Lab #3: Z-transform and Inverse Z-transform

A Matlab investigation of Z-transform, Inverse Z-transform, and Pole-zero plot;

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ECE 340 - Lab Section D11

Introduction

Causal linear time-invariant systems make up a substantial portion of systems encountered in engineering. We were provided a causal linear filter, $h(n)$, and were tasked to find the ztransfer function algebraically, and recover the impulse response, $h(n)$, numerically using Matlab.

$$h(n) = n(0.5)^n \sin\left(\frac{\pi n}{6}\right) u(n) \qquad \text{(Equ. 1)}$$

Additionally, we were provided 2 ztransfer function of causal LTI systems, *$H_1(z)$* and *$H_2(z)$*, and were tasked to find frequency response of the systems, analyse pole-zero plots, and recover their impulse response functions.

$$H_1(z) = \frac{2 + 2z^{-1}}{1 - 1.25z^{-1}} \qquad \text{(Equ. 2)}$$

$$H_2(z) = \frac{2 + 2z^{-1}}{1 - 0.75z^{-1}} \qquad \text{(Equ. 3)}$$

Methods and Results:

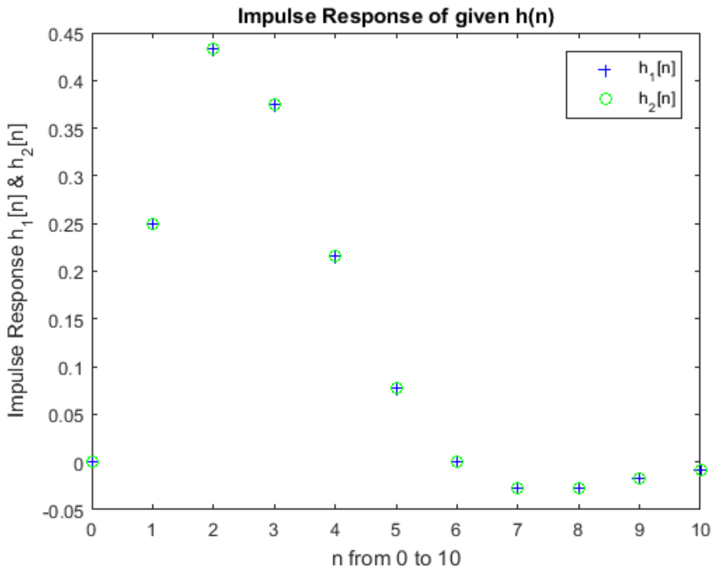
A Matlab code, see Table 1, was written to evaluate $h(n)$ --seen as $h_1[n]$ in the codes and plots-- directly over the domain $0 \le n \le 10$. Additionally, the ztransfer of $h(n)$ -- see $H(z)$ below-- was obtained using the properties of ztransfer method.

$$H(z) = \frac{4z^{-1} - z^{-3}}{16 - 16\sqrt{3}z^{-1} + 20z^{-2} - 4\sqrt{3}z^{-3} + z^{-4}} \qquad ROC: |Z| > 0.5 \qquad \text{(Equ. 4)}$$

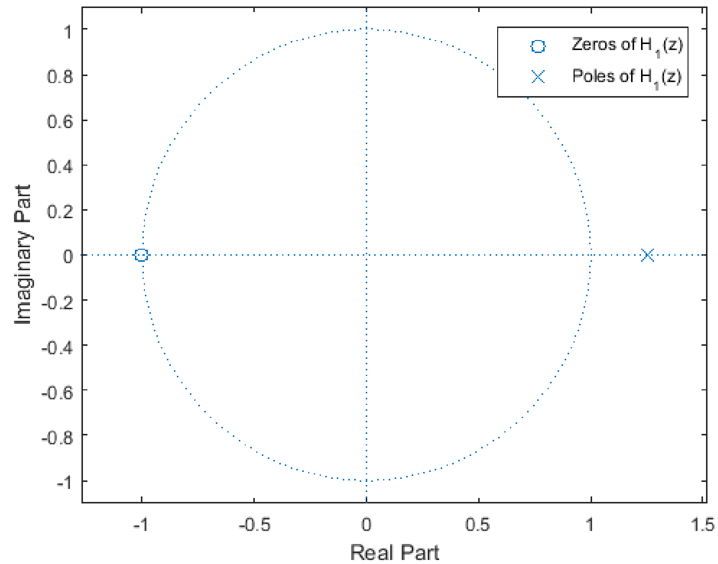
The unit impulse function was filtered using the above transfer function (see Q1_ImpulseResponse() in Table 1) and the resultant, $h_2[n]$, plotted on the same graph as the direct evaluation of h(n), $h_1[n]$. See graph 1.

