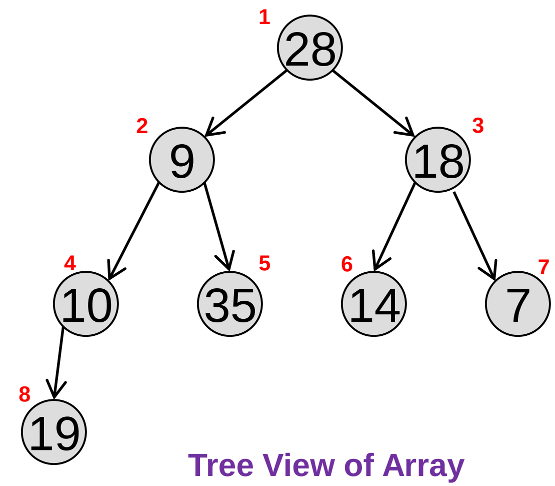
HEAPS AND TREES

Converting An Array to a Heap

- To convert an array to a heap:
- Key idea: make heaps of subtrees and combine subtrees with new root node using heapify()
- Base case: All leaf nodes are valid heaps
- Begin combining heaps with first non-leaf node

0	1	2	3	4	5	6	7	8
em	28	9	18	10	35	14	7	19

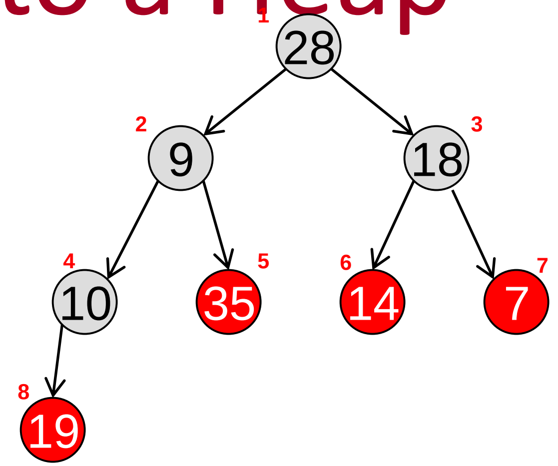
Original Array



Tree View of Array

Converting An Array to a Heap

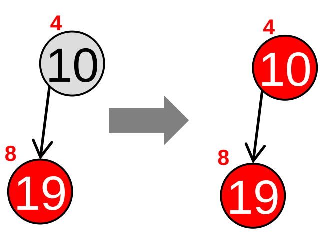
- All leaf nodes are valid heaps.
- Begin at first non-leaf node and continue to decrease location until the root, calling heapify at each location
  - Start: Heapify(Loc. 4)



Leafs are valid heaps by definition

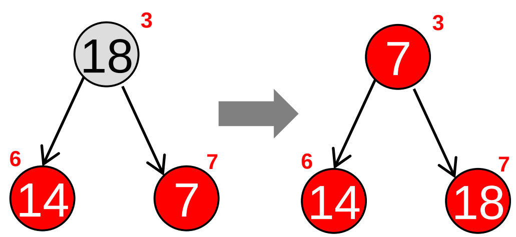
heapify(4)

Already in the right order



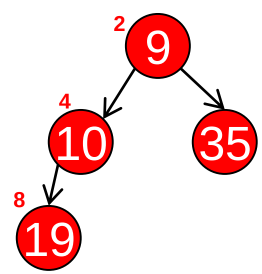
heapify(3)

Swap 18 & 7



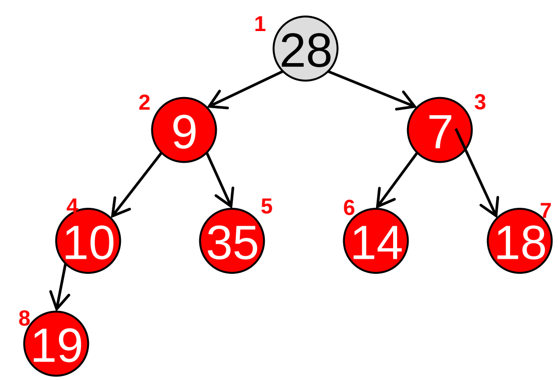
heapify(2)

Already a heap

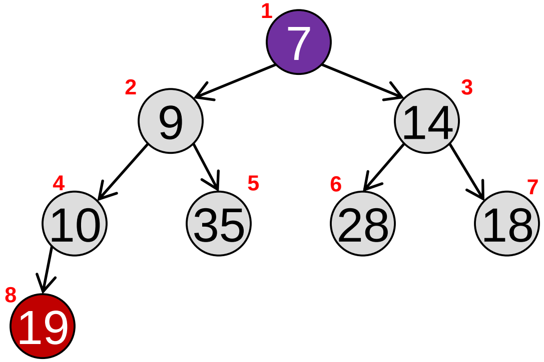


heapify(1)

Swap 28 <-> 7
Swap 28 <-> 14



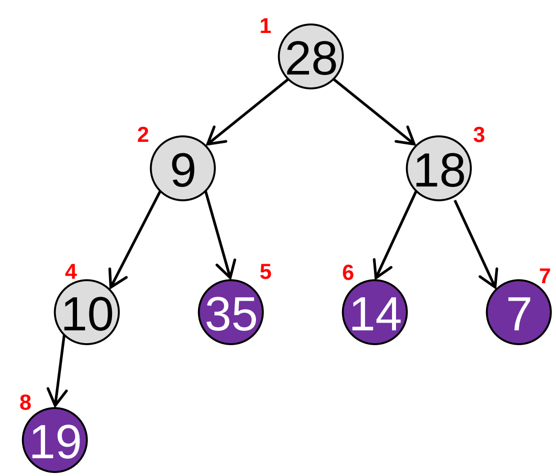
Converting An Array to a Heap



0	1	2	3	4	5	6	7	8
em	7	9	14	10	35	28	18	19

Build-Heap Run-Time

- To build a heap from an arbitrary array require n calls to heapify.
- Heapify takes O(height)
- More precisely
  - Since most of the heapify calls are shallow, this can be done in O(n)
    - n/2 calls with h=1
    - n/4 calls with h=2
    - n/8 calls with h=3
    - Totals: 1*n/2 + 2*n/4 + 3*n/8
    - T(n)=$\sum_{h=1}^{\log(n)} h * n * \left(\frac{1}{2}^{h}\right) < n * \sum_{h=1}^{\infty} \left(\frac{1}{2}^{h}\right)$
    - T(n) = $n * \theta(c) = \theta(n)$



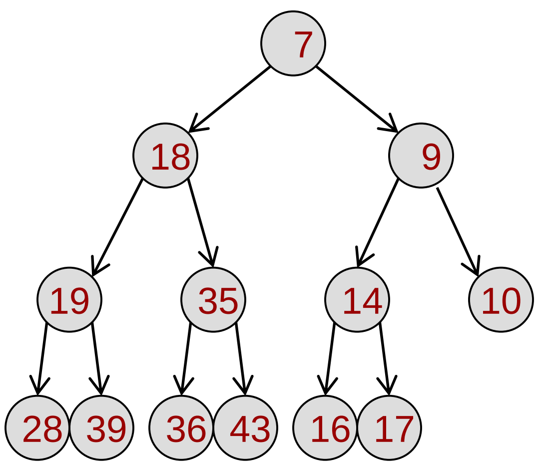
Array-based and Link-based

TREE IMPLEMENTATIONS

Array-Based Complete Binary Tree

- Binary tree that is complete (i.e. only the lowest-level contains empty locations and items added left to right) can be stored nicely in an array (let's say it starts at index 1 and index 0 is empty)
- Can you find the mathematical relation for finding node i's parent, left, and right child?
  - Parent(i) = i/2
  - Left_child(i) = 2*i
  - Right_child(i) = 2*i + 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13
em	7	18	9	19	35	14	10	28	39	36	43	16	17



parent(5) = 5/2 = 2
Left_child(5) = 2*5 = 10
Right_child(5) = 2*5+1 = 11

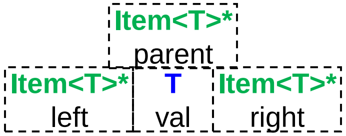
Non-complete binary trees require much more bookeeping to store in arrays…usually link-based approaches are preferred

Link-Based Approaches

- For an arbitrary (**non-complete**) d-ary tree we need to use pointer-based structures

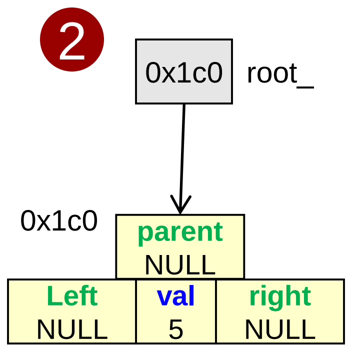
```
template <typename T>
struct Item {
T val;
shared_ptr<Item<T>> left;
shared_ptr<Item<T>> right;
shared_ptr<Item<T>> parent;
};
// Bin. Search Tree  template
<typename T>
class BinTree {
  public:
  BinTree();
  ~BinTree();
  void add(const T& v) ;;
private:
   shared_ptr<Item<T>> root;
};
```

