

$$\begin{aligned}
 E[(m_j - E[m_j])^2] &= \theta_j(1-\theta_j)^2 + (1-\theta_j)(0-\theta_j)^2 \\
 &= \theta_j(1-\theta_j)^2 + (1-\theta_j)\theta_j^2 \\
 &= 1-\theta_j(\theta_j - \theta_j^2 + \theta_j^2) = \theta_j(1-\theta_j)
 \end{aligned}$$

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1a) $y = \sum_j m_j w_j x_j$, $m_j \sim \text{Bernoulli}(\theta_j)$ eg $P(m_j=1) = \theta_j$; $P(m_j=0) = 1-\theta_j$
 $E[m_j] = \theta_j$, $\text{Var}(m_j) = \theta_j(1-\theta_j)$

$$\begin{aligned}
 E[y] &= E[\sum_j m_j w_j x_j] \\
 &= \sum_j w_j x_j E[m_j] = \sum_j w_j x_j \theta_j = \frac{1}{2} \sum w_j x_j
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}[y] &= \sum_j w_j x_j \text{Var}[m_j] \\
 &= \sum_j w_j x_j \theta_j(1-\theta_j) = \frac{1}{4} \sum w_j x_j
 \end{aligned}$$

b) Clearly $\tilde{w}_j \leftarrow \frac{1}{2} w_j$ (Recall: $\text{Var}(aX) = a^2 \text{Var}(X)$) ✓

c) $F = \frac{1}{2n} \sum E[(y_i - t_i)^2]$
 $= \frac{1}{2n} \sum E[y_i^2] - 2t_i E[y_i] + t_i^2$ $\tilde{y} = \sum \tilde{w}_j x_j = \sum \frac{1}{2} w_j x_j$
 $= \frac{1}{2} \sum w_j x_j$

Note: $E[y] = \tilde{y}$

$\text{Var}(Y) = \frac{1}{2} \tilde{y}$

Also: $\text{Var}(X) = E[X^2] - (E[X])^2$

$$\begin{aligned}
 \therefore E[y^2] &= \text{Var}(Y) + (E[Y])^2 \\
 &= \frac{1}{2} \tilde{y} + (\tilde{y})^2
 \end{aligned}$$

$$\begin{aligned}
 F &= \frac{1}{2n} \sum \left(\frac{1}{2} \tilde{y} + \tilde{y}^2 \right) - 2t_i \tilde{y} + t_i^2 \\
 &= \frac{1}{2n} \sum \tilde{y}^2 - 2t_i \tilde{y} + t_i^2 + \frac{1}{2n} \sum \frac{1}{2} \tilde{y} \\
 &= \frac{1}{2n} \sum (\tilde{y} - t_i)^2 + \frac{1}{2n} \sum \frac{1}{2} \tilde{y} \\
 &\quad \quad \quad \mathcal{L}(\tilde{w}_1, \dots, \tilde{w}_0)
 \end{aligned}$$

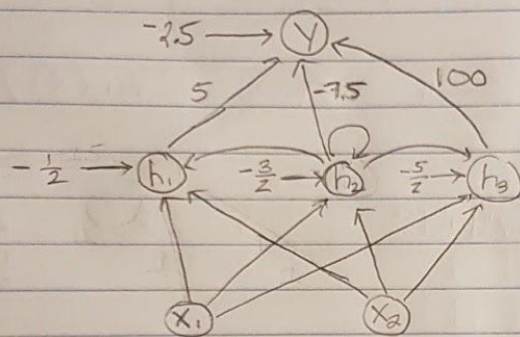
2. Three inputs: carry, input 1, input 2.

if sum is 0, output 0, carry 0

if sum is 1, output 1, carry 0

if sum is 2, output 0, carry 1

if sum is 3, output 1, carry 1



* all weights 1 unless otherwise specified.

• h_2 is 1 if incoming sum is at least 2.

It represents if "carry" is 1

• y is 1 iff sum is 3 or sum is at least 1 but not 2

• h_1, h_3 are "sum is at least 1" "sum is at least 3" respectively.