ASSIGNMENT-2

RAHUL PAI
20110155

1. We need to prove that $R S(a) R^T = S R(a)$

Let $b \in \mathbb{R}^3$ (R is a rotational matrix)

Let $R S(a) R^T b = R(a \times R^T b)$ ($S(a) b = a \times b$)

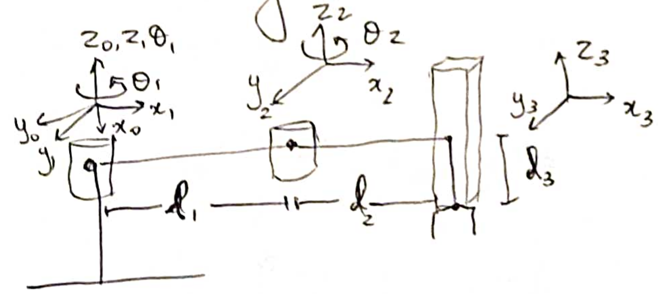
$$= (Ra) \times (R R^T b)$$

$$= (R\vec{a}) \times \vec{b}$$

$$= S(R\vec{a})\,\vec{b}$$

$$\Rightarrow \underline{R S(a) R^T = S(R\vec{a})}$$

2. SCARA Configuration:



$\theta_1 = q_1$
$\theta_2 = q_2$

$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$ $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$$R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{z,q_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

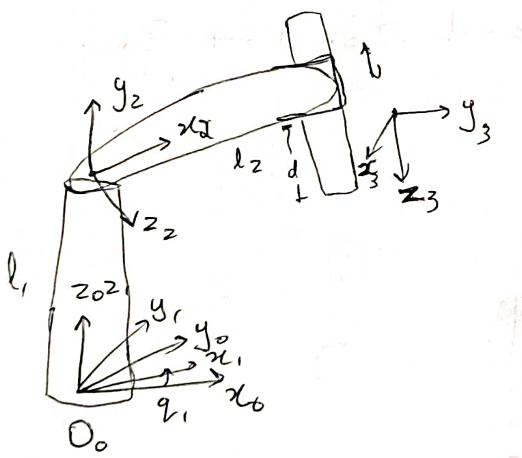
$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}$$

$R_{2,0}$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

4. STANFORD Configuration RRP



$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ 0 & 0 & -1 \\ sq_2 & cq_2 & 0 \end{bmatrix} \qquad R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_{zq_1} $\qquad R_1^2 = R_{x\frac{\pi}{2}} \cdot R_{zq_2}$

$$\begin{bmatrix} P_0^0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sq_2 & cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

5. The four transformations are

(i) z-direction at drone base → 10 m

(ii) Drone rotation about x → 30°

(iii) Drone rotation about new z → 60°

(iv) Obstacle ~~dis~~ (new z-direction) → 3 m

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_4 \\ 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

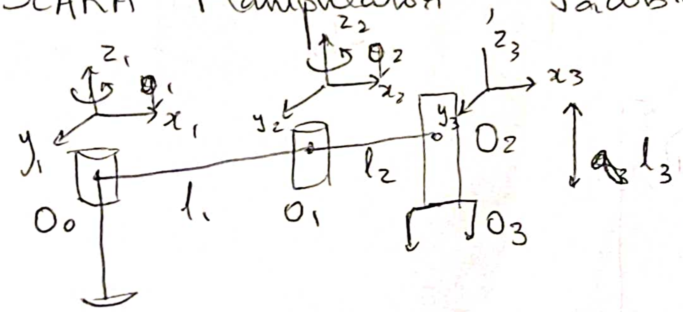
$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ \frac{3\sqrt{3}}{2} + 10 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -3/2 \\ \frac{3\sqrt{3}}{2} + 10 \end{bmatrix}$$

