

Reading Assignment & Homework 2020

To acquire credit, **homework is due on Monday the week after the homework session**. Since the homework credit is based on effort and not the correct answer, show the details of your reasoning leading to your result.

- Pls. read the entire problem before working on the answer.
- Use the computer to type your responses (scans of clearly handwritten equations and drawings by hand are acceptable).
- Make your answers complete and short; do not exceed 100 words for each answer.
- In problems where you use MATLAB, include a screenshot of the code block you used in your main document. Also include all the .m files you used in your final submission. Remember to **comment** you code.
- Make sure that all plots are provided with meaningful labeled axes and units.
- Show details and if your answer includes equations: i.e. refer to the source of the equations and use text to explain manipulations and define the variable names you use.
- In case a question seems incomplete or leaves room for different interpretations, select a solution you deem the simplest.
- Use your own words - text copied from websites or other sources will be considered incorrect.

Week 1 – April 6 2020

Read Chapters 1 - 3

1. An amplifier amplifies $10,000 \times$ and is connected to a 12 bit A/D converter with an input range of $\pm 15V$ and samples the incoming data at 1000 samples/s.

1. What is the range at the input?
2. What is the resolution of the digitized signal in time and amplitude?
3. A sampled signal consists of 100,000 samples -what is the total length of the sampled epoch expressed in s?
4. The maximum sampled value coming out of the A/D converter is 3000 – what is it's amplitude in mV?

2. For a nerve cell model we have the following parameters.

The inside and outside concentrations of sodium are 11mM and 110mM respectively. For potassium inside and outside concentrations are 90mM and 8mM respectively. In rest, the permeability of potassium is 50 larger than that of sodium.

During activation the permeability of sodium is 50 larger than that of potassium.

- a. What is the Nernst potential for sodium?
- b. What is the Nernst potential for potassium?
- c. What is the membrane potential in rest?
- d. What is the membrane potential during activation?

Week 1

3. A biopotential of 80mV is recorded with an electrode of $1.5\text{M}\Omega$ and a preamplifier with an input resistance of $10\text{M}\Omega$ (you may ignore the resistance of the ground electrode in this question).

- What is the current flowing through the electrode?
- What is the measured value of the biopotential?
- Evaluate your results

4. Evaluate the following integrals using integration limits $-\infty, \infty$

- $\int \sin(x) \delta(x) dx$
- $\int \sin(x) \delta(x - \pi/2) dx$
- $\int \sin(x) \left[\sum_{n=0}^2 \delta(x - n\pi/2) \right] dx$
- Repeat a-c for $\cos(x)$

5. You have a 9V battery and 2 k Ω resistors. Use Ohm's law to compute the current through the resistors if

- One resistor is connected between the + and – terminals of the battery
- Two resistors in series are connected between the + and – terminals of the battery
- Two resistors in parallel are connected between the + and – terminals of the battery
- In case c., show that Kirchhoff's 1st law is satisfied
- In case b., show that Kirchhoff's 2nd law is satisfied
- Now in case a., you measure the potential across the resistor at a room temperature (21°C) with a multimeter (with a bandwidth of 100Hz and a noise level of $v_{eff} = 0.3\text{mV}$). What is the noise level of your measurement?

6. Consider a signal that evolves in time according to the rule: $s_t = 4 + 0.5s_{t-1} + e_t$, where s_t is an effective signal and e_t is an additive noise component with zero mean and unit variance.

- Assuming the signal is weakly stationary (hint: $E(s_t) = E(s_{t-1}), \text{Var}(s_t) = \text{Var}(s_{t-1})$, $\forall t$), find the expectation and variance of the signal.
- What is the signal-to-noise ratio in this example? (compute in dB)

Exam Questions **READ THIS PAGE FIRST**

To acquire credit, exam assignments are due on Wednesday after the week they are posted.

Exam questions must be completed individually.

Sharing or checking of answers with others will not be allowed.

- Pls. read the entire problem before working on the answer.
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Exam Assignments start on the next page

Problem 1: Terminology and Properties

Define or describe in your own words (but less than 100 words per item) the following terms, relationships, or abbreviations.

- a. Kirchhoff's Laws
- b. Nyquist Frequency
- c. Sifting Property

Problem 2: Noise and Expectation

Consider a signal that evolves in time according to the rule: $s_t = 4 + 0.5s_{t-1} + e_t$, where s_t is an effective signal and e_t is an additive noise component with zero mean and a variance of 1.

You already did parts a. and b. for homework and do not need to resubmit your answers. The questions are repeated here for your reference.

- a. Assuming the signal is weakly stationary (hint: $E(s_t) = E(s_{t-1}), Var(s_t) = Var(s_{t-1}), \forall t$), find expectation and variance of the signal.
- b. What is the signal-to-noise ratio in this example? (compute in dB)

Complete and submit parts c. d. and e. for the exam.

- c. Simulate s_t in Matlab (for $t = 1:10000$) and verify your calculated results for the mean and variance (set $s_1 = 0$).
- d. What happens to the system if the variance of the additive noise component increases twofold? (redo a-c).
- e. Would the above reasoning for estimating Expectations work if (in the equation for s_t) the coefficient for s_{t-1} would have a value >1 rather than 0.5? Explain your answer.