11.9.4.4

EE23BTECH11027 - K RAHUL*

DERIVATIONS AND RESULTS:

Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

$$x(n) = \frac{1}{n+c}u(n), \text{ where } c \in \mathbb{R} \qquad (1)$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n)z^{-n} \qquad (2)$$

SOLUTION:

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c}z^{-n} \qquad (3)$$

$$x(n) = \frac{1}{(n+1)(n+2)}u(n) \qquad (14)$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c}z^{-(n+c)} \qquad (4)$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right)u(n) \qquad (15)$$

$$\qquad (16)$$

$$= z^c\left(-log\left(1-z^{-1}\right) - z^{-1} - \frac{z^{-2}}{2}\right.$$

Using (5) we get,

$$\left. - \frac{z^{-3}}{3} - \ldots - \frac{z^{-(c-1)}}{c-1}\right) \qquad (5)$$

$$X(z) = -z log\left(1-z^{-1}\right) + z^2 log\left(1-z^{-1}\right) + z \qquad (17)$$

$$= z(z-1) log\left(1-z^{-1}\right) + z \qquad (18)$$

$$d(n) = \frac{1}{2\pi j}\oint_C z^{n+1} log\left(1-z^{-1}\right)dz \qquad (6)$$

$$Y(z) = X(z)U(z) \qquad (19)$$

$$= \frac{-1}{2\pi j}\oint_C z^{n+1}\left(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \ldots + \frac{z^{-(n+1)}}{n+1}\right.$$

$$= z^2 log\left(1-z^{-1}\right) + \frac{z}{1-z^{-1}} \qquad (20)$$

$$\left. + \frac{z^{-(n+2)}}{n+2} + \ldots\right)dz \qquad (7)$$

$$\qquad (21)$$

$$z = e^{jt} \qquad (8)$$

Using (10) and (13),

$$= \frac{-1}{2\pi}\int_0^{2\pi} e^{(n+2)jt}\left(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3}\right.$$

$$y(n) = 1 - \frac{1}{n+1} \qquad (22)$$

$$\left. + \ldots + \frac{z^{-(n+2)}jt}{n+2} + \ldots\right)dz \qquad (9)$$

$$\qquad (23)$$

$$= \frac{-1}{n+2} \qquad (10)$$

$$d(n) = \frac{z^n}{1-z^{-1}} \qquad (11)$$

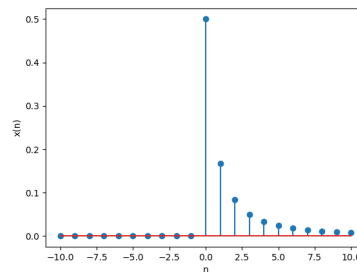$$= \lim_{x\to 1} z^{n+1}(\text{Residue Theorem}) \qquad (12)$$

$$= 1 \qquad (13)$$

Fig. 0. Stem Plot of $x(n)$ v/s n

QUESTION:

Find sum to n terms of the following series:
$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots$