11.9.4.4

EE23BTECH11027 - K RAHUL*

$$x(n) = \frac{1}{n+c}u(n), \text{where } c \in \mathbb{R} \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n)z^{-n} \tag{2}$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c}z^{-n} \tag{3}$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c}z^{-(n+c)} \tag{4}$$

$$= z^c(-log(1-Z^{-1}) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} \dots - \frac{z^{-(c-1)}}{c-1})) \tag{5}$$

$$d(n) = \frac{1}{2\pi j}\oint_C z^{n+1}\ln(1-z^{-1})dz \tag{6}$$

$$= \frac{-1}{2\pi j}\oint_C z^{n+1}(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + .. + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2}}{n+2} + ..dz \tag{7}$$

$$z = e^{jt} \tag{8}$$

$$= \frac{-1}{2\pi}\int_0^{2\pi} e^{(n+2)jt}(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + .. + \frac{z^{-(n+2}jt}{n+2} + ..dz \tag{9}$$

$$= \frac{-1}{n+2} \tag{10}$$

$$d(n) = \frac{z^n}{1-z^{-1}} \tag{11}$$

$$= \lim_{x \to 1} z^{n+1}(\text{Residue Theorem}) \tag{12}$$

$$= 1 \tag{13}$$