# Appendix 11.9.4.4

EE23BTECH11027 - K RAHUL*

$$x(n) = \frac{1}{n+c}u(n), \text{where } c \in \mathbb{R} \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \tag{2}$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \tag{3}$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \tag{4}$$

$$= z^c \Big( -log\left(1 - z^{-1}\right) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \ldots - \frac{z^{-(c-1)}}{c-1} \Big) \tag{5}$$

$$d(z) = z^2 log\left(1 - z^{-1}\right) \tag{6}$$

$$d(n) = \frac{1}{2\pi j} \oint_C z^{n+1} \log\left(1 - z^{-1}\right) dz \tag{7}$$

$$= \frac{-1}{2\pi j} \oint_C z^{n+1} \Big( z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \ldots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2)}}{n+2} + \ldots \Big) dz \tag{8}$$

$$z = e^{jt} \tag{9}$$

$$= \frac{-1}{2\pi} \int_0^{2\pi} e^{(n+2)jt} \Big( e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + .. + \frac{z^{-(n+2)} jt}{n+2} + .. \Big) dz \tag{10}$$

$$= \frac{-1}{n+2} \tag{11}$$

$$d(z) = \frac{z^k}{1 - z^{-1}}, \text{where } k \in \mathbb{R} \tag{12}$$

$$d(n) = \lim_{x \to 1} z^{n+k-1} (\text{Residue Theorem}) \tag{13}$$

$$= 1 \tag{14}$$