

# GATE 2022 ME-32

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## Question:

A rigid uniform annular disc is pivoted on a knife edge A in a uniform gravitational field as shown, such that it can execute small amplitude simple harmonic motion in the plane of the figure without slip at the pivot point. The inner radius  $r$  and outer radius  $R$  are such that  $r^2 = \frac{R^2}{2}$ , and the acceleration due to gravity is  $g$ . If the time period of small amplitude simple harmonic motion is given by  $T = \beta\pi\sqrt{\frac{R}{g}}$ , where  $\pi$  is the ratio of circumference to diameter of a circle, then  $\beta =$  (Round off to 2 decimal places)

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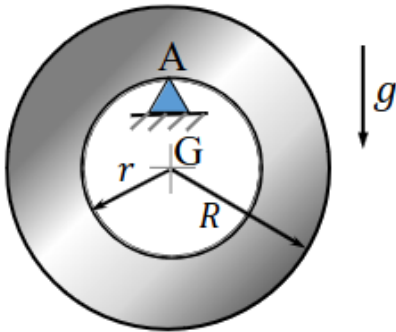


Fig. 0. Question Diagram

## Solution:

Parameters in expression		
Symbol	Description	Value
$I$	Moment of Inertia about the pivot point	$\frac{5}{4}MR^2$
$\theta(t)$	Angular displacement from vertical	?
$\alpha(s)$	Laplace Transform of $\theta(t)$	?
$r$	Distance of center of gravity from pivot point	$\frac{R}{\sqrt{2}}$

TABLE 0  
PARAMETERS

Moment of inertia of disc about pivot point is calculated as

$$I = \frac{1}{2} \left( R^2 + \frac{R^2}{2} \right) + \frac{MR^2}{2} \quad (1)$$

$$= \frac{5}{4}MR^2 \quad (2)$$

Using D' Alambert's principle,

$$I \frac{d^2\theta(t)}{dt^2} + mgr \sin(\theta(t)) = 0 \quad (3)$$

$$\Rightarrow I \frac{d^2\theta(t)}{dt^2} + mgr\theta(t) = 0, \text{ for } \theta \ll 1, \theta > 0 \quad (4)$$

Using (2) and (4), we get

$$\frac{5MR^2}{4} \frac{d^2\theta(t)}{dt^2} + Mg \frac{R}{\sqrt{2}} \theta(t) = 0 \quad (5)$$

$$\frac{d^2\theta(t)}{dt^2} + \frac{2\sqrt{2}g}{5R} \theta(t) = 0 \quad (6)$$

Taking Laplace Transform on both sides, we get

$$s^2\alpha(s) - s\theta(0) - \theta'(0) + \frac{2\sqrt{2}g}{5R}\alpha(s) = 0 \quad (7)$$

$$\alpha(s) \left( s^2 + \frac{2\sqrt{2}g}{5R} \right) = \theta'(0) \quad (8)$$

$$\alpha(s) = \frac{\theta'(0)}{\left( s^2 + \frac{2\sqrt{2}g}{5R} \right)} \quad (9)$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (10)$$

$$e^{at}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s-a} \quad (11)$$

$$\frac{(e^{jat} - e^{-jat})}{2}u(t) \longleftrightarrow \frac{a}{s^2 + a^2} \quad (12)$$

$$\sin(at) \longleftrightarrow \frac{a}{s^2 + a^2} \quad (13)$$

From (9) and (13)

$$\alpha(s) = \frac{\theta'(0) \left( \sqrt{\frac{2\sqrt{2}g}{5R}} \right)}{\left( s^2 + \frac{2\sqrt{2}g}{5R} \right)} \frac{1}{\left( \sqrt{\frac{2\sqrt{2}g}{5R}} \right)} \quad (14)$$

Taking inverse Laplace by putting  $\frac{\theta'(0)}{\left( \sqrt{\frac{2\sqrt{2}g}{5R}} \right)} = k$  and (13),

$$\theta(t) = k \sin \left( \frac{2\sqrt{2}g}{5R} t \right) \quad (15)$$

$$T = \frac{2\pi}{\frac{2\sqrt{2}g}{5R}} \quad (16)$$

$$= \sqrt{(5\sqrt{2})} \pi \sqrt{\frac{R}{g}} \quad (17)$$

Thus,

$$\beta = \sqrt{(5\sqrt{2})} \quad (18)$$

$$\beta = 2.66 \quad (19)$$