

11.9.4.4

EE23BTECH11027 - K RAHUL*

DERIVATIONS AND RESULTS:

$$x(n) = \frac{1}{n+c} u(n), \text{ where } c \in \mathbb{R} \quad (1)$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \quad (2)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \quad (3)$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \quad (4)$$

$$= z^c \left(-\log(1 - z^{-1}) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \dots - \frac{z^{-(c-1)}}{c-1} \right) \quad (5)$$

$$d(z) = z^2 \log(1 - z^{-1}) \quad (6)$$

$$d(n) = \frac{1}{2\pi j} \oint_C z^{n+1} \log(1 - z^{-1}) dz \quad (7)$$

$$= \frac{-1}{2\pi j} \oint_C z^{n+1} \left(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \dots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2)}}{n+2} + \dots \right) dz \quad (8)$$

$$z = e^{jt} \quad (9)$$

$$= \frac{-1}{2\pi} \int_0^{2\pi} e^{(n+2)jt} \left(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + \dots + \frac{e^{-(n+2)jt}}{n+2} + \dots \right) dt \quad (10)$$

$$= \frac{-1}{n+2} \quad (11)$$

$$d(z) = \frac{z^k}{1 - z^{-1}}, \text{ where } k \in \mathbb{R} \quad (12)$$

$$d(n) = \lim_{x \rightarrow 1} z^{n+k-1} \text{ (Residue Theorem)} \quad (13)$$

$$= 1 \quad (14)$$

Find sum to n terms of the following series:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$

SOLUTION:

Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

$$x(n) = \frac{1}{(n+1)(n+2)} u(n) \quad (15)$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2} \right) u(n) \quad (16)$$

$$(17)$$

Using (5) we get,

$$X(z) = -z \log(1 - z^{-1}) + z^2 \log(1 - z^{-1}) + z \quad (18)$$

$$= z(z-1) \log(1 - z^{-1}) + z \quad (19)$$

$$Y(z) = X(z) U(z) \quad (20)$$

$$= z^2 \log(1 - z^{-1}) + \frac{z}{1 - z^{-1}} \quad (21)$$

$$(22)$$

Using (11) and (14),

$$y(n) = 1 - \frac{1}{n+2} \quad (23)$$

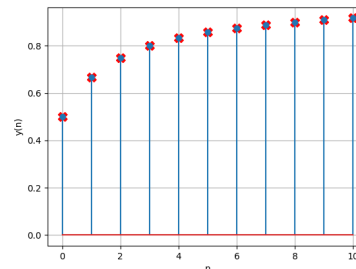


Fig. 0. Stem Plot of $y(n)$ v/s n

QUESTION: