

11.9.4.4

EE23BTECH11027 - K RAHUL*

Derivations and results:

For $w(n) = \frac{1}{n+1}u(n)$,

Symbol	Description
$x(n)$	$\frac{1}{n+c}, c \geq 2$
$w(n)$	$\frac{1}{n+1}$
$D(z)$	$z^2 \log(1 - z^{-1})$

TABLE 0
NOTATIONS

$$u(n) \xleftrightarrow{Z} \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (1)$$

$$u(n+k) \xleftrightarrow{Z} \frac{z^k}{1 - z^{-1}}, \forall k \in \mathbb{R}, \quad |z| > 1 \quad (2)$$

$$\delta(n) \xleftrightarrow{Z} 1 \quad (3)$$

$$\delta(n+k) \xleftrightarrow{Z} z^{-k}, \forall k \in \mathbb{R} \quad (4)$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \quad (5)$$

Let $x(n)$ and $w(n)$ be some expression as described in the table in discrete time domain, whose Z transform, $X(z)$ and $W(z)$ respectively, shall be obtained for future reference.

For $x(n) = \frac{1}{n+c}u(n)$, $\forall c \geq 2, c \in \mathbb{N}$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \quad (6)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \quad (7)$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \quad (8)$$

Using, (5)

$$X(z) = z^c \left(-\log(1 - z^{-1}) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \dots - \frac{z^{-(c-1)}}{c-1} \right) \forall c \geq 2, c \in \mathbb{N} \quad (9)$$

$$W(z) = \sum_{n=-\infty}^{n=+\infty} w(n) z^{-n} \quad (10)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-n} \quad (11)$$

$$= z \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-(n+1)} \quad (12)$$

Using (5),

$$W(z) = -z \log(1 - z^{-1}) \quad (13)$$

Let $D(z)$ be some expression as described in the table in Z domain, whose inverse Z transform, $d(n)$ shall be obtained for future reference.

For $D(z) = z^k \log(1 - z^{-1}) \forall k \geq 1, k \in \mathbb{Z}$

$$D(z) = \left(-z^{k-1} - \frac{1}{2}z^{k-2} - \frac{1}{3}z^{k-3} - \frac{1}{4}z^{k-4} - \dots \right) \quad (14)$$

$$d(n) = \left(-\delta(n+k-1) - \frac{1}{2}\delta(n+k-2) - \frac{1}{3}\delta(n+k-3) - \frac{1}{4}\delta(n+k-4) - \dots - \frac{1}{n+k}\delta(0) - \dots \right) \quad (15)$$

$$= \frac{-1}{n+k}u(n) \quad (16)$$

Question:

Find sum to n terms of the following series:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$

Solution:

Symbol	Description	Value
$x(n)$	n^{th} term of series	$\frac{1}{(n+1)(n+2)}u(n)$
$y(n)$	Sum of n terms of series	?

TABLE 0
PARAMETERS

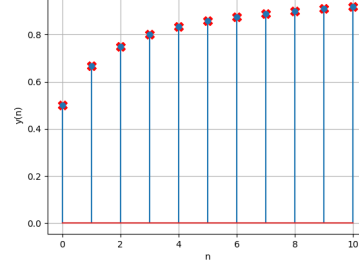


Fig. 0. Stem Plot of $y(n)$ v/s n

$$x(n) = \frac{1}{(n+1)(n+2)}u(n) \quad (17)$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2} \right) u(n) \quad (18)$$

Using (9) and (13), we get,

$$X(z) = -z \log(1 - z^{-1}) + z^2 \log(1 - z^{-1}) + z \quad (19)$$

$$= z(z-1) \log(1 - z^{-1}) + z \quad (20)$$

$$Y(z) = X(z)U(z) \quad (21)$$

$$= z^2 \log(1 - z^{-1}) + \frac{z}{1 - z^{-1}} \quad (22)$$

Using (2) and (16) ,

$$y(n) = u(n+1) - \frac{1}{n+2}u(n), \quad n \geq 0 \quad (23)$$

Since $y(n)$ is only defined for $n \geq 0$, the above expression can be equivalently written as

$$y(n) = \left(1 - \frac{1}{n+2} \right) u(n) \quad (24)$$