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Assignment - 02

1) a) Solve $y'(x) = 3x + \frac{y}{2}$, $y(0) = 1$ then find $y(0.2)$ with $h = 0.2$ using modified Euler's method.

As Given $y_0 = 1$, $x_0 = 0$, $h = 0.2$, $f(x,y) = 3x + \frac{y}{2}$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

I stage, we have Euler's formula

$y_1^{(0)} = y_0 + h.\, f(x_0, y_0)$

$= 1 + 0.2(3(0) + \frac{1}{2})$

$= 1 + 0.2(0.5)$

$y_1^{(0)} = \underline{1.1}$

We have MEF

$y_1^{(0)} = y_0 + h/2\, (f(x_0, y_0) + f(x_1, y_1^{(0)}))$

$= 1 + 0.1(0.5 + (3(0.2) + \frac{1}{2}(1.10))$

$= \underline{1.1650}$

$y_1^{(1)} = y_0 + h/2\, (f(x_0, y_0) + f(x_1, y_1^{(0)}))$

$= 1 + 0.1(0.5 + (3(0.2) + \frac{1}{2}(1.1650))$

$= \underline{1.1682}$

$y_1^{(2)} = y_0 + h/2\, (f(x_0, y_0) + f(x_1, y_1^{(1)}))$

$= 1 + 0.1(0.5 + (3(0.2) + \frac{1}{2}(1.1682))$

$= \underline{1.1684}$

$y_1^{(3)} = y_0 + h/2\, (f(x_0, y_0) + f(x_1, y_1^{(2)}))$

$= 1 + 0.1(0.5 + (3(0.2) + \frac{1}{2}(1.1684))$

$y_1^{(3)} = 1.1684$

6. Apply Runge-Kutta method of fourth order to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x+y$ and $y = 1$, when $x = 0$.

A) Given $\frac{dy}{dx} = f(x,y)$, $y_0 = 1$, $x_0 = 0$

$x_0 + h = 0.2$

$\frac{dy}{dx} = x+y$

$h = 0.2 - 0 = 0.2$

By R-KM

$y_0(x_0+h) = y_0 + h/6 (K_1 + 2K_1 + 2K_3 + K_4) \longrightarrow (1)$

$K_1 = hf(x_0, y_0)$

$= 0.2(0+1)$

$K_1 = 0.2$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$

$= 0.2f(0+0.1, 1+0.1)$

$= 0.2(0.1 + 1.1)$

$K_2 = 0.24$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$

$= 0.2f(0.1, 1+0.12)$

$= 0.2(0.1 + 1.12)$

$K_3 = 0.244$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2f(0.2, 1.244)$

$= 0.2(0.2 + 1.244)$

$K_4 = 0.2888$

$y(0+0.2) = 1 + 1/6(0.2 + 2\times0.24 + 2\times0.244 + 0.2888)$

$y(0.2) = 1.2428$

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C. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$ compute $y(0.4)$ using Milne's method.

→ Given

x	y	$y' = \frac{dy}{dx} = xy + y^2$
0	1	1
0.1	1.1169	1.3592
0.2	1.2773	1.8870
0.3	1.5049	2.7162

By Milne's PM

$$y_4^{(p)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 1 + \frac{4(0.1)}{3}\left[2 \times 1.3592 - 1.8870 + 2(2.7162)\right]$$

