Project Report

Hypersonic Scenarios in ISR360: Trajectories and Interception Systems

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Abstract

This document summarizes the features developed and integrated in the ISR360 software to simulate an end-to-end scenarios of hypersonic pursuit evasion. It involves components like target trajectory generators, seekers for the navigation, coordinated and independent guidance laws as well as dynamic systems for the interceptor models. The report is organized as follows:

- Chapter 1 describes the general format of the optimal control problem that has been used to implement two trajectory generators. Brief descriptions about their characteristics is then included followed by their implementation details and the tools utilized for their integration in ISR360. Test scenarios are then generated for both the air breathing and boost glide hypersonic vehicles to analyze their responses. The section also highlights key traits of their trajectories that make them suitable for different mission requirements.
- Chapter 2 details different guidance laws to avert hypersonic threats using multiple interception systems. The tools that are used for their implementation are also highlighted. The effectiveness of these systems is presented using two ISR360 scenarios based on air breathing and boost glide hypersonic targets. Performance of different guidance strategies is compared using their miss distances.
- Chapter 3 concludes the report and establishes the need of future work to improve the range and accuracy of the simulations of hypersonic scenarios in ISR360.

Contents

1 Hypersonic Trajectory Generators

A hypersonic aircraft due to its high mach number and high altitude holds a huge strategic importance. Ranging from time-sensitive missions of surveillance to air strikes, they provide an edge through their unpredictability and maneuverability. They also have an ability of less fuel reliance and threat identification from a long distance which they can outrun or outmaneuver using a passive defence. Realizing such advanced hypersonic systems pose challenges in the form of vehicle design considerations, constrained trajectory optimization to maintain the structural integrity of the vehicle as well as fulfill the mission requirements. A wide variety of research has thus been conducted into this constrained optimization problem. The accuracy and completeness of system dynamics and constraints are very crucial to the mission success and hence hold key importance for mission planners.

A generic framework of these optimization problems takes the form as shown in Figure 1.

- The hypersonic vehicle model contains the aerodynamics and the control system of the vehicle represented in the form of differential equations. The Earth's atmospheric model allows the calculation of quantities like vehicular lift and drag.

- Limiting constraints can be regarded as end-to-end like keeping the control commands and their rates in check to reduce the energy consumption for the mission. Mission specific constraints are imposed to meet the actual purpose of the flight like the trajectory endpoints. They also combine the requirements like no-fly-zones, maneuvers for surveillance as well as the way-points along the trajectory.

