Assignment 12

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Outline

Problem Statement

Papoulis 6.44

X and Y are independent, identically distributed binomial random variables with parameters n and p. Show that $Z = X + Y$ is also a binomial random variable. Find its parameters.

Z–transform

Let the Z–transform of the discrete random variable X be defined as

$$\mathcal{M}_X(z) = E(z^{-X}) \tag{1}$$

$$= \sum_{k=-\infty}^{\infty} z^{-k} p_X(k) \tag{2}$$

When X is a binomial random variable with parameters n and p:

$$\mathcal{M}_X(z) = \sum_{k=0}^{n} z^{-k} \binom{n}{k} p^k q^{n-k} \tag{3}$$

Solution

Events X and Y are given to be independent. Then:

$$\mathcal{M}_Z(z) = \mathcal{M}_{X+Y}(z) \tag{4}$$
$$= E(z^{-(X+Y)}) \tag{5}$$
$$= E(z^{-X})E(z^{-Y}) \tag{6}$$
$$= \left(\sum_{k=0}^{n} z^{-k}\binom{n}{k}p^k q^{n-k}\right)\left(\sum_{l=0}^{n} z^{-l}\binom{n}{l}p^l q^{n-l}\right) \tag{7}$$

Solution

Converting the summation in Equation 7 to binomials, we get:

$$\mathcal{M}_Z(z) = (q + pz^{-1})^n (q + pz^{-1})^n \tag{8}$$
$$= (q + pz^{-1})^{2n} \tag{9}$$

Expanding this, we get:

$$\mathcal{M}_Z(z) = \sum_{k=0}^{2n} z^{-k} \binom{2n}{k} p^k q^{2n-k} \tag{10}$$

Solution

Since there is a one-to-one correspondence between the Z−transform and the random variable's pmf, it follows from Equation 10 that:

$$p_Z(k) = \binom{2n}{k} p^k q^{2n-k},\ 0 \leq k \leq 2n \tag{11}$$

Therefore, it follows that Z is a binomial random variable.