

Power System Stability Enhancement over a Network with Random Delays

Raheel Javed

Department of Electrical Engineering
Pakistan Institute of Engineering and Applied Sciences
Email: msse1325@pieas.edu.pk

Ghulam Mustafa

Department of Electrical Engineering
Pakistan Institute of Engineering and Applied Sciences
Email: gm@pieas.edu.pk

Abstract—The emergence of conventional electrical grid into smart grid has enabled the use of networked controllers for power system control applications. This article discusses the rotor angle stability enhancement of a power system using networked control, where the channel has random delays. An optimal stochastic control is designed to damp the oscillations in power system and compensate the delay effects using probability distribution of delays. Due to unavailability of states, a Kalman filter like estimator is used for states estimation. Stability analysis of the closed-loop system is performed for investigating the effect of delays, delay probabilities and parametric variation on stability of the system. WSCC 3-machines, 9-bus system is used as a case study for verification of the designed observer-based control.

Keywords—Smart Grid, Networked Control System, Optimal Control, Kalman Filter, Power System, Thyristor Controlled Series Capacitor, Jump Linear System, Rotor Angle Stability, Participation Factor.

I. INTRODUCTION

There are inherent electro-mechanical oscillations in power systems, e.g., local-plant mode oscillations and inter-area mode oscillations [1]. These oscillations need to be damped; otherwise, power variations may lead to power outages. Disturbances due to load changes in the system or a momentarily fault on a tie-line may trigger these oscillations and may lead to instability if disturbance persists for a longer period. Flexible AC Transmission System (FACTS) controller, like, Thyristor Controlled Series Capacitor (TCSC) provides a mean to damp these oscillations. TCSC is a variable reactance which is controlled by the firing angle of a thyristor. It is installed at a tie-line and can control multiple modes simultaneously. Various attempts have been made to control the oscillations. In [2], authors use controlled series compensation to improve stability of multi-machine power system. It also gives a technique to select a location for series compensation based on residue analysis and design the controller using the pole placement technique. In [3], design of a Linear Quadratic Regulator (LQR) and robust Linear Quadratic Gaussian (LQG) based control for TCSC is presented to control power system oscillations. The designed controllers are verified on WSCC 9-Bus System.

The modernization of conventional electrical grid into smart grid has enabled the integration of communication and IT infrastructure [4]. It has made possible the use of networked controlled systems to enhance the stability of power systems [5]. Networked controlled systems have the

advantages of easier installation, flexibility, lower cost, easier maintenance and reliability. But the use of a networked controller poses challenges, such as, bandwidth limitations, delays in the channel, packets dropout, packets disordering and corruption, etc., which may degrade the performance of the system or cause instability. These issues have been addressed by articles like [6]-[7]. In [6], stability analysis has been performed for a networked control system on smart grid where the channel has random packets dropout, modeled by the Bernoulli's random process. A Kalman filter type estimator is designed and an LQG like control is synthesized to control TCSC for damping power system oscillations. In [8], authors have designed a speed control for networked DC motor in the presence of channel delays and packet losses. Conditions for stability along with zero steady state error are derived and estimation for distribution algorithm is used for optimization of controller parameters. In [9], an LMI based controller has been developed for power systems control based on wide area measurements. The communication effects like time delays, packets dropout and disordering have been considered by incorporating time varying delays in system model. In [7], authors have given the concept of observer-driven system copy to design a control for power system on a constrained bandwidth network. It uses the nominal system to generate measurement data when data from Phasor Measurement Units (PMUs) is not available.

There are many applications that are being implemented in smart grid like state estimation, transient stability, small signal stability and voltage stability, etc., [10]. These applications have different communication or delay requirements [11]-[12]. This paper uses [13] to design an optimal control with a Kalman filter like observer to deal with the problem of random delays that may arise in a channel. If the delays are taken as discrete and random, then the closed-loop system with observer becomes jump linear system (JLS). Stability analysis of JLS is performed to check the closed-loop system stability in case of change in parameters.

