EE605 Term Project

Design of Robust Control for Single Machine Infinite Bus System

Submitted By:

Raheel Javed

Department of Electrical Engineering
Pakistan Institute of Engineering & Applied Sciences

Course Instructor: Dr. Ghulam Mustafa

Contents

Abstract

This report presents basic concepts of robust control design, applied on electromechanical model of synchronous generator. The second order model of Single Machine Infinite Bus (SMIB) is used to develop a robust controller, by H-infinity (H_∞) synthesis techniques, which guarantee stability and performance. Using Matlab, the frequency characteristics of sensitivities functions and the mechanical power time response in closed-loop connection with designed controller have been simulated. The synchronous generators are probably most important components and design of power system stabilizers starts with a mathematical model of these. Depending on nature of study, several models of synchronous generator, having different levels of complexity have been developed. Here the input output and input-state-output mathematical model is used in its simplest form for robust control of SMIB based on mixed sensitivity techniques.

Chapter 1

Mathematical Model of Plant

From [1]. Writing swing equation for the generator,

$$J\frac{d\omega_m}{dt} = T_a = T_m - Te \tag{1.1}$$

Where : J is moment of interia of rotating mass $[Kgm^2]$; ω_m is the angular velocity of rotor $[rad/sec]$; T_a is the accelerating torque $[Nm]$; T_m is the mechanical torque; T_e is the electrical torque. Using,

$$J = \frac{2HS_b}{\omega_{om}^2}, \quad \frac{\omega_m}{\omega_{om}} = \frac{w_r}{\omega_o} \tag{1.2}$$

$$\omega_{om}\frac{T_m - T_e}{S_b} \approx P_m - P_e \tag{1.3}$$

Where: H is inertia constant $[Joule]$; S_b is the generator base rating $[VA]$; ω_{om} is the rated angular velocity. We have,

$$2H\frac{d\omega}{dt} \approx P_m - P_e \tag{1.4}$$

Where: $\omega = \frac{\omega_r}{\omega_o} - 1$. Usually, damping torque D is also added to T_a. Denoting δ as the position of rotor and considering synchronously rotating frame system. We have: $\delta = (\omega_r - \omega o)t + \delta_o$. Now using, $2H = M$ the final model can be written as,

$$\dot{x_1} = \omega_o x_2 \tag{1.5}$$

$$\dot{x_2} = -\frac{\omega_o}{M}P_e(x_1) - \frac{D}{M}x_2 + \frac{\omega o}{M}P_m \tag{1.6}$$

Where: $x = [x_1 \ x_2]^T = [\delta \ \omega]^T$
In state space form,

$$\dot{x} = Ax + Bu \tag{1.7}$$

$$y = Cx + Du \tag{1.8}$$

The parameters of the system are chosen so that the generator exhibits quite an oscillatory behaviour. The result is,

$$A = \begin{bmatrix} 0 & 100\pi \\ -0.05 & -0.01 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0 \tag{1.9}$$

Chapter 2

H∞ Problem Formulation

The system to be controlled (P), in closed-loop connection is depited in fig2.1. Where, r is

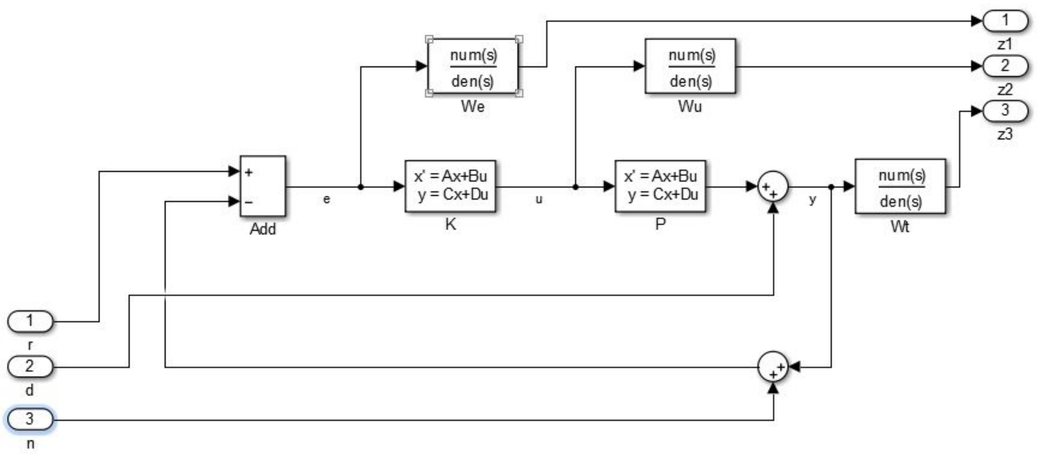

Figure 2.1: Close-loop system (mixed sensitivity)

reference input, d is output disturbance and n is measurement noise. K is controller and We, Wu and Wt represent weighting functions. The plant (P) is represented by the transfer function,

$$P = C(sI - A)^{-1}B + D \tag{2.1}$$

The sensitivity and complementary sensitivity function of feedback system is,

$$S = (1 + KP)^{-1} \tag{2.2}$$

$$T = KP(1 - KP)^{-1} = 1 - S \tag{2.3}$$

It can be shown that:

- S is transfer function from r to e, d to y and d to e. It should be high pass to reduce the error signal and attenuate the disturbance which is usually low frequency signal.
- KS is transfer function from r to u, n to u and d to u. It should be a low pass filter with controlled gain to prevent saturation and attenuate effect of noise.
- T is transfer function from r to y, n to e and n to y. It should be a low pass filter to allow tracking of reference and attenuate noise.

- Usually a trade off is made between these functions. It may happen that there is steady state error at output at the cost of low energy consumption, disturbance and noise rejection. In this case gain can be applied to reference signal without effecting over all performance of system.

The sensitivity function S depends on frequency and in ideal case should be zero. In practical applications, for certain range of frequencies, an upper bound on peak value of S is specified.

$$\|S\|_\infty = \sup_{\omega \in R} |S(j\omega)| \tag{2.4}$$

In order to reduce dependance, a weight function $We(j\omega)$ is used as,

$$\|W_e S\|_\infty = \sup_{\omega \in R} |W_e(j\omega)S(j\omega)| \tag{2.5}$$

The problem of H_∞ controller synthesis can be formulated as: Given generalized plant, exogenous inputs, outputs and control specifications, find all admissible controllers K such that the H_∞ norm of transfer matrix from exogenous inputs to outputs is minimized, subject to constraint that all K's stabilize the plant P. In practical applications it is sufficient to find a controller such that,

$$\|W_e(j\omega)S(\omega)\|_\infty < \gamma \tag{2.6}$$

and this problem is called H_∞ suboptimal problem. In design procedure it is appropriate to specify more than one performance index, expressed mathematically as,

$$\left\| \begin{matrix} W_e S \\ W_u K S \\ W_t T \end{matrix} \right\|_\infty < \gamma \tag{2.7}$$

The equation 2.7 defines problem of mixed sensitivity and it means a good tracking, limitation of control signal energy and insensitive to disturbance and noise. It can be seen that the inverse of weighting functions set design requirements for sensitivity functions. This is the problem which is solved here. For given real number (γ), there exists an admissible controller, $K_{adm} = Y.X^{-1}$, if and only if the following conditions are met:

$$H_\infty \in dom(Ric) \ and \ X_\infty = Ric(H_\infty) \geq 0 \tag{2.8}$$

$$J_\infty \in dom(Ric) \ and \ Y_\infty = Ric(J_\infty) \geq 0 \tag{2.9}$$

$$\rho(X_\infty Y_\infty) < \gamma^2 \tag{2.10}$$

Where, H_∞, J_∞ are given in [2] and Ric refers to algebric riccati equation. If conditions are met then,

$$K_{adm} = \begin{bmatrix} A_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \tag{2.11}$$

Where, $A_\infty, Z_\infty, L_\infty \ and \ F_\infty$ are given in [2].

Chapter 3

Solution to Problem

3.1 Selection of Weighting Functions

$$
\begin{aligned}
W_e &= \frac{\frac{s}{M_e} + \omega_{be}}{s + \omega_b \epsilon_e} && (3.1)\\
W_u &= \frac{s + \frac{\omega_{be}}{M_u}}{\epsilon_u s + \omega_{bu}} && (3.2)\\
W_t &= \frac{s + \frac{\omega_{bt}}{M_t}}{\epsilon_t s + \omega_{bt}} && (3.3)
\end{aligned}
$$

For inverse of sensitivity function M represents pass band peak, ω is cutoff frequency. The inverse of weighting functions serve as a bound for sensitivity functions. The parameters are chosen according to frequency band and gain requirements on sensitivity functions which are,
$M_e = 1.2,\ \epsilon_e = 0.2,\ \omega_{be} = 5rad/sec$
$M_u = 1.2,\ \epsilon_u = 0.05,\ \omega_{bu} = 12rad/sec$
$M_t = 1.2,\ \epsilon_t = 0.2,\ \omega_{bt} = 12rad/sec$

3.2 LFT of Closed Loop System

This part involves representing the closed loop system as shown in fig3.1. From [2] Choosing,

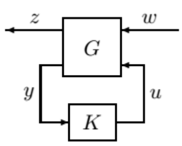

Figure 3.1: LFT of closed loop system

$z = [z_1\ z_2\ z_2\ e]^T, w = [r\ d\ n\ u]^T$ the transfer matrix T_{zw} is given as,

$$
T_{zw} = \begin{bmatrix} W_e & -W_e & -W_e & -W_e P \\ 0 & 0 & 0 & W_u \\ 0 & W_t & 0 & W_t P \\ 1 & -1 & -1 & -P \end{bmatrix} \tag{3.4}
$$

The minimal realization of T_{zw} which is equal to G is obtained using Matlab.

Chapter 4

Matlab Simulation

4.1 Simulation Results

The Matlab code for simulation is given in appendix. $hinfsyn$ command is used to find controller with γ in the range of 0-10 with tolerance of 0.0001 and γ comes out to be 5.0730. The results obtained are shown. The controller K has transfer function,

$$K = \frac{0.001737s^4 + 20.94s^3 + 4925s^2 + 378.2s + 77360}{s^5 + 122.5s^4 + 2541s^3 + 22620s^2 + 121400s + 409200} \tag{4.1}$$

