Assignment 6

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***Abstract*—This document contains the solution for Assignment 6 (NCERT Class 12 Chapter 13 Example 5)**

13 E5 [NCERT 12] : A die is thrown 3 times. Events A and B are defined as below:

A : 4 on the third throw B : 6 on the first and 5 on the second throw Find the probability of A given that B has already occurred

Solution:

Let the random variables X_i map to the set $\{0, 1\}$ as described in Table I

Variable	Event
$X_1 = 1$	A
$X_2 = 1$	B

TABLE I

(i) The sample space for 3 die throws is given by $\mathcal{S} = \{(x, y, z) : x, y, z \in \{1, 2 \dots 6\}\}$. Therefore,

$$\left|\mathcal{S}\right| = 6 \times 6 \times 6 = 216 \tag{1}$$

(ii) The sample space for event A is given by $\mathcal{S}_A = \{(x, y, 4) : x, y \in \{1, 2 \dots 6\}\}$. Therefore,

$$\left|\mathcal{S}_A\right| = 6 \times 6 = 36 \tag{2}$$

(iii) The sample space for event B is given by $\mathcal{S}_B = \{(6, 5, z) : z \in \{1, 2 \dots 6\}\}$. Therefore,

$$\left|\mathcal{S}_B\right| = 6 \tag{3}$$

(iv) The sample space for both event A and event B simultaneously occurring is given by $\mathcal{S}_{A\cap B} = \{(6, 5, 4)\}$. Therefore,

$$\left|\mathcal{S}_{A\cap B}\right| = 1 \tag{4}$$

The probabilities for different values of X_i can therefore be found.

The values are given in Table II

Now,

$$\Pr\left(X_1 = 1 | X_2 = 1\right) = \frac{\Pr\left(X_1 = 1, X_2 = 1\right)}{\Pr\left(X_2 = 1\right)} \tag{5}$$

Probability	Value
$\Pr\left(X_1 = 1\right)$	$\frac{36}{216} = \frac{1}{6}$
$\Pr\left(X_2 = 1\right)$	$\frac{6}{216} = \frac{1}{36}$
$\Pr\left(X_1 = 1, X_2 = 1\right)$	$\frac{1}{216}$
$\Pr\left(X_1 = 1 \vert X_2 = 1\right)$	?

TABLE II

Therefore, substituting the values from Table II, we have:

$$\Pr\left(X_1 = 1 | X_2 = 1\right) = \frac{\frac{1}{216}}{\frac{1}{36}} \tag{6}$$

This simplifies to:

$$\Pr\left(X_1 = 1 | X_2 = 1\right) = \frac{1}{6} \tag{7}$$