The distribution of paper is as follows:

- In Section II, a generalized model of power system is described and its simplification to the form used for design purpose is performed. It also explains the process of discretization of the plant in the presence of delays.

- Section III presents the design of an optimal networked controller that damps the oscillation of the system over the delayed network channel.
- Section IV deals with the design of an observer to give the estimate of states.
- In Section V, stability analysis of the closed-loop system with observer is presented.
- Section VI presents a case study using Western System Coordinating Council (WSCC) 3-machines, 9-bus power system to verify the damping of oscillations using the designed observer-based control.

II. PROBLEM FORMULATION

A. Mathematical Model of Power System

A generalized model of power system with actuator (FACTS controller, like, TCSC) can be represented as a set of non-linear, differential algebraic equations

$$\dot{x}_f(t) = f(x_f(t), y_f(t), u_f(t)) \quad (1)$$

$$0 = g(x_f(t), y_f(t)) \quad (2)$$

where, $x_f(t) \in R^{n_f}$ represents state vector which includes generator dynamics along with exciter and Automatic Voltage Regulator (AVR). $y_f(t) \in R^{m_f}$ represents vector of stator currents, bus voltages and phases. $u_f(t) \in R^{k_{uf}}$ represents input vector of the system and t represents time in seconds.

The location of TCSC in the system can be chosen by following the residue analysis. The procedure is mentioned in [2]. The output of plant can be selected using the concept of mode observability [14]. For design purpose, the plant is usually linearized about an operating point and its number of states is reduced keeping the modes to be controlled dominant in the reduced model [3]. The reduced model takes the form

$$\dot{x}(t) = A_c x(t) + B_c u(t) \quad (3)$$

$$y(t) = C x(t) \quad (4)$$

where, $x(t) \in R^n$ is the reduced state vector, $u(t) \in R^{k_u}$ is the control input vector and $y(t) \in R^m$ is the vector of system outputs. $A_c \in R^{n \times n}$, $B_c \in R^{n \times k_u}$, and $C \in R^{m \times n}$. To see the effects of different modes (eigenvalues) of the system on its states, participation factors are calculated as [15]

$$p_{ki} = \frac{|v_{ik}| |w_{ik}|}{\sum_{k=1}^n |v_{ik}| |w_{ik}|} \quad (5)$$

Where, w_i and v_i are the left eigenvector and right eigenvector corresponding to i th eigenvalue; respectively, and w_{ik} and v_{ik} are k th element of i th eigenvectors.

Usually there are delays in every system like the inherent delays of plant and transmission delays in case of networked control systems. A block diagram of networked controller on smart grid can be seen in Fig. 1.

It should be noted that:

- τ_{cak} and τ_{sck} are the network delays for k th measurement sample from controller to actuator and sensor to controller, respectively.

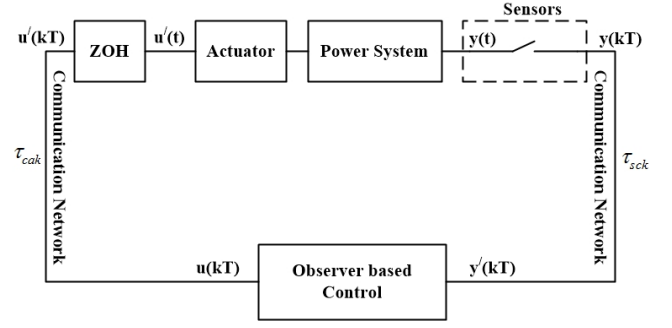


Fig. 1. Networked control of a power system on smart grid

- $\tau_{cak} + \tau_{sck} < T$. Where, T is the sampling period of the system.
- The observer-based control block is event driven. It generates the control signal at the instant when a measurement is received.

