CSC421 HW1 2019年05月11日

1. $z_1 = W_{11}^{(1)} x_1 + W_{12}^{(1)} x_2 + W_{13}^{(1)} x_3 + W_{14}^{(1)} x_4 + b_1^{(1)}$

In english: h_1 checks $x_1 < x_2$?
h_2 checks $x_2 < x_3$?
h_3 checks $x_3 < x_4$?
} all must be true.

$z_1 = W_{11}^{(1)} x_1 + W_{12}^{(1)} x_2 + b_1^{(1)}$

If $x_1 < x_2$, want $z_1 > 0$. Suppose $x_2 = x_1 + \Delta$, $\Delta > 0$

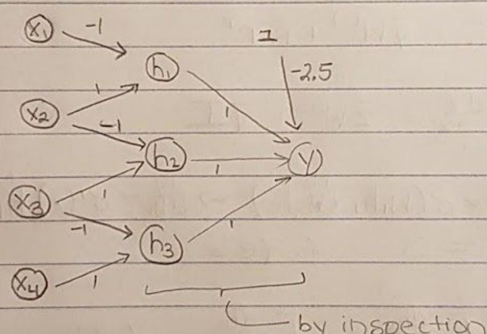
$W_{11}^{(1)} x_1 + W_{12}^{(1)} (x_1 + \Delta) + b_1^{(1)} > 0$

$(W_{11}^{(1)} + W_{12}^{(1)}) x_1 + W_{12}^{(1)} \Delta + b_1^{(1)} > 0$

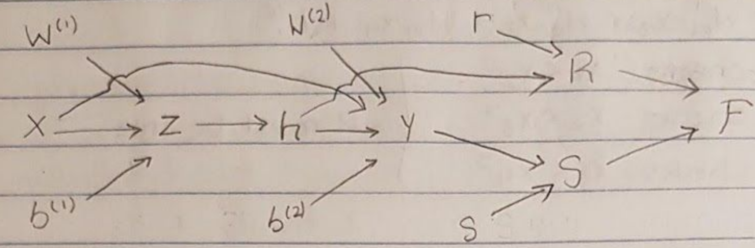
Clearly require $W_{11}^{(1)} = -W_{12}^{(1)}$ ← this was an obv. result but shows the process

$W_{12}^{(1)} > 0$

$b_1^{(1)} = 0$



2.



$\bar{F} = 1$

$\bar{R} = \bar{F}(1) = 1$

$\bar{S} = \bar{F}(1) = 1$

$\bar{r} = \bar{R}\, h^T = h^T$ ⟷ not needed though

$\bar{s} = \bar{S}(y-s)^T(-1) = -(y-s)^T$ ← not needed though.

$\bar{y} = \bar{S}\,(y-s)^T = (y-s)^T$ Note: this is a row vec.

$\bar{h} = \bar{y}\frac{dy}{dh} + \bar{R}\frac{dR}{dh} = \bar{y}W^{(2)} + \bar{R}\, r^T$

$\bar{z} = \bar{h}\sigma'(z)$

$\bar{x} = \bar{z}\frac{dz}{dx} + \bar{y}\frac{dy}{dx} = \bar{z}W^{(1)} + \bar{y}I$

3. $\frac{dL}{dw_1} = \bar{y}\frac{dy}{dw_1}$. $y = \sigma(w_1 h_1 + w_5 h_5) \Rightarrow \frac{dy}{dw_1} = \sigma'(\cdot)\cancel{h_1} = 0$ ∴ Yes

h_1 is $\varnothing$.

$\frac{dL}{dw_2} = \bar{y}\frac{dr}{dh_1}\frac{dh_1}{dw_2} = 0$ ∴ Yes

$\frac{dL}{dw_3} = \bar{h}_3\frac{dh_3}{dw_3}$

$\bar{h}_3 = \underbrace{\bar{h}_1}_{\varnothing}\frac{dh_1}{dh_3} + \bar{h}_2\frac{dh_2}{dh_3} = \bar{h}_2\frac{dh_2}{dh_3}$ ∴ No

Recall $ReLU(x) = \max(0, x)$

if $h_1 = -1$, $\sigma(h_1) = 0$, $\sigma'(h_1) = 0$

$\bar{x} = \bar{h}\sigma'(z)W^{(1)} + (y-s)^T I$

$= (\bar{y}W^{(2)} + \bar{R}r^T)\sigma'(z)W^{(1)} + (y-s)^T I$

$= (y-s)^T W^{(2)}\sigma'(z)W^{(1)} + r^T\sigma'(z)W^{(1)} + (y-s)^T I$

$= (y-s)^T[W^{(2)}\sigma'(z)W^{(1)} + I] + r^T\sigma'(z)W^{(1)}$