Assignment 9

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Outline

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Problem Statement

13.5 Q5 [NCERT 12]

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs:

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one

will fuse after 150 days of use.

Random Variable Definition

Since there are 5 bulbs, it is appropriate to define a Binomial Random Variable X as under:

Variable	**Event**
$X = 0$	0 bulbs have fused
$X = 1$	1 bulb has fused
$X = 2$	2 bulbs have fused
$X = 3$	3 bulbs have fused
$X = 4$	4 bulbs have fused
$X = 5$	5 bulbs have fused

Table 1: Random Variable X

Probability Mass Function

The probability that a bulb fuses equals $p = 0.05$.
Therefore, the probability that X maps to i is given by:

$$\Pr\left(X = i\right) = \binom{5}{i}(1-p)^{5-i}p^i,\ 0 \le i \le 5 \tag{1}$$

The values for i can be substituted in the above formula, and the graph of the PMF can be obtained.

Cumulative Distribution Function

The cumulative probability $\Pr(X \le i)$ can be defined as under:

$$\Pr(X \le i) = \sum_{k=0}^{i} \binom{5}{k} (1-p)^{5-k} p^k, \; 0 \le i \le 5 \tag{2}$$

The values of i can be substituted in the above equation, and the obtained values can be used to plot the CDF graph.

Solution

(i) The probability to be found corresponds to $\Pr(X = 0)$. Substituting $i = 0$ in Equation 1, we get

$$\Pr(X = 0) = \binom{5}{0} \times (1 - p)^{5-0} \times p^0 \tag{3}$$
$$= 1 \times (1 - 0.05)^5 \times (0.05)^0 \tag{4}$$
$$\approx 0.774 \tag{5}$$

Solution

(ii) The probability to be found corresponds to $\Pr(X = 0) + \Pr(X = 1)$. Simple addition will give the probability as the events are mutually exclusive. Substituting $i = 1$ in Equation 1, we get

$$\Pr(X = 1) = \binom{5}{1} \times (1 - p)^{5-1} \times p^1 \tag{6}$$

$$= 5 \times (1 - 0.05)^4 \times (0.05)^1 \tag{7}$$

$$\approx 0.204 \tag{8}$$

Using the value of $\Pr(X = 0)$ obtained from equation 5:

$$\Pr(X = 0) + \Pr(X = 1) \approx 0.774 + 0.204 \tag{9}$$

$$= 0.978 \tag{10}$$

Solution

(iii) The probability to be found corresponds to $\Pr(X > 1)$. This is equivalent to $1 - \Pr(X <= 1)$ (Since $X > 1$ and $X <= 1$ are mutually exclusive, and the sum of the probabilities is 1). Substituting $i = 1$ in Equation 2, we get

$$\Pr(X <= 1) = \sum_{k=0}^{1} \binom{5}{k} (1 - 0.05)^{5-k} 0.05^k \tag{11}$$

$$= 1 \times (1 - 0.05)^5 \times (0.05)^0 + 5 \times (1 - 0.05)^4 \times (0.05)^1 \tag{12}$$

$$\approx 0.774 + 0.204 \tag{13}$$

$$= 0.978 \tag{14}$$

Therefore,

$$\Pr(X > 1) \approx 1 - 0.978 \tag{15}$$

$$= 0.022 \tag{16}$$

Solution

(iv) The probability to be found corresponds to $\Pr(X >= 1)$. This is equivalent to $1 - \Pr(X < 1) = 1 - \Pr(X = 0)$ (Since $X < 1$ and $X >= 1$ are mutually exclusive).
Substituting the value found in 5:

$$1 - \Pr(X = 0) \approx 1 - 0.774 \tag{17}$$

$$= 0.226 \tag{18}$$

PMF Graph

The PMF graph is:

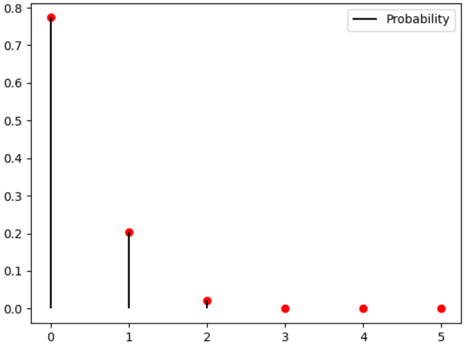

Figure 1: Probability Mass Function

CDF Graph

The CDF graph is:

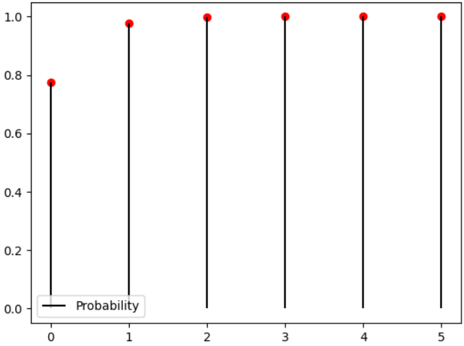

Figure 1: Cumulative Distribution Function