Automation in Construction 92 (2018) 151–165

Contents lists available at ScienceDirect

Automation in Construction

journal homepage: www.elsevier.com/locate/autcon

Enumerating generic rectangular floor plans

Krishnendra Shekhawat

BITS Pilani, Department of Mathematics, Pilani Campus, India

ARTICLE INFO

Keywords:
Adjacency
Architectural design
Dual graph
Graph theory
Algorithm
Floor plan

ABSTRACT

A *rectangular floor plan* (RFP) is a floor plan in which plan's boundary and each room is a rectangle.

The problem is to construct a RFP for the given adjacency requirements, if it exists.

In this paper, we aim to present a generic solution to the above problem by enumerating a set of RFP that topologically contain all possible RFP. This set of RFP is called generic rectangular floor plans (GRFP). Furthermore, the construction of GRFP leads us to the necessary condition for the existence of a RFP corresponding to a given graph.

1. Introduction and related work

A *floor plan* is a polygon, the plan boundary, divided by straight lines into component polygons called *rooms*. The edges forming the perimeter of each room are termed *walls*. The region not enclosed by the boundary is called *exterior*. Two rooms in a floor plan are *adjacent* if they share a wall or a section of wall; it is not sufficient for them to touch at a point only.

A *rectangular floor plan* (RFP) is a floor plan in which plan's boundary and each room is a rectangle. Any RFP with n rooms is represented by RFP(n).

Corresponding to each RFP(n), there exists a graph called *dual graph* DG_n, where each room is represented by a vertex and two vertices are adjacent if corresponding rooms are adjacent. For example, dual graph of the RFP in Fig. 1A is shown in Fig. 1B.

Definition 1. A graph for which a RFP exists is called *rectangular floor plan graph*, abbreviated as RFP_G.

For example, the graph shown in Fig. 1C is a RFP_G because of the presence of a RFP in Fig. 1D while the graph shown in Fig. 1G is not a RFP_G.

Two graphs which contain the same number of graph vertices, connected in the same way, are said to be *isomorphic* else *non-isomorphic*.

Definition 2. Two RFP(n) are said to be *distinct or non-isomorphic* if they have non-isomorphic dual graphs.

For example, the RFP in Fig. 1A and D are distinct while the RFP in Fig. 1D and I are isomorphic.

Definition 3. A RFP_G is called a *maximal* RFP_G, abbreviated as $MRFP_G$, if adding any new edge to it results in a graph that is not a RFP_G.

A RFP corresponding to a $MRFP_G$ is called *maximal rectangular floor plan*, abbreviated as MRFP.

The RFP_G in Fig. 1C is a $MRFP_G$ (refer to Section 2 and Fig. 3).

Definition 4. *Generic rectangular floor plan graphs* with n vertices, abbreviated as $GRFP_G(n)$, are represented by a set formed by all non-isomorphic $MRFP_G(n)$.

Generic rectangular floor plans with n rooms, abbreviated as GRFP(n), represent a set of all distinct MRFP(n).

For a given graph, the problem is to construct a RFP, if it exists. This problem in one of its form is known as *rectangular dualization problem* which was first studied by Bhasker & Sahni [1], and Koźmiński & Kinnen [2]. The following theorem was proposed by Koźmiński & Kinnen [2]:

Theorem 1. *A planar graph G has a RFP with four rooms on the boundary if and only if*

1 *every interior face is a triangle and the exterior face is a quadrangle,*
2 *G* has no separating triangles (a *separating triangle is a triangle whose removal separates the graph).*

In the past, many researchers have presented graph theoretical techniques for the generation of floor plans while satisfying given adjacency requirements. A brief literature review is as follows:

This approach was first presented by Levin [3] where a method for converting a graph into a spatial layout was presented. Then in 1971, Grason [4] proposed a dual graph representation of a planar graph for

E-mail address: krishnendra.shekhawat@pilani.bits-pilani.ac.in.

https://doi.org/10.1016/j.autcon.2018.03.037
Received 11 November 2017; Received in revised form 26 March 2018; Accepted 31 March 2018
0926-5805/

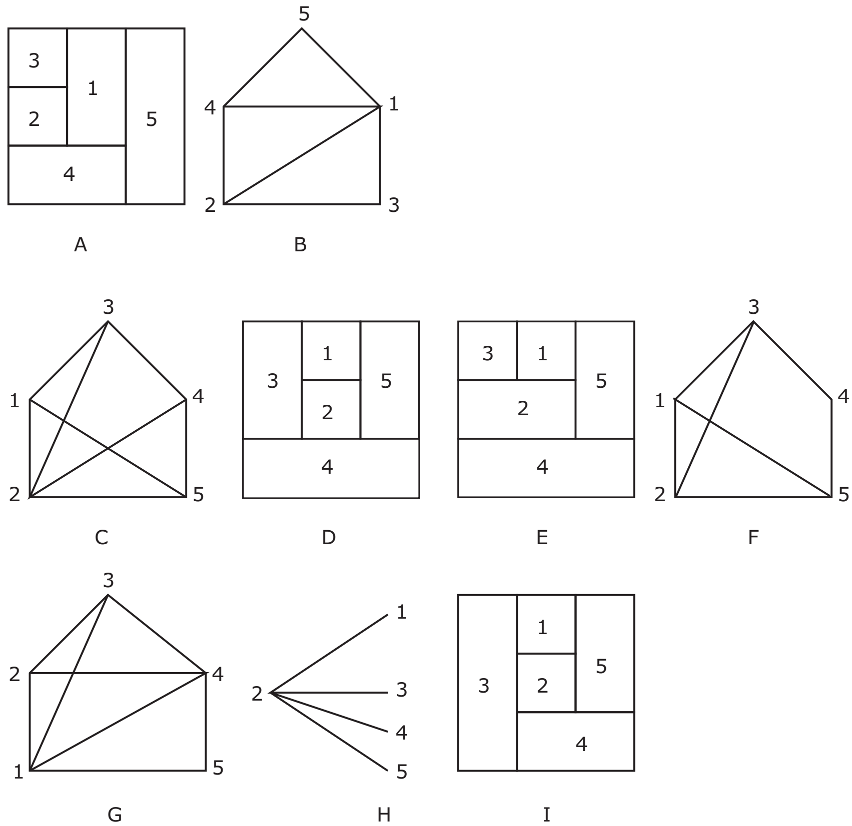

Fig. 1. Illustrating different concepts used in the paper.

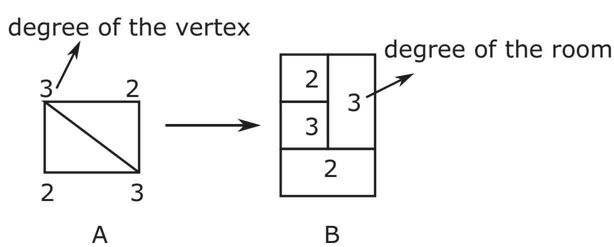

Fig. 2. $MRFP_G(4)$ and corresponding RFP.

generating a RFP. In the same year, March and Steadman [5] gave two approaches for generating floor plans, namely, an additive approach and a permutation approach. The additive approach starts with an empty floor plan and builds up a low cost layout with one solution at a time, while the other approach goes through every possible layout and searches for one having the least cost. In both the approaches, the aim is to minimize the sum of the weighted distances between different rooms, measured from the center of one room to the center of an adjacent one. The least cost solution is the one in which average sum of all the distances between adjacent rooms is minimal. In this direction, Steadman [6] exhaustively generates all topologically distinct rectangular arrangements (illustrating all possibilities up to six component rectangles). In this way, the problem of producing a plan to given specifications of adjacency becomes simply one of selection, rather than one of construction as in the approach of previous researchers. In 1977, Lynes [7] proposed that "all rooms may have windows if and only if adjacency graph is outer-planar[1]." In 1978, Gilleard [8] presented a computer-aided design package for enumerating rectangular dissections. In 1980, Baybars and Eastman [9] demonstrated a systematic procedure for obtaining an architectural arrangement (not necessarily rectangular) from a given underlying maximal planar graph (MPG)[2]. It has been shown that a given MPG with p vertices could be embedded in the plane in $2p - 4$ different ways. In 1982, Roth et al. [10] presented the construction of a dimensioned plan from a given graph. In this method, the given graph is first split into two sub-graphs by a coloring technique; each of these graphs is then converted into a dimensioned graph; the final product of the model is a set of alternative plans where the determination of the envelope's overall size is done by using the PERT algorithm [11]. In the same year, Baybars [12] presented the enumeration of floor plans with circulation spaces. In 1985, Robinson and Janjic [13] showed that, if areas are specified for rooms with a given maximal outer-planar graph, then any convex polygon with the correct area can be divided into convex rooms to satisfy both area and adjacency requirements. In 1987, Rinsma [14] showed that, for any given maximal outer-planar graph with at most four vertices of degree 2, it is not always possible to find a RFP satisfying adjacency and area conditions. In the same year, Rinsma [15] provided conditions for the existence of rectangular and orthogonal floor plans for a given tree[3]. In 1994, Schwarz et al. [16] presented a graph-theoretical model for automated building design. Here, the proposed solutions are not restricted to any shape, i.e., on the basis of given constraints, the shape of layout gets evolved. In 2000, Recuero et al. [17] presented a heuristic method for mapping a graph into rectangles so that they cover a rectangular plan.

As a recent work, in 2010, Marson and Musse [18] proposed a technique for the generation of floor plans based on squarified treemaps

[1] An undirected graph is an *outer-planar graph* if it can be drawn in the plane without crossings in such a way that all of the vertices belong to the unbounded face of the drawing.

[2] A planar graph G is *maximal* if no edges can be added to G without losing planarity.

[3] Any connected graph without cycles is a tree.

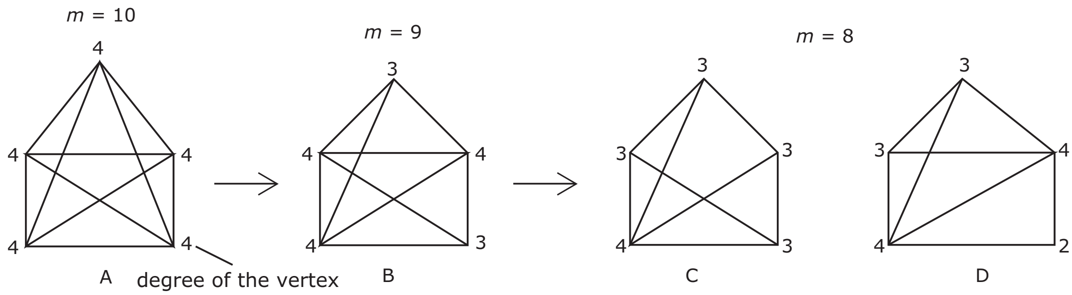

Fig. 3. Reduction of K_5 to graphs G_5^8.