$$\underline{y_4^{(p)} = 1.8352.}$$

$$y_4'^{(p)} = x_4 y_4^{(p)} + y_4^{2(p)}$$

$$= 0.4 \times 1.8852 + 1.8352^2$$

$$\underline{y_4'^{(p)} = 4.1020}$$

$$y_4^{(c)} = y_2 + h/3\left(y_2' + 4y_2' + y_4'^{(p)}\right]$$

$$= 1.2773 + 0.1/3\left[1.8870 + 4(2.7162) + 4.1020\right]$$

$$\underline{y_4^{(c)} = 1.8391.}$$

$$y_4'^{(c)} = x_4 y_4^{(c)} + y_4^{2(c)}$$

$$= 0.4 \times 1.8391 + 1.8391^2$$

$$\underline{y_4'^{(c)} = 4.1179}$$

$$y_4^{(c)} = y_2 + h/3\left[y_2' + 4y_3' + y_4'^{(c)}\right]$$

$$= 1.2773 + \frac{0.1}{3}\left[1.8870 + 4(2.7162) + 4.1179\right]$$

$y_4^{(c_1)} = 1.8396$

$y_4'^{(c_1)} = x_4 y_4^{(c_1)} + y_4^{2(c_1)}$

$= 0.4 \times 1.8396 + 1.8396^2$

$y_4'^{(c_1)} = 4.1200$

$y_4^{(c_2)} = y_2 + h/3 \left[y_2' + 4y_3' + y_4'^{(c_2)}\right]$

$= 1.2773 + \frac{0.1}{3}\left[1.8870 + 4(2.7162) + 4.1200\right]$

$y_4'^{(c_2)} = 1.8397$

$y_4'^{(c_2)} = x_4 y_4^{(c_2)} + y_4^{2(c_2)}$

$= 0.4 \times 1.8397 + 1.8397^2$

$y_4'^{(c_2)} = 4.1204$

$y_4^{(c_3)} = y_2 + h/3 \left[y_2' + 4y_3' + y_4'^{(c_2)}\right]$

$= 1.2773 + \frac{0.1}{3}\left[1.8870 + 4(2.7162) + 4.1204\right]$

$y_4^{(c_3)} = 1.8397$

2.a. Employ Taylor's series method to obtain approx. value of y at $x = 0.2$ for the differential equation $dy/dx = 2y + 3e^x$, $y(0) = 0$.

Ans TSE of $y(x)$ is given by

$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) \cdots \rightarrow ①$

$y(0) = 0$, $\Rightarrow y(x_0) = y_0$ $x_0 = 0$, $y_0 = 0$

$\frac{dy}{dx} = 2y + 3e^x$

$y' = 2y + 3e^x$

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$y'(0) = 2(0) + 3e^0 = 0 + 3 = \underline{3}$

$\frac{dy}{dx} = 2y + 3e^x$

Diff wrt x

$y'' = 2y' + 3e^x$

$y''(0) = 2y'(0) + 3e^{x(0)}$

$y''(0) = 2(3) + 3e^0$

$\underline{y''0 = 9}$

$y'' = 2y' + 3e^x$

Diff wrt x

$y''' = 2y'' + 3e^x$

$y'''(0) = 2y''(0) + 3e^{x(0)}$

$y'''(0) = 2(9) + 3e^0$

$\underline{y'''(0) = 21}$

from eq (1)

$y(0.2) = 0 + (0.2-0)3 + \frac{(0.2-0)^2}{2}9 + \frac{(0.2-0)^3}{6}21.$

$= 0 + 0.6 + 0.18 + 0.028$

$y(0.2) = \underline{0.8080}.$

or

$y(0.2) = 0 + (0.2-0)3 + \frac{(0.2-0)^2}{2}9$

$= 0 + 0.6 + 0.18$

$y(0.2) = \underline{0.78}$

6. Using the Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.

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Ans Given: $x_0 = 0$, $y_0 = 1$, $f(x_0, y_0) = \frac{y^2 - x^2}{y^2 + x^2}$

$x_0 + h = 0.2 \Rightarrow 0 + h = 0.2$

Stage I, $K_1 = hf(x_0, y_0)$

$= 0.2\, f(0, 1)$

$= 0.2 \left(\frac{1^2 - 0^2}{1^2 + 0^2}\right)$

$K_1 = 0.2$

$K_2 = hf(x_0 + h/2,\ y_0 + K_1/2)$

$K_2 = 0.2\, f(0 + 0.1,\ 1 + 0.1)$

$K_2 = 0.2 \left(\frac{1.1^2 - 0.1^2}{1.1^2 + 0.1^2}\right)$

$= 0.2\,(0.9836)$

$K_2 = 0.1967$

$K_3 = hf(x_0 + h/2,\ y_0 + K_2/2)$

$K_3 = 0.2\, f(0.1,\ 1 + 0.0984)$

$= 0.2 \left(\frac{1.0984^2 - 0.1^2}{1.0984^2 + 0.1^2}\right)$

$= 0.2\,(0.9836)$

$K_3 = 0.1967$

$K_4 = hf(x_0 + h,\ y_0 + K_3)$

$= 0.2\, f(0.2,\ 1 + 0.1967)$

$= 0.2 \left(\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2}\right)$

$= 0.2\,(0.9457)$

$K_4 = 0.1891$

By RKM $\Rightarrow y(x_0 + h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y(0 + 0.2) = 1 + \frac{1}{6}(0.2 + 2 \times 0.1967 + 2 \times 0.1967 + 0.1891)$

$y(0.2) = 1.1960$

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C. Write the Mathematical tool Codes to solve the differential equation $\frac{dy}{dx} - 2y = 3e^x$ with $y(0) = 0$ using the Taylor's series method at $x = 0.1(0.1)0.3$.

