from all devices and maximize the operation of energy storage device.
2	Minimize total energy consumption of all devices and maximize the operation of energy storage device.
3	Minimize consumer's contribution to CO_2 emissions considering an emission profile of the system.
4	Minimize the total energy costs while there is a peak power limit for electricity demand at each time interval.
5	Simultaneously minimize total costs, energy consumption, and emissions. In these simulations the weights assigned to each objective function are defined as follows: $\omega_1 = 1$, $\omega_2 = X/Y$, $\omega_3 = X/Z$, $\omega_4 = 0$, where \$$X$, Y kWh, and \$$Z$ are the objective values obtained from running the model with the individual objective functions, i.e., using J_1, J_2, and J_3, respectively.

a summary of the case studies presented and discussed here to illustrate the capabilities and performance of the developed model.

A. Input data

In order to carry out the model simulations for a residential energy hub, it is important to select appropriate model parameters which are close to those in the real world. For practical systems, most of these parameters would have to be determined by proper estimation, appliance performance tests and customer preferences. The assumed parameter settings of the devices for the purpose of this study can be found in [19].

Time-of-Use (TOU), Real-Time Pricing (RTP) and Flat Rate Pricing (FRP) for electricity, and fixed rate price for natural gas are used to calculate the total energy costs for residential customers. In Ontario the Hourly Ontario Electricity Price (HOEP) is the RTP that applies to customers who participate in the wholesale electricity market [20]; hence, this is the price used here for the study of residential customers.

Ontario's CO_2 emissions profile for a summer weekday is forecasted using the method described in Section III-D. Using the actual demand profile, the Ontario's emissions profile depicted is obtained using the proposed method; this emissions profile is used in the case studies presented here. Details of all input data can be found in [19].

B. Results

AMPL-CPLEX 12.1 [21], [22] and GLPK version 4.38 [23] were used in the present work for the purpose of simulation studies and implementation tests, respectively, to solve the proposed mathematical model. These MILP optimization models can be solved with a 0.01 MIP gap, equivalent to 99% optimality, in a fraction of a minute by both CPLEX and GLPK solvers. These problems have about 2300 variables and 2600 constraints

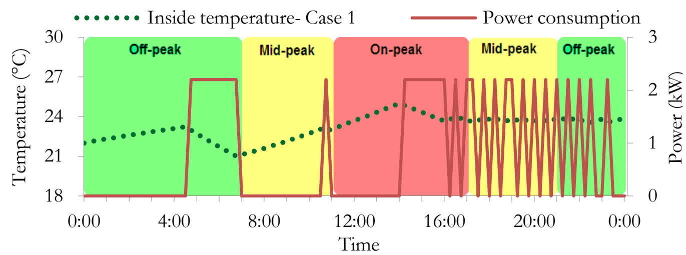

Fig. 5. Operational schedule of the AC during a typical summer day.

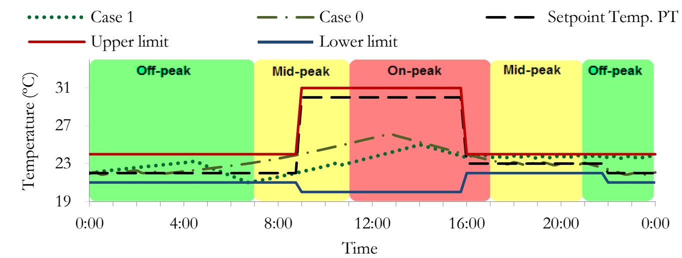

Fig. 6. Comparison of indoor household temperatures for Case 0 and Case 1.

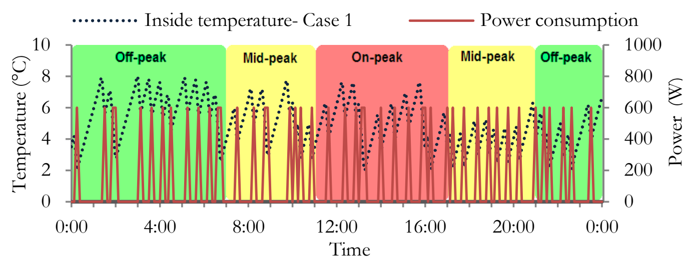

Fig. 7. Operational schedule of the fridge.

