Global Illumination

CS 481/681 Computer Graphics Rendering

University of Alaska Fairbanks

Overview

- Specular BRDFs
- Global Illumination
- Regular Expression Notation
- Path Tracing and Radiosity
- Global Illumination Techniques
- Spherical Harmonics

Midterm Procedures

- Will be posted on Thursday by 9am
- You have 48 hours to complete the exam
- It is due before 9am on Saturday morning
- Late submissions are automatically a zero, sorry
- Write your submissions in Latex by filling in the template on Overleaf
- Submit the PDF

Midterm Resources

- Whitted 1980 An Improved Illumination Model for Shaded Display
- Cook and Torrance 1982 A Reflectance Model for Computer Graphics
- Kajiya 1986 The Rendering Equation
- Hart et al 1989 Ray Tracing Deterministic 3-D Fractals
- John C. Hart 1996 Sphere tracing: a geometric method for the antialiased ray tracing of implicit surfaces
- Cook et al 1984 Distributed ray tracing
- Blinn and Newell 1976 Texture and Reflection in Computer Generated Images
- Veach and Guibas 1995 Optimally combining sampling techniques for Monte Carlo rendering
- Erik Lindholm et al 2001 A User-Programmable Vertex Engine

Regular Expression Notation

- E is eye
- L is vector
- S is specular interface
- D is diffuse interface
- LSE path is light-specular-eye path
- $L\{SD\}E$ path is light to a *single* specular/diffuse interface to eye
- $L\{SD\}^+E$ path is light to *several* specular/diffuse interfaces to eye

Path Tracing and Radiosity

- Path Tracing
  - Shoot Ray. Hit.
  - Shoot Ray in random direction. Hit.
  - If a light, then evaluate light path
  - Typically, we just average the results obtained to within a certain path depth
- Radiosity
  - Divide scene into patches.
  - Determine each patch's contribution to each other
  - Calculate transport of light from patch to patch
  - We stop after converging on answer

Global Illumination Techniques

- Photon Mapping
- Bidirectional Path Tracing
- Instant Radiosity
- Voxel Cone Tracing
- Photon Mapping
- Multiple Importance Sampling

Spherical Harmonics

- Orthogonal vectors: $\mathbf{u} \cdot \mathbf{v} = 0$
- *Function spaces* $\langle f, g \rangle = \int \overline{f(x)} g(x) dx$
- Orthogonal functions: $\langle f, g \rangle = 0$ if $f \neq g$
- These can be used to approximate other functions
- e.g. Fourier Series
  - $f : \mathbb{R} \to \mathbb{R}$
  - $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)\ dt$
  - $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)\ \cos(nt)\ dt$
  - $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)\ \sin(nt)\ dt$
  - $f \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$
- This works for 2D functions, if only we had something for spheres. . .

Legendre Polynomials

- Orthogonal function discovered by Adrien-Marie Legendre in 1782
- Rodrigues' formula: $P_\ell(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$
- First few are $1, x, \frac{1}{2}(3x^2-1), \frac{1}{2}(5x^3-3x), \ldots$
- Recursive definition
  - $(\ell - m)P_\ell^m = x(2\ell-1)P_{\ell-1}^m - (\ell+m-1)P_{\ell-2}^m$
  - $P_m^m = (-1)^m(2m-1)!!(1-x^2)^{m/2}$
  - $P_{m+1}^m = x(2m+1)P_m^m$
- Need one more step to solve Laplace's equation in spherical coordinates
- Associated Legendre Polynomials $P_\ell^m(x)$
  - $P_\ell^m(x) = (-1)^m(1-x^2)^{m/2}\frac{d^m}{dx^m(P_\ell(x))}$
  - Orthogonal: $\int P_k^m P_\ell^m = \frac{2(\ell+m)!}{(2\ell+1)(\ell-m)}\delta_{k,\ell}$
  - $-m$: $P_\ell^{-m} = (-1)^m\frac{(\ell-m)!}{(\ell+m)!}P_\ell^m(x)$
- These are the tools for Spherical Harmonics

Spherical Harmonics

- A set of orthogonal functions on the sphere
- This is an excellent way to represent low-frequency environment maps
- $Y_\ell^m(\theta,\varphi)$ is Laplace's spherical harmonics
- Approximating $f(\theta,\varphi)$
  - degree ℓ and order m: $\ell(\ell+1)$ coefficients (e.g. n^2)
  - $Y_\ell^m(\theta,\varphi) = Ne^{im\varphi}P_\ell^m(\cos\theta)$
  - $a_\ell^m = \int f(\theta,\varphi)\ Y_\ell^m(\cos\theta)$
  - $f(\theta,\varphi) \sim \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_\ell^m Y_\ell^m(\cos\theta)$

Real Spherical Harmonics

- Notice the $e^{im\varphi}$: uh oh, imaginary numbers
- Let's do this with real numbers

$$Y_\ell^m = \begin{cases} \sqrt{2}K_\ell^m\cos(m\varphi)P_\ell^m(\cos\theta) & m > 0 \\ \sqrt{2}K_\ell^m\sin(-m\varphi)P_\ell^{-m}(\cos\theta) & m < 0 \\ K_\ell^0 P_\ell^0(\cos\theta) & m = 0 \end{cases}$$

$$K_\ell^m = \sqrt{\frac{2\ell+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}$$

- Spherical coordinates $\begin{bmatrix}\sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta\end{bmatrix}^T = \begin{bmatrix}x & y & z\end{bmatrix}^T$
- How to calculate a_ℓ^m?

$$a_\ell^m = \int_0^{2\pi}\int_0^{\pi} f(\theta,\varphi)Y_\ell^m(\theta,\varphi)\sin\theta d\varphi d\theta$$