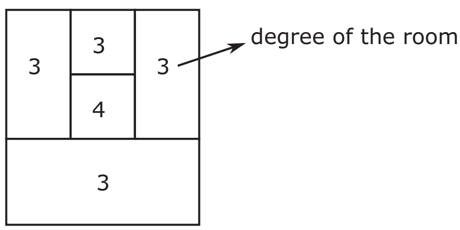

Fig. 4. MRFP(5).

algorithm[4]. In 2011, Jokar and Sangchooli [19] introduced face area concept for constructing orthogonal floor plans corresponding to a particular class of MPG. In the same year, Zhang and Sadasivam [20] studied adjacency-preserving transformations from MPG to PTP graph. In 2012, Regateiro et al. [21] proposed an approach called orthogonal compartment placement for architectural layout design problems, which is based on topological algebras and constraint satisfaction techniques. In this work, authors have proposed 169 ways in which two rooms can be considered adjacent. In 2013, Kotulski and Strug [22] used a particular class of graphs, called hypergraphs, as a means of representing building layouts. In 2014, Shekhawat [23] proposed the enumeration of a particular class of best connected RFP, i.e., the RFP having $3n-7$ edges in their dual graphs. In 2017, Ham and Lee [24] presented an algorithmic approach for quantitatively evaluating structural similarities between architectural plans and creating a phylogenetic tree of the analyzed architectural plans. In the same year, Slusarczyk [25] introduced the notion of hierarchical layout graph (HL-graph) and gave a theoretical framework for extending local graph requirements to global requirements on HL-graphs.

It can be found in the literature that most of the work done related to the existence and construction of a floor plan falls into any one of the following categories:

i. Construction of a RFP corresponding to a properly triangulated planar (PTP) graph[5] [1,23,26],
ii. Construction of a RFP corresponding to a maximal outer-planar graph [7,13],
iii. Construction of an orthogonal floor plan corresponding to a MPG [19],
iv. Transform a given graph into a MPG [9,15] or into a PTP graph [10] and construct a corresponding floor plan.

From above discussion, it is clear that most of the work related to the generation of a RFP is restricted to PTP graphs. And to the best of our knowledge, there does not exist an algorithm for checking the existence of a RFP and for constructing a RFP corresponding to graphs, other than PTP graphs. In this work, we aim to provide a MRFP (and a RFP) for any given graph, if it exists, i.e., this work is not restricted to any particular class of graphs. The steps involved in obtaining a MRFP corresponding to a given graph are as follows:

1. Check if given graph G_n is a sub-graph of any one of the $\mathrm{MRFP_G}(n)$,
2. If G_n is a sub-graph of a $\mathrm{MRFP_G}(n)$, then corresponding MRFP(n) is the required RFP with some extra connections,
3. If G_n is not a sub-graph of any of the $\mathrm{MRFP_G}(n)$, then G_n is not a $\mathrm{RFP_G}(n)$.

Once we have a MRFP(n), the extra connections can be eliminated, if feasible. For example, consider the RFP(5) in Fig. 1E which is obtained from the MRFP(5) in Fig. 1D by eliminating a connection between rooms 3 and 4. At the same time, it is interesting to note that the connection between rooms 2 and 4 in Fig. 1D cannot be removed while preserving rectangularity and other adjacencies, i.e., the graph in Fig. 1F, obtained from the graph in Fig. 1C by deleting an edge between vertices 2 and 4, is not a $\mathrm{RFP_G}(5)$.

Considering a MRFP corresponding to a given graph assures that the adjacency constraints set by the user are satisfied. In addition, it reduces the complexity of computation involved in producing a RFP for a given graph, i.e., it is not required to look for a RFP corresponding to the given graph, each time we change the adjacency constraints. Hence, in this paper, we present an algorithm to enumerate GRFP(n). In addition, we can identify that Theorem 1 is restricted to PTP graphs only. In this work, we present a necessary condition for the existence of a RFP corresponding to a given graph.

Remark 1. There exist many planar graphs that are not $\mathrm{RFP_G}$. For example, the graph in Fig. 1H is planar but it is not a $\mathrm{RFP_G}$. Hence, it is not possible to construct a RFP corresponding to them. On the other hand, if any one of these graphs is a sub-graph of a $\mathrm{MRFP_G}$, then we can have a MRFP corresponding to it satisfying all of its adjacency requirements. For example, Fig. 1D is a MRFP for the graph in Fig. 1H.

The flow of the paper is as follows:

Section 2 computes GRFP(n) for $n = 4, 5, 6$, to prove that a $\mathrm{MRFP_G}(n)$ only belongs to any one of the groups:

i. $\mathrm{RFP_G}(n)$ with $3n-7$ edges,
ii. W_n (*wheel graph* formed by connecting a single vertex to all vertices of a cycle).

Section 3 describes best connected RFP and derives GRFP(7) and GRFP(8). Section 4 presents a method to check if two RFP(n) are topologically distinct which would further be used to examine the isomorphism of any two RFP(n). Section 5 provides an algorithm for obtaining GRFP(n) from GRFP($n-1$). Section 6 presents a method for examining if any graph G_n is a sub-graph of any of the $\mathrm{MRFP_G}(n)$. Section 7 provides an elaborated example for demonstrating the results

[4] Treemaps subdivide an area into small pieces to represent the importance of each part in the hierarchy whereas squarified treemaps are used to generate rooms with aspect ratios close to one.

[5] A *properly triangulated planar* (PTP) graph, G, is a connected planar graph that satisfies the following properties:

i. Every face (except the exterior) is a triangle (i.e., bounded by three edges),
ii. All internal vertices have degree ≥ 4,
iii. All cycles that are not faces have length ≥ 4.

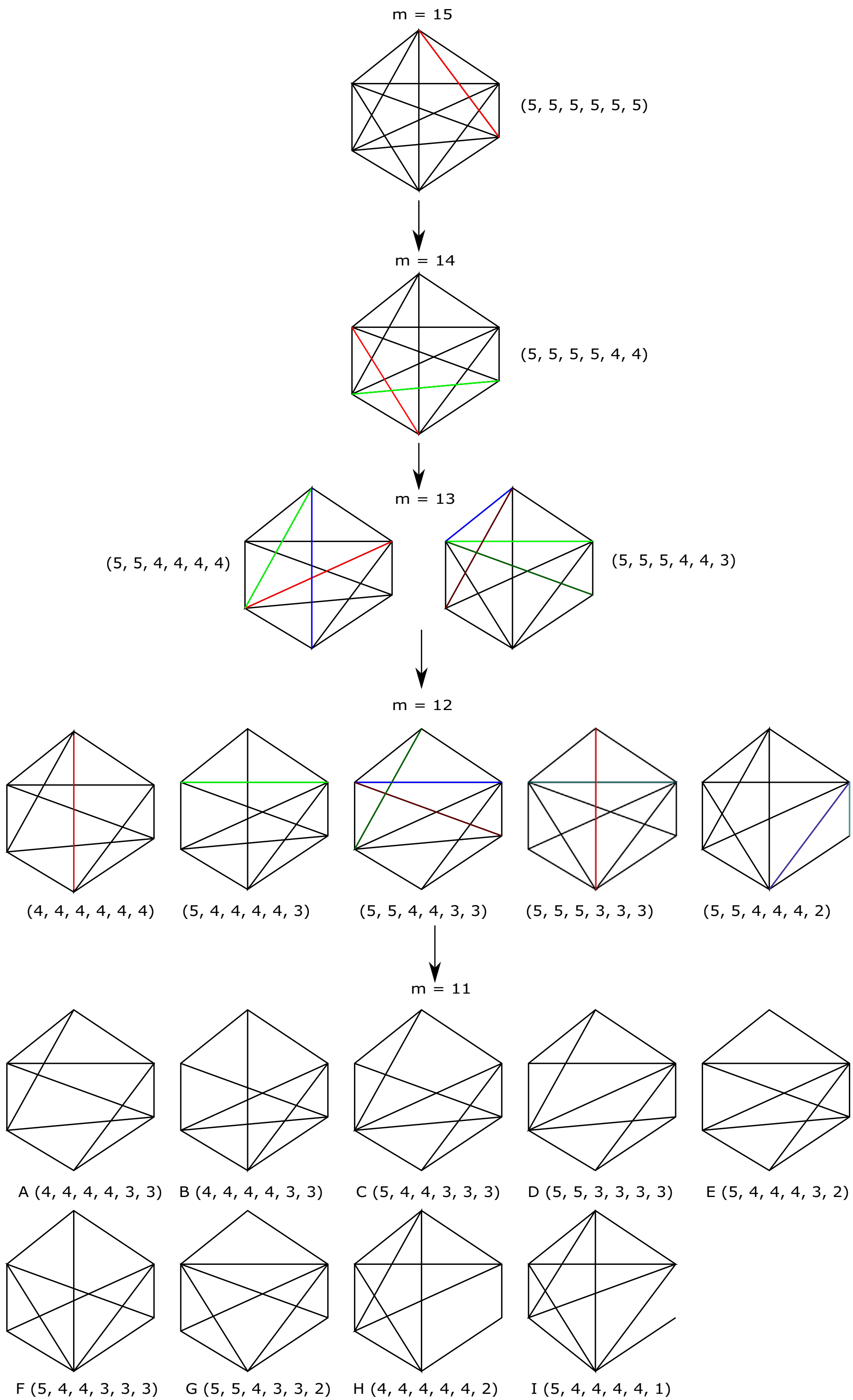

Fig. 5. Deriving all non-isomorphic G_6^{11} from K_6 (to obtain G_6^{m-1} from G_6^m, distinct colored edges in all of the G_6^m were deleted). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

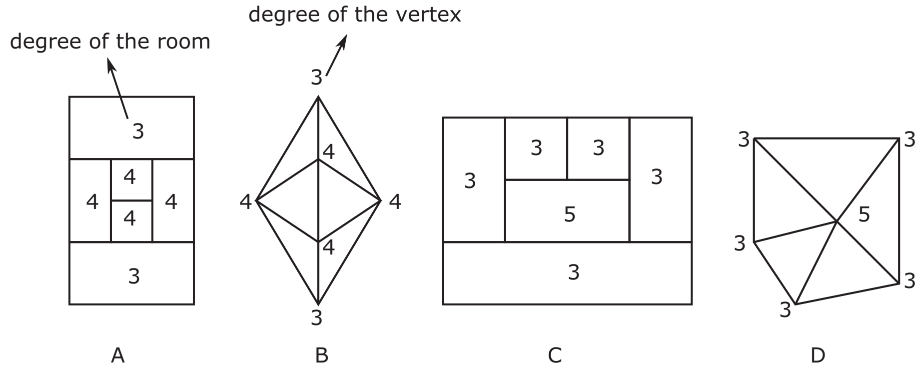

Fig. 6. All distinct MRFP(6) and their dual graphs.

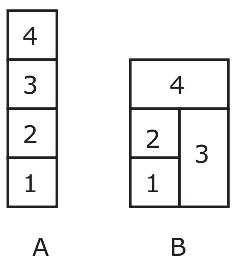

Fig. 7. Rooms sharing a full wall.

of the paper. In the end, Section 8 is about the conclusion and future work.