1) Optimal Operational Schedules of Devices: The operational schedules of various devices generated in Case 1 for a typical summer day are presented and discussed next for TOU pricing. Figure 5 shows the operational schedule of the AC and resulting indoor temperature for Case 1, and Fig. 6 shows a comparison of indoor temperature obtained from Case 0 and Case 1. In Case 1, the user defined temperature variation range is wider during working hours, when probably no one is present in the house, and narrower at other times. Similarly, the temperature set points are higher during working hours in Case 0. Observe that in both cases, indoor temperature varies within the user defined ranges.

The operational schedule and inside temperature generated by the optimization model for the fridge in Case 1 are depicted in Fig. 7. Observe that when the activity level increases during the evening hours, the fridge needs to operate more often to keep the inside temperature within the user defined ranges. Inside fridge temperatures obtained from the model in Case 1 and Case 0 are shown in Fig. 8. In Case 0, the temperature tracks the user defined set point (3.5 °C) while in Case 1 the temperature varies within the user defined upper and lower limits.

Figure 9 compares the scheduling of the dishwasher be-

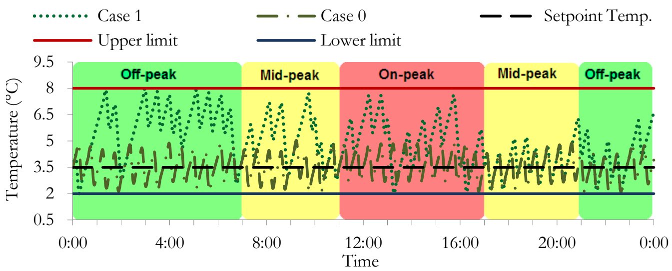

Fig. 8. Comparison of inside fridge temperature in Case 1 and Case 0.

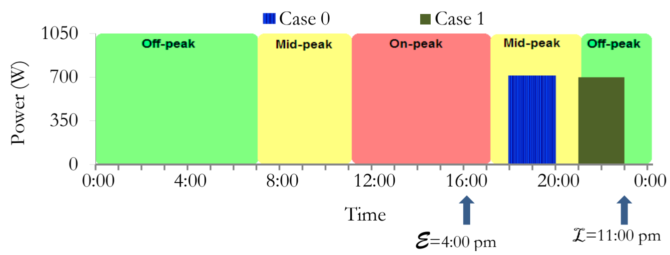

Fig. 9. Comparison of operational schedule of dishwasher in Case 1 and Case 0.

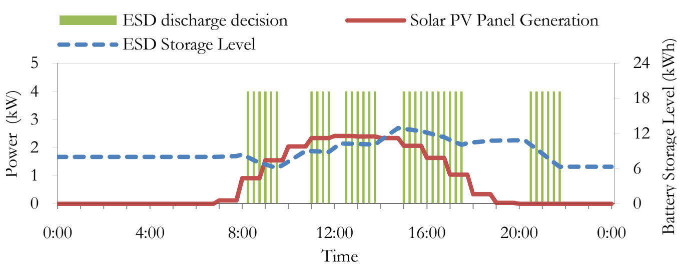

Fig. 10. Operational schedule and storage level of the ESD in Case 1.

tween Case 1 and Case 0, considering that the user defined operation interval is between 4:00 PM and 11:00 PM. Case 0 schedules the dishwasher during mid-peak period, while in Case 1 the dishwasher is scheduled during off-peak price periods to achieve cost savings. Figure 10 demonstrates the operation of the ESD including its discharge and storage levels, and solar PV panel generation at each interval. The model tries to maximize the battery discharge in order to sell power to the grid, while maintaining a minimum level of battery storage. Notice that in this simulations, since the energy price is fixed, the schedules are independent of TOU price.

2) Comparison of Cases: Results of Case 0 for individual devices on a summer day for a programmable thermostat (PT) and a fixed temperature setting (FTS) are presented in Table II. Both energy consumptions and energy costs are less for the AC when there is a programmable thermostat. Since the temperature settings only influence the operation of air conditioning (heating) system, the operation of other devices is the same for both PT and FTS.

Table III presents a summary results of all cases for a typical summer day. Observe that Case 0 has the highest

TABLE II
RESULTS OF CASE 0 FOR INDIVIDUAL DEVICES FOR A SUMMER DAY.

