Random Numbers

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Outline

Problem Statement

(2.5)

Find the mean and variance of $p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$

Solution

We have:

$$E[X] = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{1}$$

$$= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)\Bigg|_{-\infty}^{\infty} \tag{2}$$

$$= 0 \tag{3}$$

Also,

$$E[X^2] = \int_{-\infty}^{\infty} \frac{x^2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{4}$$

$$= -\frac{x}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)\Bigg|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{5}$$

$$= 0 + \frac{1}{\sqrt{2\pi}} \times \sqrt{2\pi} \tag{6}$$

$$= 1 \tag{7}$$

Hence,

$$\text{var}(X) = E[X^2] - E[X]^2 \tag{8}$$

$$= 1 \tag{9}$$