11.9.4.4

EE23BTECH11027 - K RAHUL*

QUESTION:
The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases} \tag{1}$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

  A $y(t) = 0$ for all $t < 0$

  B $y(t) = 0$ for all $t > 1$

  C $y(t) = 0$ for all $t > 3$

  D $\int_{0.1}^{0.75} \frac{dy(t)}{dt} \, dt \ne 0$

SOLUTION:

$$y(t) = x(t) * x(t) \tag{2}$$

$$Y(s) = X(s)X(s) \tag{3}$$

$$= (\frac{1 - e^{-s}}{s})^2 \tag{4}$$

$$= \frac{1 + e^{-2s} - 2e^{-s}}{s^2} \tag{5}$$

Using (**??**) and (**??**),

$$y(t) = tu(t) + (t - 2)u(t - 2) - 2(t - 1)u(t - 1) \tag{6}$$

For $t < 0$, $u(t), u(t - 1), u(t - 2)$ all are zero, hence $y(t) = 0$. Thus, option A is right.

If $1 < t < 2$, $u(t)$ and $u(t - 1)$ are equal to unity while $u(t - 2)$ is equal to 0. Thus, $y(t) = 2 - t$. Thus, option B is wrong.

If $t > 3$, $u(t), u(t - 1), u(t - 2)$ are all equal to unity. Thus, $y(t) = 0$. Thus, option C is right.

$$\int_{0.1}^{0.75} dy(t) = y(0.75) - y(0.1) \tag{7}$$

$$= 0.75 - 0.1 \tag{8}$$

$$= -0.25 \tag{9}$$

Thus, option D is right.