7. SCARA Manipulator, Jacobian can be written as.



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = d_0^1 = \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \end{bmatrix} \qquad O_2 = R_0^1 d_1^2 + d_0^1$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_2 \\ l_2 sq_2 \\ 0 \end{bmatrix} + \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 c(q_1+q_2) + l_1 cq_1 \\ l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^1 R_1^2 d_2^3 + R_0^1 d_1^2 + d_0^1$$

$$R_1^2 = I \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix} + O_2 = \begin{bmatrix} 0 \\ 0 \\ l_3 \end{bmatrix} + O_2$$

$$= \begin{bmatrix} l_2 c(q_1+q_2) + l_1 cq_1 \\ l_2 s(q_1+q_2) + l_1 sq_1 \\ -l_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (O_3 - O_0) \\ z_0 \end{bmatrix} \longrightarrow \text{revolute joint.}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad O_3 - O_0 = \begin{bmatrix} l_2 c(q_1+q_2) + l_1 cq_1 \\ l_2 s(q_1+q_2) + l_1 sq_1 \\ -l_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_1 sq_1) \\ l_2 c(q_1+q_2) + l_1 cq_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (O_3 - O_1) \\ z_1 \end{bmatrix} \longrightarrow \text{Revolute joint} \qquad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad O_3 - O_1 = \begin{bmatrix} l_2 c(q_1+q \\ l_2 s(q_1+q \\ -l_3 \end{bmatrix}$$

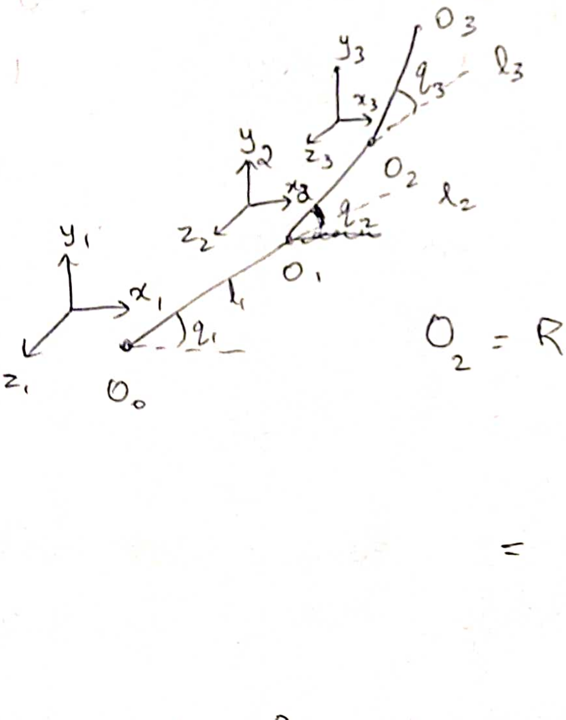
$$= \begin{bmatrix} -l_2 s(q_1+q_2) \\ +l_2 c(q_1+q_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix} \rightarrow \text{prismatic joint} \qquad z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow J_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} -(l_2 s(q_1+q_2) + l_1 sq_1) & -l_2 s(q_1+q_2) & 0 \\ l_2 c(q_1+q_2) + l_1 cq_1 & l_2 c(q_1+q_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

9.



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = d_0^1 = \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_2 cq_2 \\ l_2 sq_2 \\ 0 \end{bmatrix}$$

$$O_2 = R_0^1 \times d_1^2 + d_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_2 \\ l_2 sq_2 \\ 0 \end{bmatrix} + \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 c(q_1+q_2) + l_1 cq_1 \\ l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^2 \times d_2^3 + R_0^1 \times d_1^2 + d_0^1 = R_0^1 \times R_1^2 \times d_2^3 + O_2$$

$$= \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 & -sq_1 & 0 \\ sq_1 & q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 cq_3 \\ l_3 sq_3 \\ 0 \end{bmatrix} + \begin{bmatrix} l_2 c(q_1+q_2) + l_1 cq_1 \\ l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 cq_1 \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & J_2 & J_2 \end{bmatrix} = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_1) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$= \begin{bmatrix} -(l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 s q_1) & -(l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2)) & -l_3 s(q_1+q_2+q_3) \\ l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 c q_1 & l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

6(a) Some types of gearboxes are :

(i) Spur gearbox

- They are very cheap & easy to handle & is used in low speed machines
- But they dont work with high speed & torque & the reductions are low

(ii) Planetary gearbox.

- They have a small size & have high efficiency.
- High torque can be applied & large reductions are possible.
- They are complex gearbox.

(iii) Work Gearbox

- Large reductions are possible & high thermal & shock load capacity
- It is costly & large. It's efficiency is moderate.

(iv) 6 speed gearbox

- 6 choice of gear ratio's can be created.
- But it is moderately costly.

(b) Drones mostly dont use gearboxes. It is required when motor power needs to be converted to high torque but drones require high speed not high torque.
Some large drones with large size propellers use gearboxes.