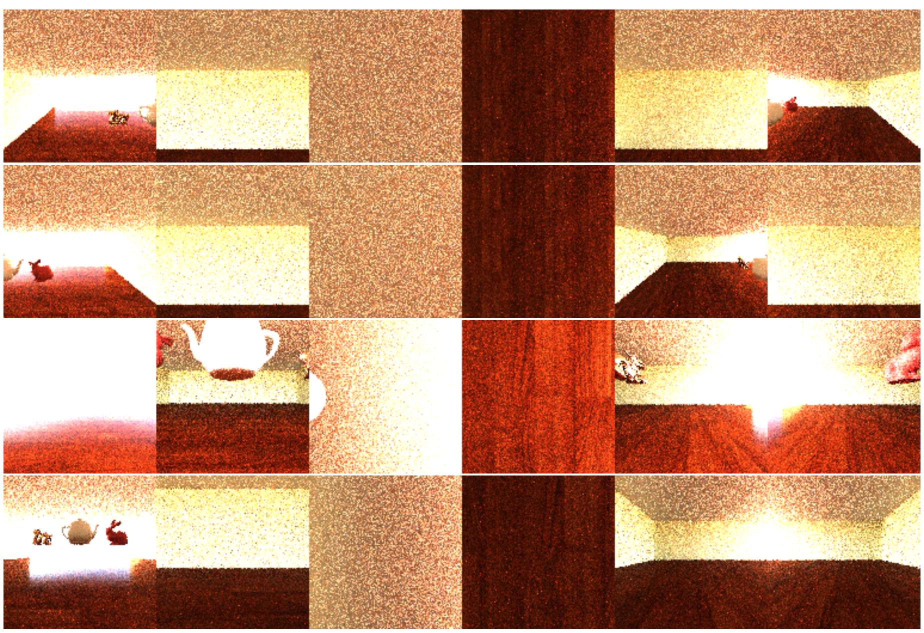
Monte Carlo Estimator

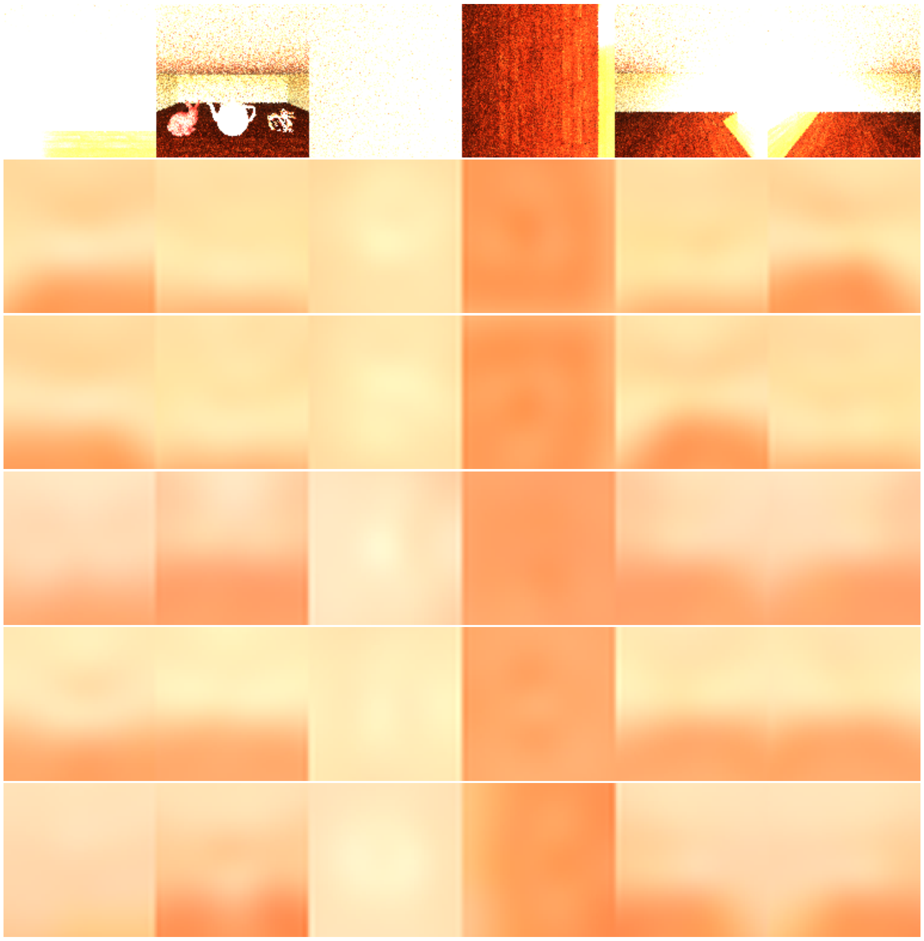
- $\int_\Omega g \; d\omega \approx \frac{1}{N} \sum_{j=1}^{N} g(x_j) w(x_j)$
- $a_\ell^m = \frac{p(\omega)}{N} \sum_{j=1}^{N} f(\theta, \varphi) \; Y_\ell^m(\theta, \varphi) \frac{1}{p(\omega)}$
- Since $p(\omega)$ is constant, or 4π, we can move it outside the sum
- Process
  - Take high resolution environment map
  - Pick several samples from it
  - Create coefficients using Monte Carlo
  - In our shader, use our coefficients to recalculate the image
  - Now, we have band limited shading!

Global Illumination

- Greger et al 1998 The Irradiance Volume
- Ramamoorthi and Hanrahan 2001 An Efficient Representation for Irradiance Environment Maps
- Sloan et al 2002 Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments
- Robin Green 2003 Spherical Harmonic Lighting: The Gritty Details

SPH





Hybrid Topics

- None this week because ...
- Time Reserved for Midterm Exam