11.9.4.4

EE23BTECH11027 - K RAHUL*

DERIVATIONS AND RESULTS:

$$x(n) = \frac{1}{n+c}u(n), \text{ where } c \in \mathbb{R} \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n} \tag{2}$$

$$= \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-n} \tag{3}$$

$$= z^{c} \sum_{n=0}^{n=+\infty} \frac{1}{n+c} z^{-(n+c)} \tag{4}$$

$$= z^{c}\left(-log\left(1-z^{-1}\right) - z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} - \ldots - \frac{z^{-(c-1)}}{c-1}\right) \tag{5}$$

$$d(n) = \frac{1}{2\pi j}\oint_C z^{n+1}\log\left(1-z^{-1}\right)dz \tag{6}$$

$$= \frac{-1}{2\pi j}\oint_C z^{n+1}\left(z^{-1} + \frac{z^{-2}}{2} + \frac{z^{-3}}{3} + \ldots + \frac{z^{-(n+1)}}{n+1} + \frac{z^{-(n+2}}{n+2} + \ldots\right)dz \tag{7}$$

$$z = e^{jt} \tag{8}$$

$$= \frac{-1}{2\pi}\int_0^{2\pi} e^{(n+2)jt}\left(e^{-jt} + \frac{e^{-2jt}}{2} + \frac{e^{-3jt}}{3} + .. + \frac{z^{-(n+2}jt}{n+2} + ..\right)dz \tag{9}$$

$$= \frac{-1}{n+2} \tag{10}$$

$$d(n) = \frac{z^n}{1-z^{-1}} \tag{11}$$

$$= \lim_{x\to 1} z^{n+1} \text{(Residue Theorem)} \tag{12}$$

$$= 1 \tag{13}$$

QUESTION:

Find sum to n terms of the following series:
$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots$

Symbol	Description	Value
$x(n)$	n^{th} term of series	

TABLE 0
PARAMETERS

SOLUTION:

$$x(n) = \frac{1}{(n+1)(n+2)}u(n) \tag{14}$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right)u(n) \tag{15}$$

$$\tag{16}$$

Using (5)we get,

$$X(z) = -zlog\left(1-z^{-1}\right) + z^2 log\left(1-z^{-1}\right) + z \tag{17}$$

$$= z(z-1)\,log\left(1-z^{-1}\right) + z \tag{18}$$

$$Y(z) = X(z)\,U(z) \tag{19}$$

$$= z^2 log\left(1-z^{-1}\right) + \frac{z}{1-z^{-1}} \tag{20}$$

$$\tag{21}$$

Using (10) and (13),

$$y(n) = 1 - \frac{1}{n+1} \tag{22}$$

$$\tag{23}$$

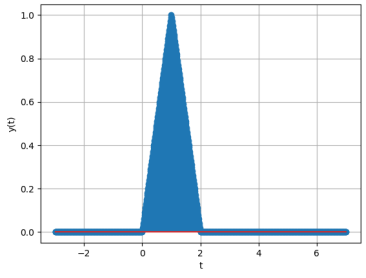

Fig. 0. Stem Plot of $y(t)$ v/s t