10.5.3

EE23BTECH11027 - K RAHUL*

QUESTION:

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

SOLUTION:

Parameters in expression		
Symbol	Description	Value
$x(n)$	n^{th} term of series	
$x(l)$	Last(l^{th}) term of series	350
$x(0)$	Starting (0^{th}) term of series	17
d	Common difference of AP	9

TABLE 0
PARAMETERS

$$x(n) = (x(0) + nd)u(n) \tag{1}$$
$$x(l) = (17 + 9l)u(l) \tag{2}$$

Thus,

$$l = 37 \tag{3}$$

Using (**??**),

$$X(z) = (17 - 8z^{-1})((1 - z^{-1})^{-2}, \quad |z| > |1| \tag{4}$$
$$y(n) = x(n) * u(n) \tag{5}$$
$$\implies Y(z) = X(z)U(z) \tag{6}$$
$$= \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} \tag{7}$$

Using contour integral to find Z transform, we get

$$y(37) = \frac{1}{2\pi j} \oint_C Y(z)z^{36}dz \tag{8}$$
$$= \frac{1}{2\pi j} \oint_C \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} z^{36}dz \tag{9}$$

Now, using Cauchy's residual theorem and observing the fact that 3 repeated poles exist at z = 1,

$$R = \frac{1}{(k-1)!} \lim_{z \to c} \frac{d^{k-1}}{dz^{k-1}}((z-c)^k f(z)) \tag{10}$$
$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^{k-1}}{dz^{k-1}}((z-1)^3 \frac{(17 - 8z^{-1})}{(1 - z^{-1})^3} z^{36}) \tag{11}$$
$$= \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2}(17z^{39} - 8z^{38}) \tag{12}$$
$$= 6973 \tag{13}$$

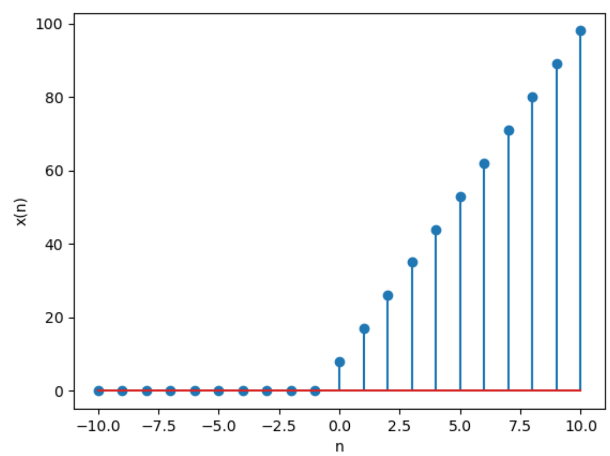

Fig. 0. Stem Plot of $x(n)$ v/s n