Study Project
Math F266

Under the Guidance of
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Title: Enumeration of Maximal Rectangular Floor Plans

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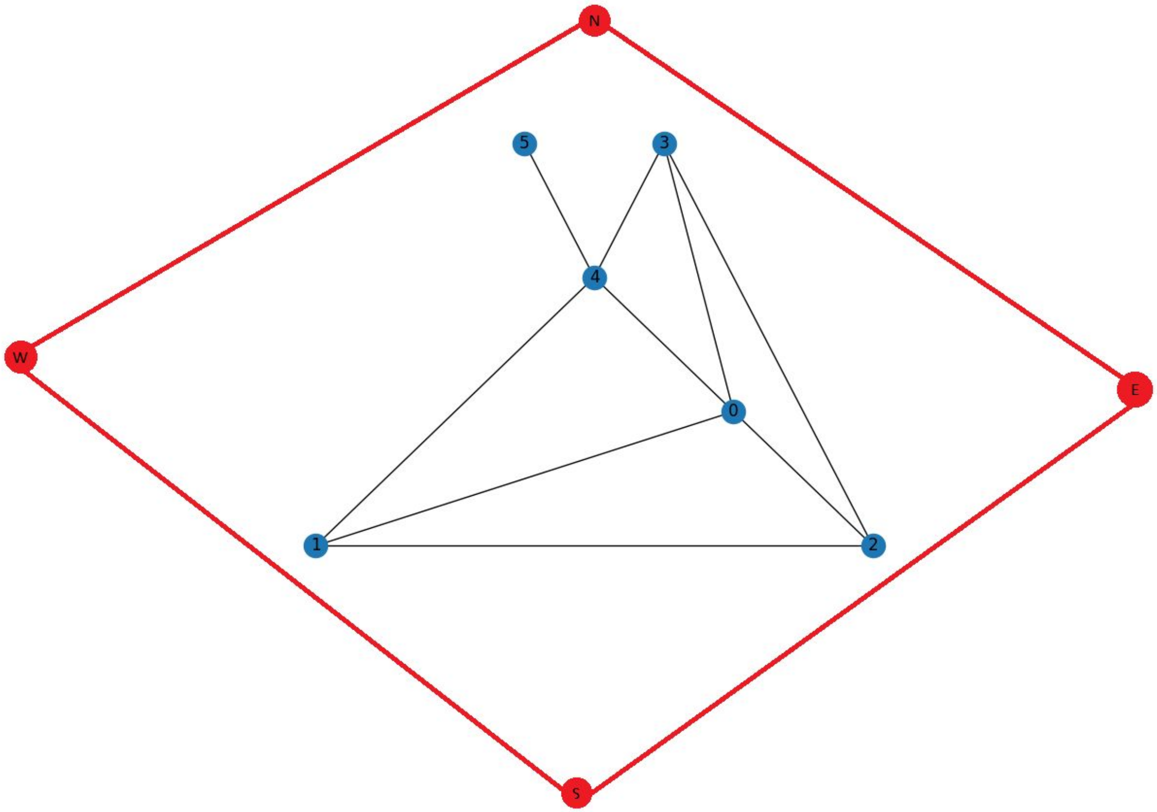
IO Method

Consider the Inner Graph given below. The task is to add 4 outer boundaries to it.



Definitions

1. Inner Graph:-The initially given dual graph of the given RFP on which we have to add four rooms.

2. NESW vertices:-The four vertices to be added are names as NESW being ‘North’ , ‘East’ , ‘South’ ,’West’ respectively.

3. Outer Graph:- The final graph after adding the NESW vertices is called the outer graph

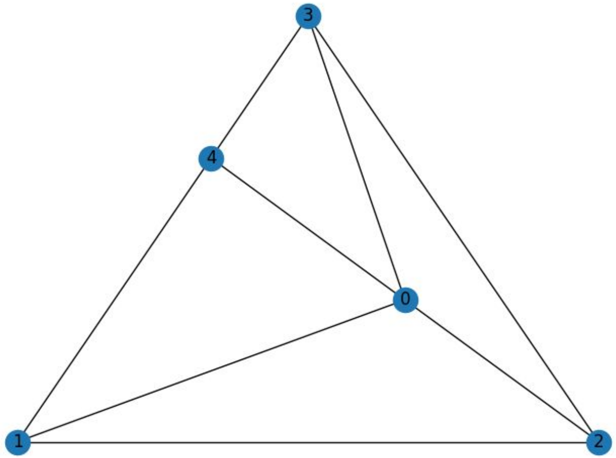
4. I-I edges:- Edges of the given inner graphs.
5. O-O edges:- Edges between NESW vertices.
6. I-O edges:- Edges between the vertices of inner graph and NESW vertices.