Notations:

RFP	rectangular floor plan/s
RFP(n)	RFP with n rooms
n	the number of rooms in a RFP or the number of vertices in a graph
m	the number of edges in a graph
G_n	n-vertex simple connected graph
G_n^m	G_n with m edges
RFP_G	rectangular floor plan graph
DG	dual graph of a RFP
$MRFP_G$	maximal RFP_G
MRFP	maximal RFP
BCRFP	best connected RFP (a maximal RFP with $3n - 7$ rooms)
v_i	ith vertex of a graph
R_i	ith room of a RFP
W_n	wheel graph with n vertices

2. Generic rectangular floor plans when $n = 4, 5, 6$

In this section, we present a method to enumerate GRFP for $n = 4$, 5, 6. Also, we are going to show that a $MRFP_G(n)$ only belongs to any one of the groups (see Proposition 5):

- RFP_G with $3n - 7$ edges,
- W_n.

In addition, we present a necessary condition for a graph to be a RFP_G (see Proposition 6).

Definition 5. Given an undirected graph, a *degree sequence* is a monotonic non-increasing sequence of the vertex degrees of its graph vertices.

We say that a degree sequence $\{a_1, a_2, \ldots, a_n\}$ is *contained* in a degree sequence $\{b_1, b_2, \ldots, b_n\}$ if and only if $a_i \leq b_i \ \forall \ \ i = 1, 2, \ldots, n$.

For example, the degree sequence of the graph in Fig. 1B, i.e., {4,3,3,2,2} is contained in the degree sequence of the graph in Fig. 1C, i.e., {4,3,3,3,3}.

Proposition 1. *The number of distinct $MRFP_G(4)$ is one.*

Proof. For any complete graph K_n, we have $m = \frac{n(n-1)}{2}$ and for any dual graph DG_n, we have $m \leq (3n - 7)$ provided that $n > 3$. Hence, for K_4 and for any DG_4, we have $m = (4 \times 3)/2 = 6$ and $m \leq (3 \times 4) - 7 = 5$ respectively. Clearly, there exists only one graph G_4^5 that can be derived from K_4 by deleting any one of its edges. A G_4^5 and corresponding RFP are illustrated in Fig. 2. Hence, this graph is a $MRFP_G$. Also, it is easy to verify that there does not exist a $RFP_G(4)$ whose degree sequence is not contained in the degree sequence of $RFP_G(4)$ in Fig. 2. This concludes the required proof.

Now, for K_5, $m = (5 \times 4)/2 = 10$ and for any DG_5, we have $m \leq (3 \times 5) - 7 = 8$. All distinct graphs G_5^8 can be derived from K_5 by deleting two of its edges, as shown in Fig. 3A–D. In Fig. 3, to derive G_5^9 from G_5^{10}, any one of the edges of G_5^{10} can be deleted because the degree of each vertex in Fig. 3A is same; but to derive G_5^9 from G_5^8, there are two ways for deleting an edge, i.e., delete an edge joining the vertices with degree 4 (see Fig. 3C), or delete an edge joining the vertices with degree 3 and degree 4 (see Fig. 3D), which leads us to two non-isomorphic G_5^8.

From Fig. 3, there exist only two distinct graphs with $3n - 7 = (3 \times 5) - 7 = 8$ edges and for both of them, there does not exist a super-graph which is a $RFP_G(5)$. A RFP for G_5^8 in Fig. 3C is illustrated in Fig. 4 but the G_5^8 in Fig. 3D is not a RFP_G (because of the presence of complete graph K_4, see Proposition 2). Also, we can verify that there does not exist a $RFP_G(5)$ whose degree sequence is not contained in the G_5^8 in Fig. 3C. This suggests that the G_5^8 in Fig. 3C is the only $MRFP_G(5)$.

Proposition 2. *The complete graph K_4 is not a RFP_G.*

Proof. From Theorem 1, there does not exist a RFP corresponding to the complete graph K_4 because its outer face is not a quadrangle. This implies that K_4 is not a RFP_G.

For K_6, $m = (6 \times 5)/2 = 15$ and for any DG_6, we have $m \leq (3 \times 6) - 7 = 11$. All non-isomorphic G_6^{11} derived from K_6, are shown in Fig. 5 Now, we will show that there is only one $MRFP_G$ among the nine G_6^{11} in Fig. 5, as illustrated in Fig. 5A (see Proposition 3).

Proposition 3. *The graph G_6 in Fig. 5A is a $MRFP_G(6)$.*

Proof. From Kuratowski's theorem[6], it can be easily verified that the following graphs in Fig. 5 are non-planar:

6

Theorem 2. *(Kuratowski's theorem [27]) A finite graph is planar if and only if it does not contain a sub-graph that is a subdivision of K_5 (the complete graph on five vertices) or of $K_{3,3}$ (complete bipartite graph on six vertices).*

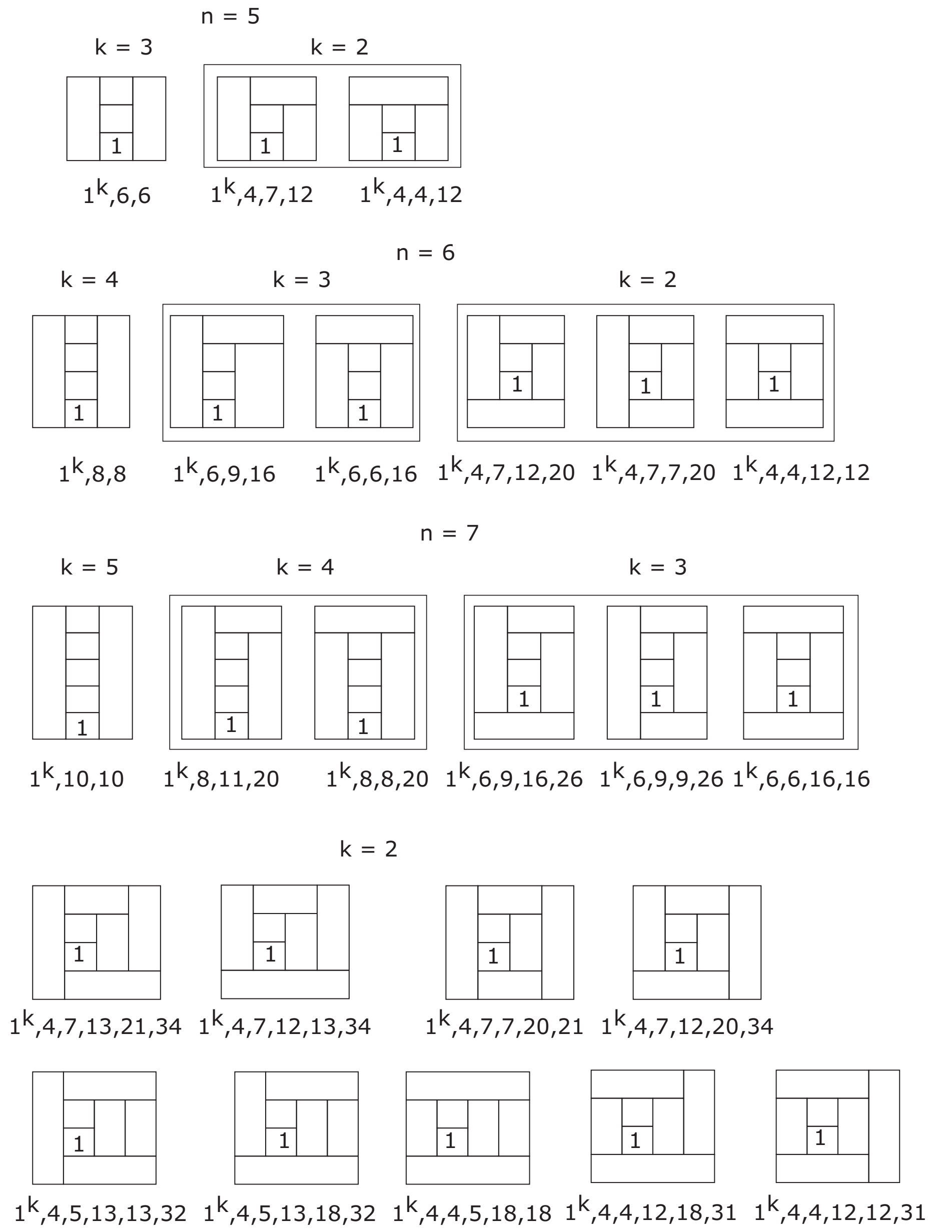

Fig. 8. All topologically distinct BCRFP with five, six and seven rooms respectively.

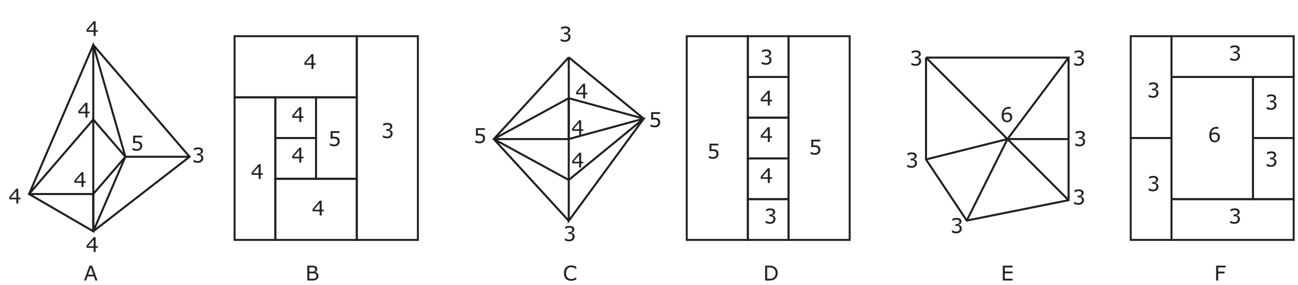

Fig. 9. All distinct $MRFP_G(7)$ and corresponding RFP.

- Fig. 5B and C (because of the presence of $K_{3,3}$)
- Fig. 5H (because of the presence of a subdivision of K_5)
- Fig. 5I (because of the presence of K_5)

Also, the graphs in Fig. 5D– G are not RFP_G because of the presence of K_4.

From above discussion, there exists only one G_6^{11}, i.e., Fig. 5A, which is a $RFP_G(6)$ (see Fig. 6A). This concludes the proof.

From Proposition 3, there exists only one $RFP_G(6)$ with 11 edges but it can be noticed that the degree sequence of a graph G_6 having at least one vertex of degree 5 cannot be contained in the degree sequence {4,4,4,4,3,3}. It leads us to the following results, where $\deg R_i$ is the degree of room R_i.

