Data Structures

Sec 3 : Big O

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MOTIVATIONS FOR COMPLEXITY ANALYSIS

- There are often **many different algorithms** which can be used to solve the **same problem**. Thus, it makes sense to develop techniques that **allow us to**:
  - **compare** different algorithms with respect to their “efficiency”
  - **choose** the most efficient algorithm for the problem
- The **efficiency** of any algorithmic solution to a problem is a **measure** of the:
  - **Time efficiency:** the time it takes to execute.
  - **Space efficiency:** the space (primary or secondary memory) it uses.
- We will focus on an algorithm's efficiency **with respect to time**.

MACHINE INDEPENDENCE

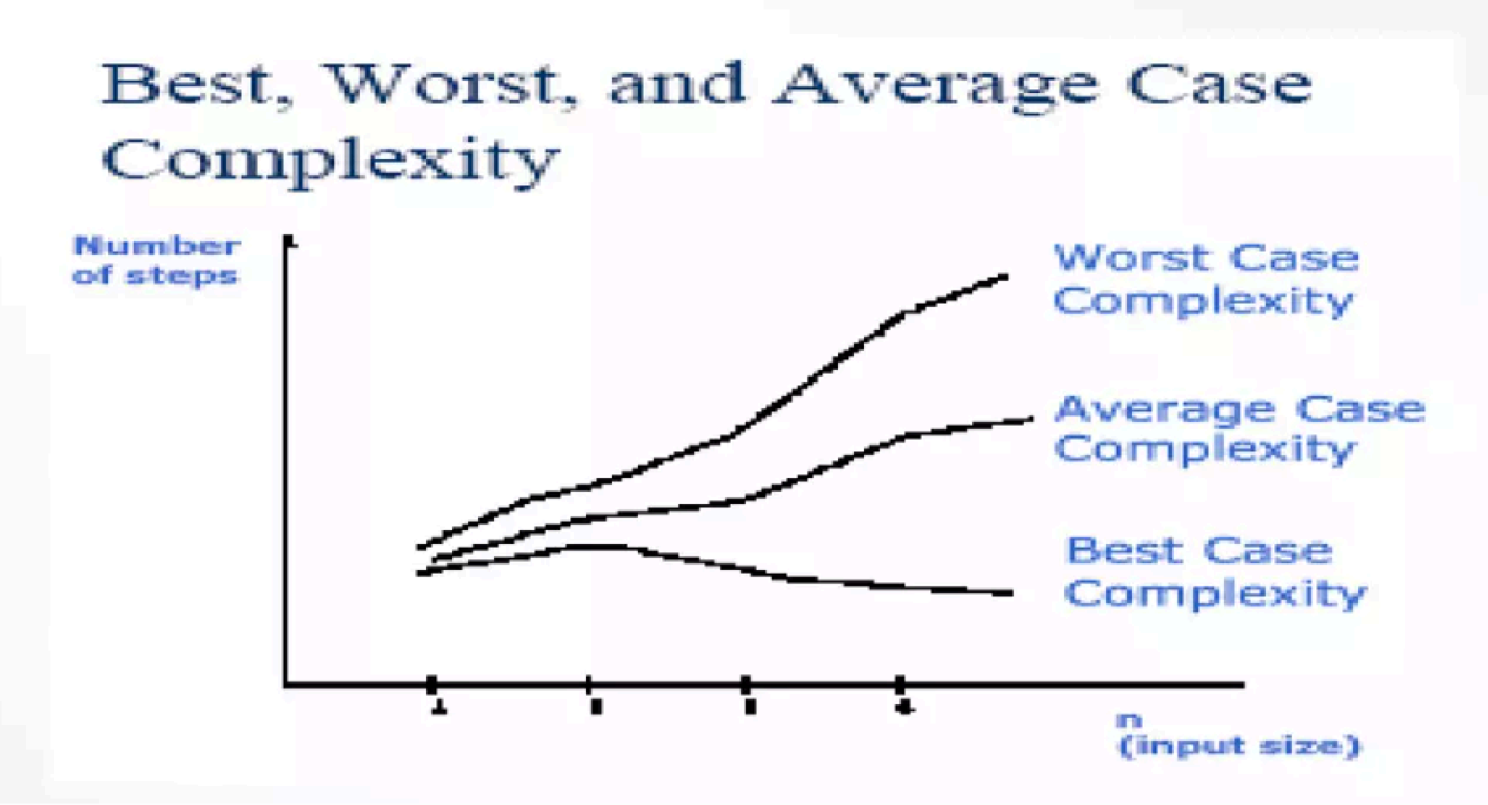
- The evaluation of efficiency should be **as machine independent as possible.**
- It is **not useful** to measure **how fast** the algorithm runs as this depends on which particular computer, OS, programming language, compiler, and kind of inputs are used in testing.
- **Instead**, we count the **number** of basic operations the algorithm performs.
- we calculate how this number depends on the size of the input.