Definitions

4. Wrapped Vertices:-

Those vertices which have a cycle in the subgraph formed from its neighbors.In the figure, the vertex 0 is a wrapped vertex because the subgraph formed from its

neighbors 1,2,3,4, has a cycle i.e, 1-2-3-4-1.



Proposition 1 : A maximal rectangular dual graph has 3n-7 edges

Proposition 2 : A degree 1 vertex must have exactly 3 I-O edges

Proposition 3 : A cut vertex must have exactly 2 I-O edges.

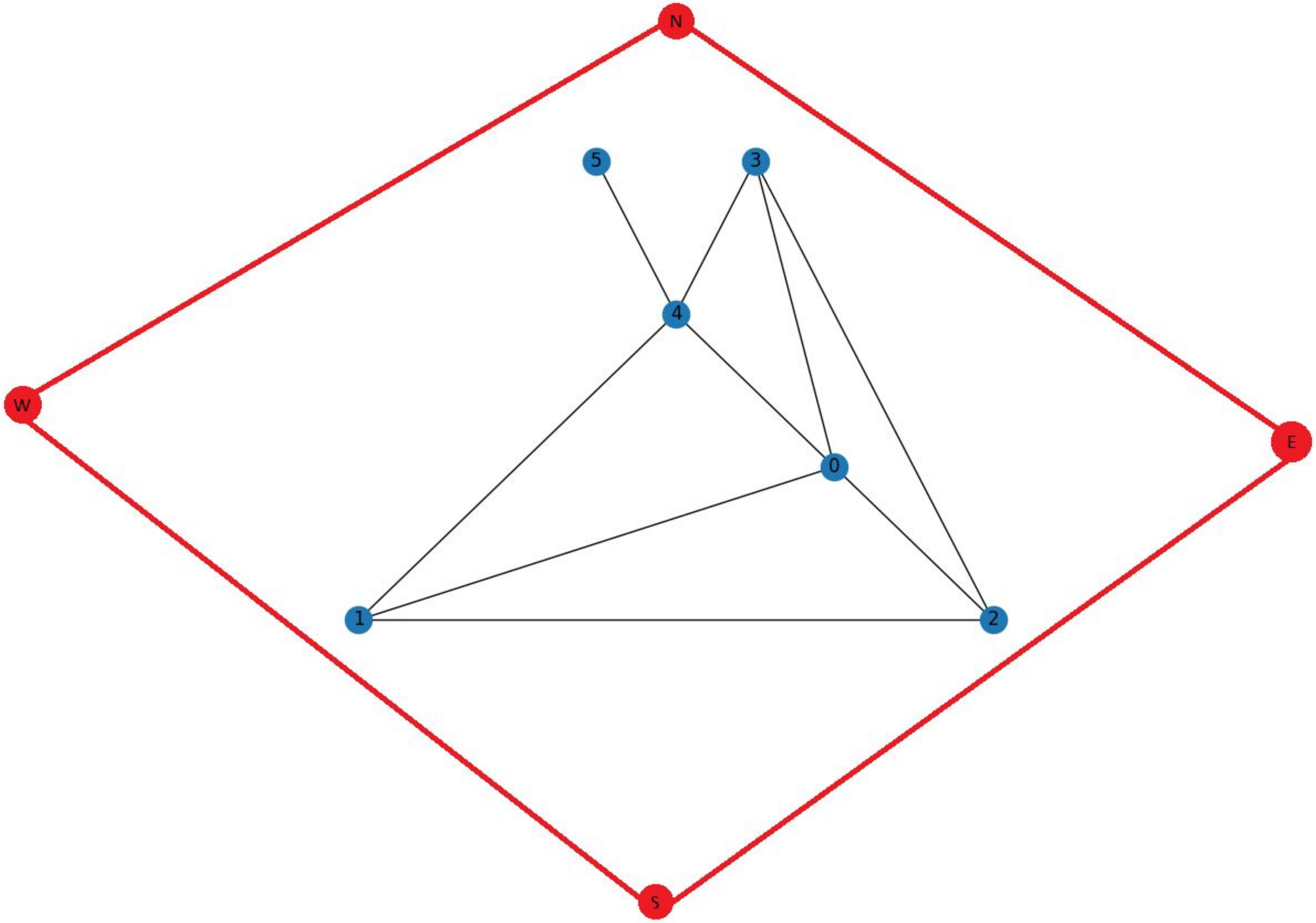
Proposition 4 : All cut vertices must be connected to the same 2 opposite NESW vertices.
WLoG let us take it East and West.