Next the poles and zeroes of $H_1(z)$ and $H_2(z)$ were found using Matlab and they are present in graphs 2 and 3. As seen from the graphs and equations 2 and 3, the poles of the $H_1(z)$ and $H_2(z)$ are located at $z = 1.25$ and $z = 0.75$, respectivly. Thus, since $z = 1.25$ is outside the unit circle, therefore $H_1(z)$ is an unstable system. On the contrary, $H_2(z)$ is expected to be a stable system.

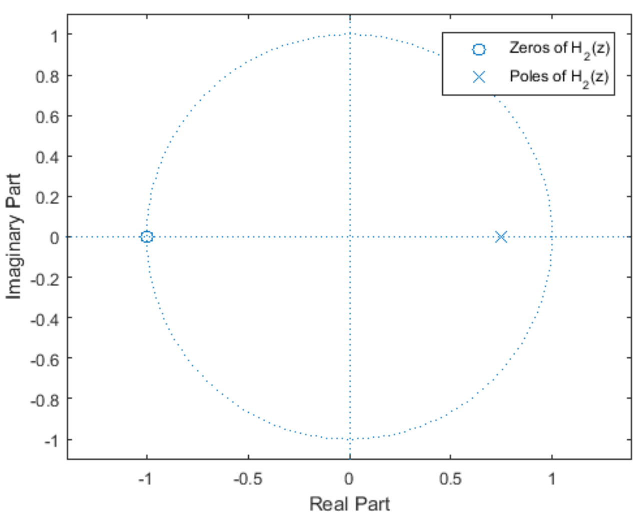
Next, the magnitude and phase of both systems were calculated, by replacing the *z* with e^{iw} and the results are present in graphs 4 and 5. Finally, using Matlab's built-in symbolic logic and *iztrans* function (inverse z-transform command) the inverse z-transform of *H* was calculated and evaluated (see Table 1) over the region $0 \le 0 \le 25$. These transform functions represent the impulse response of the system, see graphs 6 and 7. As seen, $H_1(z)$ is non-stable because $h_1(n)$ grows without bounds while $h_2\ (n)$ is stable. This conclusion agrees with the pole-zero plot prediction.



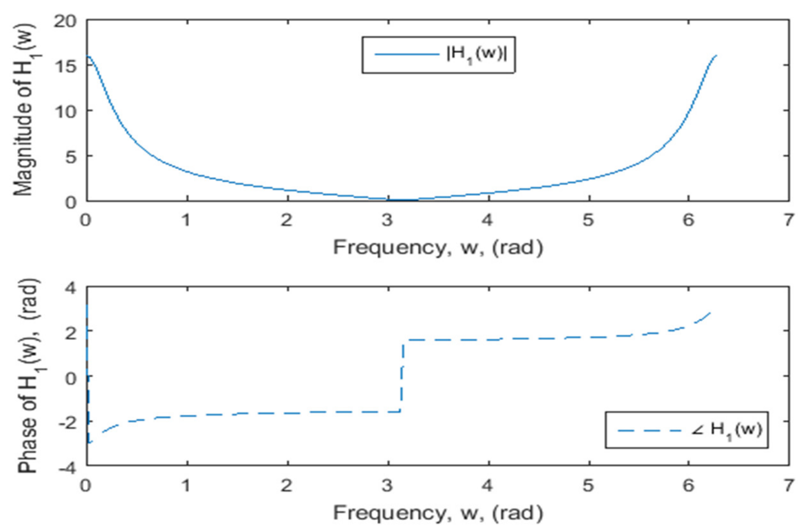
Graph 1: Impulse response of *h(n)* evaluated directly, $h_1[n]$, and indirectly using Matlab built-in function, $h_2[n]$, is shown above. The two evaluations of it match perfectly.



