

# 10.5.3

EE23BTECH11027 - K RAHUL\*

## QUESTION:

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

## SOLUTION:

Parameters in expression		
Symbol	Description	Value
$x(n)$	$n^{th}$ term of series	
$x(l)$	Last( $l^{th}$ ) term of series	350
$x(0)$	Starting ( $0^{th}$ ) term of series	17
d	Common difference of AP	9

TABLE 0  
PARAMETERS

$$x(n) = (x(0) + nd)u(n) \quad (1)$$

$$x(l) = (17 + 9l)u(l) \quad (2)$$

Thus,

$$l = 37 \quad (3)$$

If  $|Z| > 1$ , then

$$X(z) = (17 - 8z^{-1})(1 - z^{-1})^{-2} \quad (4)$$

$$y(n) = x(n) * u(n) \quad (5)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (6)$$

$$= \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} \quad (7)$$

Using contour integral to find Z transform, we get

$$y(37) = \frac{1}{2\pi j} \oint_C Y(z) z^{36} dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} z^{36} dz \quad (9)$$

Now, using Cauchy's residual theorem and observing the fact that 3 repeated poles exist at  $z = 1$ ,

$$R = \frac{1}{(k-1)!} \lim_{z \rightarrow c} \frac{d^{k-1}}{dz^{k-1}} ((z-c)^k f(z)) \quad (10)$$

$$= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^{k-1}}{dz^{k-1}} ((z-1)^3 \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} z^{36}) \quad (11)$$

$$= \frac{1}{2} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (17z^{39} - 8z^{38}) \quad (12)$$

$$= 6973 \quad (13)$$

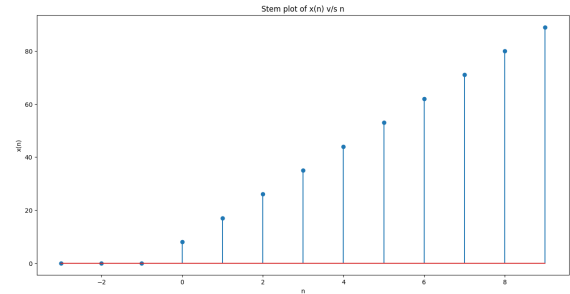


Fig. 0. Stem Plot of  $x(n)$  v/s  $n$