Proposition 5 : A wrapped vertex has 0 I-O edges.

Proposition 6 : The vertices between 2 cut vertices must have exactly 1 I-O edge

Proposition 7 : As the inner graph is connected, each vertex has at least one of its sides block by the blocks of the inner graph. So as each vertex has 4 sides to be covered, the maximum number of I-O edges that can be added to the rest of the vertices is 3.

Proposition 8 : Assume a generic vertex v, which doesn't come under the above criteria of vertices. Let the degree of v be d. Now the maximum number of sides the neighbors of v can take is d. So the minimum number of I-O edges for v to be added must be 4-d. Now if d is greater than or equal to 4, then the minimum number must be zero.


N
5
3
4
W
E
0
1
2
S

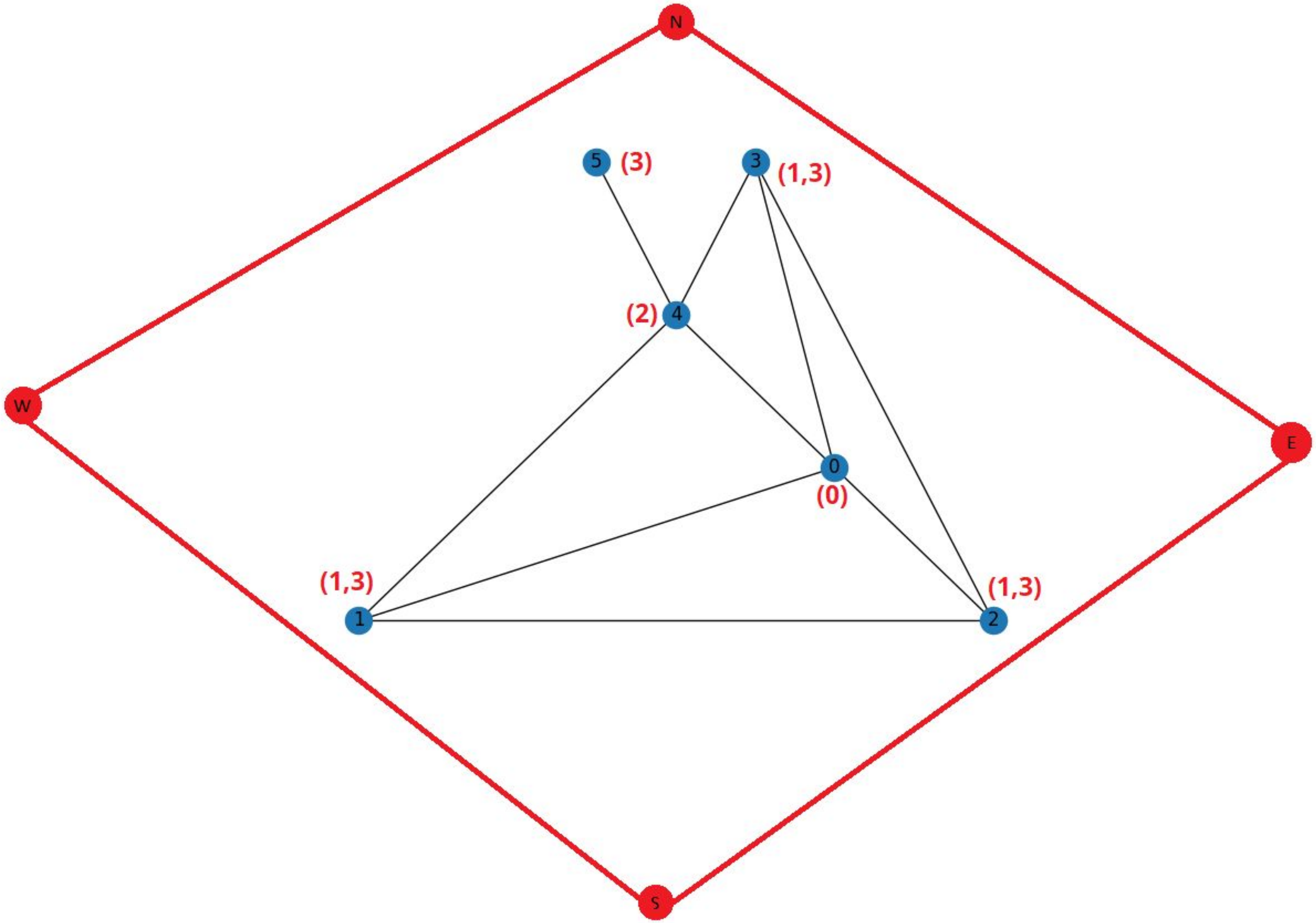
- Basics of Edge counting
  Let n be the number of vertices in the inner graph

