Assignment 10

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Outline

Problem Statement

Papoulis 4.27

A coin is tossed an infinite number of times. Show that the probability that k heads are observed at the n^{th} toss but not earlier equals $\binom{n-1}{k-1}p^k q^{n-k}$

Events Definition

Let events L, M and N be defined as under:

Event	**Description**
L	k^{th} head occurs on the n^{th} toss
M	The n^{th} toss is heads
N	$k-1$ heads have occurred in $n-1$ tosses

Random Variable Definition

Define a binomial random variable X with parameters m and p.
We then have:

$$p_X(i) = \binom{m}{i} \times p^i \times q^{m-i},\ 0 \le i \le m \tag{1}$$

In the present problem, X can be used to find probabilities relevant to event N.

Solution

Observe that we are required to find $\Pr(L)$. Also observe that

$$L = MN \tag{2}$$

Since M and N are independent events, we have:

$$\begin{aligned} \Pr(L) &= \Pr(MN) && (3) \\ &= \Pr(M)\Pr(N) && (4) \end{aligned}$$

Solution

$\Pr(N)$ corresponds to $p_X(k-1)$ where the parameters of X are $(n-1, p)$. Therefore, using Equation 1:

$$\Pr(N) = p_X(k-1) \tag{5}$$

$$= \binom{n-1}{k-1} \times p^{k-1} \times q^{n-k} \tag{6}$$

Solution

Now,

$$\Pr(M) = p \tag{7}$$

Therefore, substituting the corresponding values in Equation 4:

$$\Pr(L) = p \times \binom{n-1}{k-1} p^{k-1} q^{n-k} \tag{8}$$

$$= \binom{n-1}{k-1} p^{k} q^{n-k} \tag{9}$$