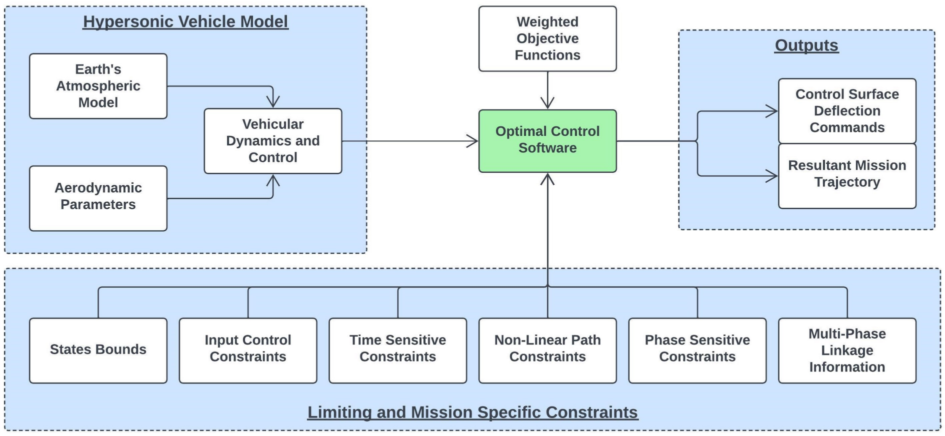

Figure 1: Optimization Problem for the Hypersonic Trajectory Generation

1.1 Multi-Phase Boost Glide Vehicle

The hypersonic trajectory presented in [8] has been implemented and added to the ISR360 platform. It depicts a multi-phase vehicle that vertically boosts to the exo-atmosphere and then re-enters the atmosphere to glide towards the destination. In the last phase, the vehicle descends and performs a dive attack. The major use of such vehicle thus lies in the aerial strikes. The problem consists of 8 phases in total with constraints involving dynamic pressure, heating load and rate as well as phase specific constraints and linkages. Phases 1 to 4 comprise boost stage of the trajectory where the vehicle escapes the atmosphere. In phase 5, the vehicle reaches the peak altitude and its descend starts in phase 6 till it reaches the piercing point of the atmosphere. Phase 7 and phase 8 involve low and high dynamic pressure flight, respectively guided to the destination at the end of phase 8. The objective function of this problem limits the control surface deflections by minimizing the control commands which can be written as [8]:

$$\begin{aligned} f &= \sum_{p=1}^{8} f^{(p)} \\ f^{(p)} &= \int_{t_0^{(p)}}^{t_f^{(p)}} [(\frac{\alpha - \bar{\alpha}}{\alpha_{max}})^2 + (\frac{u_\alpha}{u_{\alpha,max}})^2 + (\frac{u_\sigma}{u_{\sigma,max}})^2] dt \end{aligned} \tag{1}$$

Where, p = phase number, α = angle of attack, $\bar{\alpha}$ = nominal angle of attack, u_α = rate of angle of attack and u_σ = rate of bank angle. The optimization problem is constrained by the aerodynamics in [4] for phases 2 to 7. Due to singularity in the dynamics during vertical flight phases 1 and 8 are represented by [7]. The constrained optimal control problem is solved using

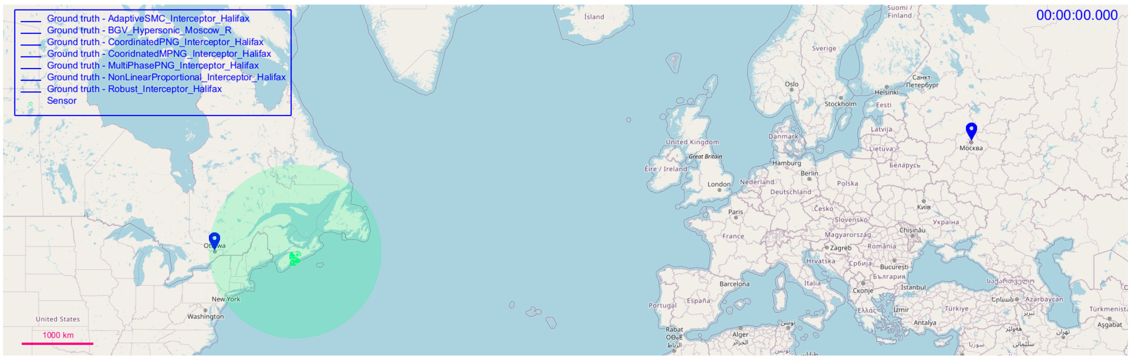

Figure 2: ISR360 Scenario for Boost Glide Hypersonic Ascent Entry Vehicle

IPOPT [12] in Julia with the linear solver **HSL MA27**. The code has been implemented in Windows Subsystem for Linux (WSL) and then integrated via a MATLAB plugin into the ISR360. ISR360 allows full configuration for the trajectory generator including paramterization of different dynamics and constraints as well as the trajectory endpoints.

