

Assignment 8

Rahul Ramachandran

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Problem Statement

13.2 Q17 [NCERT 12]

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is?

Random Variable Definition

In this experiment, there are two consecutive Bernoulli trials. Therefore, it is appropriate to define a Binomial Random Variable X as under:

Variable	Event
$X = 0$	0 even prime numbers are obtained
$X = 1$	1 even prime number is obtained
$X = 2$	2 even prime numbers are obtained

Table 1: Random Variable X

Probability Mass Function

The probability of success (assuming a fair die) is $p = \frac{1}{6}$.
Therefore, the probability that X maps to i is given by:

$$\Pr(X = i) = \binom{2}{i} (1 - p)^{2-i} p^i, \quad 0 \leq i \leq 2 \quad (1)$$

The values for i can be substituted in the above formula, and the graph of the PMF can be obtained.

Cumulative Distribution Function

The cumulative probability $\Pr(X \leq i)$ can be defined as under:

$$\Pr(X \leq i) = \sum_{k=0}^i \binom{2}{k} (1-p)^{2-k} p^k, \quad 0 \leq k \leq 2 \quad (2)$$

The values of i can be substituted in the above equation, and the obtained values can be used to plot the CDF graph.

Solution

The probability to be found corresponds to the case $i = 2$. Substituting $i = 2$ in Equation 1, we get

$$\Pr(X = 2) = \binom{2}{2} \times (1 - p)^{2-2} \times p^2 \quad (3)$$

$$= 1 \times \left(1 - \frac{1}{6}\right)^0 \times \left(\frac{1}{6}\right)^2 \quad (4)$$

$$= \frac{1}{36} \quad (5)$$

PMF Graph

The PMF graph is:

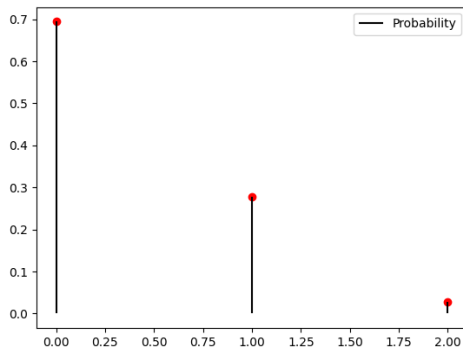


Figure 1: Probability Mass Function

CDF Graph

The CDF graph is:

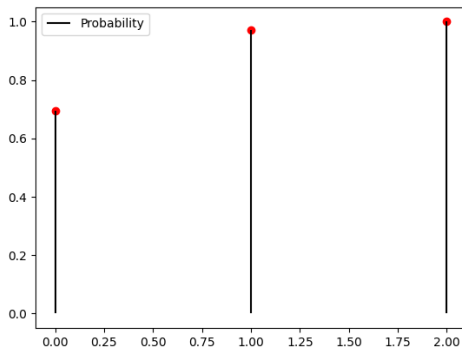


Figure 1: Cumulative Distribution Function