Devices		With PT		With FTS	
		Energy consumption (kWh)	Energy costs ($)	Energy consumption (kWh)	Energy costs ($)
AC		20.90	2.28	22.55	2.45
Water heater	Elec.	2.45	0.26	2.45	0.26
	Gas	4.90	1.44	4.90	1.44
Fridge		3.53	0.36	3.53	0.36
Lighting		12.04	1.32	12.04	1.32
Stove		4.50	0.50	4.50	0.49
Dishwasher		1.40	0.16	1.40	0.16
Washer		0.90	0.10	0.90	0.10
Dryer		2.20	0.18	2.20	0.18
Tub water heater		1.50	0.17	1.50	0.15
Pool pump		7.50	0.91	7.50	0.90

energy costs, energy consumption and emissions among all cases. In terms of total energy costs, Case 1 and Case 5 have almost the same amount of savings as compared to Case 0. In Case 2, the total emission is the high-est among all cases, whereas energy consumption is the least; note that since the model minimizes total energy consumption, it may shift energy consumption from low- to high-price hours, as prices are not considered in the objective function. Case 3 and Case 5 have approximately the same amount of total emissions, which are the lowest emissions among all cases. Case 4 with maximum feasible peak power constraint shows 50% less peak demand as compared to Case 0, while the total energy costs is also less. Gas consumptions in all cases remain almost the same but slightly lower than in Case 0, while electricity consumption is considerably less. Case 3 has the lowest amount of emissions, and it is significantly less compared to Case 0. In general, significant benefits are achieved by deploying the model to optimize the operational schedule of devices in a residential energy hub, i.e., up to $36/month energy cost reductions (Case 1); up to 225 kWh/month energy consumption reductions (Case 2); and up to 34.8 kg/month of emissions reductions (Case 3).

3) Effects on Household's Demand: Figure 11 depicts the effects of peak power constraints on the power consumption profile of the household for a summer day. In this figure the demand of the household for Case 0, Case 1, and Case 4 are illustrated. Observe that the peak demand of the household is shifted from the higher peak-price hours (evening hours) in Case 0 to lower off-peak price hours (after 10:00 PM) in Case 1. The results obtained for Case 4 with no peak power cap (which is the same as Case 1), Intermediate Peak Load (IPL) cap, and the maximum feasible peak load (MFPL) cap shows that the peak demand of the household is significantly reduced without any major increase in total energy costs and energy consumption.

4) Effect of Energy Pricing Schemes: The effects of TOU, RTP and FRP schemes on the operational schedules of the devices for Case 1 and Case 0 are shown in Fig. 12. The results show that the total energy costs are the highest for TOU, although the energy consumptions are lower compared

TABLE III
SUMMARY RESULTS OF ALL CASES FOR A SUMMER DAY.

Case		Energy costs ($)	Energy consumption (kWh)	Gas costs ($)	ESD revenue ($)	ESD energy supply (kWh)	CO_2 emissions (kg)	Peak demand (kW)
0	PT	6.24	56.91	1.44	16.84	21.00	4.96	7.10
	FTS	6.37	58.56	1.44	16.84	21.00	5.07	6.05
1		5.03	49.96	1.35	19.85	24.75	3.98	7.45
2		5.46	49.41	1.35	19.85	24.75	4.27	6.05
3		5.05	49.96	1.35	19.85	24.75	3.80	7.15
4	IPL	5.07	49.96	1.35	19.85	24.75	4.04	5.35
	MFPL	5.32	50.51	1.35	19.85	24.75	4.23	3.55
5		5.04	49.96	1.35	19.85	24.75	3.82	7.75

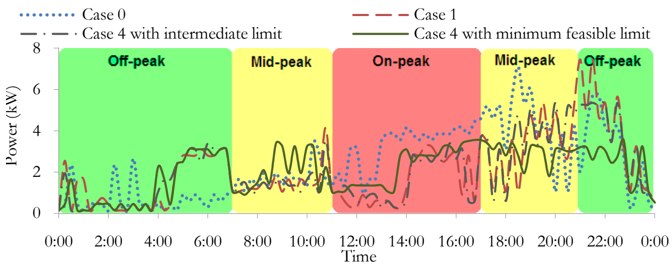

Fig. 11. Effect of peak power constraint on household demand.

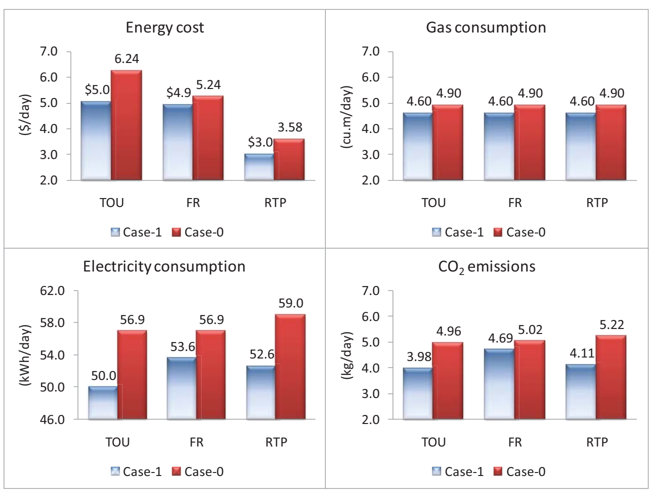

Fig. 12. Comparison of energy cost, energy consumption, gas consumption and emissions for TOU, FRP and RTP tariffs for a summer day in Case 1 and Case 0.