B. Sampling of the Continuous Time Model

Assume that the network delays are random, discrete, and can take N values. Then they can be modelled as discrete random variables and the probability mass functions can be obtained by pre-testing the channel as shown in Fig. 3. The delays τ_{cak} and τ_{sck} can take values d_1, d_2, \dots, d_N . If

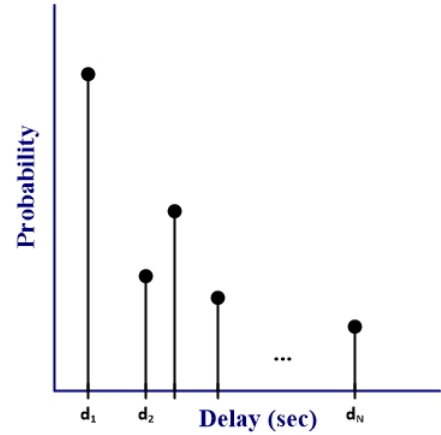


Fig. 2. General delay profile of network

the delay values while testing the channel are closer, then they can be lumped to an average value for simplicity. To implement the observer-based control for the system, the previous values of delays are required. The delay can be measured online by time stamping the packets, synchronizing the clocks and using the method as given in Fig. 3.

The model is discretized at sampling period T . The discretization process is performed by solving the continuous time differential equation (3) [16] and is summarized as:

$$x(kT + T) = e^{A_c T} x(kT) + \int_{kT}^{kT+T} (e^{A_c(kT+T-\tau')} B_c u(\tau' - \tau_k) d\tau') \quad (6)$$

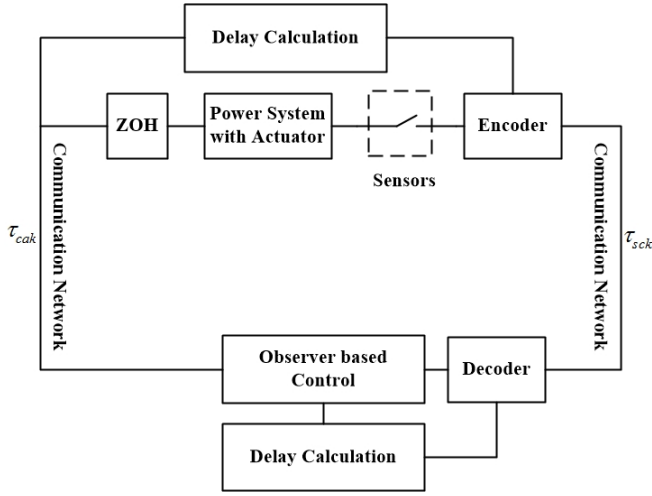


Fig. 3. Online measurement of network delay

where, $\tau_k = \tau_{cak} + \tau_{sck}$. After change of variables and simplification, a compact form can be obtained as follows:

$$x(kT + T) = \phi x(kT) + \Gamma_0(\tau_k)u(kT) + \Gamma_1(\tau_k)u(kT - T) \quad (7)$$

Where,

$$\begin{aligned} \phi &= e^{A_c T} \\ \Gamma_0(\tau) &= \int_0^{T-\tau} e^{A_c \tau'} B_c d\tau' \\ \Gamma_1(\tau) &= e^{A_c (T-\tau)} \int_0^{\tau} e^{A_c \tau'} B_c d\tau' \end{aligned}$$

After adding actuator and sensor noises as v_k and ω_k , respectively, the system can be written as,

$$x_{k+1} = \phi x_k + \Gamma_0(\tau_k)u_k + \Gamma_1(\tau_k)u_{k-1} + v_k \quad (8)$$

$$y_k = Cx_k + \omega_k \quad (9)$$

where, v_k and ω_k are uncorrelated Gaussian white noises with zero mean and covariance matrices R_1 and R_2 , respectively. Note that, in the absence of delays, the pair (ϕ, Γ_0) should be stabilizable and the pair (ϕ, C) should be detectable to have stabilizing observer based control.

III. CONTROLLER DESIGN

The optimal controller which is LQ-controller, is designed to damp the power system oscillations. To design the control law for the delayed system, the following assumptions are made:

- The overall delay from sensor to actuator is always less than sampling period, i.e., $\tau < T$.
- Old time delays i.e. the set $(\tau_{sc0}, \dots, \tau_{sck}, \tau_{ca0}, \dots, \tau_{ca(k-1)})$ is known at the time of calculation of u_k .

