

GATE BM Q49

EE23BTECH11027 - K RAHUL*

DERIVATIONS AND RESULTS:

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (1)$$

$$tu(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s^2} \quad (2)$$

$$y(t) = x(t) * x(t) \quad (5)$$

$$Y(s) = X(s)X(s) \quad (6)$$

$$= \left(\frac{1 - e^{-s}}{s}\right)^2 \quad (7)$$

$$= \frac{1 + e^{-2s} - 2e^{-s}}{s^2} \quad (8)$$

Using (2) and (3),

$$f(t) \xleftrightarrow{\mathcal{L}} F(s) \implies f(t+a)u(t-a) \xleftrightarrow{\mathcal{L}} e^{as}F(s) \quad (3)$$

$$y(t) = tu(t) + (t-2)u(t-2) - 2(t-1)u(t-1) \quad (9)$$

For $t < 0$, $u(t)$, $u(t-1)$, $u(t-2)$ all are zero, hence $y(t) = 0$. Thus, option A is right.

QUESTION:

The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (4)$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

A $y(t) = 0$ for all $t < 0$

B $y(t) = 0$ for all $t > 1$

C $y(t) = 0$ for all $t > 3$

D $\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt \neq 0$

If $1 < t < 2$, $u(t)$ and $u(t-1)$ are equal to unity while $u(t-2)$ is equal to 0. Thus, $y(t) = 2 - t$. Thus, option B is wrong.

If $t > 3$, $u(t)$, $u(t-1)$, $u(t-2)$ are all equal to unity. Thus, $y(t) = 0$. Thus, option C is right.

$$\int_{0.1}^{0.75} dy(t) = y(0.75) - y(0.1) \quad (10)$$

$$= 0.75 - 0.1 \quad (11)$$

$$= -0.25 \quad (12)$$

Thus, option D is right.

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SOLUTION:

| Symbol | Description |
|--------------|--------------------------------------------------|
| $X(s)$ | Laplace transform of $x(t)$ |
| $Y(s)$ | Laplace transform of $y(t)$ |
| $u(t - t_0)$ | Unit step function, $u(t - t_0) = 1, t \geq t_0$ |

TABLE 4
PARAMETERS

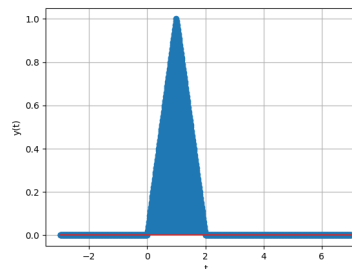


Fig. 4. Stem Plot of $y(t)$ v/s t