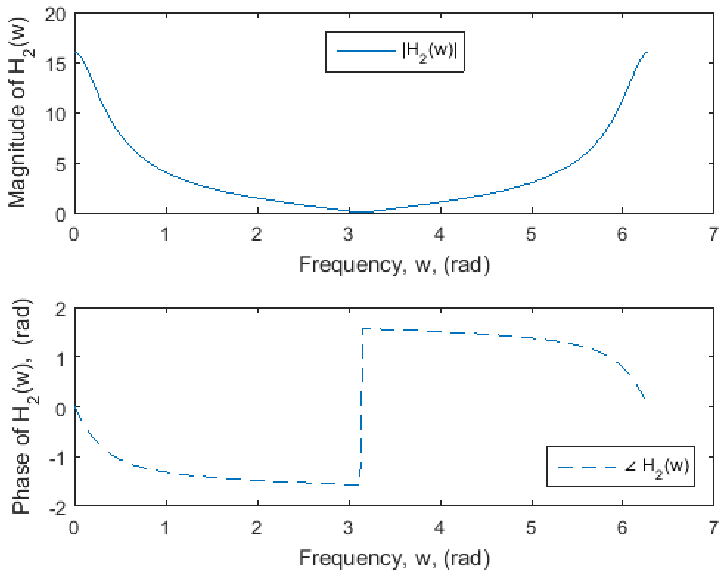
Graph 2: The pole-zero plot of $H_1(z)$ is shown above. The plot suggests a non-stable output response.



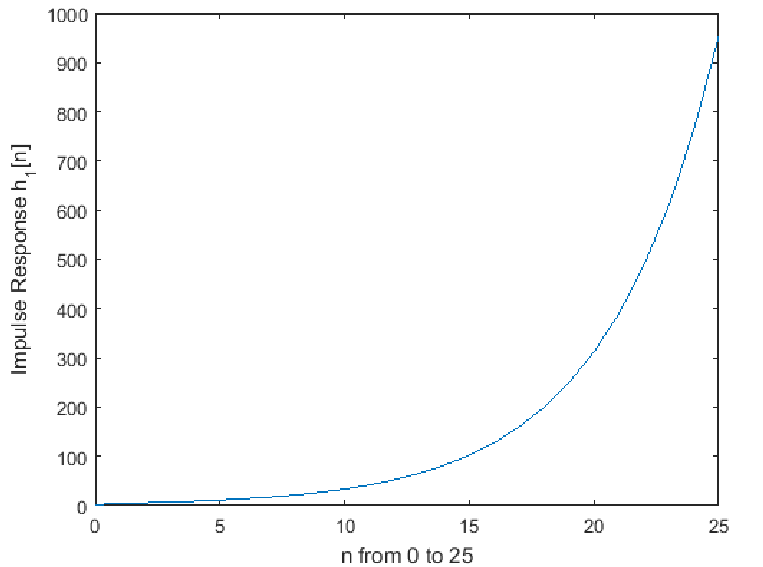
Graph 3: The pole-zero plot of $H_1(z)$ is shown above. The plot suggests a stable output response.



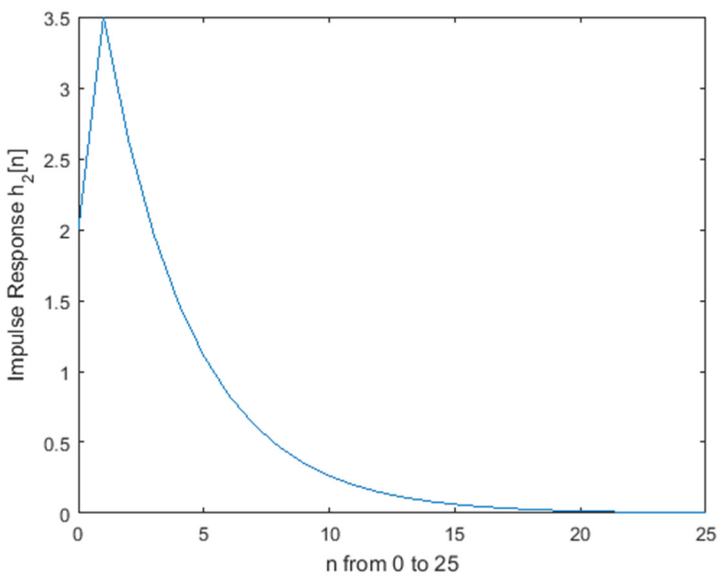
Graph 4: The frequency response of $H_1(z)$ over the frequency region $0 \leq w \leq 2\pi$ is shown above.



Graph 5: The frequency response of *H₂(z)* over the frequency region $0 \leq w \leq 2\pi$ is shown above.



Graph 6: The impulse response of $H_1(z)$ over the region $0 \leq n \leq 25$ is shown above. As seen from the graph, the system is not stable and output grows without any bounds.



Graph 7: The impulse response of $H_2(z)$ over the region $0 \leq n \leq 25$ is shown above. The system seems to be stable as impulse-response decays to zero.