1.1.1 Test Scenario

Figure 2 depicts a test scenario for a multi-phase boost glide hypersonic vehicle. The blue points of interest in Figure 2 represent end points of the trajectory. The results of the optimization

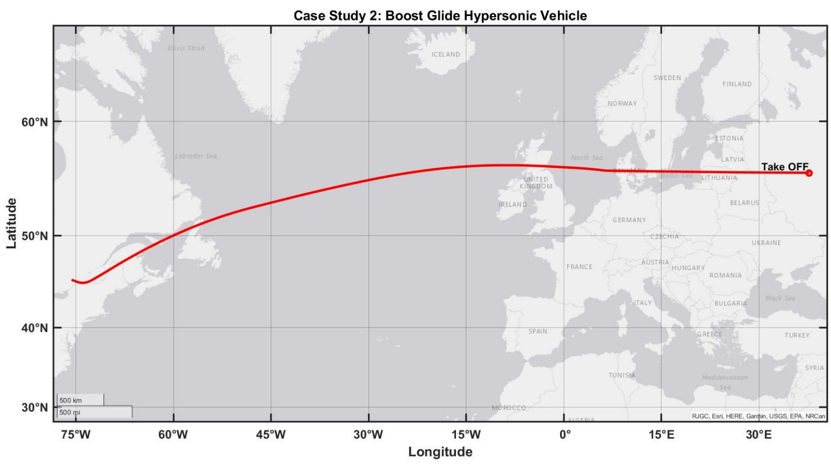

Figure 3: Trajectory of the Boost-Glide Vehicle

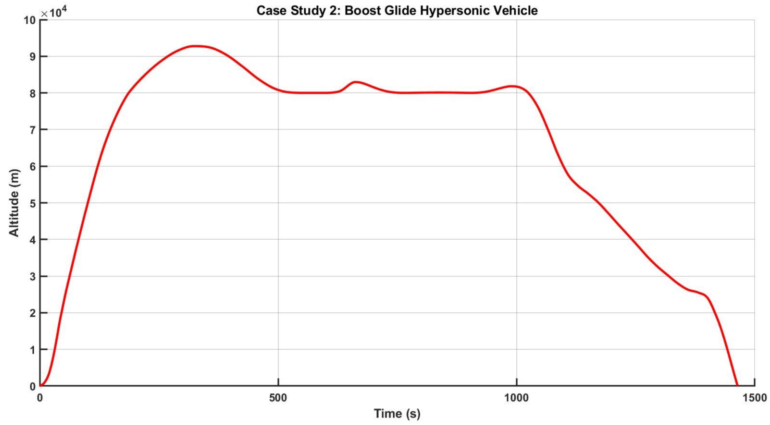

Figure 4: Altitude of the Boost-Glide Vehicle

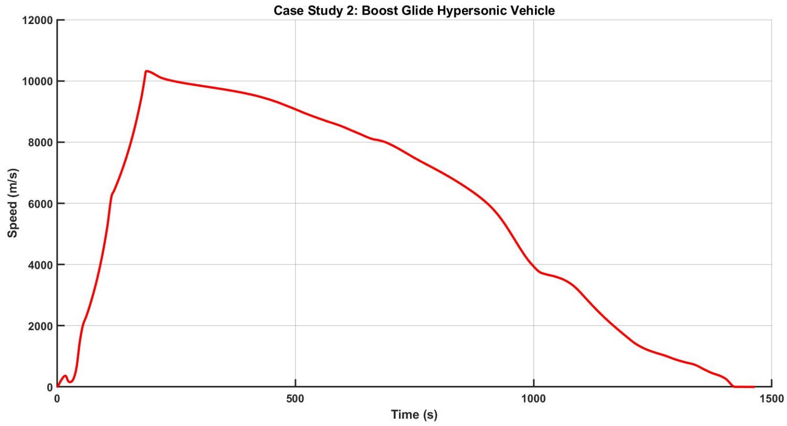

Figure 5: Speed of the Boost-Glide Vehicle

problem can be seen in Figures 3, 4 and 5. It can be seen that the maximum speed is achieved when the altitude is at the peak and the vehicle exits the atmosphere. After this phase, the vehicle looses the speed continuously as it descends to re-enter the atmosphere. It then glides at almost a constant altitude until it reaches its target, where the vehicles descends again and goes into a vertical dive attack.

1.2 Air Breathing Hypersonic Reconnaissance Vehicle

It can be observed that the multi-phase boost glide vehicle does not include way-points and no-fly-zones but instead it achieves a very high altitude and speed by escaping the Earth's atmosphere and then re-entering. It also does not use any fuel other than the boost phase and it uses its high speed and altitude to outmaneuver any threats. This section describes the implementation of the vehicle presented in [6].

