Random Numbers

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Outline

Problem Statement

(1.5)

Verify the results for the mean and variance of a uniform distribution theoretically given that $E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x)$

Solution

Since

$$dF_U(x) = p_U(x)dx \tag{1}$$

we have:

$$E[U^k] = \int_{-\infty}^{\infty} x^k p_U(x)dx \tag{2}$$

Also,

$$p_U(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ 1, & x \in (0, 1) \\ 0, & x \in (1, \infty) \end{cases} \tag{3}$$

Solution

Therefore, from Equations 2 and 3, we have:

$$E[U^2] = \int_{-\infty}^{\infty} x^2 p_U(x) dx \tag{4}$$
$$= \int_0^1 x^2 dx \tag{5}$$
$$= \frac{1}{3} \tag{6}$$

Similarly,

$$E[U^2] = \int_{-\infty}^{\infty} x p_U(x) dx \tag{7}$$
$$= \int_0^1 x dx \tag{8}$$
$$= \frac{1}{2} \tag{9}$$

Solution

Therefore, the mean is $\frac{1}{2}$, and the variance equals:

$$E[U^2] - E[U]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 \tag{10}$$

$$= \frac{1}{12} \tag{11}$$