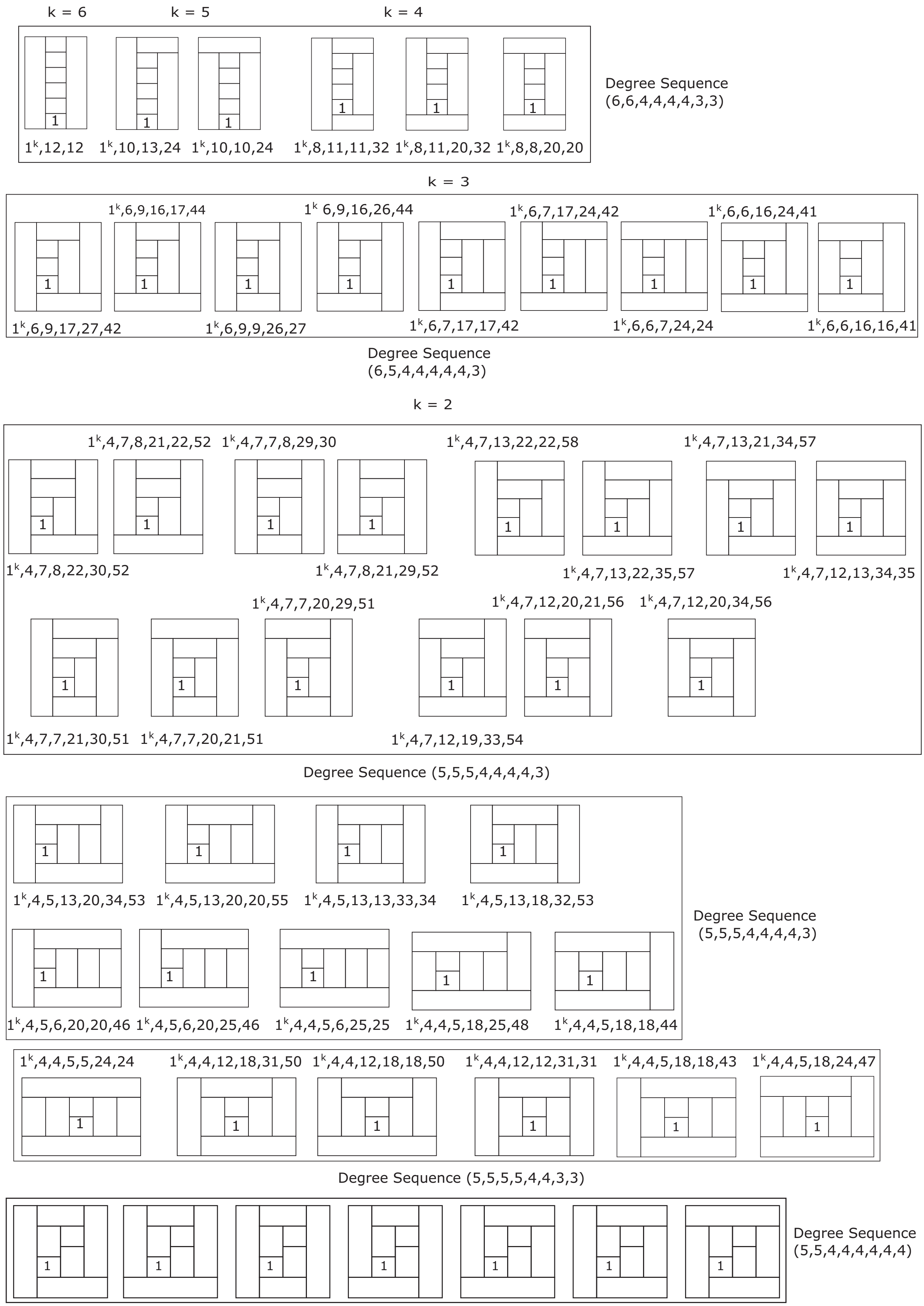

Fig. 10. All topologically distinct BCRFP(8) (here, there are five distinct boxes with BCRFP, where any two BCRFP in two different boxes are distinct).

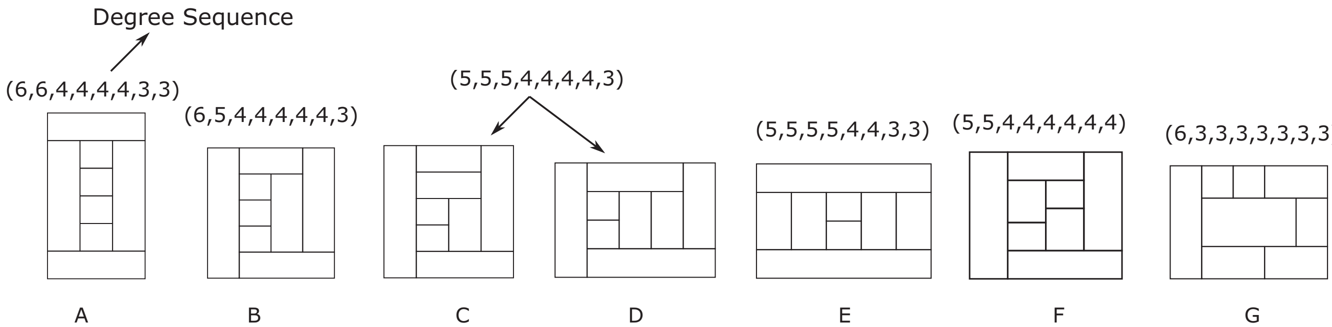

Fig. 11. All distinct MRFP(8).

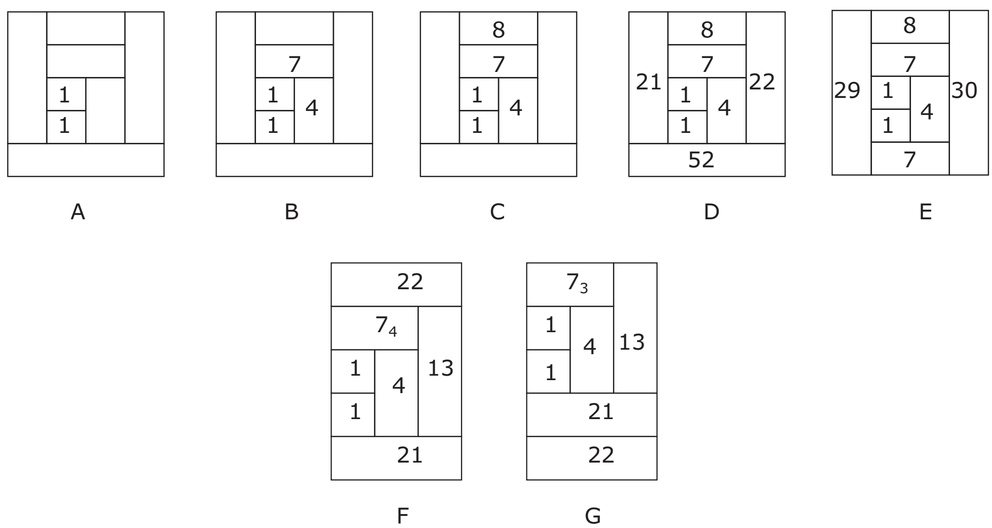

Fig. 12. Labeling of a RFP.

Proposition 4. *If* $\deg R_i = n - 1$ *,* $n > 4$ *for some* $1 \leq i \leq n$*, then* $\deg R_j \leq 3$ *, for all* $1 \leq j \leq n, j \neq i$*.*

Proof. Let $\deg R_A = n - 1$ and R_B is adjacent to R_A. Clearly, it is not possible to draw a RFP(n) having three rooms (other than R_A and R_B) adjacent to both R_A and R_B, i.e., if all three rooms are adjacent to R_A, then at most two of the rooms can be adjacent to R_B. This concludes the proof.

Corollary 1. *W_n, $n > 4$, is a MRFP$_G$.*

Proof. It is always possible to construct a RFP(n), $n > 4$, corresponding to W_n; it can be verified by drawing a RFP(n) with a room, say R_A, in the center and $n - 1$ rooms adjacent to R_A such that there is at least one room adjacent to each of the four sides of R_A. As an example, a RFP for W_6 is illustrated in Fig. 6C. Hence, proof directly follows from Proposition 4.Propositions 1–4 and Corollary 1 lead us to the following two results.

Proposition 5. *For* $n > 3$*, a MRFP$_G$(n) only belongs to any one of the groups:*

i *RFP$_G$(n) with* $3n - 7$ *edges,*
ii *W_n.*

Proposition 6. *There exists a MRFP(n) corresponding to any given labeled graph G_n if and only if its underlying unlabeled graph is a sub-graph of any one of the MRFP$_G$(n). Hence, there does not exist a RFP(n) for a given graph G_n if its underlying unlabeled graph is not a sub-graph of any of the MRFP$_G$(n).*

3. Best connected rectangular floor plans (BCRFP)

In this section, we enumerate GRFP(7) and GRFP(8).

Definition 6. Consider a room with horizontal sides a (lower side), a' (upper side) and vertical sides b (left side), b' (right side), and represent it as $R_{a,b,a',b'}$. Two adjacent rooms $R_{a,b,a',b'}$ and $R_{c,d,c',d'}$ *share a full wall* if any one of the following holds:

$$a' = c;\ a = c';\ b' = d;\ b = d'.$$

A room $R_{e,f,e',f'}$, adjacent to rooms $R_{a,b,a',b'}$ and $R_{c,d,c',d'}$, shares a full wall with $R_{a,b,a',b'}$ and $R_{c,d,c',d'}$ if any one of the following holds:

$$a' + c' = e;\ a + c = e';\ b' + d' = f;\ b + d = f'.$$

In symbol, if rooms R_1 and R_2 share a full wall, we write $R_1|R_2$. If for n given rooms, $R_1, R_2, ..., R_n$, each pair of rooms R_i and R_{i+1} share a full wall, we write $R_1|R_2|...|R_n$. If R_i shares a full wall with R_j and R_k, we write $R_i|\{R_j, R_k\}$.

For example, in Fig. 7A, we have $R_1|R_2|R_3|R_4$ and in Fig. 7B, we have $R_1|R_2$, $R_4|\{R_2, R_3\}$ and $R_3|\{R_1|R_2\}$.

Definition 7. A RFP(n), $n > 3$, is called *best connected* if its dual graph has $3n - 7$ edges. A best connected RFP is represented as BCRFP. A RFP$_G$(n) with $3n - 7$ edges is called *best connected* RFP$_G$(n), denoted by BCRFP$_G$(n).

For example, Fig. 1C is BCRFP$_G$(5) while Fig. 1D and I illustrate BCRFP(5).

