Random Numbers

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Outline

1. Problem Statement

2. Solution

Problem Statement

(2.5)

Find a theoretical expression for $F_V(x)$, where $V = -2\ln(1 - U)$.

Solution

We have:

$$
\begin{align}
F_V(x) &= \Pr(V \le x) \tag{1}\\
&= \Pr(-2\ln(1-U) \le x) \tag{2}\\
&= \Pr\left(1 - U \ge \exp\left(-\frac{x}{2}\right)\right) \tag{3}\\
&= \Pr\left(U \le 1 - \exp\left(-\frac{x}{2}\right)\right) \tag{4}\\
&= F_U\left(1 - \exp\left(-\frac{x}{2}\right)\right) \tag{5}
\end{align}
$$

Solution

Therefore,

$$F_V(x) = \begin{cases} 0, & 1 - \exp\left(-\frac{x}{2}\right) \in (-\infty, 0) \\ 1 - \exp\left(-\frac{x}{2}\right), & 1 - \exp\left(-\frac{x}{2}\right) \in (0, 1) \\ 1, & 1 - \exp\left(-\frac{x}{2}\right) \in (1, \infty) \end{cases} \quad (6)$$

From this we get:

$$F_V(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ 1 - \exp\left(-\frac{x}{2}\right), & x \in (0, \infty) \end{cases} \quad (7)$$