Assignment 8

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Abstract—**This document contains the solution for Assignment 8 (NCERT Class 12 Chapter 13 Exercise 13.2 Q17)**

13.2 Q17 [NCERT 12] : The probability of obtaining an even prime number on each die, when a pair of dice is rolled is:

Solution: The sample space $\mathcal{S}$ for a single die throw is $\{1, 2, 3, 4, 5, 6\}$. Define random variables X_1 and X_2 for the first and second die throws respectively:

$$X_1 : \mathcal{S} \to \mathbb{R} \tag{1}$$
$$X_2 : \mathcal{S} \to \mathbb{R} \tag{2}$$

Further, let:

$$X_i(\omega_i) = \omega_i \tag{3}$$

for event $\omega_i \in \mathcal{S}$.

Relevant random variable values are given in Table I:

Variable	Event
$X_1 = 2$	1$^{\text{st}}$ number is 2
$X_2 = 2$	2$^{\text{nd}}$ number is 2

TABLE I

Let event E represent the first die throw resulting in a 2, and let event F be defined analogously for the second die throw. We are required to find the intersection of the two events, EF. The relevant probabilities are given in Table II:

Event	Probability	Value
E	$\Pr(X_1 = 2)$	$\frac{1}{6}$
F	$\Pr(X_2 = 2)$	$\frac{1}{6}$
EF	$\Pr(X_1 = 2, X_2 = 2)$	?

TABLE II

Since the die throws are independent, the probability can be found:

$$\Pr(EF) = \Pr(E)\Pr(F) \tag{4}$$
$$= \frac{1}{6} \times \frac{1}{6} \tag{5}$$
$$= \frac{1}{36} \tag{6}$$

The claim that the events are independent can be verified by directly computing $\Pr(EF)$:

$$\Pr(EF) = \frac{n(EF)}{n(\mathcal{S} \times \mathcal{S})} \tag{7}$$
$$= \frac{1}{6 \times 6} \tag{8}$$
$$= \frac{1}{36} \tag{9}$$

Therefore, since $\Pr(EF) = \Pr(E)\Pr(F)$, the events are independent.

It follows then that the answer to the question is $\boxed{\frac{1}{36}}$