GATE BM Q49

EE23BTECH11027 - K RAHUL*

Question:

The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases} \tag{1}$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

(A) $y(t) = 0$ for all $t < 0$

(B) $y(t) = 0$ for all $t > 1$

(C) $y(t) = 0$ for all $t > 3$

(D) $\int_{0.1}^{0.75} \frac{dy(t)}{dt}\,\mathrm{dt} \neq 0$

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Solution:

Symbol	Description
$X(s)$	Laplace transform of $x(t)$
$Y(s)$	Laplace transform of $y(t)$
$u(t-t_0)$	Unit step function, $u(t-t_0) = 1, t \ge t_0$

TABLE 4
PARAMETERS

$$y(t) = x(t) * x(t) \tag{2}$$

$$Y(s) = X(s)X(s) \tag{3}$$

$$= \left(\frac{1-e^{-s}}{s}\right)^2 \tag{4}$$

$$= \frac{1+e^{-2s}-2e^{-s}}{s^2} \tag{5}$$

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \tag{6}$$

$$tu(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^2} \tag{7}$$

$$f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s) \implies f(t+a) \overset{\mathcal{L}}{\longleftrightarrow} e^{as}F(s) \tag{8}$$

Using (7) and (8),

$$y(t) = tu(t) + (t-2)u(t-2) - 2(t-1)u(t-1) \tag{9}$$

Checking (9) with every option,

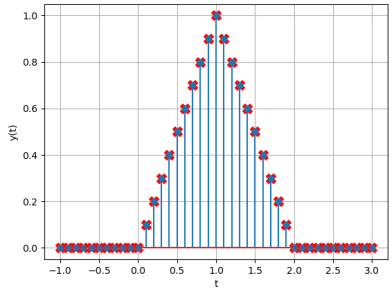

Fig. 4. Stem Plot of $y(t)$ v/s t

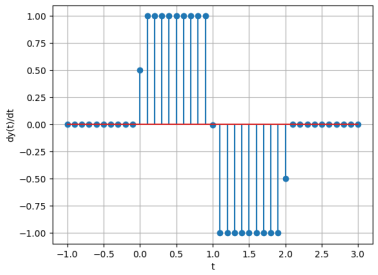

Fig. 4. Stem Plot of $dy(t)/dt$ v/s t

(A) From Fig: 4 , $y(t) = 0, \forall t < 0$, hence it is true

(B) From Fig: 4 ,$y(t) \neq 0, \forall t \in [1,2]$, hence it is false

(C) From Fig: 4, $y(t) = 0, \forall t > 3$, hence it is true

(D) From Fig: 4, area under graph is non-zero, hence it is true