The vehicle referred to as air breathing uses fuel for its scram jet to propel it to the hypersonic speeds. The trajectory of this hypersonic vehicle includes way-points that can cause an altitude change and speed variation as well as no-fly-zones that the vehicle avoids as a defence measure. Such trajectories are useful for reconnaissance missions where a certain position and orientation may be required to point specialized sensors for gathering surveillance data. The optimization problem for this type of vehicle includes dynamic pressure and temperature constraints as well. Additional constraints are introduced when multiple way-points are required for the mission which makes the problem a multi-phase problem. Fuel injection rate serves as an additional input for air breathing vehicle unlike a gliding vehicle. The objective function then takes the following form:

$$f = \epsilon(t_f - t_0) + (1 - \epsilon) \int_{t_0}^{t_f} \|u(t)\|_2^2 dt \tag{2}$$

Where, ϵ = relative weighting factor, $(t_f - t_0)$ = flight duration and u = vector of input commands that include angle of attack, banking angle and fuel injection rate. The objective function thus minimizes a combination of total flight time as well as control efforts to meet the mission criteria. The optimal control problem is solved using ICLOCS[9] that uses pseudospectral methods for the non-linear optimization. The trajectory generator is integrated into the ISR360 as a MATLAB plugin. The optimization problem uses 1976 U.S. Standard Atmosphere as Earth's atmosphere where as preset aerodynamic and propulsion model are used from [14] and [13], respectively. The trajectory parametrization includes configuring the end-points, waypoints and no-fly-zones.

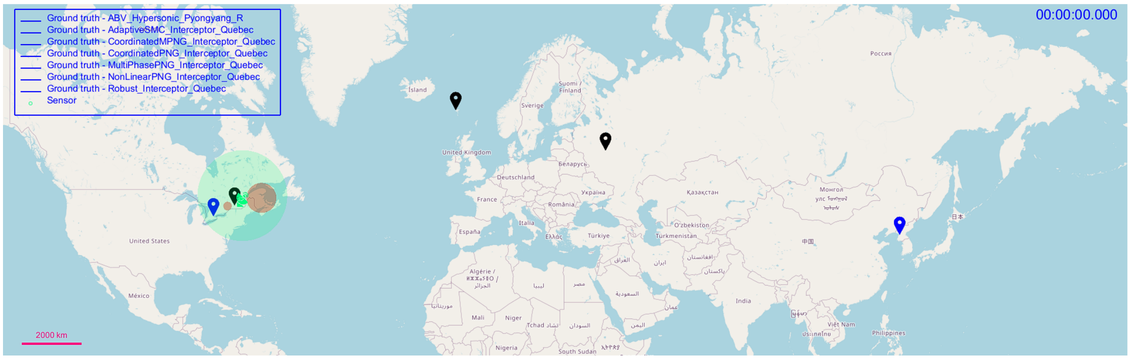

Figure 6: ISR360 Scenario for Air Breathing Hypersonic Reconnaissance Vehicle

1.2.1 Test Scenario

Consider the test scenario for the air breathing vehicle shown in Figure 6. The blue points of interest show trajectory endpoints and black points represent the way-points vehicles would be taking to reach the destination. The regions highlighted by the red circles are considered as no-fly-zones. The trajectory output can be seen in Figure 7 and the speed and altitude responses can be observed from Figures 8 and 9, respectively. It can be seen that the vehicle takes-off and cruises at a constant altitude until it reaches a way-point where it descends and reduces its speed as well. This trait can be attributed to the thicker atmosphere at lower altitude that limits the speed as more resistance is offered by the atmosphere. Since this vehicle uses fuel for propulsion which limits the distance it can cover, low altitude way-points can also act as in-flight refuelling options[6] to extend the range of the vehicle as per the mission requirements. This feature has also been incorporated in the ISR360.

