10.5.3

EE23BTECH11027 - K RAHUL*

I. Question:

A. Question statement

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

B. Solution

The expression to find the n^{th} term of the Arithmetic Progression is given by

table 1

Parameters in expression		
Symbol	Description	Value
n	The nth term of series	To be computed
a_n	nth term of series	350
a_0	0th term of series	17
d	Common difference of AP	9

$$a_n = a_0 + nd \tag{1}$$

The question asked for the number of terms present in the AP, hence from **??**

$$n = \frac{a_n - a_0}{d} \tag{2}$$

Thus, plugging the values into equation **??**

$$\boxed{n = 37}$$

Now, the expression for sum of numbers in an Arithmetic Progression is given by

$$S_n = (\frac{n+1}{2})(a_1 + a_n) \tag{3}$$

Where S_n is the sum of n+1 terms of AP and the rest of the symbols carry the same meaning as mentioned above.
Plugging the values into equation, we get

$$S_n = 6973$$

Thus, the number of terms present in the Arithmetic Progression is 38 and sum of terms of the Arithmetic Progression is 6973.

II. Question 2

A. Question statement

Express nth term as x(n) in terms of u(n) and find it's Z transform. Additionally, plot the graph of x(n) using python and give the ROC of X(Z).

B. Solution

The formula for sum of numbers in an AP is

$$x_n = x_0 + nd$$

Thus, plugging the values as given in the question

table 1

Parameters in expression		
Symbol	Description	Value
n	The nth term of series	N/A
a_n	nth term of series	To be computed
a_0	0th term of series	17
d	Common difference of AP	9

$$x_n = 8 + 9n \tag{4}$$

Now, the function x(n) is in terms of u(n) as well. Thus from equation **??**,

$$x(n) = (8 + 9n)u(n) \tag{5}$$

where u(n) is the unit step function
Now, it's Z transform has to be computed. The Z

transform for a general function f(n) is calculated by using the expression

$$X(Z) = \mathcal{Z}(x|n|) = \sum_{n=-\infty}^{\infty} x[n]Z^{-n} \tag{6}$$

Where

- X(Z) is the function as asked in the question.
- $\mathcal{Z}(x|n|)$ is the Z transform.
- x[n] is the value of the function defined in equation **??** at a value n.

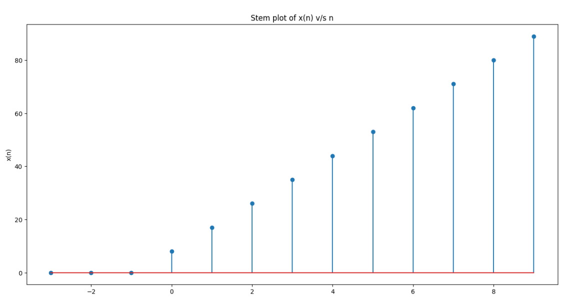
Now, plugging in the function defined in equation **??** into expression **??**, we get

$$X(z) = \sum_{n=0}^{\infty}(8 + 9n)z^{-n} \tag{7}$$

Now, this expression can be evaluated only when $|z| > 1$(Thus, making $|z| > 1$ the region of convergence).
If $|Z| > 1$, then expression **??** simplifies to

$$X(z) = (8 + z^{-1})((1 - z^{-1})^{-2} \tag{8}$$

Fig. 1. Stem Plot of $x(n)$ v/s n



Now, to find the Z transform of the AP Sum function, we take the Z transform of the convolution between the AP term function **??** and the unit step function. Let this new function be $Y(z)$.

$$Y(z) = x(k) \star u(k) \tag{9}$$

Now, knowing that Z transform converts from time domain to frequency domain, we can say that **??** can be written as a product of the Z-transforms of the two functions, as a convolution in the time domain corresponds to multiplication in the frequency domain. Thus,

$$Y(z) = X(z)U(z) \tag{10}$$

Where:

- $X(z)$ is the Z-transform of the AP Term function $x(n)$
- $U(z)$ is the Z-transform of the unit step function.

$$U(z) = \sum_{k=0}^{k=+\infty} z^{-n}$$

$$U(z) = \frac{1}{1 - z^{-1}} \tag{11}$$

Thus, from equation **??** and equation **??**, we get

$$Y(z) = \frac{(8 + z^{-1})}{(1 - z^{-1})^3}$$

Now, taking the inverse Z-transform of this would give the sum of the AP upto n terms.
First, partial fraction has to be taken.

$$Y(z) = \frac{-1}{(1 - z^{-1})^2} + \frac{9}{(1 - z^{-1})^3}$$

Now, the Z-transform of $u(n)$ is given in equation **??**
Let the inverse Z-transform be denoted as $\mathcal{Z}^{-1}$(X), where X is a function in Z.
Remember that from equation **??**, U(z) is the Z transform of the unit step function

$$\mathcal{Z}^{-1}\left(\frac{1}{(1 - Z^{-1})^2}\right) = \mathcal{Z}^{-1}(U(z) * U(z))) \tag{12}$$
$$= \mathcal{Z}^{-1}(U(z)) \star \mathcal{Z}^{-1}(U(z)) \tag{13}$$

$$\mathcal{Z}^{-1}\left(\frac{1}{(1 - z^{-1})^2}\right) = (u(n)) \star (u(n)) \tag{14}$$

Equation **??** stems from the fact that the inverse Z-transform changes the function from frequency domain to time domain, and multiplication in frequency domain is equivalent to convolution in time domain.

$$\mathcal{Z}^{-1}\left(\frac{1}{(1 - z^{-1})^2}\right) = \sum_{k=0}^{n} u(n)u(n - k) \tag{15}$$
$$= (n + 1)u(n) \tag{16}$$

Similarly,

$$\mathcal{Z}^{-1}\left(\frac{1}{(1 - z^{-1})^3}\right) = \mathcal{Z}^{-1}((U(z))^3) \tag{17}$$
$$= ((n + 1)u(n)) \star (u(n)) \tag{18}$$

Thus,

$$\mathcal{Z}^{-1}(\frac{1}{(1-z+^{-1})^3}) = \frac{(n+1)(n+2)}{2}u(n) \qquad (19)$$

Thus, from equations **??** and **??**, we get:

$$Sum = (\frac{9(n+1)(n+2)}{2} - (n+1))u(n)$$

$$Sum = \frac{(n+1)(9n+16)}{2}u(n)$$