So

```
from numpy import array, zeros, exp
def taylor (deriv, x, y, xStop, h):
        X=[ ]
        Y=[ ]
        X.append(x)
        Y.append(y)
        while x < xStop:
            D = deriv(x,y)
            H=1.0
            for j in range(3):
                H= H*h/(j+1)
                y=y+D[j]*H
            x=x+h
            X.append(x)
            Y.append(y)
        return array(X), array(Y)

def deriv(x,y):
    D=zeros((4,1))
    D[0]=[2*y[0]+3*exp(x)]
    D[1]=[4*y[0]+9*exp(x)]
    D[2]=[8*y[0]+21*exp(x)]
    D[3]=[16*y[0]+45*exp(x)]
    return D
```

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```
x=0.0
xStop=0.3
y=array([0.0])
h=0.1
X,Y=taylor(deriv, x, y, xStop, h)

Print("The required values are : at x=%0.2f, y=%0.5f,
                                  x=%0.2f, y=%0.5f,
                                  x=%0.2f, y=%0.5f,
                                  x=%0.2f, y=%0.5f"
                                  %(X[0], Y[0], X[1], Y[1],
                                  X[2], Y[2], X[3] Y[3]))
```

3.a By Taylor's series method, find the value of y at $x=0.1$ and $x=0.2$ to 5 places of decimals from $dy/dx = x^2y-1$, $y(0)=1$.

Ans Given $y(0)=1 \Rightarrow y(x_0)=y_0 \Rightarrow y_0=1,\ x_0=0$.

$$\frac{dy}{dx} = x^2y-1$$

$y' = (x^2y-1)$

$y'(x_0) = (x_0^2 y_0 - 1)$

$y'(0) = (0^2 \cdot 1 - 1)$

$\underline{y'(0) = -1}$

$y' = (x^2y-1)$

Diff wrt x

$y'' = 2xy + x^2y'$

$y''(x_0) = 2x_0y_0 + x_0^2 y'(x_0)$

$y''(0) = 2.0.1 + 0^2(-1)$

$\underline{y''(0) = 0}$

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$y'' = 2xy + x^2y'$

Diff w.r.t x

$y''' = 2y + 2xy' + 2xy' + x^2y''$

$y'''(x_0) = 2y_0 + 2x_0y'(x_0) + 2x_0y'(x_0) + x_0^2y''(x_0)$

$y'''(x_0) = 2.1 + 2\cdot0.-1 + 2.0.-1 + 0^2.0$

$\underline{y'''(x_0) = 2}$

By TSM

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) \cdots \quad -(2)$$

from Eq (1)

$$y(0.1) = 1 + (0.1-0)-1 + \frac{(0.1-0)^2}{2}0 + \frac{(0.1-0)^3}{3\times2}2$$

$y(0.1) = 1 + (-0.1) + (0.005) + (0.0003)$

$\underline{y(0.1) = 0.9053.}$

b. Using the Runge-Kutta method of fourth order, find $y(0.1)$ given that $dy/dx = 3e^x + 2y$, $y(0) = 0$, taking $h = 0.1$

A: Given $\frac{dy}{dx} = 3e^x + 2y$, $y(0.1) = ?$, $y_0 = 0$, $x_0 = 0$, $h = 0.1$

$f(x_0, y_0) = 3e^x + 2y$

$K_1 = hf(x_0, y_0)$

$= 0.1(3e^0 + 2(0))$

$\underline{K_1 = 0.3}$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$

$= 0.1 f(0.05, 0.15)$

$\underline{K_2 = 0.3454}$

$K_3 = hf(x_0 + h/2,\ y_0 + K_2/2)$

$= 0.1\ f(0.05,\ 0.1577)$

$K_3 = 0.3469$

$K_4 = hf(x_0 + h,\ y_0 + K_3)$

$= 0.1\ f(0.10,\ 0.3469)$

$K_4 = 0.4015$

By R-KM

$y(x_0+h) = y_0 + 1/6\,(K_1 + 2K_2 + 2K_3 + K_4)$

$y(0+0.1) = 0 + 1/6\,(0.3 + 2\times0.354 + 2\times0.3469 + 0.4015)$

$y(0.1) = 0.3487.$

C. Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0)=0$, $y(0.2)=0.02$, $y(0.4)=0.0795$, $y(0.6)=0.1762$. compute y at $x=0.8$ by applying Milne's method.

Ans Given

x	y	$y' = dy/dx = x - y^2$
0	0	0
0.2	0.02	0.1996
0.4	0.0795	0.3937
0.6	0.1762	0.5690

$h = 0.2$

By Milne's PM

$y_4^{(P)} = y_0 + 4h/3\,(2y_1' - y_2' + 2y_3')$

$= 0 + \frac{4(0.2)}{3}\,[2(0.1996) - 0.3937 + 2(0.5690)]$

$y_4^{(P)} = 0.3049.$

$y_4'^{(P)} = x_4 - y_4^{2\,(P)}$

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$y'^{(c)} = 0.8 - (0.3049)^2 = 0.707$