Table 1: Matlab codes used during this lab is provide in the following table. "ECE340_Lab3.m" is a single m-file that is broken into multiple independent functions used to investigate various aspect of sampling and aliasing.

```
%% Main Function
%% Main Function
function ECE340_Lab2()
   Q1_ImpulseResponse();
   Q2_LTI_systems();
end

%%Q1
function Q1_ImpulseResponse()
   %defining the impulse response function h(n)
   function result=h(n)
      if(n<=0)      %h(n) is causal and one-sided with h(0)=0
         result=0;
      else
         PI_6=pi/6;
         result=n.*sin(PI_6 .*n)./(2.^n);
      end
   end

   %Computing the impulse response of h(n) directly
   h1=h(0:10);

   %Computing the impulse response of h(n) indirectly using built-in
   %ztransform filter function
   N=[0 4 0 -1 0];
   D=[16  -16*sqrt(3)  20  -4*sqrt(3)  1];
   h2=filter(N,D,[1 zeros(1,10)]);

   %Plotting the impulse response functions on the same plot
   plot(0:10,h1,'b+');
   hold on
   plot(0:10,h2,'go')
   xlabel('n from 0 to 10');
   ylabel('Impulse Response h_1[n] & h_2[n]');
   title('Impulse Response of given h(n)');
   legend('h_1[n]','h_2[n]');
end
```

```
%%Q2
function Q2_LTI_systems()
  %Anonymous functions that returns the frequency response of H1 and H2
  %transfer functions
  H1_frequency_Response=@(w) (2+2.*exp(-1i.*w))./(1-1.25.*exp(-1i.*w));
  H2_frequency_Response=@(w) (2+2.*exp(-1i.*w))./(1-0.75.*exp(-1i.*w));

  %Definning the numerator and denominators of H1 and H2
  H1_N=[2 2];
  H1_D=[1 -1.25];
  H2_N=[2 2];
  H2_D=[1 -0.75];

 %Finding the poles and zeroes of the ztranfer functions
 [Z1 P1 K1]=tf2zpk(H1_N, H1_D);
 [Z2 P2 K2]=tf2zpk(H2_N, H2_D);

 %Plotting the poles and zeroes of the ztransfer functions
 figure
 zplane(Z1,P1);
 legend('Zeros of H_1(z)','Poles of H_1(z)');
 figure
 zplane(Z2,P2);
 legend('Zeros of H_2(z)','Poles of H_2(z)');

 %Computing the frequency response of the systems
 w=linspace(0,2*pi,200);
 H1_freq_resp_mag=abs(H1_frequency_Response(w));
 H1_freq_resp_phase=angle(H1_frequency_Response(w));
 H2_freq_resp_mag=abs(H2_frequency_Response(w));
 H2_freq_resp_phase=angle(H2_frequency_Response(w));

 %Plotting the frequency responses of the systems
 figure
 subplot(2,1,1)
 plot(w,H1_freq_resp_mag,'-');
  xlabel('Frequency, w, (rad)');
  ylabel('Magnitude of H_1(w)');
  legend('|H_1(w)|','location','north');
 subplot(2,1,2)
 plot(w,H1_freq_resp_phase,'--');
  xlabel('Frequency, w, (rad)');
  ylabel('Phase of H_1(w),  (rad)');
  legend('\angle H_1(w)','location','southeast');
 figure
 subplot(2,1,1)
 plot(w,H2_freq_resp_mag,'-');
  xlabel('Frequency, w, (rad)');
```

```
    ylabel('Magnitude of H_2(w)');
    legend('|H_2(w)|','location','north');
   subplot(2,1,2)
   plot(w,H2_freq_resp_phase,'--');
    xlabel('Frequency, w, (rad)');
    ylabel('Phase of H_2(w),  (rad)');
    legend('\angle H_2(w)','location','southeast');

   %Using inverse-ztransform to compute impulse response
   syms z n;
   H1=(2+2*z^-1)/(1-1.25*z^-1);
   H2=(2+2*z^-1)/(1-0.75*z^-1);
   h1=iztrans(H1,z,n);
   h2=iztrans(H2,z,n);

   %Evaluate and plot the impulse responses
   n=0:25;
   figure
   plot(n,subs(h1));
    xlabel('n from 0 to 25');
    ylabel('Impulse Response h_1[n]');
   figure
   plot(n,subs(h2));
    xlabel('n from 0 to 25');
    ylabel('Impulse Response h_2[n]');
end
```