$$E[(m_j - E[m_j])^2] = \theta_j(1-\theta_j)^2 + (1-\theta_j)(0-\theta_j)^2$$
$$= \theta_j(1-\theta_j)^2 + (1-\theta_j)\theta_j^2$$
$$= 1-\theta_j(\theta_j - \theta_j^2 + \theta_j^2) = (\theta_j)(1-\theta_j)$$

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1a) $y = \sum_j m_j w_j x_j$, $m_j \sim Bernoulli(\theta_j)$ eg $P(m_j=1) = \theta_j$; $P(m_j=0) = 1$

$E[m_j] = \theta_j$, $Var(m_j) = (\theta_j)(1-\theta_j)$

$$E[Y] = E[\sum m_j w_j x_j]$$
$$= \sum_j w_j x_j E[m_j] = \sum w_j x_j \theta_j = \frac{1}{2}\sum w_j x_j$$

$Var[Y] = \sum_j w_j^2 x_j^2 Var[m_j]$ — by indep of each example

$$= \sum_j w_j^2 x_j^2 \theta_j(1-\theta_j) = \frac{1}{4}\sum w_j^2 x_j^2 =$$

b) Clearly $\tilde{w}_j \leftarrow \frac{1}{2}w_j$ (Recall: $Var(aX) = a^2 Var(X)$) ✓

c) $F = \frac{1}{2N}\sum E[(y^i - t^i)^2]$

$= \frac{1}{2N}\sum E[y^{i2}] - 2t^i E[y^i] + t^{i2}$

$\tilde{y} = \sum \tilde{w}_j x_j = \sum \frac{1}{2} w_j x_j$
$= \frac{1}{2}\sum w_j x_j$

= Note: $E[Y] = \frac{1}{2}\sum w_j x_j$, $Var(Y) = \frac{1}{4}\sum w_j^2 x_j^2$

Also: $Var(X) = E[X^2] - (E[X])^2$

$\therefore E[Y^2] = Var(Y) + (E[Y])^2$

$$F = \frac{1}{2N}\sum_i \underbrace{\frac{1}{4}\sum_j (w_j x_j^i)^2} + \underbrace{(\underbrace{\tfrac{1}{2}\sum w_j x_j^i}_{\tilde{y}^i})^2 - 2t^i(\underbrace{\tfrac{1}{2}\sum w_j x_j^i}_{\tilde{y}^i}) + t^{i2}}$$

Recall: $\tilde{y} = \frac{1}{2}\sum w_j x_j$

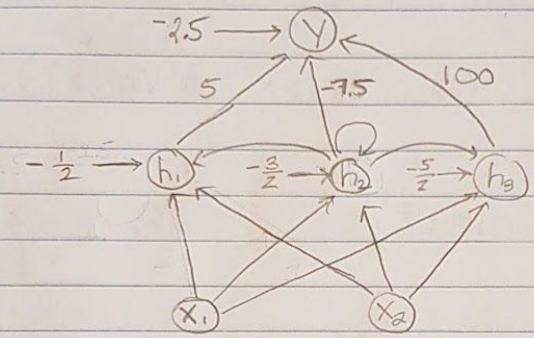
$$= \frac{1}{2N}\sum_i \sum_j \left(\frac{1}{2}w_j x_j\right)^2 + \frac{1}{2N}\sum_i (\tilde{y}^i - t^i)^2$$

$$= \underbrace{\frac{1}{2N}\sum_i \sum_j (\tilde{w}_j x_j)^2}_{R} + \frac{1}{2N}\sum_i (\tilde{y}^i - t^i)^2$$

Note: x_j depends on i so cannot be factored out.

2. Three inputs: carry, input 1, input 2.

if sum is 0, output 0, carry 0
if sum is 1, output 1, carry 0
if sum is 2, output 0, Carry 1
if sum is 3 output 1, carry 1



* all weights 1 unless otherwise specified.

- h_2 is 1 if incoming sum is at least 2. It represents if "carry" is 1
- y is 1 iff sum is 3 or sum is at least 1 but not 2
- h_1, h_3 are "sum is at least 1", "sum is at least 3" respectively.

$$U = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad V = [5 \quad -7.5 \quad 100]$$

$$W = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad b_h = \begin{bmatrix} -1/2 \\ -3/2 \\ -5/2 \end{bmatrix}$$

$$b_y = -2.5$$