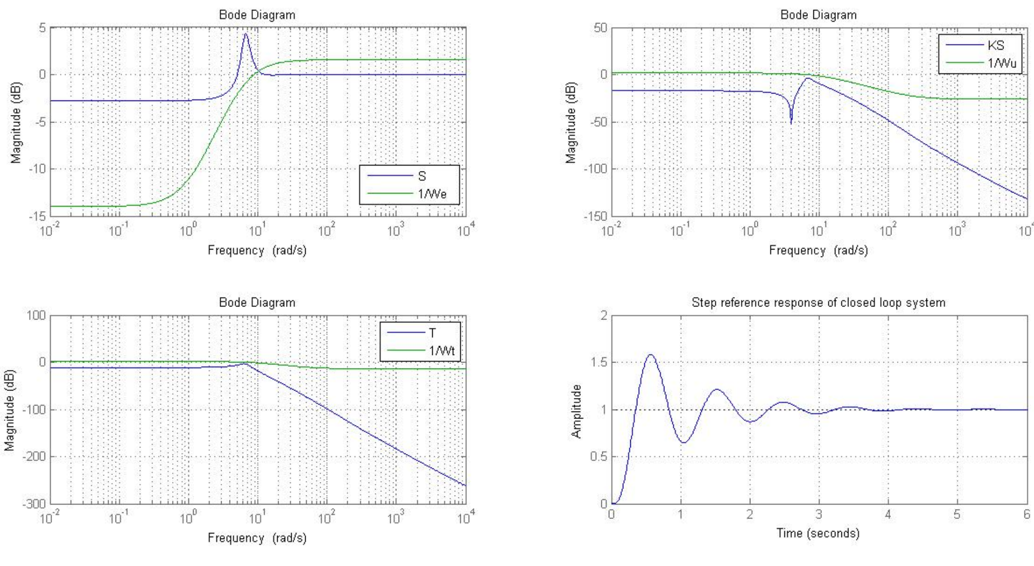

Figure 4.1: Simulation Results

4.2 Comments

- The inverses of weighting functions are upper bound for sensitivity functions except for S but We*S is still less than γ.
- The output follows reference input with suitable gain applied to r. Gain is needed because in an attempt to reduce noise and disturbance at output reference input is also attenuated.

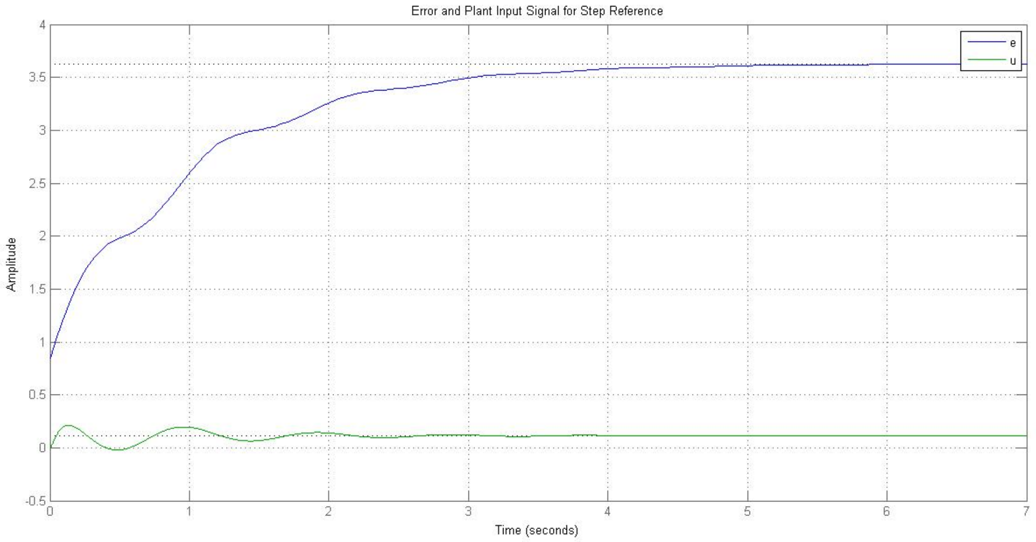

Figure 4.2: Error and plant input signal for step reference input

- Due to the trade off made between sensitivity functions energy consumption is small at the cost of larger error signal.
- From plot of weighted sensitivity functions it can be seen that maximum peak is bounded by γ.