to the RTP and FRP cases. Note that RTP results in the least energy costs, whereas total energy consumption is not the least. Observe that Case 1 yields lower energy costs, CO_2 emissions, and energy consumption for all pricing schemes.

VI. IMPLEMENTATION ASPECTS

A. Parameters Setting Calculations and Procedures

It is important to properly calculate the mathematical model parameters as these have direct impact on the results. It was found that while some parameters in the mathematical model can be easily calculated based on thermodynamic analyses, other parameters are more difficult to determine. Therefore, practical procedures are proposed here to calculate these parameters.

1) Activity Level: An average predictor of the activity level has been used to construct the Activity Level profiles. Since the household occupancy and activity is affected by the season of the year and the day of the week, the measured data of the previous week can be used to predict the energy consumption on a similar day of the present week. The measured or forecasted data is normalized with respect to the total energy consumption of the day, which is assumed to be 100%. Thus, the following formula is used to calculate the Activity Level at time interval t of day i:

$$A(i,t) = \frac{P(i-7,t)}{\sum_{t=1}^{t=96} P(i-7,t)} \tag{15}$$

where $P(i,t)$ represents the measured demand of the household at time interval t of day i.

The effect of household activity on the fridge's energy consumption is modeled using an AL_{fr} index. The minimal value of total energy consumption on a day usually occurs during time periods of inactivity inside the house. Thus any load that is less than this base load will not contribute to the fridge activity. To calculate the activity level for the fridge model, the following formula is proposed:

$$A(i,t) = \frac{P(i-7,t) - P_{base}(i)}{\sum_{t=1}^{t=96} P(i-7,t)} \tag{16}$$

where in the present work it is assumed that the base load is 50% of the average hourly household electrical energy consumption of the same day on the last week.

2) Warming Effect of Fridge OFF State (Ψ_{fr}): To calculate the heat loss during an OFF state of the fridge the following formula can be used:

$$\Psi_{fr} = \Delta\theta_{fr}/\Delta t \tag{17}$$

where $\Delta\theta_{fr}$ is the fridge temperature change during the measurement time interval Δt, when the compressor is OFF.

3) Cooling Effect of Fridge ON State (Φ_{fr}): The cooling effect of an ON state of the fridge can be calculated using the following formula:

$$\Phi_{fr} = \frac{\Delta\theta_{fr}}{\Delta t} + \Psi_{fr} \tag{18}$$

4) Warming Effect of Activity Level on Fridge (Ω_{fr}): To calculate Ω_{fr}, which represents the weight attached to the activity level on the refrigerator temperature increment, one first needs to calculate the Activity Level $A_{fr}(t)$; thus, the term $\Omega_{fr}A_{fr}(t)$ will represent the temperature rise due to refrigerator usage. It was found that daily door openings for a family usage will give 20% to 32% energy consumption increase over the rated value. To calculate Ω_{fr}, the total number of people in the house, n, is needed. The following procedure is then used to calculate Ω_{fr}:

1) Run the optimization model with Ω_{fr}=0, and find the total energy consumption of the fridge over the scheduling horizon, E_0.
2) Calculate the coefficient of house occupancy on energy consumption of the fridge; $G_{fr} = 0.02 + (n-2) \times 0.02$
3) Calculate $E^{max} = (1 + G_{fr}) \times E_0$
4) Change Ω_{fr} by trial and error to find a value that provides an energy consumption of the fridge over the scheduling horizon equal to E^{max}.

5) Effect of Outdoor Temperature (Υ_{ac}): The effect of outdoor temperature on the inside temperature can be calculated using the following formula:

$$\Upsilon_{ac} = \frac{\Delta\theta_{in}}{(\theta_{out} - \theta_{in})} \times \Delta t \tag{19}$$

where $\Delta\theta_{in} = \theta_{in}(t) - \theta_{in}(t-1)$ is the inside temperature change during the measurement time interval Δt when the AC is OFF.

6) Warming effect of Water Heater ON State (Φ_{wh}) : The parameter Φ_{wh} represents the rate at which water temperature increases when the heating element is ON for one time interval of Δt minutes. It can be calculated using thermodynamic equations as follows:

$$H_i = mC_p\Phi_{wh} \tag{20}$$

where H_i is the amount of heat (in Joules) injected into the water when the heater is ON during one time interval; C_p is the specific heat of water = 4185 J/kg °C; Φ_{wh} is the amount of water temperature rise (in °C) per time interval; and m is the mass of the water inside the water heater tank in kg. The amount of heat injected to the water when the heating element is ON for Δt seconds can be also calculated using:

$$H_i = P\Delta t \tag{21}$$

where P is the heating element power rating in watts and Δt is the time interval in seconds. From the above it follows that:

$$\Phi_{wh} = \frac{P\Delta t}{mC_p} \tag{22}$$

7) Water Heater Heat Loss (Ω_{wh}): The parameter Ω_{wh} represents the temperature drop (in °C) due to heat losses through tank walls during Δt seconds. The lost (or rejected) heat (in joules) from a water tank is a function of the hot water temperature, ambient temperature surrounding the water tank, and the water tank shield resistance, and is given by:

$$H_r = mC_p\Omega_{wh} \tag{23}$$

thus,

$$\Omega_{wh} = \frac{H_r^*\Delta t}{mC_p} \tag{24}$$

where H_r^* represents the amount of water tank heat loss to the ambient in joules per second.

B. Experimental Validation and Pilots

An experiment was set up to show the practical determination of the Φ_{fr}, Ψ_{fr}, and Ω_{fr} parameters and to validate the mathematical model of the refrigerator. A small 4.3 ft^3 refrigerator was used for these purposes, and it was loaded with twenty 0.5 liter water bottles to simulate the presence of food inside the refrigerator. To estimate Ψ_{fr}, the refrigerator's inside temperature was measured every minute using a temperature data logger. The power consumption of the fridge was also logged at the same time using an energy logger. The empirical values of the fridge parameters, calculated from the measurements, were used to determine the optimal fridge operation schedule using the developed mathematical model.

The developed mathematical models have been implemented and tested on a single board computer, showing that they can be solved efficiently for real-time applications. The MILP problems were solved using the GLPK solver to obtain the optimal operational schedule of various devices. These models are being implemented in various pilot locations in Ontario to carry out field tests, monitor the performance of the provided system, and study the effectiveness of the models. From preliminary results, it should be mentioned the models presented here are predicting well the actual observed behavior of the different household appliances, particularly for the AC and furnace.

To cope with forecast errors, in pricing and other parameters such as outside temperatures, in the implementation of the model it is considered that if there are large discrepancies between forecasted values and observed ones, i.e. large deviations in the expected objective function values and its actual values, the model will be re-solved using the updated values to minimize the impact of forecasting errors. This approach is somewhat similar to the Model Predictive Control approach used by system operators in generation dispatch (e.g., see [24]), in which the system model model is continuously solved in real time (e.g., every 5 minutes) with the latest information to avoid large objective function deviations from the actual optimal due to forecasting errors.

Finally, it should be mentioned that the proposed model may actually be infeasible and thus yield no solutions, which has been occasionally observed during the current deployment phase. This may occur when over-constraining the model, as in Case 4 in Section V for a peak demand limit greater than 50%, or very narrow temperature ranges for the cooling/heating system. In this case, the infeasible constraints may be relaxed by steadily increasing the constraint limits until a feasible and reasonable solution is obtained.

VII. Conclusions

Optimization models of residential energy hubs which can be readily integrated into household automation systems and

EMSs to increase their functionality, improve their effectiveness, and ensure total energy costs and emissions reduction for customers while considering their preferences and comfort have been presented. Mathematical models of major household demands, i.e., fridge, freezer, dishwasher, washer and dryer, stove, water heater, hot tub, pool pumps, lighting, heating and air conditioning, and generic models for solar PV panels and energy generation/storage devices in a typical house were developed. Based on these models, an MILP optimization problem for the optimal operation scheduling of residential energy hubs to minimize demand, total cost of electricity and gas, emissions and peak load of the hubs was formulated. The developed models incorporate electricity and gas energy carriers, give the priority to customer's preferences, and take into account human comfort factors and CO_2 emissions, thus facilitating the integration of residential customers into Smart Grids. The applications of these models to a real household considering a number of simulation case studies, and a discussion of various implementation aspects have been presented, where it is shown that savings of up to 20% on energy costs and 50% on peak demand can be achieved, while maintaining the household owner's desired comfort levels.

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