  Total edges in the outer graph is 3*(n+4)-7 = K

  There are some edges which are evident so we start forming the outer graph by counting these vertices.

1. I-I edges
2. O-O edges
3. I-O edges
   a. Edges from cut vertices = 2
   b. Edges from degree 1 vertices = 3
   c. Edges from wrapped vertices = 0
   d. Edges from vertices between 2 consecutive cut vertices = 1
   e. Edges from the minimum number of generic vertices

   If K = 0 after this process then there is a unique solution else



N
W
E
S
5
(3)
3
(1,3)
(2)
4
0
(0)
(1,3)
1
(1,3)
2

Example of edge counting :

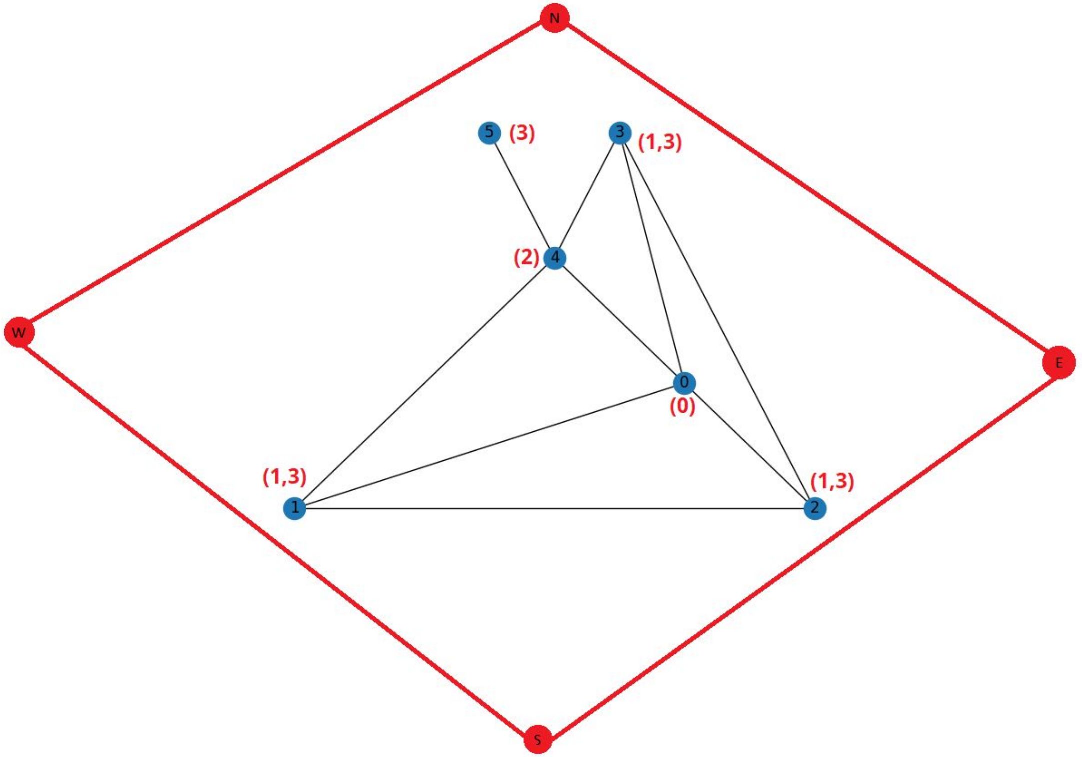
K = 3 * (6+4) - 7
K = 23

1. K- I-I
   K= 23- 9 = 14

2. K - O-O
   K = 14 - 4 = 10

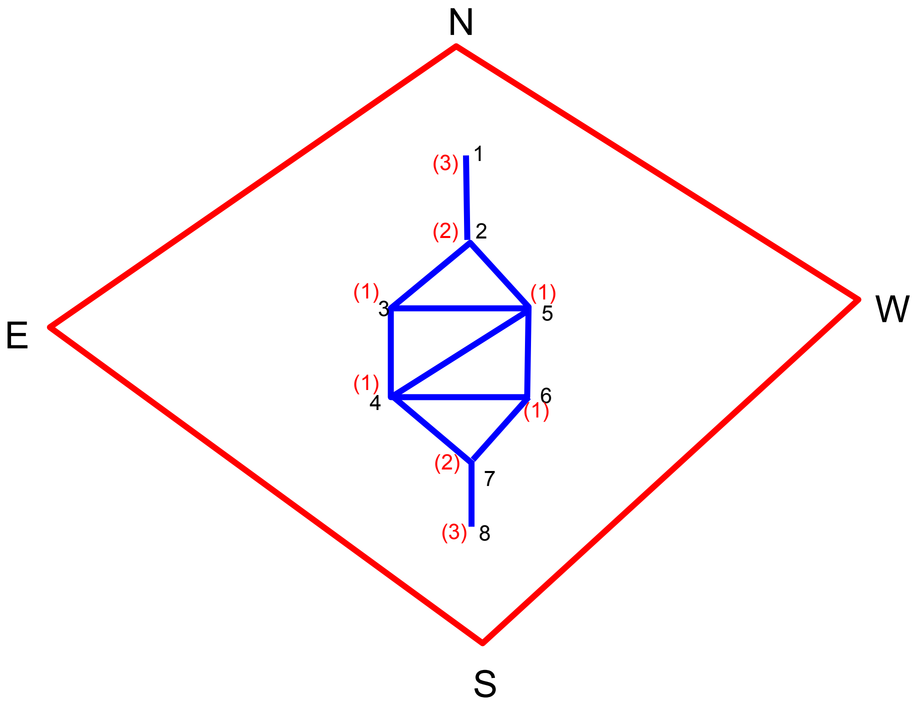
3. K - I-O
   K = 10 - 8 = 2

Remaining edges is 2
Possible non-isomorphic combinations for 2 edges are adding I-O to
1,1
1,2
1,3
2,2.



Completion Problem

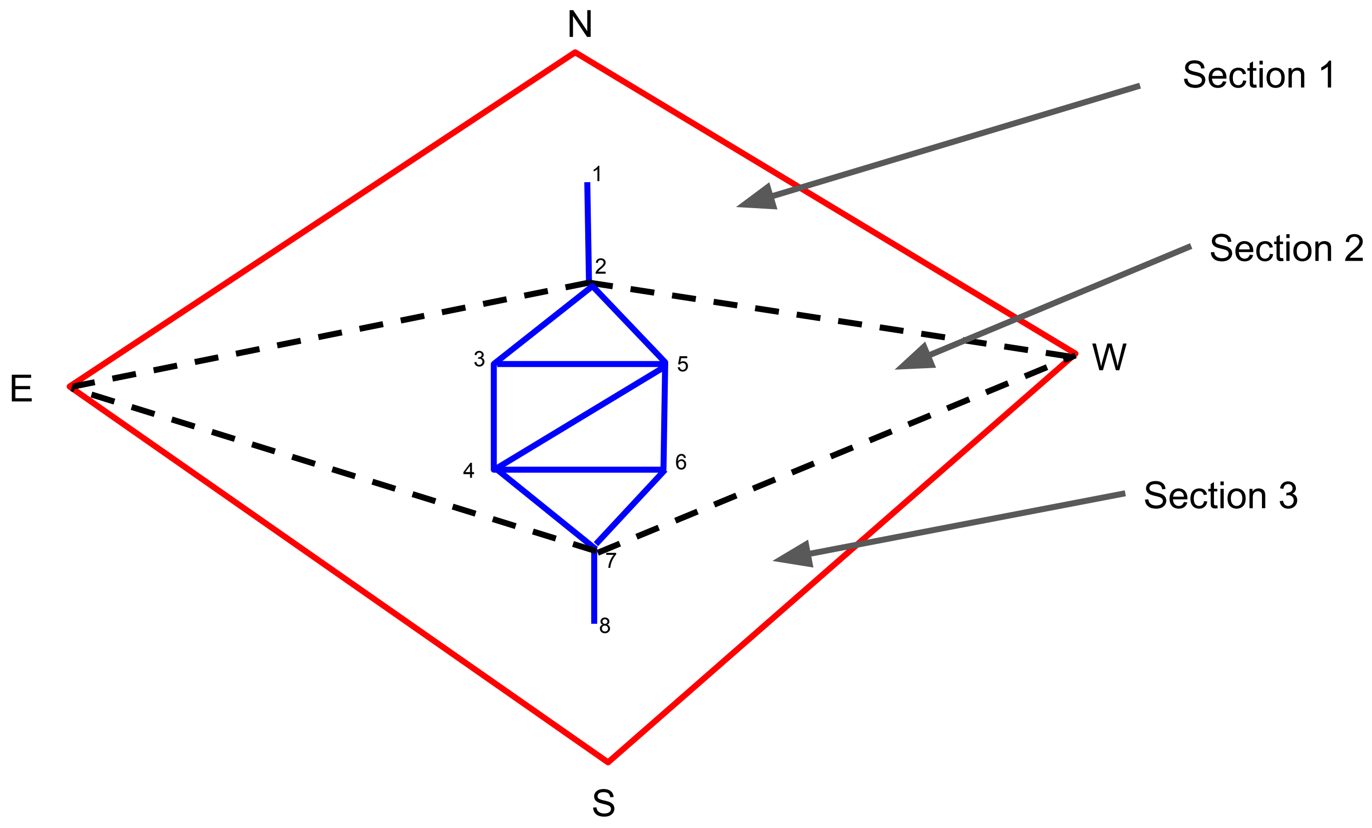
Let us consider the outer graph with I-O edges after edge counting. Let this graph be G. Figure represents G