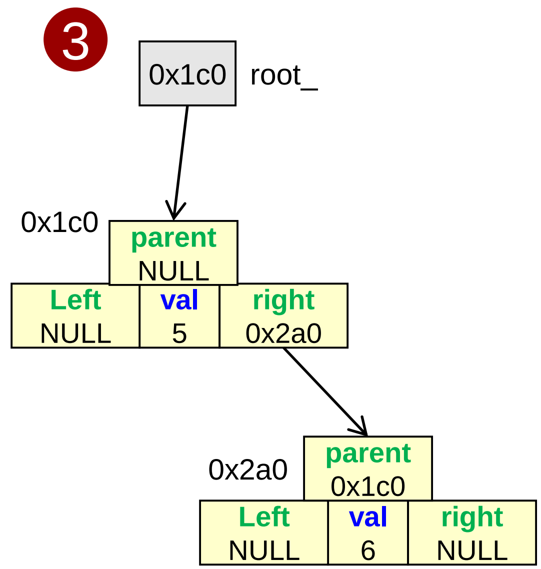
Item<T> blueprint:

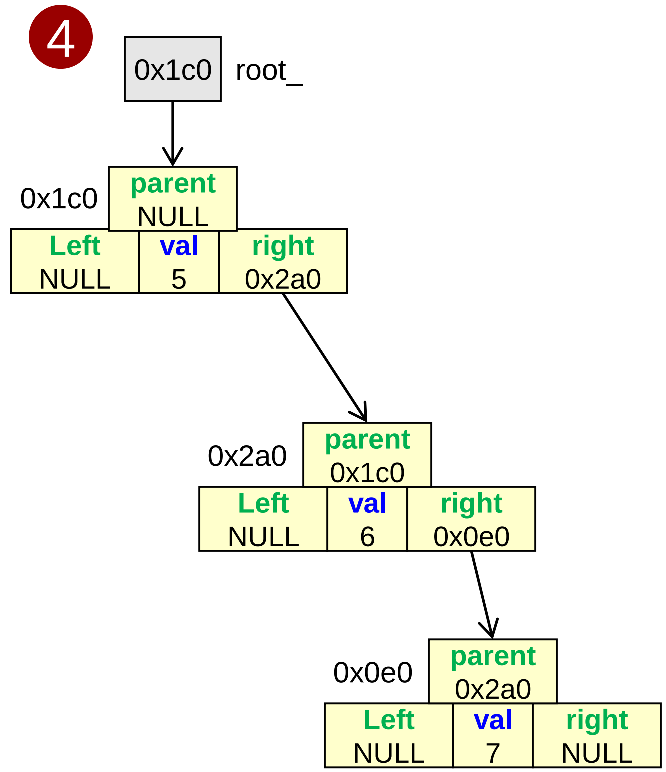


Link-Based Approaches

- Add(5)
- Add(6)
- Add(7)



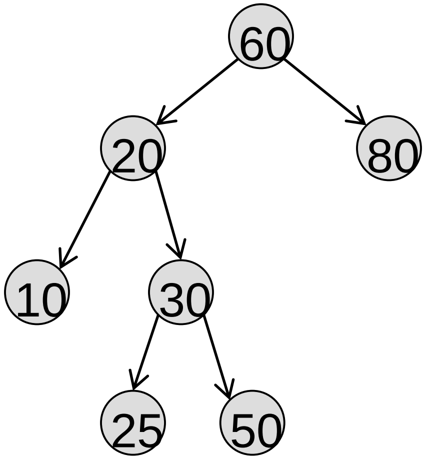




Recursive Tree Traversals

- A traversal iterates over all nodes of the tree
  - Usually using a depth-first, recursive approach
- Three general traversal orderings
  - Pre-order [Process root then visit subtrees]
  - In-order [Visit left subtree, process root, visit right subtree]
  - Post-order [Visit left subtree, visit right subtree, process root]

```
// Node definition
struct TNode
{
  int val;
  TNode *left, *right;
};
```




```
Preorder(TNode* t)
{   if t == NULL return
    process(t) // print val.
    Preorder(t->left)
    Preorder(t->right)
}
```

60 20 10 30 25 50 80

```
Inorder(TNode* t)
{   if t == NULL return
    Inorder(t->left)
    process(t) // print val.
    Inorder(t->right)
}
```

10 20 25 30 50 60 80

```
Postorder(TNode* t)
{   if t == NULL return
    Postorder(t->left)
    Postorder(t->right)
    process(t) // print val.
}
```

10 25 50 30 20 80 60