Definition 8. Two RFP(n) are said to be *topologically distinct* (or

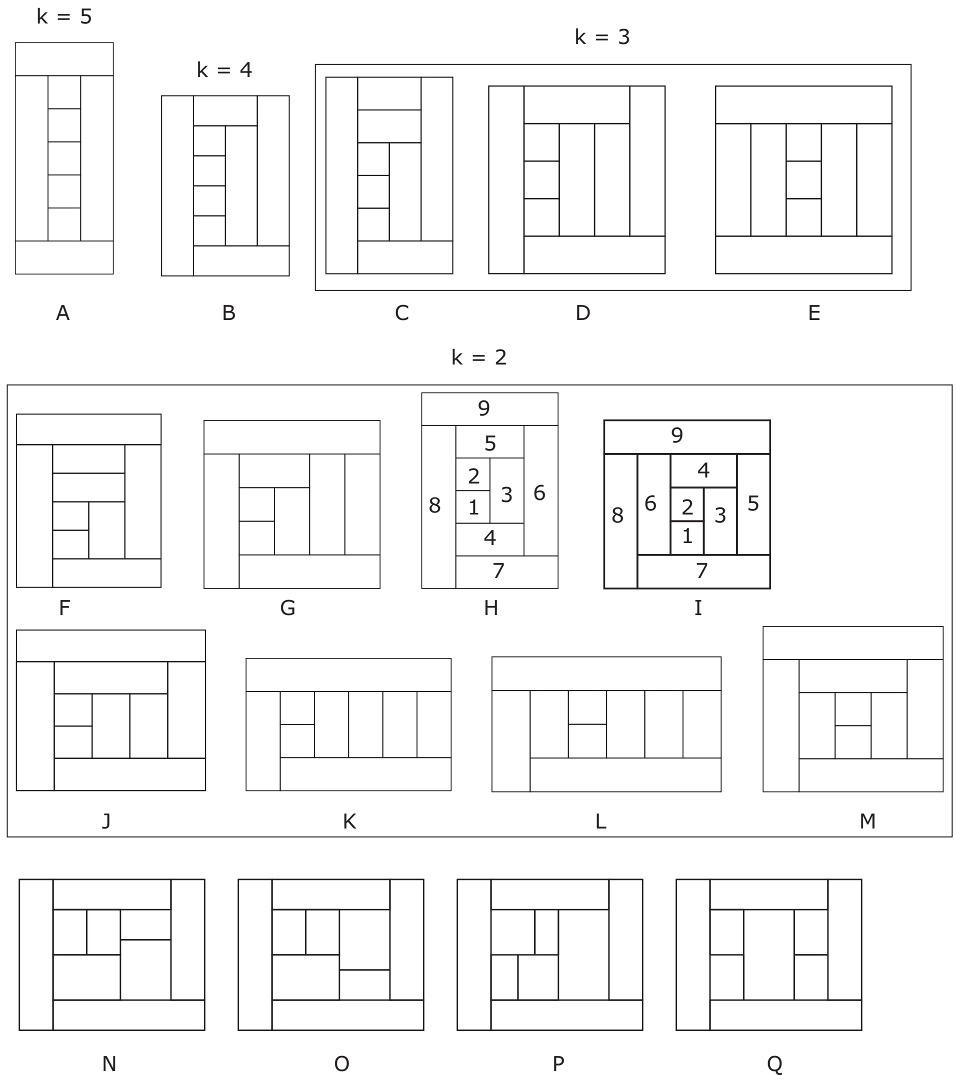

Fig. 13. All distinct BCRFP(9).

topologically non-isomorphic) if and only if one cannot be derived from other using the following transformations [28]:

1 Translation
2 Rotation
3 Reflection
4 Scaling
5 Any composition of the above

For example, the RFP in Fig. 1D and I are topologically distinct.

We know that W_n is unique for a fixed value of n. From Proposition 5, we can say that, to have GRFP(n), we need to compute all BCRFP$_G$(n). But the procedure used in Section 2 to compute BCRFP$_G$ involves a lot of computation and there may be a chance of an error. To cross-check the results presented in Section 2, and to derive GRFP(7) and GRFP(8), we introduce a new methodology, which involves less computation, as follows:

Construct all topologically distinct BCRFP(n) for $4 < n < 9$ and among them, choose all distinct BCRFP(n).

For a better understanding, refer to Fig. 8 where all topologically distinct BCRFP with five, six and seven rooms respectively, have been demonstrated. In Figs. 8 and 10, corresponding to each BCRFP, a labeling has been assigned where 1^k represents 1, 1, …,1 (k times)). Also, to have the compositional schema of these BCRFP, the first room to be placed is the one with number '1' written in its center.

It can be verified from Fig. 8 that for $n = 5$, 6, the number of non-isomorphic BCRFP$_G$ are 1, 1 respectively, as proved in Section 2. All non-isomorphic MRFP$_G$(7) and corresponding RFP are demonstrated in Fig. 9, where panels Band D in Fig. 9 are BCRFP(9).

In a BCRFP(n) where the composition of all drawn rooms after drawing each room is rectangular, k represents first k allocated rooms such that $R_1|R_2|...|R_k$. For constructing it, first we allocate k rooms such that $R_1|R_2|...|R_k$, for example, in Fig. 13A and B, we have $k = 5$, 4 respectively. Then each of the remaining rooms is allocated in such a way that each drawn room must share a full wall with at least one of the existing rooms and last drawn room must be adjacent to exactly 3 existing rooms. For example, refer to Fig. 13H and I where rooms are allocated in the order of their numbering starting from 1. For a better understanding of the construction of BCRFP with different values of k, refer to [29].

For $n = 8$, there exist 51 topologically distinct BCRFP(8) (refer to Fig. 10), out of which there are only six distinct BCRFP(8), as shown in Fig. 11A–F.

Let BCRFP(n)[$k > 2$] and BCRFP(n)[$k = 2$] stand for BCRFP(n) (where the composition of all drawn rooms after drawing each room is rectangular) when $k > 2$ and $k = 2$ respectively.

Observation 1. Two BCRFP[$k = 2$](n), $n > 7$, are distinct if the corresponding BCRFP[$k = 2$]($n - 4$), obtained by deleting all four

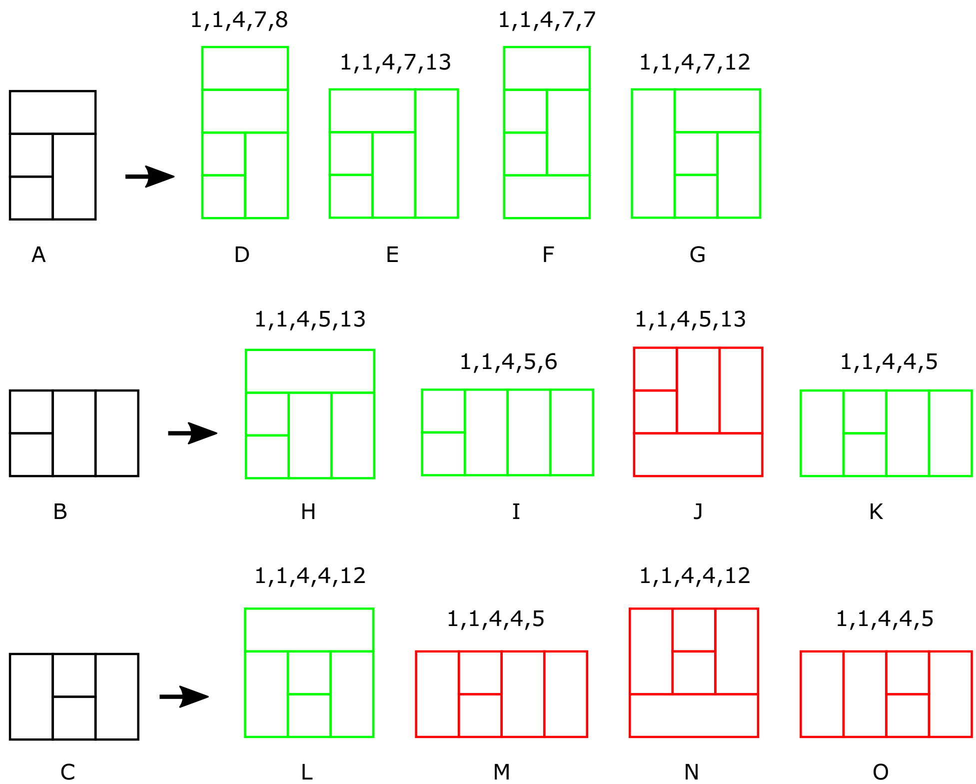

Fig. 14. Construction of BCRFP(n − 4)[k = 2] from BCRFP(n − 5)[k = 2].

boundary rooms of BCRFP[k = 2](n), are topologically distinct.

Observation 2. Two BCRFP(n), where the composition of all drawn rooms after drawing each room cannot be rectangular, are distinct if and only if corresponding RFP(n − 4), obtained by deleting all four boundary rooms of BCRFP(n), are distinct.

To understand the result of Observation 1, refer to Fig. 10 and consider BCRFP[k = 2](8). We can see that BCRFP[k = 2](4), obtained by deleting the boundary rooms of BCRFP[k = 2](8), inside any of the 3 boxes are isomorphic while any two BCRFP[k = 2](4) in two different boxes are non-isomorphic. Similarly, it can be verified from Fig. 13N to Q that all BCRFP(9) are distinct if and only if corresponding RFP (n − 4), i.e., RFP(5) are distinct.

4. Labeling of best connected rectangular floor plans

Here we consider those BCRFP(n) where the composition of all drawn rooms after drawing each room is rectangular. In this section, we present a method to examine if two given BCRFP(n) are topologically distinct or not.

It can be verified visually that the BCRFP in Fig. 10 are topologically distinct but in the literature, we did not find a procedure for checking it. In this section, the idea is to associate a unique representation or labeling to a BCRFP which would be used to check if the two given BCRFP (n) are topologically distinct or not. The steps used for deriving a labeling for a BCRFP are as follows (a label is assigned in the order of allocation of the rooms):

1. Assign number ‘1’ to each of the first k rooms (see Fig. 12A). Let us call the room to which a labeling has been assigned as labeled room otherwise an unlabeled room.
2. The next room to be labeled is the unlabeled room that shares a full wall only with the labeled rooms. The number assigned to the next unlabeled room (other than the first k rooms) is equal to the sum of the numbers assigned to the labeled rooms that are adjacent to it, plus the number of labeled rooms that are adjacent to it (see Fig. 12B– D).
3. After associating a label to all the rooms, a monotonic increasing sequence of the numbers associated with the rooms is the required labeling. If two BCRFP(n) have different labeling, then they are topologically distinct (for example, labeling of Fig. 12D and E are (1,1,4,7,8,21,22,52) and (1,1,4,7,7,8,29,30) respectively, hence they are topologically distinct).
4. If labeling of two BCRFP obtained in above steps are equal, then we redefine the labeling by associating a subscript with each of the numbers of the labeling, that is equivalent to the degree of the corresponding room. If two RFP have different labeling, then, they are topologically distinct. For example, initially Fig. 12F and G have the same labeling but after adding subscripts, we can see that they are topologically distinct.

