Assignment 13

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Outline

Problem Statement

Papoulis 8.10

Among 4000 newborns, 2080 are male. Find the 0.99 confidence interval of the probability $p = P(\text{male})$

Discussion

We are given a sample space of babies, and are required to provide an estimate of parameter p: the proportion of babies in a population that are male.
Let the random variable X map to 0 when the baby is female, and 1 otherwise. Let the average value of X_i for the given sample space be $\hat{p}$.
We can use $\hat{p} = 1 - \hat{q}$ to estimate an interval that p is likely to lie in.

Solution

Since $\hat{p} = \frac{\sum X_i}{n}$, and since n is large, the sampling distribution of sample proportion can be approximated to a normal distribution, by the Central Limit Theorem.
To find the confidence interval, we assume that the mean of this normal distribution is $\hat{p}$, and that the standard deviation is $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Solution

We have

$$\hat{p} = 2080/4000 \tag{1}$$
$$= 0.52 \tag{2}$$

Also,

$$\sigma_{\hat{p}} = \sqrt{\frac{0.52(1-0.52)}{4000}} \tag{3}$$
$$= 0.0079 \tag{4}$$

Solution

To find the interval, we use the $z-$score, which tells us the number of standard deviations between the end-points of the confidence interval and the mean. Since we are interested in the 0.99 confidence interval, we have

$$
\begin{aligned}
\gamma &= 0.99 && (5)\\
\implies \delta &= 1 - \gamma && (6)\\
&= 0.01 && (7)
\end{aligned}
$$

Therefore, we have to find z corresponding to $\delta = 0.01$, which from the z-score table equals 2.58

Solution

Therefore, it follows that

$$p_u = \mu + z\sigma \tag{8}$$
$$= 0.52 + 2.58 \times 0.0079 \tag{9}$$
$$= 0.54 \tag{10}$$

where p_u is the upper limit of the interval. Similarly,

$$p_l = \mu - z\sigma \tag{11}$$
$$= 0.52 - 2.58 \times 0.0079 \tag{12}$$
$$= 0.49 \tag{13}$$

Therefore, the 0.99 confidence interval for p is $[0.49, 0.54]$