- A **basic operation** is an operation which takes a **constant** amount of time to execute.
- Hence, the **efficiency of an algorithm is the number of basic operations it performs**. This number is a **function of the input size n**.

BEST, AVERAGE, AND WORST CASE COMPLEXITIES

- We are usually interested in the **worst case** complexity: what are the **most** operations that might be performed for a given problem size. We will not discuss the other cases -- best and average case.
- **Best case** depends on the input
- **Average case** is difficult to compute
- So we usually focus on **worst case analysis**

1. **Easier** to compute
2. Usually **close to the actual** running time
3. Crucial to **real-time** systems (e.g. air-traffic control)



SIMPLE COMPLEXITY ANALYSIS: LOOPS (WITH <)

In the following for-loop:

```
for (int i = k; i < n; i = i + m) {
    statement1;
    statement2;
}
```

- The **number of iterations** is: **(n - k) / m**
- The **initialization** statement, i = k, is executed **one time**.
- The **condition**, i < n, is executed **(n - k) / m + 1 times**.
- The **update** statement, i = i + m, is executed **(n - k) / m** times.
- **Each** of **statement1** and **statement2** is executed **(n - k) / m** times.

SIMPLE COMPLEXITY ANALYSIS: LOOPS (WITH <=)

In the following for-loop:

```
for (int i = k; i <=n; i = i + m) {
    statement1;
    statement2;
}
```

- The **number of iterations** is: **(n - k) / m+1**
- The **initialization** statement, i = k, is executed **one time**.
- The **condition**, i < =n, is executed **(n - k) / m + 2 times**.
- The **update** statement, i = i + m, is executed **(n - k) / m+1** times.
- **Each** of **statement1** and **statement2** is executed **(n - k) / m+1** times.

Find the exact number of basic operations in the following program fragment:

```
double x, y;
x = 2.5 ; y = 3.0;
for(int i = 0; i < n; i++){
    a[i] = x * y;
    x = 2.5 * x;
    y = y + a[i];
}
```

- There are **2 assignments** outside the loop ⇒ **2 operations**.
- The for loop actually comprises:
- an assignment **(i=0)** ⇒ **1 operation**
- a test **(i<n)** ⇒ **n+1** operations
- an increment **(i++)** ⇒ **2n** operations
- The **loop body** that has three assignments, two multiplications, and an addition ⇒ **6n** operations
- **Thus the total number of basic operations is 6n+2n+(n+1)+3 = 9n+4.**

SIMPLE COMPLEXITY ANALYSIS: LOOPS WITH LOGARITHMIC ITERATIONS

- **In the following for-loop: (with <)**

```
for (int i = k; i < n; i = i * m) {
    statement1;
    statement2;
}
```

The number of iterations is: $\lfloor(\log_m(n/k))\rfloor$

- **In the following for-loop: (with ≤)**

```
for (int i = k; i <= n; i = i * m) {
    statement1;
    statement2;
}
```

The number of iterations is: $\lfloor(\log_m(n/k))\rfloor + 1$

ASYMPTOTIC NOTATIONS

Following are the commonly used asymptotic notations to calculate the running time complexity of an algorithm.