5. Maximal rectangular floor plans for any *n*

A RFP corresponding to a W_n can be constructed using Corollary 1. Using the concept of Observations 1 and 2, in this section, we present an algorithm for computing all distinct BCRFP(n) from all distinct BCRFP(n − 1) when $n > 6$. The steps of the algorithm are as follows:

1. Increase k by one in all distinct BCRFP(n − 1) to obtain all distinct BCRFP(n)[$k > 2$]. For example, all distinct BCRFP(9)[$k > 2$] obtained from BCRFP(8) are illustrated in Fig. 13A–E.
   Next is to compute all distinct BCRFP(n)[k = 2] using the concept of Observation 1. Let the number of distinct BCRFP(n − 1)[k = 2] be r.
2. Delete all four boundary rooms of all r BCRFP(n − 1)[k = 2] to obtain r distinct BCRFP(n − 5)[k = 2]. For example, 3 distinct BCRFP(4)[k = 2] obtained from BCRFP(8)[k = 2] are shown in Fig. 14A– C.
3. Consider each of the r BCRFP(n − 5)[k = 2] one by one and add a room to each of them at four different positions, i.e., above, right, below and left to the BCRFP(n − 5)[k = 2] to obtain $4r$ BCRFP

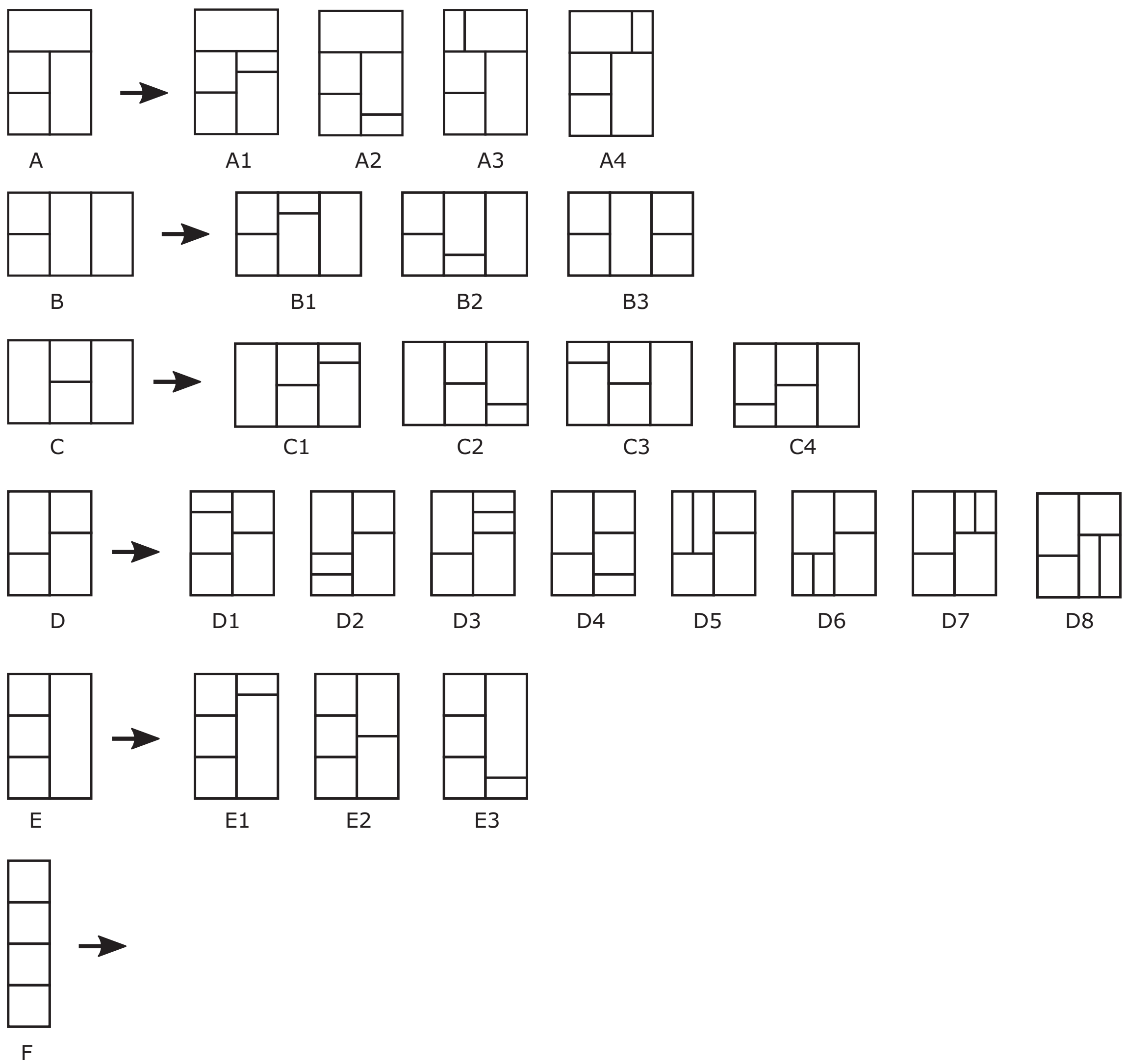

Fig. 15. Dissecting each room of BCRFP($n-5$) to have all possible RFP($n-4$).

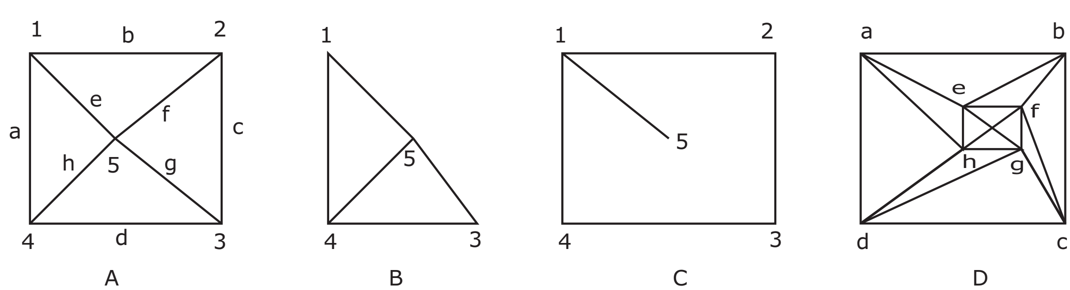

Fig. 16. Illustrating the concept of a vertex induced sub-graph, an edge induced sub-graph, and a line graph.

($n-4$)[$k=2$]. For example, 12 BCRFP(5)[$k=2$] are shown in Fig. 14D– O.

4. Label each of the obtained 4r BCRFP($n-4$)[$k=2$]. For example, a labeling has been associated to all BCRFP(5)[$k=2$] in Fig. 14.
5. Among the 4r BCRFP($n-4$)[$k=2$], pick all topologically distinct BCRFP($n-4$)[$k=2$]. For example, out of 12 BCRFP(5)[$k=2$] in Fig. 14D– O, 8 BCRFP(5)[$k=2$] are topologically distinct, that are shown in green.
6. Add four boundary rooms to all topologically distinct BCRFP($n-4$) [$k=2$] to have all distinct BCRFP(n)[$k=2$] (four boundary rooms are added to the four sides of BCRFP($n-4$)[$k=2$] respectively). Corresponding to 8 topologically distinct BCRFP(5)[$k=2$] in Fig. 14, 8 distinct BCRFP(9)[$k=2$] are illustrated in Fig. 13F– M.
7. Delete all four boundary rooms in all BCRFP($n-1$) to obtain all BCRFP($n-5$). For example, 6 distinct BCRFP(4) obtained from BCRFP(8) are shown in Fig. 15A– F.
8. Dissect each room of the obtained BCRFP($n-5$), horizontally and vertically, to obtain RFP($n-4$), where the composition of all drawn rooms after drawing each room cannot be rectangular. For example, 22 RFP(5) obtained from 6 BCRFP(4) are illustrated in Fig. 15.
9. Among all RFP($n-4$) obtained in above step, pick the distinct ones and add four boundary rooms to them to have distinct BCRFP(n)

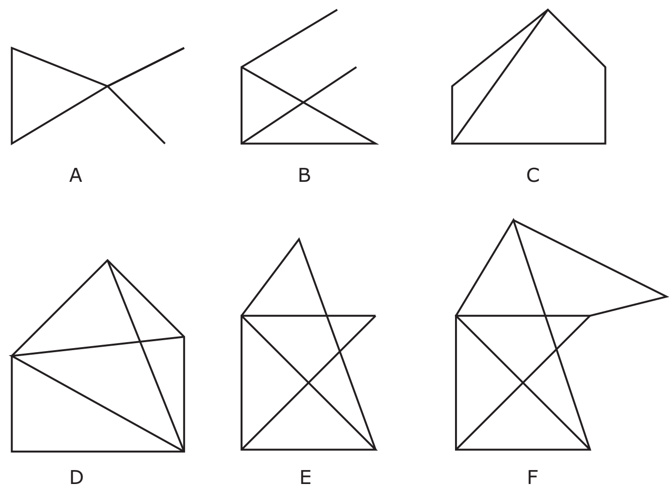

Fig. 17. Edge induced sub-graph isomorphism.

(four boundary rooms are added to the four sides of RFP($n-4$) respectively). Among all 22 RFP(5) in Fig. 15, there are 4 distinct RFP(5) that leads to 4 distinct BCRFP(9), shown in Fig. 13N– Q.

6. Induced sub-graph isomorphism

Definition 9. Let $G=(V,E)$ be any graph and $S \subseteq V$. The *vertex induced sub-graph* is the graph with vertex set S and whose edge set consists of all of the edges in E that have both endpoints in S.

Definition 10. An *edge induced sub-graph* is a subset of the edges of a graph G together with any vertices that are their endpoints.

Refer to Fig. 16 where Fig. 16B is a vertex induced sub-graph of Fig. 16A while Fig. 16C is an edge induced sub-graph of Fig. 16A.

Definition 11. A *line graph* of a graph G is another graph $L(G)$ that represents the adjacencies between edges of G. Any two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G have a vertex in common.

For example, Fig. 16D is the line graph of Fig. 16A.

Once we have GRFP$_G$, our aim is to construct a MRFP for a given graph G_n. And to have a MRFP corresponding to G_n, we first need to test if G_n is a sub-graph of at least one of the MRFP$_G(n)$.

To check if any graph G_1 is a sub-graph of a graph G_2, we can use VF2 algorithm [30], which works for induced sub-graph isomorphism, i.e., using VF2 algorithm, we can check that if G_1 is a vertex induced sub-graph of G_2 or not. In our case, G_1 and G_2 are of same order. Hence, G_1 is a vertex induced sub-graph of G_2 only if G_1 and G_2 are isomorphic. Therefore, in this case, we need to check if G_1 is an edge induced sub-graph of graph G_2 or not, i.e., we cannot use VF2 algorithm.

To examine edge induced isomorphism and to use VF2 algorithm, we first compute the line graphs of G_1 and G_2. Let these line graphs be represented as G_1^L and G_2^L respectively. Now we apply VF2 algorithm to G_1^L and G_2^L which gives vertex induced sub-graph isomorphism of G_1^L and G_2^L. If G_1^L is vertex induced sub-graph of G_2^L, G_1 is an edge induced sub-graph of G_2. Hence, G_1 is a sub-graph of G_2. The validity of this method follows from Whitney Graph Isomorphism Theorem [31].

