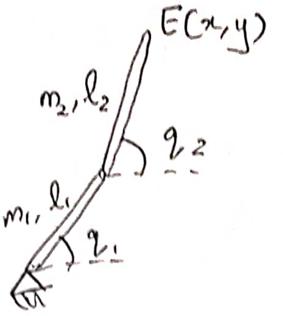
ITR



Tasks:

T_1 : Trajectory following

T_2 : Apply force on the wall

T_3 : Act like a spring

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$
— ①

Cos – c , Sin – s

Differentiating,

$$\dot{x} = -l_1 s q_1 \dot{q}_1 - l_2 s q_2 \dot{q}_2$$
$$\dot{y} = l_2 c q_1 \dot{q}_1 + l_2 c q_2 \dot{q}_2$$

∴ End effector (E) velocity:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_2 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— ②}$$

We need the reverse relation, for given x, y we need a function for q_1, q_2

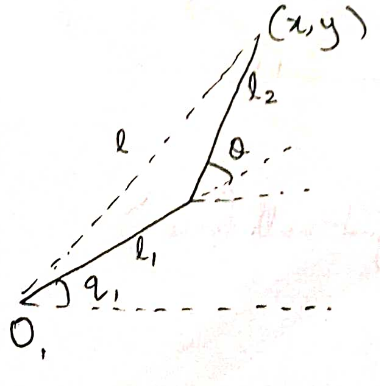
(i) Solving Numerically

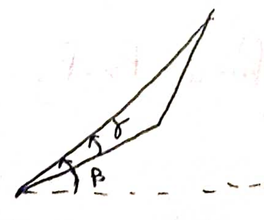
(ii) Derive a closed form expression - Hard in general, Multiple solutions.

$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

(Cosine rule)





$$q_1 = \beta - \gamma$$

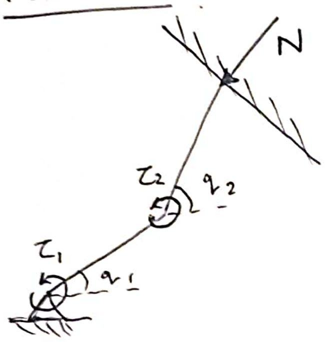
$$\Rightarrow q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

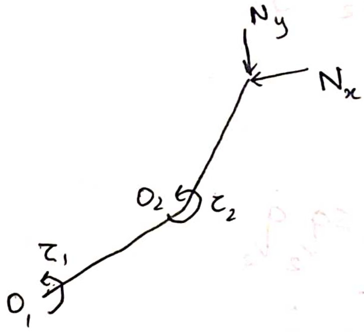
$$q_2 = q_1 + \theta$$

(3)

The desired values are x_d, y_d, q_{1d} & q_{2d}

Task - 2





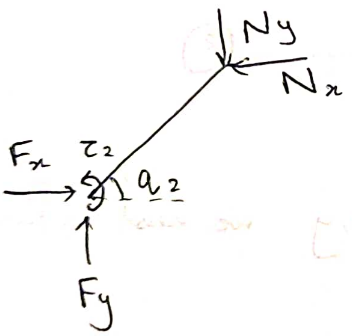
Force applied by manipulator

$F_x = -N_x$, $F_y = -N_y$

We neglect gravity and it is in static equilibrium.

FBD for each link seperately.

(2)

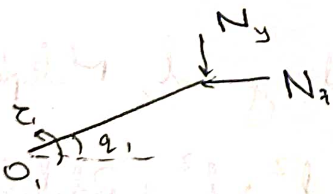


$\Sigma M_{O_2} = 0$

CCW $\Rightarrow$ +ve

$$\Rightarrow N_y l_2 c q_2 - N_x l_2 s q_2 = \tau_2$$

(1)



$\Sigma M_{O_1} = 0$

$$N_y l_1 c q_1 - N_x l_1 s q_1 = \tau_1$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad (4)$$

This is valid for any force F_x, F_y (not just wall forces)

Task - 3 :

Lagranges Equation : $L = K - V$

K - Kinetic Energy
V - Potential Energy.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i' \quad (5)$$

Q_i' - Generalised forces derived using principles of virtual work.

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) q_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) q_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

(Pure rotation of l_1) (rotation about CoM) v_{c_2} - Velocity of CoM of l_2

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

Solving for (5),

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_2 g \frac{l_2}{2} s q_2 = \tau_2 \quad (6)$$