

# Assignment 9

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May 20, 2022

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# Problem Statement

## 13.5 Q5 [NCERT 12]

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs:

- (i) none
  - (ii) not more than one
  - (iii) more than one
  - (iv) at least one
- will fuse after 150 days of use.

# Random Variable Definition

Since there are 5 bulbs, it is appropriate to define a Binomial Random Variable  $X$  as under:

Variable	Event
$X = 0$	0 bulbs have fused
$X = 1$	1 bulb has fused
$X = 2$	2 bulbs have fused
$X = 3$	3 bulbs have fused
$X = 4$	4 bulbs have fused
$X = 5$	5 bulbs have fused

Table 1: Random Variable  $X$

# Probability Mass Function

The probability that a bulb fuses equals  $p = 0.05$ .

Therefore, the probability that  $X$  maps to  $i$  is given by:

$$p_X(i) = \binom{5}{i}(1-p)^{5-i}p^i, 0 \leq i \leq 5 \quad (1)$$

The values for  $i$  can be substituted in the above formula, and the graph of the PMF can be obtained.

# Cumulative Distribution Function

The cumulative probability  $\Pr(X \leq i)$  can be defined as under:

$$F_X(i) = \sum_{k=0}^i \binom{5}{k} (1-p)^{5-k} p^k, \quad 0 \leq i \leq 5 \quad (2)$$

The values of  $i$  can be substituted in the above equation, and the obtained values can be used to plot the CDF graph.

# Solution

- (i) The probability to be found corresponds to  $\Pr(X = 0)$ . From the PMF Graph, we get:

$$\Pr(X = 0) = p_X(0) \quad (3)$$

$$= 0.774 \quad (4)$$

# Solution

- (ii) The probability to be found corresponds to  $F_X(0)$ . Therefore, from the CDF graph:

$$\Pr(X = 1) + \Pr(X = 2) = F_X(1) \quad (5)$$

$$= 0.978 \quad (6)$$



# Solution

- (iii) The probability to be found corresponds to  $\Pr(X > 1)$ . This is equivalent to  $1 - F_X(1)$ . Therefore,

$$\Pr(X > 1) = 1 - F_X(1) \quad (7)$$

$$\approx 1 - 0.978 \quad (8)$$

$$= 0.022 \quad (9)$$

# Solution

(iv) The probability to be found corresponds to  $\Pr(X \geq 1)$ . This is equivalent to  $1 - \Pr(X < 1) = 1 - p_X(0)$  Therefore,

$$\Pr(X \geq 1) = 1 - p_X(0) \quad (10)$$

$$= 1 - 0.774 \quad (11)$$

$$= 0.226 \quad (12)$$

# PMF Graph

The PMF graph is:

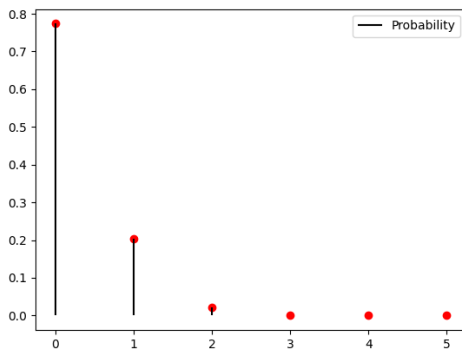


Figure 1: Probability Mass Function

# CDF Graph

The CDF graph is:

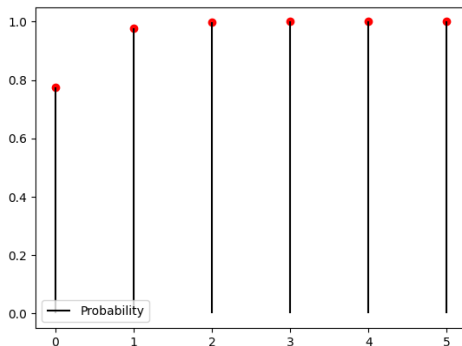


Figure 1: Cumulative Distribution Function