Assignment 9

Rahul Ramachandran

May 20, 2022

Outline

Problem Statement

13.5 Q5 [NCERT 12]

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs:

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one

will fuse after 150 days of use.

Random Variable Definition

Since there are 5 bulbs, it is appropriate to define a Binomial Random Variable X as under:

Variable	Event
$X = 0$	0 bulbs have fused
$X = 1$	1 bulb has fused
$X = 2$	2 bulbs have fused
$X = 3$	3 bulbs have fused
$X = 4$	4 bulbs have fused
$X = 5$	5 bulbs have fused

Table 1: Random Variable X

Probability Mass Function

The probability that a bulb fuses equals $p = 0.05$.
Therefore, the probability that X maps to i is given by:

$$p_X(i) = \binom{5}{i}(1-p)^{5-i}p^i,\ 0 \leq i \leq 5 \tag{1}$$

The values for i can be substituted in the above formula, and the graph of the PMF can be obtained.

Cumulative Distribution Function

The cumulative probability $\Pr(X \leq i)$ can be defined as under:

$$F_X(i) = \sum_{k=0}^{i} \binom{5}{k} (1-p)^{5-k} p^k, \ 0 \leq i \leq 5 \tag{2}$$

The values of i can be substituted in the above equation, and the obtained values can be used to plot the CDF graph.

Solution

(i) The probability to be found corresponds to $\Pr(X = 0)$. From the PMF Graph, we get:

$$\Pr(X = 0) = p_X(0) \tag{3}$$

$$= 0.774 \tag{4}$$

Solution

(ii) The probability to be found corresponds to $F_X(0)$. Therefore, from the CDF graph:

$$\Pr(X = 0) + \Pr(X = 1) = F_X(1) \tag{5}$$

$$= 0.978 \tag{6}$$

Solution

(iii) The probability to be found corresponds to $\Pr(X > 1)$. This is equivalent to $1 - F_X(1)$. Therefore,

$$\Pr(X > 1) = 1 - F_X(1) \tag{7}$$
$$\approx 1 - 0.978 \tag{8}$$
$$= 0.022 \tag{9}$$

Solution

(iv) The probability to be found corresponds to $\Pr(X >= 1)$. This is equivalent to $1 - \Pr(X < 1) = 1 - p_X(0)$ Therefore,

$$\Pr(X >= 1)1 - p_X(0) \tag{10}$$
$$= 1 - 0.774 \tag{11}$$
$$= 0.226 \tag{12}$$

PMF Graph

The PMF graph is:

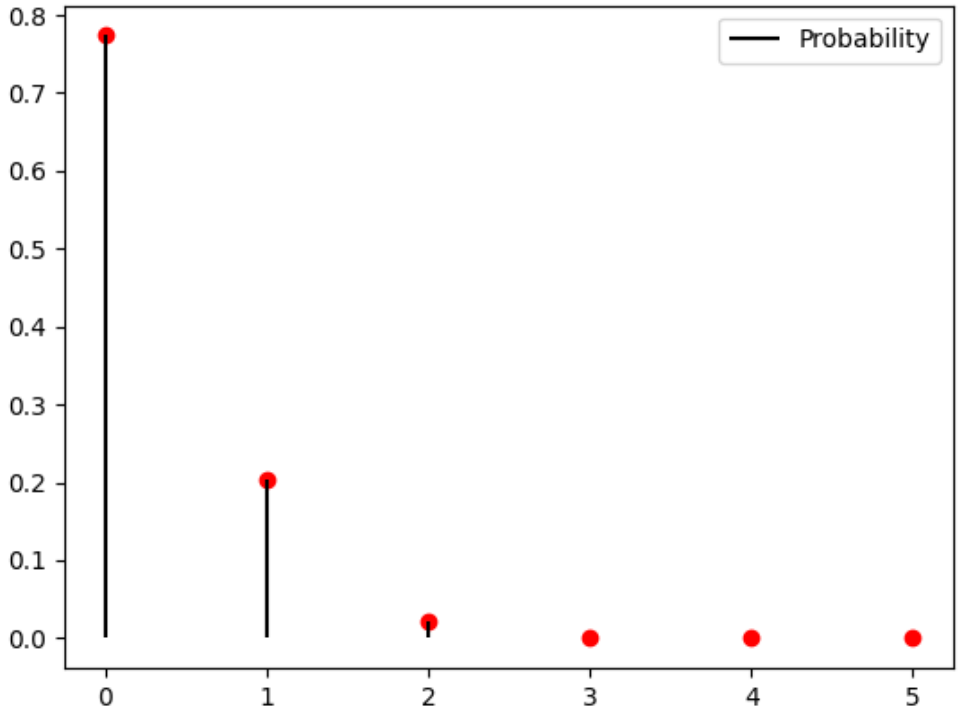

Figure 1: Probability Mass Function

CDF Graph

The CDF graph is:

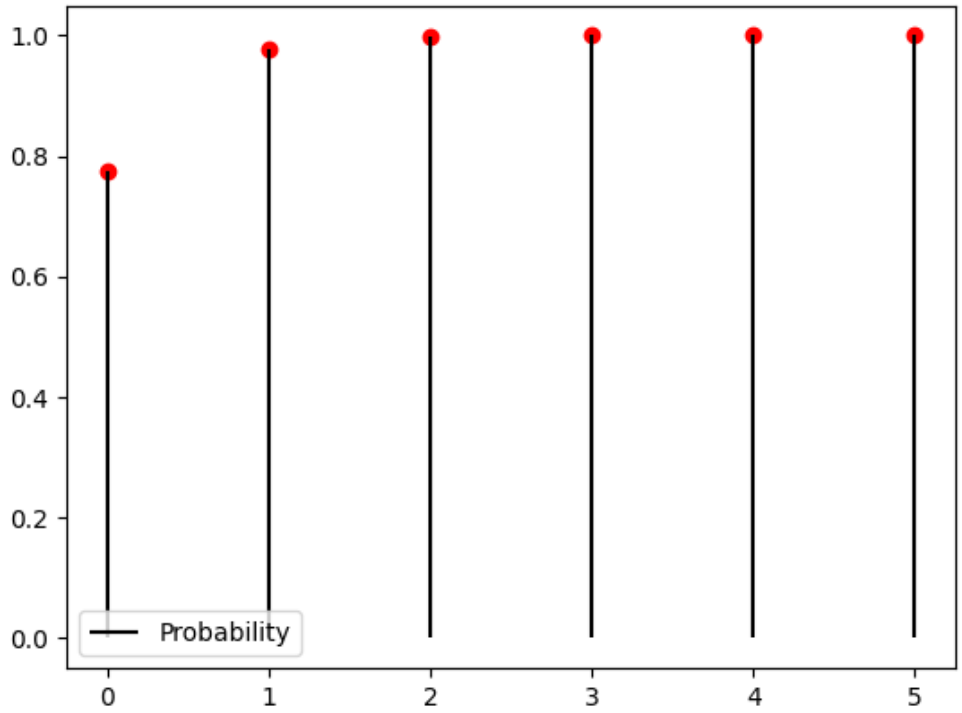

Figure 1: Cumulative Distribution Function