Assignment 2

Rahul Ramachandran (cs21btech11049)

***Abstract*—This document contains the solution for Assignment 2 (ICSE Class 12 Maths 2018 Q.17(a))**

17(a) [ICSE 12 2018]: Draw a rough sketch of the curve and find the area of the region bounded by curve $y^2 = 8x$ and the line $x = 2$

Solution: The given parabola $y^2 = 8x$ can be expressed using the parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = -\begin{pmatrix} 4 \\ 0 \end{pmatrix}, f = 0 \tag{1}$$

The line $x = 2$ can be represented by the equation

$$\mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \tag{2}$$

Where $\mathbf{q}$ is a point and $\mathbf{m}$ is the direction vector of the line.

Choosing $\mathbf{q}$ as $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, we get:

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

The intersection of this line with the parabola is given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \tag{4}$$

Where κ_i is given by

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u})]^2 - (\mathbf{q}^T \mathbf{V} \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f)(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \tag{5}$$

Substituting the values, the intersection parameters κ_i for the line are

$$\kappa = \pm 4 \tag{6}$$

These values can also be found by substitution the vector $\mathbf{x}$ in the equation of the parabola.

Using these values of κ, the intersection points are

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ -4 \end{pmatrix} \tag{7}$$

From the figure, it is then clear that the desired area equals

$$2 \int_0^2 2\sqrt{2} \times \sqrt{x} = \frac{8\sqrt{2}}{3} \left[x^{\frac{3}{2}} \right]_0^2 = \boxed{\frac{32}{3}} \tag{8}$$

The blue shaded region in the figure represents this area.

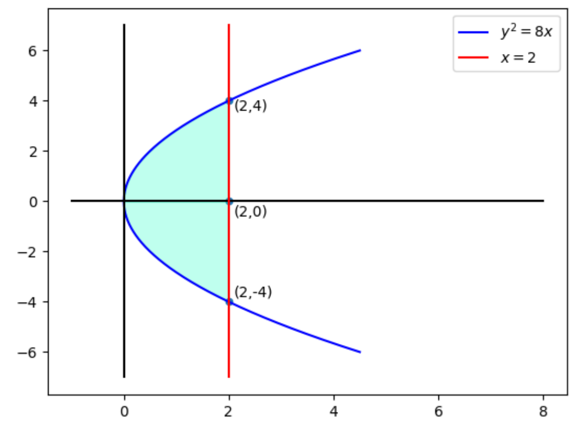

Fig. 1. Plot of the parabola and the shaded area