11.9.4.4

EE23BTECH11027 - K RAHUL*

DERIVATIONS AND RESULTS:

$$x(n) = \frac{1}{n+c} u(n)\,, \text{where } c \in \mathbb{R} \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n)\, z^{-n} \tag{2}$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \tag{3}$$

$$= z^{c} \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \tag{4}$$

$$= z^{c}\Big(-log\left(1 - z^{-1}\right) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \ldots - \frac{z^{-(c-1)}}{c-1}\Big) \tag{5}$$

$$d(z) = z^{2} log\left(1 - z^{-1}\right) \tag{6}$$

$$d(n) = \frac{1}{2\pi j} \oint_C z^{n+1} \log\left(1 - z^{-1}\right) dz \tag{7}$$

$$= \frac{-1}{2\pi j} \oint_C z^{n+1}\Big(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \ldots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2)}}{n+2} + \ldots\Big) dz \tag{8}$$

$$z = e^{jt} \tag{9}$$

$$= \frac{-1}{2\pi} \int_0^{2\pi} e^{(n+2)jt}\Big(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + .. + \frac{z^{-(n+2)}jt}{n+2} + ..\Big) dz \tag{10}$$

$$= \frac{-1}{n+2} \tag{11}$$

$$d(z) = \frac{z^{k}}{1 - z^{-1}}, \text{where } k \in \mathbb{R} \tag{12}$$

$$d(n) = \lim_{x \to 1} z^{n+k-1} (\text{Residue Theorem}) \tag{13}$$

$$= 1 \tag{14}$$

QUESTION:

Find sum to n terms of the following series: $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

SOLUTION:

Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

$$x(n) = \frac{1}{(n+1)(n+2)} u(n) \tag{15}$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right) u(n) \tag{16}$$

$$\tag{17}$$

Using (5)we get,

$$X(z) = -zlog\left(1 - z^{-1}\right) + z^{2}log\left(1 - z^{-1}\right) + z \tag{18}$$

$$= z(z-1)\, log\left(1 - z^{-1}\right) + z \tag{19}$$

$$Y(z) = X(z)\, U(z) \tag{20}$$

$$= z^{2} log\left(1 - z^{-1}\right) + \frac{z}{1 - z^{-1}} \tag{21}$$

$$\tag{22}$$

Using (11) and (14),

$$y(n) = 1 - \frac{1}{n+2} \tag{23}$$

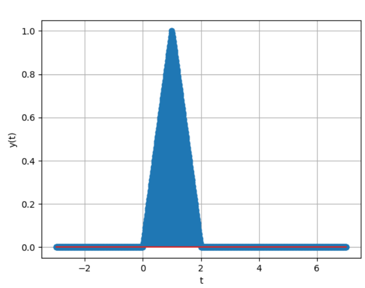

Fig. 0. Stem Plot of $y(t)$ v/s t