The controller can be designed by following similar procedure as given in [13]. The cost function which is minimized is given as

$$J_N = E[x_N^T Q_N x_N] + E \left[\sum_{k=0}^{N-1} \begin{bmatrix} x_k \\ u_k \end{bmatrix}^T Q \begin{bmatrix} x_k \\ u_k \end{bmatrix} \right] \quad (10)$$

where,

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$$

also, Q is symmetric and $Q \geq 0$ but $Q_{22} > 0$. The solution minimizing the performance index can be obtained as follows:

$$u_k = -L_k(\tau_{sck}) \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} \quad (11)$$

where,

$$\begin{aligned} L_k(\tau_{sck}) &= (Q_{22} + \tilde{S}_{k+1}^{22})^{-1} [Q_{12}^T + \tilde{S}_{k+1}^{21} \tilde{S}_{k+1}^{23}] \\ \tilde{S}_{k+1}(\tau_{sck}) &= E_{\tau_{cak}} [G(\tau_k)^T S_{k+1} G(\tau_k)] \\ G(\tau_k) &= \begin{bmatrix} \phi & \Gamma_0(\tau_k) & \Gamma_1(\tau_k) \\ 0 & I & 0 \end{bmatrix} \\ S_k &= E_{\tau_{sck}} [F_1^T(\tau_{sck}) Q F_1(\tau_{sck}) + F_2^T(\tau_{sck}) \tilde{S}_{k+1} F_2(\tau_{sck})] \\ F_1(\tau_{sck}) &= \begin{bmatrix} I & 0 \\ -L_k(\tau_{sck}) \end{bmatrix} \\ F_2(\tau_{sck}) &= \begin{bmatrix} I & 0 \\ -L_k(\tau_{sck}) \\ 0 & I \end{bmatrix} \\ S_N &= \begin{bmatrix} Q_N & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

and, \tilde{S}_{k+1}^{ij} is the (i, j) th block of symmetric matrix \tilde{S}_{k+1} and Q_{ij} is the (i, j) th block of Q . $E_{\tau_{sck}}$ and $E_{\tau_{cak}}$ are the expectations w.r.t the random variables τ_{sck} and τ_{cak} , respectively. The relations to calculate S_k from S_{k+1} is given by a stochastic Riccati equation solved backwards in time. A steady state value S_∞ of S_k can be obtained by iteratively solving the Riccati equation, which will give a stationary values of control law L_1, L_2, \dots, L_N for τ_{sck} values d_1, d_2, \dots, d_N , respectively. The controller thus switching in nature which depends upon the values of network delay from sensor to controller.

Since, all states of the system are not measured, an observer is designed in the next section.

IV. OBSERVER DESIGN

It is impossible most of the times to have full information of states. Thus, state estimators are used. Here, Kalman filter type estimator seems to be a good choice, since, the random delays entering the system in non-linear fashion are known upto time $k-1$ (except $\tau_{ca}(k-1)$) at time k and x_k depends upon delays till $k-1$ as shown in (8). The state estimator which minimizes the error covariance for the plant (8)-(9) is given as follows [13]:

$$\hat{x}_k = \hat{x}_{k|k-1} + \bar{K}_k(y_k - C\hat{x}_{k|k-1}) \quad (12)$$

and,

$$\begin{aligned}
\hat{x}_{k+1|k} &= \phi \hat{x}_k + \Gamma_0(\tau_k)u_k + \Gamma(\tau_k)u_{k-1} + K_k(y_k - C\hat{x}_k) \\
\hat{x}_{0|-1} &= E[x_0] \\
P_{k+1} &= \phi P_k \phi^T + R_1 - \phi P_k C^T [C P_k C^T + R_2]^{-1} C P_k \phi^T \\
P_0 &= R_0 = E[x_0 x_0^T] \\
K_k &= \phi P_k C^T [C P_k C^T + R_2]^{-1} \\
\bar{K}_k &= P_k C^T [C P_k C^T + R_2]^{-1}
\end{aligned}$$

where, \bar{K}_k and K_k are the gain matrices for the k th sample of time and P_k represents the covariance of Gaussian error with zero mean. A stationary solution for gain matrices can be obtained by iteratively solving error covariance equation with the gain equations. It can be seen that the estimator uses a delayed system model to accurately estimate state in the presence of network delays. One important point to consider is that, since $\tau_{ca(k-1)}$ is unknown, its previous value can be used in observer model.