The worst case time complexity of the proposed method to check sub-graph isomorphism is $O(2^m)$. But in the case of floor plan design, generally m is not very large, hence we can use this method to check if given graph G_n is a sub-graph of at least one of the MRFP$_G(n)$.

For a better understanding, refer to Fig. 17 where we have given three graphs in Fig. 17A–C and we need to check if graphs in Fig. 17A, B are sub-graphs of the graph in Fig. 17C. Clearly, none of them is a vertex induced sub-graph of Fig. 17C. Hence, we look for edge induced sub-graph isomorphism by computing the line graphs of Fig. 17A–C, which are illustrated in Fig. 17D–F. It can be easily verified that Fig. 17D is not an edge induced sub-graph of Fig. 17F while Fig. 17E is an edge induced sub-graph of Fig. 17F. This implies that Fig. 17A is not a sub-graph of Fig. 17C but Fig. 17B is a sub-graph of Fig. 17C.

7. Results

In this paper, we aim to provide a MRFP and a RFP for any given graph, if they exist. The steps involved in obtaining them are illustrated using Fig. 18. For a better understanding, let us consider that we have been given five graphs as shown in Fig. 18A to E. To proceed further, we would go through the following steps:

1. Check if given graph G_n is a sub-graph of any of the MRFP$_G(n)$.
2. If G_n is a sub-graph of a MRFP$_G(n)$, then the corresponding MRFP(n) is the required MRFP with some extra connections.
   This can be verified that
   (a) The underlying graph of G_6 in Fig. 18A is a sub-graph of MRFP$_G$(6) in Fig. 6B. Hence, the required MRFP(6) is given by Fig. 18F with extra connections between rooms R_2 and R_6, R_1 and R_4, R_1 and R_5.
   (b) The underlying graph of G_8 in Fig. 18B is a sub-graph of MRFP$_G$(8) in Fig. 11B. Hence, the required MRFP(8) is given by Fig. 18G with extra connections between rooms R_1 and R_3, R_3 and R_5, R_4 and R_7.
   (c) The underlying graph of G_{10} in Fig. 18C is a sub-graph of one of the MRFP$_G$(10). Hence, the required MRFP(10) is given by Fig. 18H with extra connections between rooms R_1 and R_9, R_2 and R_9, R_3 and R_9, R_7 and R_9.
   (d) The underlying graph of G_6 in Fig. 18D is a sub-graph of MRFP$_G$(6) in Fig. 6B. Hence, the required MRFP(6) is given by Fig. 18I with extra connections between rooms R_2 and R_4, R_3 and R_6.
3. Once we have the required MRFP(n), next is to eliminate the extra connections, if feasible.
   (a) In Fig. 18N– R, the elimination of extra connections of MRFP(6) in Fig. 18F, has been demonstrated, i.e., Fig. 18K is the required RFP(6) corresponding to G_6 in Fig. 18A.
   (b) Similarly, RFP(8) and RFP(10) corresponding to G_8 and G_{10} in Fig. 18B and C are shown in Fig. 18L and M respectively.
   (c) It is interesting to note that there does not exist a RFP(6) for G_6 in Fig. 18D, i.e., for MRFP(6) in Fig. 18I, it is not possible to eliminate the extra connections between rooms R_2 and R_4, R_3 and R_6, while preserving other adjacencies in the MRFP(6).
4. If G_n is not a sub-graph of any of the MRFP$_G(n)$, then G_n is not a RFP$_G(n)$.
   For example, G_6 in Fig. 18E is not a RFP$_G$(6). The connections asked by the G_6 can only be met by considering one of the rooms non-rectangular, as shown in Fig. 18J.

8. Conclusion and future work

In this paper, we describe GRFP and present an algorithm for their enumeration. The contribution of this work in three different fields can be seen as follows:

1. Mathematics: To the best of our knowledge, there does not exist a mathematical theory for checking the existence of a RFP for a given graph, other than the PTP graphs [2]. Using the graph theoretical tools, we presented a necessary and sufficient condition for the existence of a MRFP for a given graph G_n, which further leads to a necessary condition for the existence of a RFP for G_n. Furthermore, for a graph G_n, if it is not feasible to eliminate all extra connections

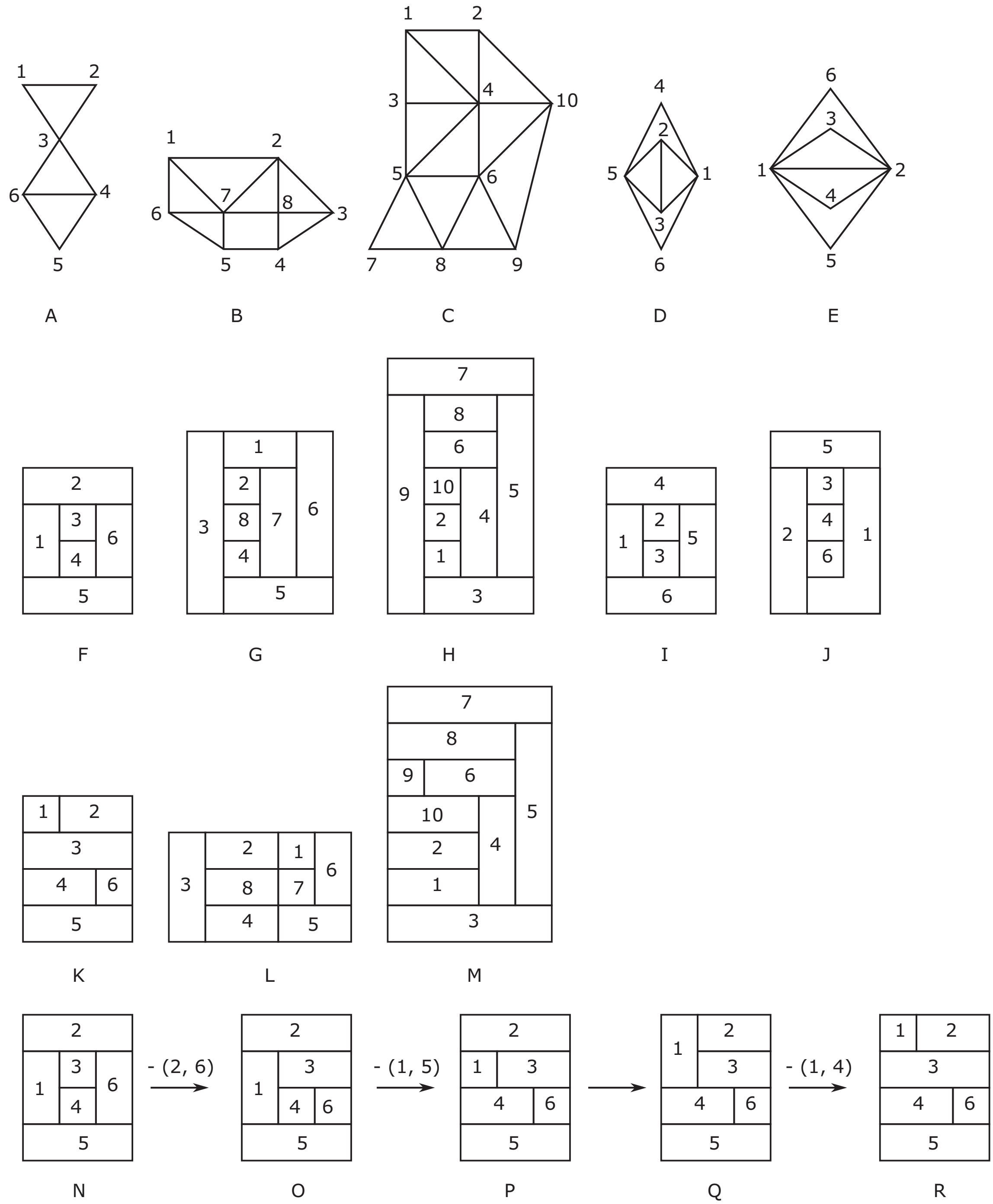

Fig. 18. Example demonstrating the results of this paper.

from the corresponding MRFP(n), then G_n is not a $\mathrm{RFP_G}(n)$ (an algorithm for transforming a MRFP(n) into the required RFP(n) will be presented in future work).

2. Computer science: Once GRFP are described mathematically, next is to construct them. In this paper, we have presented an algorithm for enumerating GRFP(n) for any n.
3. Architecture: Architecturally, the idea is to reduce a large and complex problem, i.e., designing a RFP for a given adjacency graph, into a simple problem, i.e., look for a $\mathrm{MRFP_G}(n)$ corresponding to it. This leads to a class of RFP that is independent of adjacency constraints. Furthermore, once we have a MRFP, we can derive a RFP corresponding to a given graph, if it exists. Most importantly, using the concept of GRFP, a RFP(n) isomorphic to an existing RFP(n) can easily be derived. For example, refer to Fig. 19A and B, illustrating the RFP corresponding to subtype Aa and Bb of Siza's houses at Malagueira [32,33]. The RFP isomorphic to Fig. 19A and B have been derived from corresponding MRFP, as shown in Fig. 19C–F and G–M respectively.

As for future work, we aim to construct floor plans corresponding to the graphs for which RFP and MRFP do not exist. In this case, we need to make some rooms non-rectangular. For example, for G_6 in Fig. 18E, there does not exist a RFP and a MRFP but a corresponding orthogonal floor plan can be obtained by making R_1 non-rectangular, as shown in Fig. 18J. Also, in architectural design, a multitude of aspects with