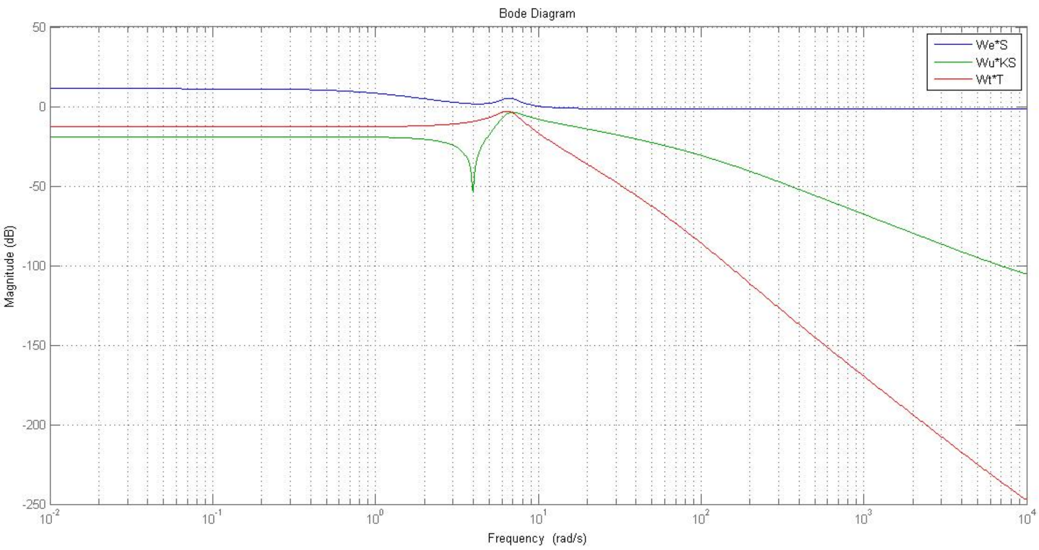

Figure 4.3: Weighted sensitivity functions

Appendix A

Matlab Code for Simulation

```
clear
clc
close all

%Plant
Ap=[0 100*pi; -0.05 -0.01];
Bp=[0; 0.1];
Cp=[1 0];
Dp=0;
Ps=ss(Ap, Bp, Cp, Dp);
P=tf(Ps)

%We
Me=1.2;                 %Peak of 1/We
ee=0.2;
wbe=5;                  %Cutoff frequency
num=[1/Me wbe];
den=[1 wbe*ee];
We=tf(num, den)

%Wu
r1=1;                   %Comparative weighting
Mu=1.2;                 %Peak of 1/Wu
eu=0.05;
wbu=12;                 %Cutoff frequency
num=r1*[1 wbu/Mu];
den=[eu wbu];
Wu=tf(num, den)

%Wt
r2=1;                   %Comparative Weighting
Mt=1.2;                 %Peak of 1/Wt
et=0.2;
wbt=12;                 %Cutoff frequency
num=r2*[1 wbt/Mt];
den=[et wbt];
Wt=tf(num, den)
```

```
%G
Gt=[We -We -We -We*P;
    0    0   0    Wu;
    0   Wt   0  Wt*P
    1  -1   -1     P];

G=ss(Gt);
G=minreal(G)

[K, Tzw, ro]=hinfsyn(G, 1, 1, 0, 10, 0.0001);
K=tf(K)

%Sensitivity functions
S=1/(1+K*P);
T=1-S;
KS=K*S;
w1=-2;
w2=4;
w=logspace(w1, w2, 1000);

%Weighting functions plot
figure
bodemag(1/We, 1/Wt, 1/Wu, w);
grid

%Sensitivity functions plot
figure
subplot(2, 2, 1)
bodemag(S, 1/We, w);
grid
subplot(2, 2, 2)
bodemag(KS, 1/Wu, w);
grid
subplot(2, 2, 3)
bodemag(T, 1/Wt, w);
grid

%Weight For Reference Input
Wr=1/0.229;          %To remove steady state error
subplot(2, 2, 4)
step(T*Wt*Wr);
title('Step reference response of closed loop system')
grid

%Weighting Sensitivity functions plot
figure
bodemag(We*S, Wu*KS, Wt*T, w);
grid

%e and u for step input r
figure
```

```
step(We*S, Wu*KS);
grid
title('Error and plant input signal for step reference')
```

Bibliography

[1] Mircea Dulau and Dorin Bica. Design of robust control for single machine infinite bus system. *Procedia Technology*, 19(0):657 – 664, 2015. 8th International Conference Interdisciplinarity in Engineering, INTER-ENG 2014, 9-10 October 2014, Tirgu Mures, Romania.

[2] Kemin Zhou and John Comstock Doyle. *Essentials of robust control*, volume 180. Prentice hall Upper Saddle River, NJ, 1998.