Note, that the separation principle holds here, so, observer and controller can be designed independently.

V. STABILITY ANALYSIS OF THE CLOSED-LOOP SYSTEM

Stability analysis is performed in this section using the Lyapunov theory to investigate the effects of change in delays, delay probabilities and system or controller parameters on the stability of the closed-loop system with observer.

If the noise is not taken into account, then, the system and observer equations (8) and (12) can be written as,

$$x_{k+1} = \phi x_k + \Gamma_0(\tau_k)u_k + \Gamma_1(\tau_k)u_{k-1} \quad (13)$$

$$\hat{x}_{k+1} = \hat{x}_{k+1|k} + \bar{F}_k(y_{k+1} - C\hat{x}_{k+1|k}) \quad (14)$$

$$\hat{x}_{k+1|k} = \phi \hat{x}_k + \Gamma_0(\tau_k)u_k + \Gamma(\tau_k)u_{k-1} + F_k(y_k - C\hat{x}_k) \quad (15)$$

The output equation can be given by,

$$y_{k+1} = C\phi x_k + C\Gamma_0(\tau_k)u_k + C\Gamma_1(\tau_k)u_{k-1} \quad (16)$$

Let, the steady state solutions of controller and observer gain matrices be $L(\tau_{sck})$, \bar{K} and K . Use of control law in above equations gives the following form:

$$\tilde{x}_{k+1} = \mathcal{A}(\tau_k)\tilde{x}_k \quad (17)$$

where

$$\begin{aligned}
\tilde{x}_k &= \begin{bmatrix} x_k \\ \hat{x}_k \\ \hat{x}_{k-1} \end{bmatrix} \\
\mathcal{A}(\tau_k) &= \begin{bmatrix} \phi & -\Gamma_0(\tau_k)L(\tau_{sck}) & -\Gamma_1(\tau_k)L(\tau_{sck}) \\ \mathcal{A}_1 & \mathcal{A}_2 & -\Gamma_1(\tau_k)L(\tau_{sck}) \\ 0 & I & 0 \end{bmatrix} \\
\mathcal{A}_1 &= \bar{K}C(\phi - KC) + KC \\
\mathcal{A}_2 &= \phi - \Gamma_0L(\tau_{sck}) - \bar{K}C(\phi - KC) - KC
\end{aligned}$$

Now, $\mathcal{A}(\tau_k)$ can take N^2 values based on τ_{cak} and τ_{sck} with probability distribution $P_{\tau_{ca}}P_{\tau_{cs}}$. Where, $P_{\tau_{ca}}$ and $P_{\tau_{sc}}$ are probability distributions of τ_{cak} and τ_{sck} , respectively. The

system now becomes a jump linear system (JLS) because of change in system matrix by delay values. Let, $\mathcal{A}(\tau_{krs})$ represent a mode of the system for the case $\tau_{sck} = d_r$ and $\tau_{cak} = d_s$ with probability $P_{\tau_{sc}}(d_r)P_{\tau_{ca}}(d_s)$. Then, stability of the system can be verified as follows:

Consider a Lyapunov function,

$$\begin{aligned}
V_k &= \tilde{x}_k^T P_{ij} \tilde{x}_k \\
V_{k+1} &= \tilde{x}_{k+1}^T P_{gh} \tilde{x}_{k+1}
\end{aligned}$$

where, $i, j, g, h = 1, 2, 3, \dots, N$. P_{ij} is symmetric and $P_{ij} > 0$. The system in (17) will be stochastic stable, i.e., $\lim_{k \rightarrow \infty} E[\tilde{x}_k] = 0$ if $E[\Delta V_k] < 0$.