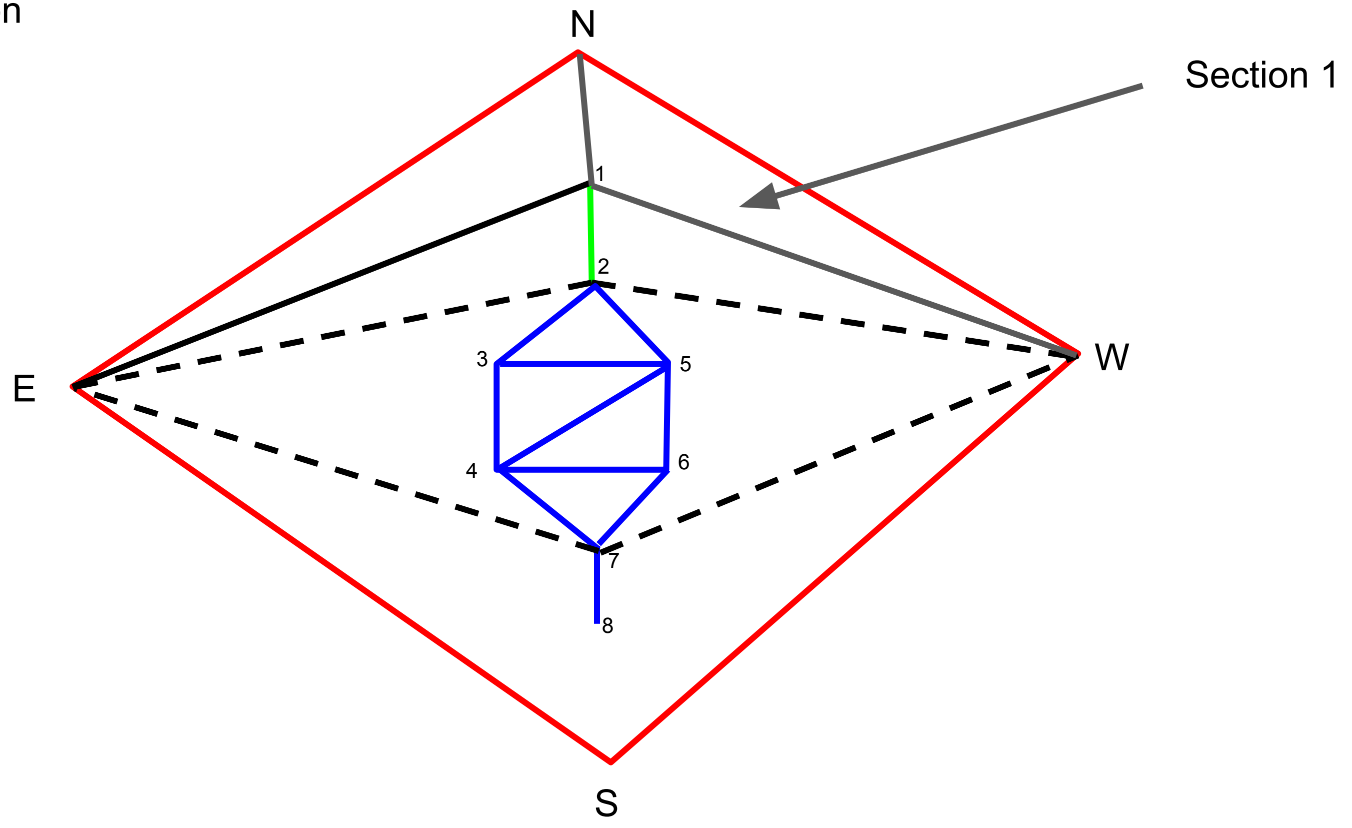
Steps to be followed for completion

1. Find the set of cut vertices.
2. Connect the Cut vertices to East and West.
3. The graph is divided into sections of biconnected subgraphs.
4. Find the outer boundary for each biconnected subgraph.
5. Start connecting.

