

GATE 2022 ME-32

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Question:

A rigid uniform annular disc is pivoted on a knife edge A in a uniform gravitational field as shown, such that it can execute small amplitude simple harmonic motion in the plane of the figure without slip at the pivot point. The inner radius r and outer radius R are such that $r^2 = \frac{R^2}{2}$, and the acceleration due to gravity is g . If the time period of small amplitude simple harmonic motion is given by $T = \beta\pi\sqrt{\frac{R}{g}}$, where π is the ratio of circumference to diameter of a circle, then $\beta =$ (Round off to 2 decimal places)

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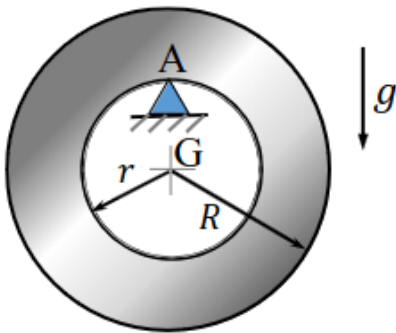


Fig. 0. Question Diagram

Solution:

Moment of inertia of disc about pivot point is calculated as

$$I = \frac{1}{2} \left(R^2 + \frac{R^2}{2} \right) + \frac{MR^2}{2} \quad (1)$$

$$= \frac{5}{4} MR^2 \quad (2)$$

Using D' Alambert's principle,

$$I \frac{d^2\theta(t)}{dt^2} + mgr \sin(\theta(t)) = 0 \quad (3)$$

$$\Rightarrow I \frac{d^2\theta(t)}{dt^2} + mgr\theta(t) = 0, \text{ for } \theta \ll 1, \theta > 0 \quad (4)$$

Parameters in expression		
Symbol	Description	Value
I	Moment of Inertia about the pivot point	$\frac{5}{4}MR^2$
$\theta(t)$	Angular displacement from vertical	?
$\theta(0)$	Value of $\theta(t)$ at $t = 0$	0
$\Theta(s)$	Laplace Transform of $\theta(t)$?
r	Distance of center of gravity from pivot point	$\frac{R}{\sqrt{2}}$

TABLE 0
PARAMETERS

Using (2) and (4), we get

$$\frac{5MR^2}{4} \frac{d^2\theta(t)}{dt^2} + Mg \frac{R}{\sqrt{2}} \theta(t) = 0 \quad (5)$$

$$\frac{d^2\theta(t)}{dt^2} + \frac{2\sqrt{2}g}{5R} \theta(t) = 0 \quad (6)$$

Taking Laplace Transform on both sides, we get

$$s^2\Theta(s) - s\theta(0) - \theta'(0) + \frac{2\sqrt{2}g}{5R}\alpha(s) = 0 \quad (7)$$

$$\Theta(s) \left(s^2 + \frac{2\sqrt{2}g}{5R} \right) = \theta'(0) \quad (8)$$

$$\Theta(s) = \frac{\theta'(0)}{\left(s^2 + \frac{2\sqrt{2}g}{5R} \right)} \quad (9)$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (10)$$

$$e^{at}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s-a} \quad (11)$$

$$\frac{(e^{jat} - e^{-jat})}{2} u(t) \longleftrightarrow \frac{a}{s^2 + a^2} \quad (12)$$

$$\sin(at) \longleftrightarrow \frac{a}{s^2 + a^2} \quad (13)$$

From (9)

$$\Theta(s) = \frac{\theta'(0) \left(\sqrt{\frac{2\sqrt{2}g}{5R}} \right)}{\left(s^2 + \frac{2\sqrt{2}g}{5R} \right)} \frac{1}{\left(\sqrt{\frac{2\sqrt{2}g}{5R}} \right)} \quad (14)$$

Taking inverse Laplace by putting $\frac{\theta'(0)}{\left(\sqrt{\frac{2\sqrt{2}g}{5R}} \right)} = k$ and (13),

$$\theta(t) = k \sin \left(\frac{2\sqrt{2}g}{5R} t \right) \quad (15)$$

$$T = \frac{2\pi}{\frac{2\sqrt{2}g}{5R}} \quad (16)$$

$$= \sqrt{(5\sqrt{2})} \pi \sqrt{\frac{R}{g}} \quad (17)$$

Thus,

$$\beta = \sqrt{(5\sqrt{2})} \quad (18)$$

$$\beta = 2.66 \quad (19)$$

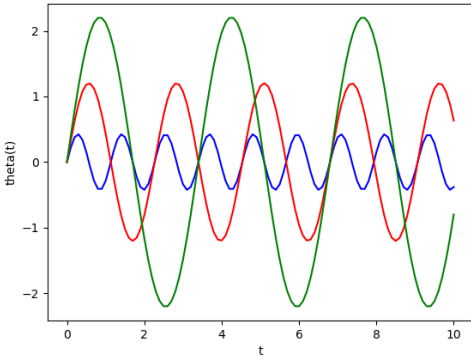


Fig. 0. Plot of $\theta(t)$