

# GATE BM Q49

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RESULTS:

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (1)$$

$$tu(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s^2} \quad (2)$$

$$y(t) = x(t) * x(t) \quad (5)$$

$$Y(s) = X(s)X(s) \quad (6)$$

$$= \left( \frac{1 - e^{-s}}{s} \right)^2 \quad (7)$$

$$= \frac{1 + e^{-2s} - 2e^{-s}}{s^2} \quad (8)$$

Using (2) and (3),

$$f(t) \xleftrightarrow{\mathcal{L}} F(s) \implies f(t+a)u(t-a) \xleftrightarrow{\mathcal{L}} e^{as}F(s) \quad (3)$$

$$y(t) = tu(t) + (t-2)u(t-2) - 2(t-1)u(t-1) \quad (9)$$

For  $t < 0$ ,  $u(t), u(t-1), u(t-2)$  all are zero, hence  $y(t) = 0$ . Thus, option A is right.

QUESTION:

The continuous time signal  $x(t)$  is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (4)$$

If  $y(t)$  represents  $x(t)$  convolved with itself, which of the following options is/are TRUE?

A  $y(t) = 0$  for all  $t < 0$

B  $y(t) = 0$  for all  $t > 1$

C  $y(t) = 0$  for all  $t > 3$

D  $\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt \neq 0$

If  $1 < t < 2$ ,  $u(t)$  and  $u(t-1)$  are equal to unity while  $u(t-2)$  is equal to 0. Thus,  $y(t) = 2 - t$ . Thus, option B is wrong.

If  $t > 3$ ,  $u(t), u(t-1), u(t-2)$  are all equal to unity. Thus,  $y(t) = 0$ . Thus, option C is right.

$$\int_{0.1}^{0.75} dy(t) = y(0.75) - y(0.1) \quad (10)$$

$$= 0.75 - 0.1 \quad (11)$$

$$= -0.25 \quad (12)$$

Thus, option D is right.

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SOLUTION:

Symbol	Description
$X(s)$	Laplace transform of $x(t)$
$Y(s)$	Laplace transform of $y(t)$
$u(t - t_0)$	Unit step function, $u(t - t_0) = 1, t \geq t_0$

TABLE 4  
PARAMETERS

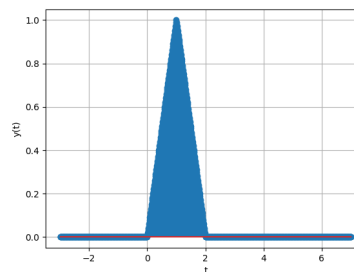


Fig. 4. Stem Plot of  $y(t)$  v/s  $t$