- O Notation
- Ω Notation
- θ Notation

DETERMINING COMPLEXITY OF CODE STRUCTURES

Loops:

- Complexity is determined by the number of iterations in the loop times the complexity of the body of the loop.

Examples:

```
for (int i = 0; i < n; i++)
     sum = sum - i;
```

$O(n)$

```
for (int i = 0; i < n * n; i++)
     sum = sum + i;
```

$O(n^2)$

```
int i=1;
while (i < n) {
     sum = sum + i;
     i = i*2
}
```

$O(\log n)$

```
for(int i = 0; i < 100000; i++)
     sum = sum + i;
```

$O(1)$

DETERMINING COMPLEXITY OF CODE STRUCTURES

Nested independent loops:

- Complexity of inner loop * complexity of outer loop.

Examples:

```
int sum = 0;
for(int i = 0; i < n; i++)
    for(int j = 0; j < n; j++)
        sum += i * j ;
```

$O(n^2)$

```
int i = 1, j;
while(i <= n) {
   j = 1;
   while(j <= n){
      statements of constant complexity
      j = j*2;
   }
   i = i+1;
}
```

$O(n \log n)$

DETERMINING COMPLEXITY OF CODE STRUCTURES

Nested dependent loops:

Examples:

```
int sum = 0;
for(int i = 1; i <= n; i++)
    for(int j = 1; j <= i; j++)
        sum += i * j ;
```

Number of repetitions of the inner loop is: 1 + 2 + 3 + . . . + n = n(n+2)/2
Hence the segment is O(n2)

```
int sum = 0;
for(int i = 1; i <= n; i++)
    for(int j = i; j < 0; j++)
        sum += i * j ;
```

Number of repetitions of the inner loop is: 0
The outer loop iterates n times.
Hence the segment is O(n)

```
int n = 100;
// . . .
for(int i = 1; i <= n; i++)
    for(int j = 1; j <= n; j++)
        sum += i * j ;
```

An important question to consider in complexity analysis is whether the problem size is a variable or a constant.

DETERMINING COMPLEXITY OF CODE STRUCTURES

If Statement

O(max(O(condition1), O(condition2), . . . ,

O(branch1), O(branch2), . . ., O(branchN))

```
char key;
........
if(key == '+')  {            O(1)
   for(int i = 0; i < n; i++)              O(n²)
     for(int j = 0; j < n; j++)
        C[i][j] = A[i][j] + B[i][j];
}

else if(key == 'x')  O(1)
   C = matrixMult(A, B, n);  O(n³)

else                                              O(1)
   System.out.println("Error! Enter '+' or 'x'!");
```

Overall complexity $O(n^3)$

O(if-else) = Max[O(Condition), O(if), O(else)]

```
int[] integers = new int[100];
// n is the problem size, n <= 100
........
if(hasPrimes(integers, n) == true)
      integers[0] = 20;            → O(1)
else
      integers[0] = -20;           → O(1)


public boolean hasPrimes(int[] x, int n) {
      for(int i = 0; i < n; i++)
                  ..........
                  ..........       O(n)
}
```

$\boxed{O(\text{if-else}) = O(\text{Condition}) = \mathbf{O(n)}}$

Switch: Take the complexity of the most expensive case including the default case

```
char key;
int[] x = new int[100];
int[][] y = new int[100][100];
........
 // n is the problem size (n <= 100)
switch(key)  {
    case 'a':
       for(int i = 0; i < n; i++)
         sum += x[i];
       break;
    case 'b':
       for(int i = 0; i < n; j++)
         for(int j = 0; j < n; j++)
              sum += y[i][j];
       break;
  }
```

→ $o(n)$

→ $o(n^2)$

Overall Complexity: $o(n^2)$

THANK YOU!