$$\begin{aligned}
E[\Delta V_k] &= \tilde{x}_k^T (E[\mathcal{A}(\tau_{kgh})^T P_{gh} \mathcal{A}(\tau_{kgh})] - P_{ij}) \tilde{x}_k \\
E[\Delta V_k] &= \tilde{x}_k^T \left(\sum_{g=1, h=1}^N P_{\tau_{cs}}(d_g) P_{\tau_{ca}}(d_h) \mathcal{A}^T(\tau_{kgh}) P_{gh} \right. \\
&\quad \left. \mathcal{A}(\tau_{kgh}) - P_{ij} \right) \tilde{x}_k
\end{aligned}$$

After applying Schur's complement, following LMI constraints are obtained:

$$\begin{bmatrix} P_{ij} & \mathcal{A}^T(\tau_{k11}) & \dots & \mathcal{A}^T(\tau_{kNN}) \\ \mathcal{A}(\tau_{k11}) & P_{\tau}(d_1, d_1)P_{11} & \dots & 0 \\ \vdots & 0 & \dots & 0 \\ \mathcal{A}(\tau_{kNN}) & 0 & \dots & P_{\tau}(d_N, d_N)P_{NN} \end{bmatrix} > 0$$

Where, $P_{\tau}(d_g, d_h) = P_{\tau_{cs}}(d_g)P_{\tau_{ca}}(d_h)$ and $i, j = 1, 2, 3, \dots, N$.

VI. CASE STUDY

Consider the problem of small signal rotor-angle stability for the power system shown in Fig. 4 [15]. To represent the dynamics of the generators, two-axis model is used with the IEEE type-I exciter. The details of the model and the parameters involved can be seen in [15]. The TCSC is installed between line 6 and 9 (from residue analysis) and its dynamics are represented as [2],

$$\dot{X}_c = \frac{1}{T_c}(-X_c + X_u), \quad -0.073p.u. < X_c < 0.073p.u. \quad (18)$$

where, X_c is reactance of TCSC, T_c is the time constant equal to 0.1sec and X_u is the input signal to TCSC. The model of power system and TCSC is combined and then linearized. A state of the linearized system can be reduced by referring the rotor angles of machine 2 and 3 (δ_2 and δ_3) to the rotor angle of machine 1 (δ_1). The eigenvalues analysis of the system shows that the system is stable but two pairs of eigenvalues are poorly damped causing oscillatory behavior as shown in Table VI. δ_{21} and δ_{31} have high participation factors in these modes. So, the modes need to be damped to control rotor angle deviations. The outputs of the system are chosen here to be the rotor angles of machines 2 and 3, i.e., δ_{21} and δ_{31} . The linearized model is not reduced.

Here, $n = 21$, $m = 2$, $k_u = 1$. The system is discretized at sampling period T which is taken as 0.2s. The sampling time is calculated by the Nyquist criterion applied to the modes that need to be controlled. The network delays (both τ_{sck} and τ_{cak}) can take values 0.03s, 0.06s and 0.09s with probability

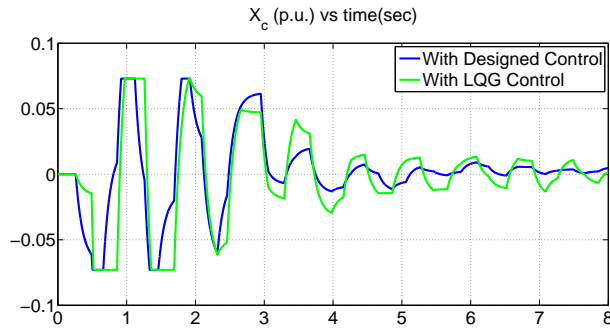


Fig. 7. Reactance of TCSC

the design is that, the overall network delay should be less than the sampling period. So, delay compensation techniques employing time varying sampling periods to minimize the effects of delays can be used for power system oscillations control, when the network delays are greater than the sampling time of the system as given in [17], [18] and [19].

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