

11.9.4.4

EE23BTECH11027 - K RAHUL*

Derivations and results:

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \quad (1)$$

Let $x(n)$ be some expression in discrete time domain, whose Z transform, $X(z)$ shall be obtained for future reference.

For $x(n) = \frac{1}{n+c}u(n)$, where $c \geq 2$, $c \in \mathbb{N}$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \quad (2)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \quad (3)$$

$$= z^c \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \quad (4)$$

(5)

Using, (1)

$$X(z) = z^c \left(-\log(1-z^{-1}) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \dots - \frac{z^{-(c-1)}}{c-1} \right) \quad (6)$$

For $x(n) = \frac{1}{n+1}u(n)$,

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \quad (7)$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-n} \quad (8)$$

$$= z \sum_{n=0}^{n=+\infty} \frac{1}{n+1} z^{-(n+1)} \quad (9)$$

(10)

Using (1),

$$X(z) = -z \log(1-z^{-1}) \quad (11)$$

Let $D(z)$ be some expression in Z domain, whose inverse Z transform, $d(n)$ shall be obtained for future reference.

For $D(z) = z^2 \log(1-z^{-1})$

$$d(n) = \frac{1}{2\pi j} \oint_C z^{n+1} \log(1-z^{-1}) dz \quad (12)$$

$$= \frac{-1}{2\pi j} \oint_C z^{n+1} \left(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \dots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2)}}{n+2} + \dots \right) dz \quad (13)$$

(14)

Making the substitution $z = e^{jt} \implies dz = je^{jt}$

$$d(n) = \frac{-1}{2\pi} \int_0^{2\pi} e^{(n+2)jt} \left(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + \dots + \frac{z^{-(n+2)}jt}{n+2} + \dots \right) dz \quad (15)$$

$$= \frac{-1}{n+2} \quad (16)$$

For $D(z) = \frac{z^k}{1-z^{-1}}$, where $k \in \mathbb{R}$

$$d(n) = \frac{1}{2\pi j} \oint_C \frac{z^{n+k-1}}{1-z^{-1}} dz \quad (17)$$

Using Residue Theorem,

$$d(n) = \lim_{z \rightarrow 1} \left(\frac{z^{n+k-1}}{1-z^{-1}} \right) (1-z^{-1}) \quad (18)$$

$$= 1 \quad (19)$$

Question:

Find sum to n terms of the following series:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$

Solution:

Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

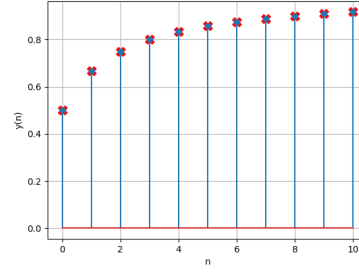


Fig. 0. Stem Plot of $y(n)$ v/s n

$$x(n) = \frac{1}{(n+1)(n+2)} u(n) \quad (20)$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2} \right) u(n) \quad (21)$$

$$(22)$$

Using (6) and (11), we get,

$$X(z) = -z \log(1 - z^{-1}) + z^2 \log(1 - z^{-1}) + z \quad (23)$$

$$= z(z-1) \log(1 - z^{-1}) + z \quad (24)$$

$$Y(z) = X(z) U(z) \quad (25)$$

$$= z^2 \log(1 - z^{-1}) + \frac{z}{1 - z^{-1}} \quad (26)$$

$$(27)$$

Using (16) and (19),

$$y(n) = 1 - \frac{1}{n+2} \quad (28)$$