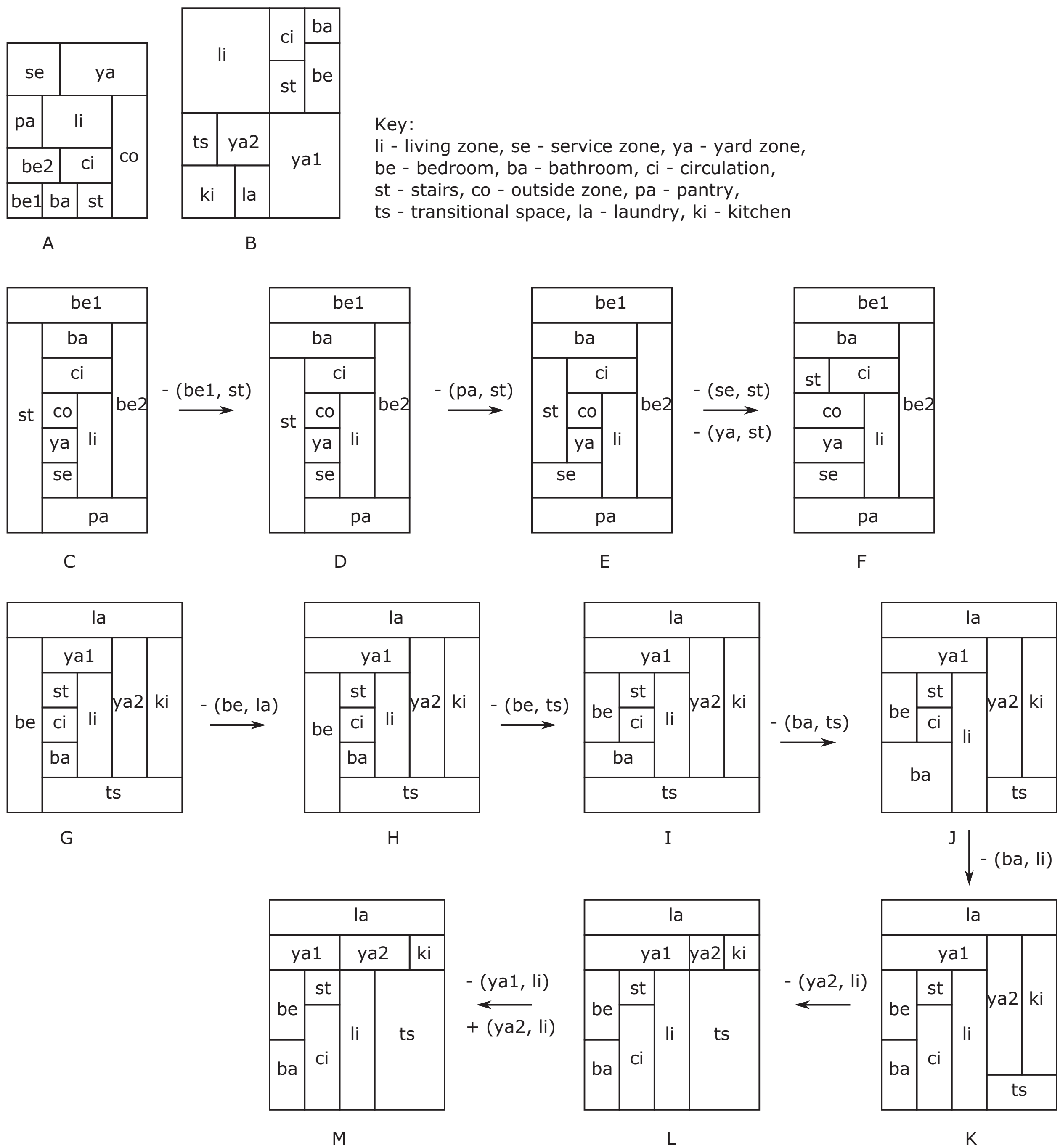

Fig. 19. Deriving existing RFP from corresponding MRFP.

different nature need to be considered. In this paper, we are dealing with adjacency requirements in the strict sense only. In the future, we will cover other aspects, like (aesthetic) composition, style, functionality, access to light, social meaning, and contextual constraints.

As a concluding remark, it might be interesting to note that this work may find its applications in many other fields, apart from architectural design. To list few of them, RFP can be used as rectangular cartograms in cartography [34], as floor plans in VLSI design [35-37], and as facility layouts [19-38].

Acknowledgment

The research described in this paper evolved as part of the research project Mathematics-aided Architectural Design Layouts (File Number: ECR/2017/000356) funded by the Science and Engineering Research Board, India.

References

[1] J. Bhasker, S. Sahni, A linear time algorithm to check for the existence of a rectangular dual of a planar triangulated graph, Networks 17 (3) (1987) 307–317, http://dx.doi.org/10.1002/net.3230170306.
[2] K. Koźmiński, E. Kinnen, Rectangular dual of planar graphs, Networks 5 (1985) 145–157, http://dx.doi.org/10.1002/net.3230150202.
[3] P.H. Levin, Use of graphs to decide the optimum layout of buildings, Archit. J. 140 (15) (1964) 809–817.
[4] J. Grason, An approach to computerized space planning using graph theory, DAC '71 Proceedings of the 8th Design Automation Workshop, USA, 1971, pp. 170–178, , http://dx.doi.org/10.1145/800158.805070.
[5] L. March, P. Steadman, The Geometry of Environment (An Introduction to Spatial Organization in Design), Methuen & Co Ltd, London, 1971 ISBN-10: 0262630559.

[6] J.P. Steadman, Architectural Morphology: An Introduction to the Geometry of Building Plans, Pion Press, 1983 ISBN-10: 0850861438.
[7] J.A. Lynes, Windows and floor plans, Environ. Plann. B. 4 (1977) 51–55, http://dx.doi.org/10.1068/b040051.
[8] J. Gilleard, Layout - a hierarchical computer model for the production of architectural floor plans, Environ. Plann. B. 5 (2) (1978) 233–241, http://dx.doi.org/10.1068/b050233.
[9] I. Baybars, C.M. Eastman, Enumerating architectural arrangements by generating their underlying graphs, Environ. Plann. B. 7 (1980) 289–310, http://dx.doi.org/10.1068/b070289.
[10] J. Roth, R. Hashimshony, A. Wachman, Turning a graph into a rectangular floor plan, Build. Environ. 17 (3) (1982) 163–173, http://dx.doi.org/10.1016/0360-1323(82)90037-3.
[11] P.E. Radcliffe, D.E. Kawal, R.J. Stephenson, Critical Path Method, Cahner, Chicago, Ill, (1967) ISBN-10: 0843601035.
[12] I. Baybars, The generation of floor plans with circulation spaces, Environ. Plann. B. 9 (1982) 445–456, http://dx.doi.org/10.1068/b090445.
[13] D.F. Robinson, I. Janjic, The constructability of floorplans with certain given outerplanar adjacency graph and room areas, Ars Comb. 20B (1985) 133–142.
[14] I. Rinsma, Nonexistence of a certain rectangular floorplan with specified areas and adjacency, Environ. Plann. B. Plann. Des. 14 (1987) 163–166, http://dx.doi.org/10.1068/b140163.
[15] I. Rinsma, Rectangular and orthogonal floorplans with required room areas and tree adjacency, Environ. Plann. B. Plann. Des. 15 (1988) 111–118, http://dx.doi.org/10.1068/b150111.
[16] A. Schwarz, D.M. Berry, E. Shaviv, On the use of the automated building design system, Comput. Aided Des. 26 (10) (1994) 747–762, http://dx.doi.org/10.1016/0010-4485(94)90013-2.
[17] A. Recuero, O. Río, M. Alvarez, Heuristic method to check the realisability of a graph into a rectangular plan, Adv. Eng. Softw. 31 (2000) 223–231, http://dx.doi.org/10.1016/S0965-9978(99)00023-X.
[18] F. Marson, S.R. Musse, Automatic real-time generation of floor plans based on squarified treemaps algorithm, Int. J. Comput. Games Technol. (2010) 2010, http://dx.doi.org/10.1155/2010/624817.
[19] M.R.A. Jokar, A.S. Sangchooli, Constructing a block layout by face area, Int. J. Adv. Manuf. Technol. 54 (2011) 801–809, http://dx.doi.org/10.1007/s00170-010-2960-4.
[20] H. Zhang, S. Sadasivam, Improved floor-planning of graphs via adjacency-preserving transformations, J. Comb. Optim. 22 (2011) 726–746, http://dx.doi.org/10.1007/s10878-010-9324-8.
[21] F. Regateiro, J. Bento, J. Dias, Floor plan design using block algebra and constraint satisfaction, Adv. Eng. Inform. 26 (2012) 361–382, http://dx.doi.org/10.1016/j.aei.2012.01.002.
[22] L. Kotulski, B. Strug, Supporting communication and cooperation in distributed representation for adaptive design, Adv. Eng. Inform. 27 (2013) 220–229, http://dx.doi.org/10.1016/j.aei.2012.10.002.
[23] K. Shekhawat, Algorithm for constructing an optimally connected rectangular floor plan, Front. Archit. Res. 3 (3) (2014) 324–330, http://dx.doi.org/10.1016/j.foar.2013.12.003.
[24] S. Ham, G. Lee, Time-based joining method for generating phylogenetic trees of architectural plans, J. Comput. Civ. Eng. 31 (2) (2017) 04016055, http://dx.doi.org/10.1061/(ASCE)CP.1943-5487.0000626.
[25] G. Slusarczyk, Graph-based representation of design properties in creating building floorplans, Comput. Aided Des. 95 (2018) 24–39, http://dx.doi.org/10.1016/j.cad.2017.09.004.
[26] X. He, On finding the rectangular duals of planar triangular graphs, SIAM J. Comput. 22 (1993) 1218–1226, http://dx.doi.org/10.1137/0222072.
[27] C. Kuratowski, Sur le problème des courbes gauches en topologie, Fundam. Math. 15 (1) (1930) 271–283 http://eudml.org/doc/212352.
[28] G. Stiny, Introduction to shape and shape grammars, Environ. Plann. B. 7 (1980) 343–351, http://dx.doi.org/10.1068/b070343.
[29] K. Shekhawat, J.P. Duarte, Automated best connected rectangular floorplans, in: J. Gero (Ed.), Design Computing and Cognition '16, Springer International Publishing, 2017, pp. 495–511, , http://dx.doi.org/10.1007/978-3-319-44989-0_27.
[30] L.P. Cordella, P. Foggia, C. Sansone, M. Vento, A (sub)Graph isomorphism algorithm for matching large graphs, IEEE Trans. Pattern Anal. Mach. Intell. 26 (10) (2004) 1367–1372, http://dx.doi.org/10.1109/TPAMI.2004.75.
[31] H. Whitney, Congruent graphs and the connectivity of graphs, Am. J. Math. 54 (1) (1932) 150–168, http://dx.doi.org/10.2307/2371086.
[32] J.P. Duarte, Customizing Mass Housing: A Discursive Grammar for Siza's Malagueira Houses (PhD Thesis), Massachusetts Institute of Technology, 2001, http://hdl.handle.net/1721.1/8189.
[33] J.P. Duarte, A discursive grammar for customizing mass housing: the case of Siza's houses at Malagueira, Autom. Constr. 14 (2005) 265–275, http://dx.doi.org/10.1016/j.autcon.2004.07.013.
[34] M. v. Kreveld, B. Speckmann, On rectangular cartograms, Comput. Geom. 37 (2007) 175–187, http://dx.doi.org/10.1016/j.comgeo.2006.06.002.
[35] Y.T. Lai, S.M. Leinwand, Algorithms for floorplan design via rectangular dualization, IEEE Trans. Comput. Aided Des. Integr. Circuits Syst. 7 (12) (1988) 1278–1289, http://dx.doi.org/10.1109/43.16806.
[36] K. Tani, I. Shirakawa, S. Tsukiyama, An algorithm to enumerate all rectangular dual graphs, Electron. Commun. Jpn. Part III 72 (1989) 34–46, http://dx.doi.org/10.1002/ecjc.4430720304.
[37] D. Eppstein, E. Mumford, B. Speckmann, K. Verbeek, Area-universal and constrained rectangular layouts, SIAM J. Comput. 41 (3) (2009) 537–564, http://dx.doi.org/10.1137/110834032.
[38] K.H. Watson, J.W. Giffin, The vertex splitting algorithm for facilities layout, Int. J. Prod. Res. 35 (9) (1997) 2477–2492, http://dx.doi.org/10.1080/002075497194615.