2 Hypersonic Interception Systems

The hypersonic aircrafts as mentioned in the previous chapter find many applications in surveillance and aerial support missions which calls for exploring deterrence techniques if they are used by hostile elements. The traditional interception strategies fall short in case of hypersonic vehicles due to their high altitude, high speed and maneuvering capabilities. So, improved interception systems are required for effective threat elimination of such advanced weapons. To complete the end-to-end hypersonic scenarios, ISR360 includes multiple independent and coordinated interceptors along with their guidance laws and navigation.

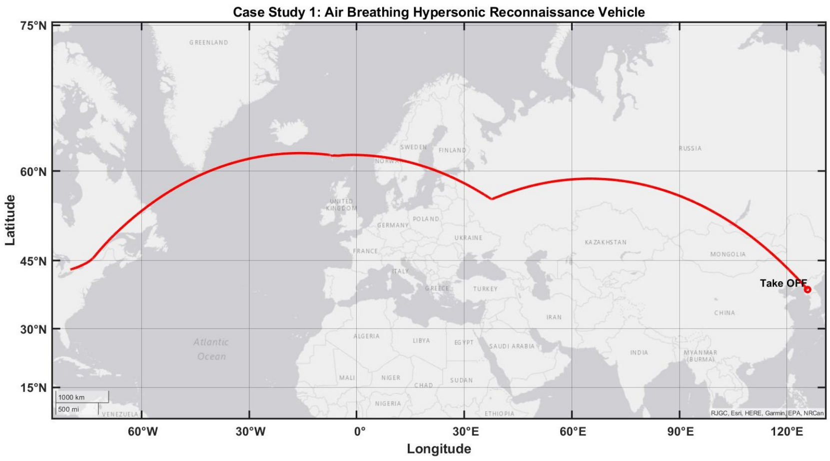

Figure 7: Trajectory of the Air-Breathing Vehicle

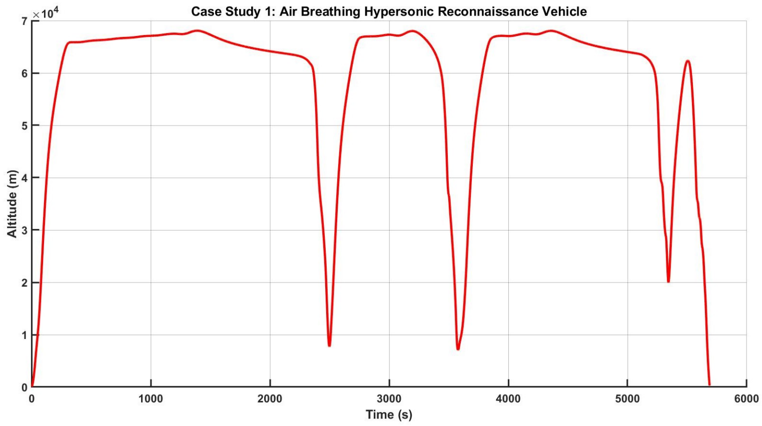

Figure 8: Altitude of the Air-Breathing Vehicle

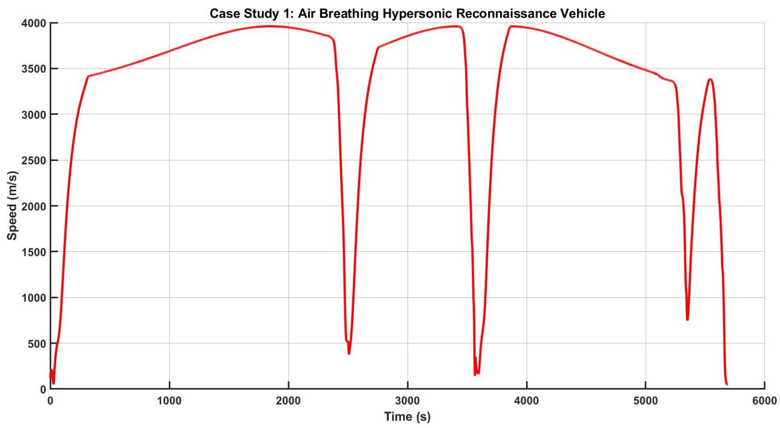

Figure 9: Speed of the Air-Breathing Vehicle

2.1 Guidance Laws

2.1.1 Independent Guidance Algorithms

- Multi-Phase Boost Glide Interceptor with Proportional Guidance:

A distinct trait of hypersonic vehicles is their gliding behaviour at high altitude. This interception technique as described in [5] aim at intercepting hypersonic targets during their gliding phase where their maneuverability is limited as compared to their vertical dive attack phase. This guidance law comprises four phases:

  - It starts with a boosting phase to achieve a high altitude, high speed and a desired orientation.
  - It then enters the gliding phase which makes the rate of line of sight angle constant to ensure the same heading direction as the target.
  - The scenario then enters the aiming phase where the interceptor utilizes proportional guidance to chase the target.
  - When the target is in close vicinity, booster rockets are utilized for a proper orientation and intercepting the target in the hit-to-kill phase.

- Robust Fractional Calculus based Proportional Guidance Law:

  It can be observed that hypersonic aircrafts maintain a constant altitude during most of the flight and thus require relatively lesser efforts for interception in the longitudinal plane. This interception strategy [2] caters for the maneuvers in the lateral plane and uses rate as well as the acceleration of the line-of-sight angle. This allows the interceptor to respond quickly to target maneuver and improves the killing accuracy. Due to a very high sensitivity of the acceleration and the guidance law's dependency on it, fractional derivatives are employed to enhance the robustness of the guidance law which then takes the form of a proportional guidance law.

- Non-Linear Proportional Guidance Law:

  This interception law is also described in [2]. It uses the similar proportional guidance but the acceleration and rate of line-of-sight angle is calculated using a dynamic tracker.

- Adaptive Reaching Law based Guidance:

  Since hypersonic vehicles attain a very high speed and need a complicated design to meet speed requirements. So, having an interceptor reaching a comparable speed also require meticulous construction that may not be always possible or offer huge budget for realization. [11] designs a guidance law for relatively slow speed interception which is achieved in three phases:

  - In the first phase, the interceptor uses proportional guidance to approach the target.
  - The interceptor then takes a turn and moves away from the target.
  - In the terminal phase, since the interceptor moving away from the target at a slower speed, the target approaches the interceptor and the interception is achieved.

2.1.2 Coordinated Guidance Algorithms

- Coordinated Guidance Strategy for Heterogeneous Missiles

  The hypersonic vehicles due to their maneuverability and evasion often make it difficult for a single interceptor for a complete defence strategy. Multiple coordinated interceptor system can thus prove as a valuable asset for this deterrence. [10] describes a cooperative guidance strategy based on leader follower technique that communicate using a minimum spanning tree structure. It ensures that all the members are fully connected for data exchange as only the leader have access to a seeker to pursue the target while followers share the data to chase the leader. The guidance law used by the leader is a modified proportional guidance that makes of target overload in lateral plane to respond effectively to maneuvers. The cooperative guidance ensures that the followers converge to the location of the leader. This guidance law is also developed in 2D- lateral plane.

2.2 Implementation Details

All the interceptors discussed above have been implemented as a single C++ library that has been integrated via a C++ plugin of ISR360. A single GUI can be used to configure and utilize these range of interceptors. A number of third-party tools are utilized to incorporate features for these interception systems in the ISR360 including **GeographicLib** for calculating geodesic distances and angles for the navigation, **Ascent** for solving dynamic systems to find states of the interceptors and **Eigen**[3] to handle matrices and vectors in case of coordinated guidance laws.