1. Cut vertex set = { 2 , 7 }
2. Cut vertex connection.

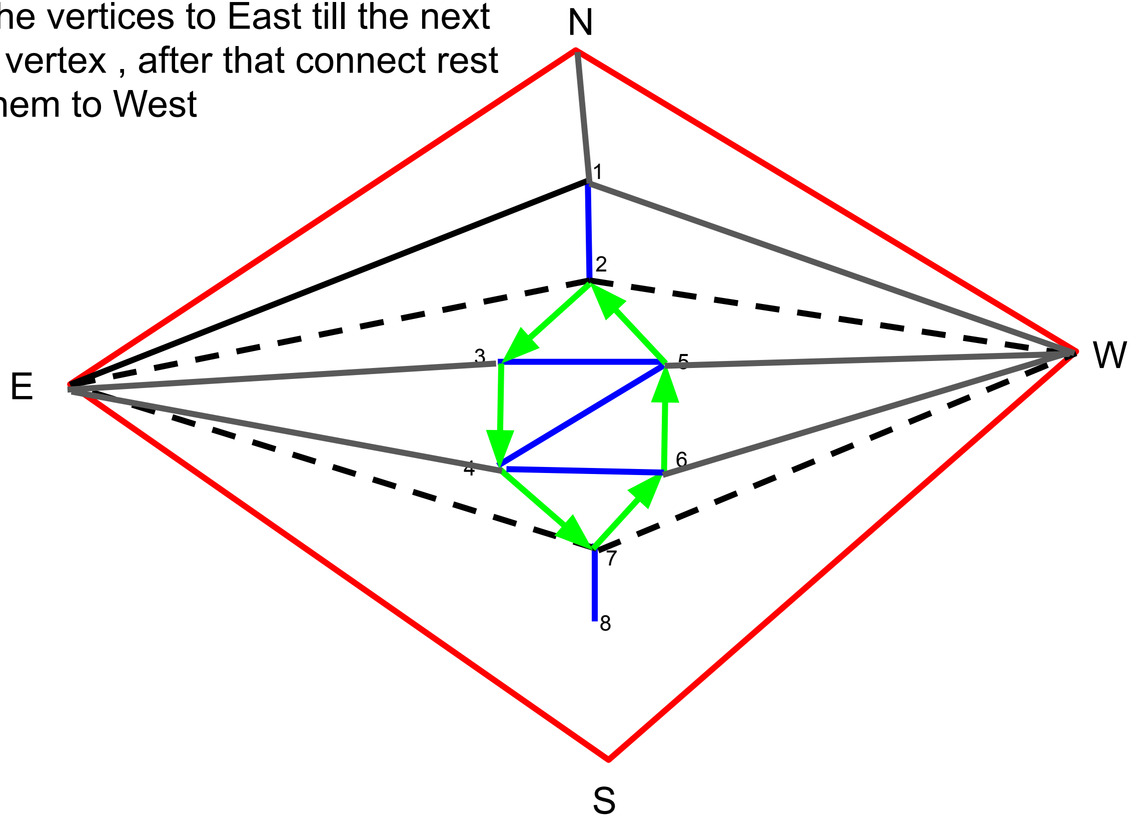


Steps: connection of first and last section is done in a circular fashion



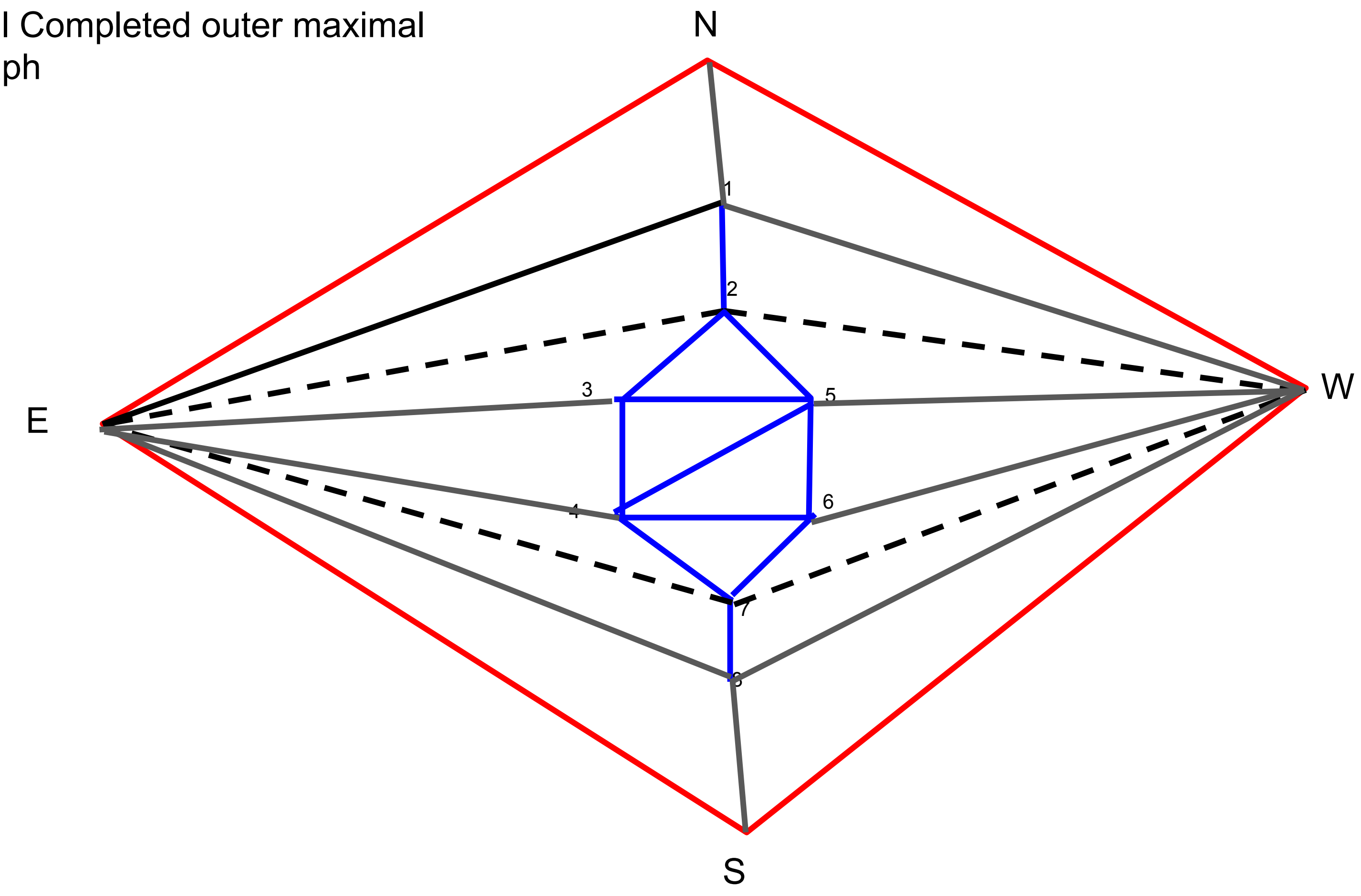
Steps:

1. Find the outer boundary
2. Start from a cut vertex and connect all the vertices to East till the next Cut vertex , after that connect rest of them to West



Section 2

The final Completed outer maximal
dual graph



Thank You