11.9.4.4

EE23BTECH11027 - K RAHUL*

QUESTION:
Find sum to n terms of the following series:
$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots$

SOLUTION:

Parameters in expression		
Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

$$x(n) = \frac{1}{(n+1)(n+2)}u(n) \tag{1}$$

$$\tag{2}$$

Using (**??**),

$$X(z) = z(z-1)log(1-z^{-1}) + z, \quad |z| > |1| \tag{3}$$
$$y(n) = x(n) * u(n) \tag{4}$$
$$\implies Y(z) = X(z)U(z) \tag{5}$$
$$= z^2 \ln(1-z^{-1}) + \frac{z^2}{z-1} \tag{6}$$

Using contour integral to find Z transform, we get

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{7}$$
$$= \frac{1}{2\pi j}\oint_C z^{n+1}\ln(1-z^{-1}) + \frac{z^{n+1}}{z-1}dz \tag{8}$$

Now, using Cauchy's residual theorem for the second term of the integral(=R,say), and observing the fact that one pole exists at z = 1,

$$R = \lim_{z\to c}(z-c)f(z) \tag{9}$$
$$= \lim_{z\to 1} z^{n+1} \tag{10}$$
$$= 1 \tag{11}$$

Thus, The first term of the integral(=d(n),say) can be calculated directly.

$$d(n) = \frac{1}{2\pi j}\oint_C z^{n+1}\ln(1-z^{-1})dz \tag{12}$$
$$= \frac{-1}{2\pi j}\oint_C z^{n+1}(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + .. + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2}}{n+2} + ..dz \tag{13}$$
$$z = e^{jt} \tag{14}$$
$$= \frac{-1}{2\pi}\int_0^{2\pi} e^{(n+2)jt}(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + .. + \frac{z^{-(n+2}jt}{n+2} + ..dz \tag{15}$$
$$= \frac{-1}{n+2} \tag{16}$$

Thus,

$$y(n) = 1 - \frac{1}{n+2} \tag{17}$$

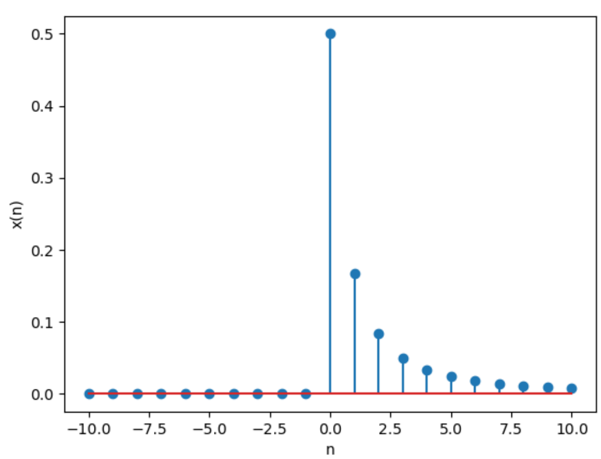

Fig. 0. Stem Plot of $x(n)$ v/s n