GATE BM Q49

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Question:

The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases} \quad (1)$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

A $y(t) = 0$ for all $t < 0$

B $y(t) = 0$ for all $t > 1$

C $y(t) = 0$ for all $t > 3$

D $\int_{0.1}^{0.75} \frac{dy(t)}{dt} \, dt \ne 0$

<div style="text-align:center">GATE 2023 BM- Q 49</div>

Solution:

Symbol	Description
$X(s)$	Laplace transform of $x(t)$
$Y(s)$	Laplace transform of $y(t)$
$u(t - t_0)$	Unit step function, $u(t - t_0) = 1, t \ge t_0$

TABLE 4
PARAMETERS

$$y(t) = x(t) * x(t) \quad (2)$$

$$Y(s) = X(s)X(s) \quad (3)$$

$$= \left(\frac{1 - e^{-s}}{s} \right)^2 \quad (4)$$

$$= \frac{1 + e^{-2s} - 2e^{-s}}{s^2} \quad (5)$$

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \quad (6)$$

$$tu(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^2} \quad (7)$$

$$f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s) \implies f(t + a) \overset{\mathcal{L}}{\longleftrightarrow} e^{as}F(s) \quad (8)$$

Using (7) and (8),

$$y(t) = tu(t) + (t - 2)u(t - 2) - 2(t - 1)u(t - 1) \quad (9)$$

Checking (9) with every option,
For option (A) ,

$$t < 0 \implies y(t) = 0 \quad (10)$$

For option (B) ,

$$1 < t < 2 \quad (11)$$

$$\implies u(t) = u(t - 1) = 1, u(t - 2) = 0 \quad (12)$$

$$\implies y(t) = 2 - t \quad (13)$$

For option (C) ,

$$t > 3 \quad (14)$$

$$\implies u(t) = u(t - 1) = u(t - 2) = 0 \quad (15)$$

$$\implies y(t) = 0 \quad (16)$$

For option (D) ,

$$\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt = \int_{0.1}^{0.75} dy(t) \quad (17)$$

$$= y(0.75) - y(0.1) \quad (18)$$
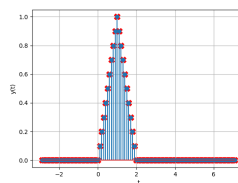
$$= 0.75 - 0.1 \quad (19)$$

$$= -0.25 \quad (20)$$

Thus, options $(A),(C)$ and (D) are true.

Fig. 4. Stem Plot of $y(t)$ v/s t