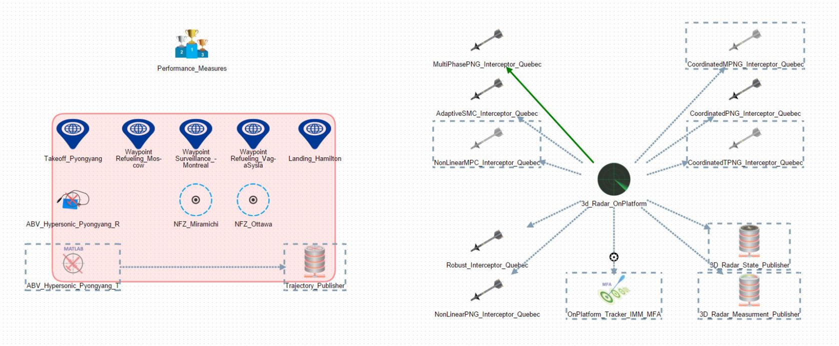

Figure 10: Designer window for the scenario of Figure 6 (Case 1)

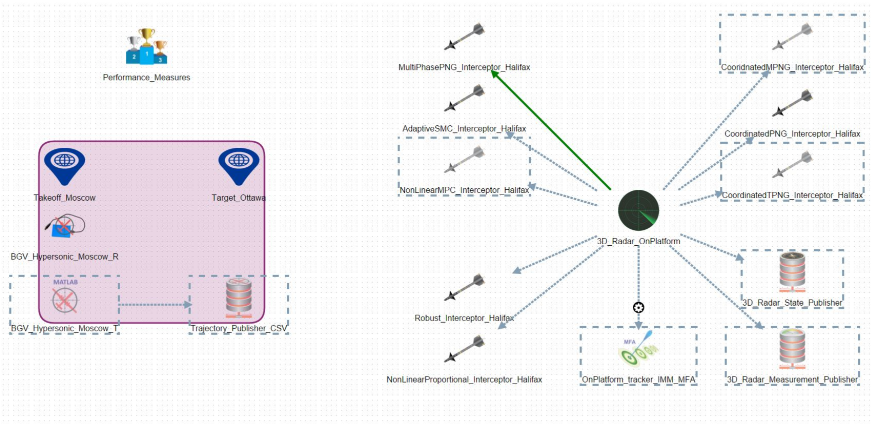

Figure 11: Designer window for the scenario of Figure 2
(Case 2)

2.3 Case Studies

Consider the scenarios of Figure 6 and 2 termed as case study 1 and 2 from now and on, respectively. The green region in the Figures 6 and 2 on the map represents the area covered by the interceptors. This also acts as the detection range of the sensor and the pursuit of the target begins as soon as it enters this region. The trajectories of the interceptors for case study 1 and 2 are shown in Figure 12 and 13, respectively. It can be seen that the interceptors takes different trajectories based on their distinct guidance strategies and response to target maneuvers. An important factor to consider before analyzing the trajectories is that

