

11.9.4.4

EE23BTECH11027 - K RAHUL*

Derivations and results:

Symbol	Description
$x(n)$	$\frac{1}{n+c}, c \geq 2$
$w(n)$	$\frac{1}{n+1}$
$D(z)$	$z^2 \log(1 - z^{-1})$

TABLE 0
NOTATIONS

$$\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \quad (1)$$

Let $x(n)$ and $w(n)$ be some expression as described in the table in discrete time domain, whose Z transform, $X(z)$ and $W(z)$ respectively, shall be obtained for future reference.

For $x(n) = \frac{1}{n+c}u(n)$, $\forall c \geq 2, c \in \mathbb{N}$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \quad (2)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \quad (3)$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \quad (4)$$

Using, (1)

$$X(z) = z^c \left(-\log(1 - z^{-1}) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \dots - \frac{z^{-(c-1)}}{c-1} \right) \forall c \geq 2, c \in \mathbb{N} \quad (5)$$

For $w(n) = \frac{1}{n+1}u(n)$,

$$W(z) = \sum_{n=-\infty}^{n=+\infty} w(n) z^{-n} \quad (6)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-n} \quad (7)$$

$$= z \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-(n+1)} \quad (8)$$

Using (1),

$$W(z) = -z \log(1 - z^{-1}) \quad (9)$$

Let $D(z)$ be some expression as described in the table in Z domain, whose inverse Z transform, $d(n)$ shall be obtained for future reference.

For $D(z) = z^2 \log(1 - z^{-1})$

$$d(n) = \frac{1}{2\pi j} \oint_C z^{n+1} \log(1 - z^{-1}) dz \quad (10)$$

$$= \frac{-1}{2\pi j} \oint_C z^{n+1} \left(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \dots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2)}}{n+2} + \dots \right) dz \quad (11)$$

Making the substitution $z = e^{jt} \Rightarrow dz = je^{jt} dt$

$$d(n) = \frac{-1}{2\pi} \int_0^{2\pi} e^{(n+2)jt} \left(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + \dots + \frac{z^{-(n+2)}jt}{n+2} + \dots \right) dt \quad (12)$$

$$= \frac{-1}{n+2} u(n) \quad (13)$$

$$u(n) \xleftrightarrow{\mathcal{L}} \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (14)$$

$$u(n+1) \xleftrightarrow{\mathcal{L}} \frac{z}{1 - z^{-1}}, \quad |z| > 1 \quad (15)$$

Question:

Find sum to n terms of the following series:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$

Solution:

Symbol	Description	Value
$x(n)$	n^{th} term of series	$\frac{1}{(n+1)(n+2)}u(n)$
$y(n)$	Sum of n terms of series	?

TABLE 0
PARAMETERS

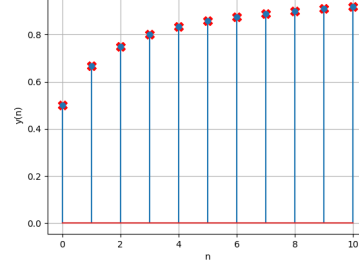


Fig. 0. Stem Plot of $y(n)$ v/s n

$$x(n) = \frac{1}{(n+1)(n+2)}u(n) \quad (16)$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2} \right) u(n) \quad (17)$$

Using (5) and (9), we get,

$$X(z) = -z \log(1 - z^{-1}) + z^2 \log(1 - z^{-1}) + z \quad (18)$$

$$= z(z-1) \log(1 - z^{-1}) + z \quad (19)$$

$$Y(z) = X(z)U(z) \quad (20)$$

$$= z^2 \log(1 - z^{-1}) + \frac{z}{1 - z^{-1}} \quad (21)$$

Using (13) and (15),

$$y(n) = u(n+1) - \frac{1}{n+2}u(n), \quad n \geq 0 \quad (22)$$

Since $y(n)$ is only defined for $n \geq 0$, the above expression can be equivalently written as

$$y(n) = \left(1 - \frac{1}{n+2} \right) u(n) \quad (23)$$