$[0.3937 + 4\times 0.5690 + 0.707]$

$y_4^{(c)} = 0.3046$

$y_4'^{(c)} = x_4 - y_4^{2(c)}$

$= 0.8 - (0.3046)^2$

$y_4'^{(c)} = 0.7072$

$y_4^{(c)} = y_2 + h/3\left[y_2' + 4y_3' + y_4'^{(c)}\right]$

$= 0.0795 + \frac{0.2}{3}\left[0.3937 + 4\times 0.5890 + 0.7072\right]$

$\therefore \boxed{y(0.1) = 0.3046}$

Using the modified Euler's method, find y(0.1) given that

Given : $dy/dx = x^2+y$, $y_0 = 1$, $x_0 = 0$, $h = 0.05$

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9a. Continue

Stage I:-

$$y_{(n+1)} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.5(0^2+1) = \underline{1.05}$$

now By modified EM

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$\underline{y_1^{(1)} = 1.05131}.$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$\underline{y_1^{(2)} = 1.05135}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$\underline{y_1^{(3)} = 1.05135}.$$

Stage II:- Let $x_0 = 0.05$, $y_0 = 1.05135$, $h = 0.05$

$f(x, y) = x^2 + y$, $x_1 = x_0 + h = 0.05 + 0.05 = 0.1$

by Euler's Method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1.05135 + 0.05 f(0.05)^2 + 1.05135)$$

$$y_1^{(0)} = 1.10404.$$

By MEM,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1.05135 + \left(\frac{0.05}{2}\right)\left[1.05385 + \left((0.1)^2 + 1.10404\right)\right]$$

$$y_1^{(1)} = 1.10555$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1.05135 + \left(\frac{0.05}{2}\right)\left(1.05385 + \left((0.1)^2 + 1.10555\right)\right)$$

$$y_1^{(2)} = 1.10559.$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1.05135 + \left(\frac{0.05}{2}\right)\left(1.05385 + \left((0.1)^2 + 1.10559\right)\right)$$

$$\therefore y_1^{(3)} = 1.10559.$$

6. Using the Runge-Kutta method of fourth order, find $y(0.2)$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0.1) = 1.0912$, taking $h = 0.1$.

Given : $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$y(x_0 + h) = y(0 + 0.1) = y(0.1)$

$f(x_0, y_0) = \frac{y-x}{y+x}$

Stage I, $K_1 = h f(x_0, y_0)$

$= 0.1 f(0, 1)$

$= 0.1 \left(\frac{1-0}{1+0}\right)$

$K_1 = 0.1$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$

$= \left(0+0.05, 1+0.05\right)$

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$K_4 = 0.0832$

By R-KM, $y(x_0+h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y(0+0.1) = 1 + \frac{1}{6}(0.1 + 2\times 0.0909 + 2\times 0.0909 + 0.0832)$

$y(0.1) = 1.0911$.

II Stage, $y_0 = 1.0911$, $x_0 = 0.1$, $h = 0.1$

$K_1 = hf(x_0, y_0)$

$= 0.1\left(\frac{1.0911 - 0.1}{1.0911 + 0.1}\right)$

$K_1 = 0.0832$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$

$= 0.1\, f(0.15, 1.1327)$

$= 0.1\left(\frac{1.1327 - 0.15}{1.1327 + 0.15}\right)$

$K_2 = 0.0766$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$

$= 0.1\, f(0.15, 1.1294)$

$= 0.1\left(\frac{1.1294 - 0.15}{1.1294 + 0.15}\right)$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.1\, f(0.2, 1.1677)$

$= 0.1\left(\frac{1.1677 - 0.2}{1.1677 + 0.2}\right)$

$K_4 = 0.0708$

By R-KM; $y(x_0+h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

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$y(0.2) = 1.0911 + 1/6(0.0832 + 2\times0.0766 + 2\times0.0766 + 0.0708)$

$y(0.2) = 1.1678$

Using Mathematical tools, write the code to find the solution of $dy/dx = 1 + y/x$ at $y(2)$ taking $h = 0.2$. Given that $y(1) = 2$ [illegible]

```
from sympy import *
import numpy as np
def RungeKutta(g, x0, h, y0, xn):
    x, y = symbols('x, y')
    [illegible]
    xt = x0+h
    Y = [y0]
    while xt <= xn:
        k1 = h * f(x0, y0)
        [illegible]
        k3 = h * f(x0+h/2, y0+k2/2)
        [illegible]
        [illegible] + (1/6) * (k1 + 2*k2 + 2*k3 + k4)
        [illegible]
        x0 = xt
        y0 = y1
        xt = xt + h
    return np.round(Y, 2)
RungeKutta('1+(y/x)', 1, 0.2, 2, 2)
```

VEMANA IT

5a. Find y at $x=5$ if $y(1)=-3$, $y(3)=9$, $y(4)=30$, $y(6)=132$ using lagrange's interpolation formula.

Ans

x	1	3	4	6
y	-3	9	30	132

here $x_0=1$ $x_1=3$ $x_2=4$ $x_3=6$ $x=5$

$y_0=-3$ $y_1=9$ $y_2=30$ $y_3=132$ $y=?$

By L.I.F

$$y=f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_0)(x_1-x_3)}y_1$$

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$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$=\frac{(5-3)(5-4)(5-6)}{(1-3)(1-4)(1-6)}3+\frac{(5-1)(5-4)(5-6)}{(3-1)(3-4)(3-6)}9$$

$$+\frac{(5-1)(5-3)(5-6)}{(4-1)(4-3)(4-6)}30+\frac{(5-1)(5-3)(5-4)}{(6-1)(6-3)(6-4)}132$$

$f(5) = -0.2 - 6 + 40 + 35.2$

$f(5) = \underline{\underline{69}}$

---

b. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ by taking 7 ordinates and by using Simpson's 3/8 rule.

A) Seven ordinates means that the given interval [2,8] must be divided into 6 equal parts ($n=6$)

$x_0 = a,\quad x_0+nh = 1\quad a=0\quad b=1$

(or) $0+6h=1$

$h = \frac{b-a}{n} = h = \frac{1-0}{6} = \frac{1}{6}$

$x = 0,\ \frac{1}{6},\ 2/6,\ 3/6,\ 4/6,\ 5/6,\ 6/6$

$x = 0,\ 1/6,\ 1/3,\ 1/2,\ 2/3,\ 5/6,\ 1.$

By data, $y = \frac{1}{1+x^2}$

At $x=0$, $y = \frac{1}{1+0} = 1 \Rightarrow y_0$

$x = 1/6$, $y = \frac{1}{1+(1/6)^2} = 0.85729 \Rightarrow y_1$

$x = 1/3$, $y = \frac{1}{1+(2/3)^2} = 0.75 \Rightarrow y_2$

$x = 3/6, \quad y = \frac{1}{1+(3/6)} = 0.66 \Rightarrow y_3$

$x = 2/3, \quad y = \frac{1}{(1+(2/3))} = 0.6 \Rightarrow y_4$

$x = 5/6, \quad y = \frac{1}{1+(5/6)} = 0.54 \Rightarrow y_5$

$x = 1, \quad y = \frac{1}{1+1} = 0.5 \Rightarrow y_6$

By Simpson ($3/8^{th}$) rule

$= \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$

$= \frac{3(1/6)}{8}\left[(1 + 0.5) + 3(2.7524) + 2(0.6)\right]$

$= \frac{3}{48}\left[1.5 + 8.2572 + 1.20\right]$

$= \frac{3}{48} \times 10.9572$

$\therefore \int_0^1 \frac{dx}{1+x} = 0.6848.$

C. Evaluate $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ by using the Trapezoidal rule by taking 7 Ordinates.

A) $n = 6 \qquad x_0 + nh = \pi/2 \qquad x_0 = 0$

$6h = \pi/2 - 0$

$h = \pi/12$

$x_0 = 0, \quad y_0 = 1$

$x = \pi/2 \quad y_{\pi/2} = 0.9828 \Rightarrow y_1$

D D M M Y Y Y Y

$x = 2\pi/_{12}$, $y_{2\pi/12} = 0.9306 \Rightarrow y_2$

$x = 3\pi/_{12}$, $y_{3\pi/12} = 0.8409 \Rightarrow y_3$

$x = 4\pi/_{12}$, $y_{4\pi/12} = 0.707 \Rightarrow y_4$

$x = 5\pi/_{12}$, $y_{5\pi/12} = 0.5087 \Rightarrow y_5$

$x = 6\pi/_{12}$, $y_{\pi/2} = 0 \Rightarrow y_6$

By Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$f(x) = \frac{\pi}{24}\left[(1+0) + 2(0.9828+0.9306+0.8409+0.707+0.5087)\right]$$

$\therefore$ Trapezoidal of $\int_0^{\pi/2} \sqrt{\cos x}\,.dx = 1.1740$

6a. Using Newton's divided difference formula, evaluate f(8) from the following.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

As

x	y	Ist D.D	2nd D.D	3rd D.D	4th D.D
4	48	52	15	1	0
5	100	97	21	1	0
7	294	202	27	1	
10	900	310	33		
11	1210	409			
13	2028				

We have NDDF

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) +$$

D D M M Y Y Y Y

$(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)$
$(x-x_2)(x-x_3)f(x_1,x_2,x_3,x_4)$.

$= 48 + (8-4)52 + (8-4)(8-5)15 + (8-4)(8-5)(8-7)$
$(1) + (8-4)(8-5)(8-7)(8-10)(0)$.
$= 48 + 208 + 180 + 12 + 0$
$= \underline{976}$.

6. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using the Trapezoidal rule by taking 6 division.

Ans

$n+1 = 6$ $\quad$ $x_0 + nh = 1$
$\boxed{n=5}$ $\quad$ $6h = 1$
$h = 1/6$

$x = 0, \quad y_0 = 1$
$x_1 = 1/6, \quad y_{1/6} = 0.973 \Rightarrow y_1$
$x_2 = 2/6, \quad y_{2/6} = 0.9 \Rightarrow y_2$
$x_3 = 3/6, \quad y_{3/6} = 0.8 \Rightarrow y_3$
$x_4 = 4/6, \quad y_{4/6} = 0.6923 \Rightarrow y_4$
$x_5 = 5/6, \quad y_{5/6} = 0.5902 \Rightarrow y_5$
$x_6 = 1, \quad y_1 = 0.5 \Rightarrow y_6$

Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+y_3+y_4+y_5)\right]$$

$$f(x) = \frac{1}{12}\left[(1+0.5) + 2(0.973+0.9+0.8+0.6923+0.5902)\right]$$

$$= \frac{1}{12}\left[1.5 + 2(3.955)\right]$$

D D M M Y Y Y Y

$$\int_0^1 \frac{1}{1+x^2} = \frac{9.411}{12}$$

$$\int_0^1 \frac{1}{1+x^2} = 0.7843$$

C. Evaluate $\int_0^3 \frac{1}{4x+5}\,dx$ by using simpson's $1/3^{rd}$ rule by taking 7 ordinates.

Ans Given $n+1=7 \Rightarrow n=6$

$$I = \int_0^3 \frac{1}{4x+5} \qquad a=0 \qquad b=3$$

$$h = b-a/n = 3-0/6 = 1/2 = 0.5$$

$x=0$, $y_0 = 0.2 \longrightarrow y_0$
$x_1=0.5$, $y_{0.5} = 0.1429 \longrightarrow y_1$
$x_2=1$, $y_1 = 0.1111 \longrightarrow y_2$
$x_3=1.5$, $y_{1.5} = 0.0909 \longrightarrow y_3$
$x_4=2.0$, $y_2 = 0.0769 \longrightarrow y_4$
$x_5=2.5$, $y_{2.5} = 0.0667 \longrightarrow y_5$
$x_6=3$, $y_3 = 0.0588 \longrightarrow y_6$

By simpson's $(1/3^{rd})$ rule

$$I = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.5}{3}\left[(0.2+0.0588) + 4(0.1429+0.0909+0.0667) + 2(0.1111+0.0769)\right]$$

$$= 0.1667\left[0.2588 + 4(0.3005) + 2(0.1880)\right]$$

$$= 0.1667\left[0.2588 + 1.2020 + 0.3760\right]$$

$$= 0.1667\,[1.8368]$$

$$I = 0.3062$$