

GATE BM Q49

EE23BTECH11027 - K RAHUL*

Question:

The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (1)$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

(A) $y(t) = 0$ for all $t < 0$

(B) $y(t) = 0$ for all $t > 1$

(C) $y(t) = 0$ for all $t > 3$

(D) $\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt \neq 0$

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Solution:

Symbol	Description
$X(s)$	Laplace transform of $x(t)$
$Y(s)$	Laplace transform of $y(t)$
$u(t - t_0)$	Unit step function, $u(t - t_0) = 1, t \geq t_0$

TABLE 4
PARAMETERS

$$\begin{aligned} y(t) &= x(t) * x(t) \\ Y(s) &= X(s) X(s) \\ &= \left(\frac{1 - e^{-s}}{s} \right)^2 \\ &= \frac{1 + e^{-2s} - 2e^{-s}}{s^2} \end{aligned}$$

$$\begin{aligned} u(t) &\xleftrightarrow{\mathcal{L}} \frac{1}{s} \\ tu(t) &\xleftrightarrow{\mathcal{L}} \frac{1}{s^2} \end{aligned}$$

$$f(t) \xleftrightarrow{\mathcal{L}} F(s) \implies f(t+a) \xleftrightarrow{\mathcal{L}} e^{as} F(s) \quad (8)$$

Using (7) and (8),

$$y(t) = \begin{cases} 1 - |1 - t|, & \text{if } 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

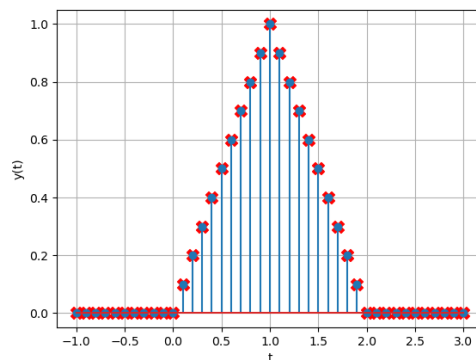


Fig. 4. Stem Plot of $y(t)$ v/s t

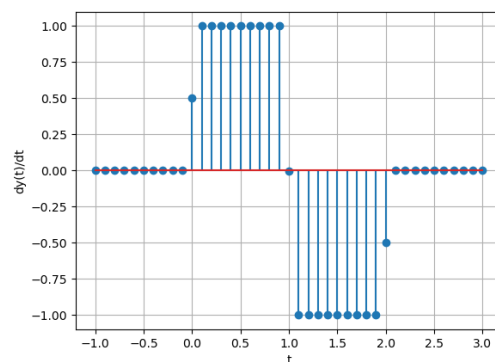


Fig. 4. Stem Plot of $dy(t)/dt$ v/s t

Checking (9) with every option,

- (6) (A) From Fig: 4, $y(t) = 0, \forall t < 0$, hence (A) is true
- (7)

- (B) From Fig: 4 , $y(t) \neq 0, \forall t \in [1, 2]$, hence (B) is false
- (C) From Fig: 4 , $y(t) = 0, \forall t > 3$, hence (C) is true
- (D) From Fig: 4 , area under graph between 0.75 and 0.1 is non-zero, hence (D) is true