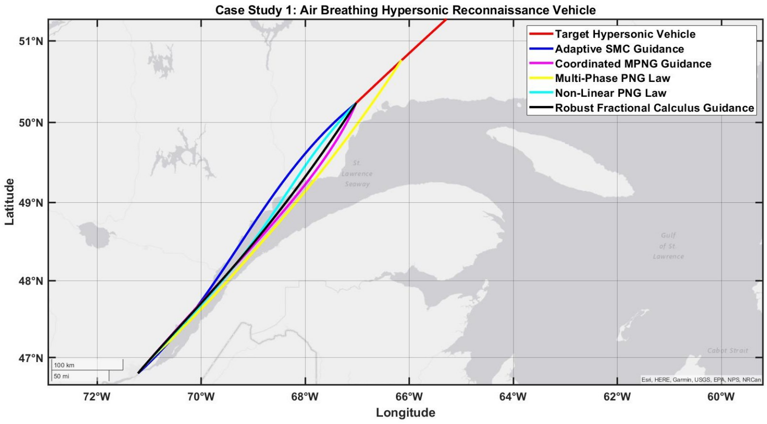

Figure 12: Interceptor Trajectories in the Scenario of Figure 6

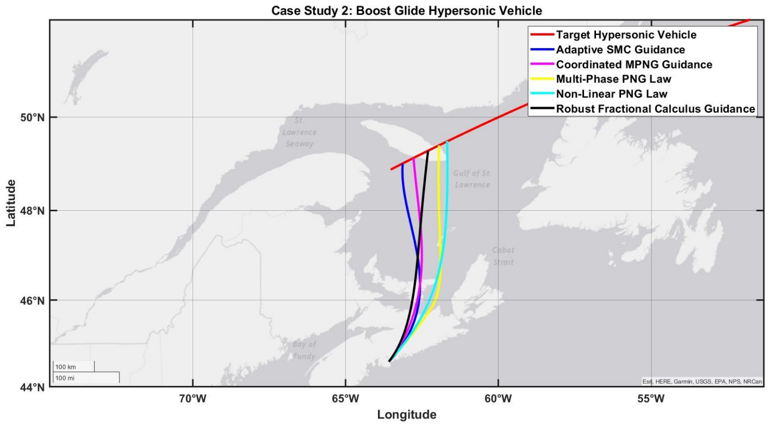

Figure 13: Interceptor Trajectories in the Scenario of Figure 2

the detection system (radar) has a sampling time of $1s$. It means that the guidance commands are also updated once every second and thus the miss distances can be a few hundred meters as the speed of chase exceeds $2000\frac{m}{s}$. The sampling time is set to larger value to increase the speed of the simulation. ISR360 allows the configuration of the distance for interception individually for each interceptor system.

2.3.1 Performance Analysis

The performance of an interceptor can be gauged using different metrics but since the primary purpose is the interception of the hypersonic target, a comparison here is made using their miss distances. Miss distance can be regarded as the minimum distance between the interceptor and the target that is achieved during the pursuit. The lesser the miss distance, the more accurate is the interceptor system to eliminate the hypersonic threat. Table 1 summarizes the performance of different interceptor systems available in ISR360 for both the case studies involving air breathing hypersonic vehicle as well as boost glide vehicle.

- The adaptive reaching law has the highest miss distances for both the scenarios. It can be partly attributed to the utilization of a slower speed for interception as compared to other strategies.
- The coordinated guidance shows improvement as expected but it still does not match the performance of other interceptor systems. This degradation can be attributed to a

Table 1: Performance of Different Guidance Strategies for Hypersonic Target Interception

Guidance Law	Case 1: Miss Distance (m)	Case 2: Miss Distance(m)
Adaptive Reaching Law	574.189	398.511
Multi-Phase Boost Glide	386.183	242.311
Coordinated Guidance	522.964	311.758
Robust Fractional Calculus	58.7599	22.5966
Non-Linear PNG	117.043	498.005

number of factors including tuning of the leader since followers rely on its trajectory for convergence. Number of interceptors and their positions relative to the leader are also a major factor in determining the overall interception performance.

- Multi-phase boost glide uses a hypersonic interceptor to pursue a hypersonic target and uses a dynamic model for the interceptor. This can be attributed to the enhanced performance since the interceptor can now imitate the maneuvers of the target that aids the interception process.

- The best response is demonstrated by the fractional calculus guidance which can be a direct consequence of utilizing line-of-sight angle's acceleration to better predict and cater to the target manuevers. The improvement is by far impressive than the rest of the interception strategies.

3 Conclusion

This report summarizes the hypersonic trajectory generators and interception systems integrated into the ISR360. It also details the performance of these systems via two scenarios including two different varieties of hypersonic vehicles. The simulation demonstrates the possibility of intercepting hypersonic weapons as well as different strategies for it. The available capabilities though provide means for an end-to-end scenario to evaluate a hypersonic pursuit-evasion but it still lacks some aspects that can increase the range of scenarios as well as the accuracy of the simulations.

So, the future work includes the addition of more hypersonic trajectory generators like the boost-skipping trajectory as presented in [1]. Another key improvement would be to include dynamic models and controllers for the interception systems for a more realistic simulation experience. The future work also includes addition of more independent and coordinated guidance techniques for the interceptor systems to provide a variety of selection for deterrence schemes suiting for different hypersonic invasion scenarios.

References

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